MTH301: Analysis I
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Assignment 10
Question 3

Q3 If M is complete, is every nonempty open set a second category set?

Solution:

Yes, the given statement is true. We prove by contradiction.

Given M is complete, consider $A \subseteq M$ s.t. A is open and non-empty. Let us assume that A is a first category set.

A is open, so $A^{\complement}$ will be closed.

So, by corollary of Baire's category theorem, we can show that $A^{\complement}$ is dense in M.

$\implies \overline{A^{\complement}} = M$.

But since $A^{\complement}$ is closed, $\overline{A^{\complement}} = A^{\complement}$

$\implies A^{\complement} = M$.

$\implies A = \emptyset$

But A was a non-empty i.e. $A \neq \emptyset$. This is a contradiction. Hence, our assumption was false, that is A cannot a first category set. Therefore, A is a second category set.

Hence, proved. ∎