CS302-Modulation and Simulation

Lab 6

Undertaken By

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Contents

1 Heat Diffusion on a 2-dimensional plate

Heat diffusion occurs due to the process of thermal conduction. During the collision of articles from two temperature areas, the particles at higher temperature transfer some of their energy to the particles at lower energy areas.

1.1 Problem Statement

We want to model the heat diffusion through a thin metal piece that has a constant application of heat and cold at designated locations on the bar. We also want to develop an animated scientific visualization for the diffusion process.

1.2 Assumptions

1. Each internal point on a line perpendicular to the top surface of the bar has the same temperature. If a point on the top surface has temperature 25 °C, then every point directly below that location is at 25 °C.
2. bar is in a still room and that the immediate surroundings are at the same temperatures as the bar.
3. Temperature diffuses within the bar, but external conditions do not affect the temperatures.
4. The bar is modelled in two dimensions, length and width.

1.3 Implementation

The system is implemented as a grid of m*n matrix. Each cell in the matrix represents the average temperature of that point in degree Celsius. The ambient temperature is set to 25 °C. The hot and cold values are set to 50 °C and 0 °C. The top, right and left sides of the bar are set to hot whereas the bottom one is set to cold temperature. All the other cells in the middle would be having ambient temperature in the beginning.

For simulating the heat diffusion we have taken the Moore neighbourhood - that is it will take into consideration all the eight neighbouring cells for calculating the average value of itself.

The model is based on Newton's law of cooling which states - "Rate of change of temperature of an object is proportional to the difference between the difference of it's temperature and the surroundings. Hence the change in temperature of a given site can be given as:

$$\Delta site = r \sum_{i=1}^{8} (neighbor_i - site)$$

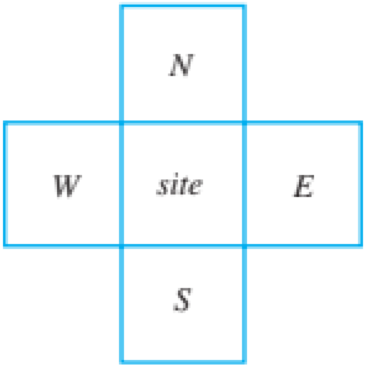

Figure 1: Von Neumann's neighbourhood which takes into account all neighbouring 8 sites

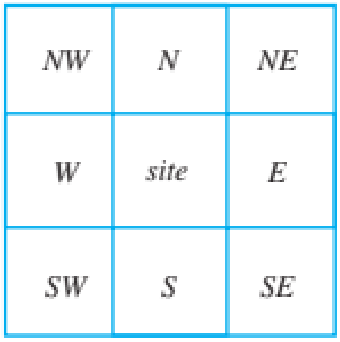

Figure 2: Moore's neighbourhood which takes into account only four neighbouring sites

The temperature of a given site will be equal to :

$$site + \Delta site = site + r\sum_{i=1}^{8}(neighbor_i - site)$$

We have used an reflecting boundary condition that is we have given the new formed ghost cells the values of their immediate neighbour. Hence, the values of the first row appears again on the newly formed ghost cells and similarly for all the boundaries. The reason for doing this is, in case of heat diffusion the air around the bar mimics the temperature of the bar itself because the air is still around it.

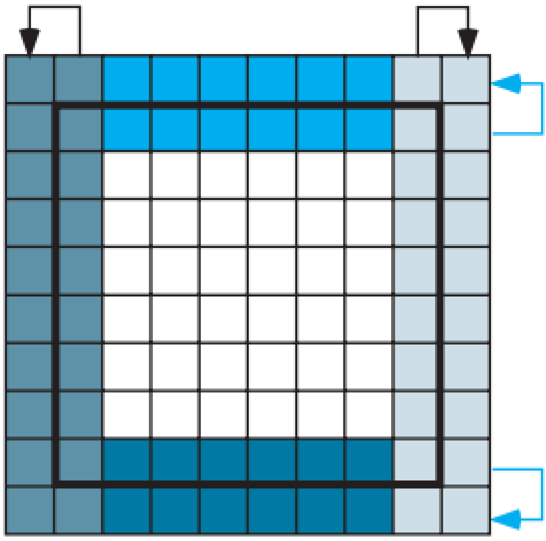

Figure 3: Reflecting Boundary Conditions

1.4 Computational Analysis

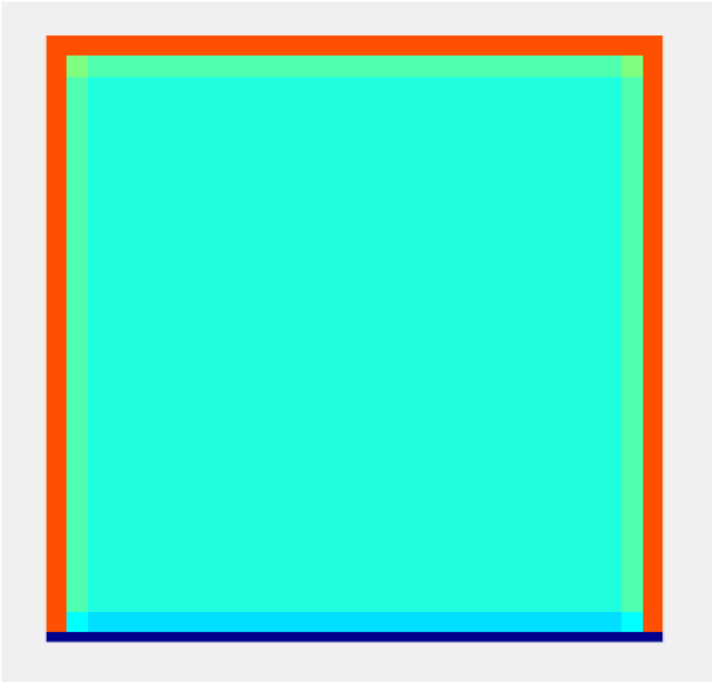

Figure 4: The initial conditions of the bar where the right, left and top bars are initialized to 50 °C.

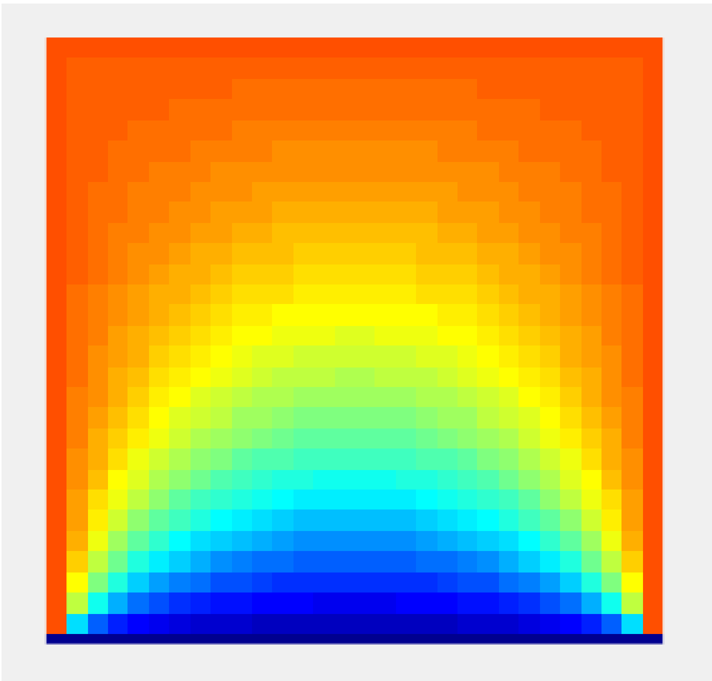

Figure 5: The final condition of the bar after 500 time steps where the heat diffusion is allowed according to the rules stated above.

1.5 Observations

As seen below, if we increase the diffusion rates then the time which is taken by the bar to come to a steady state (a state where the temperature of neighbouring cells is comparable and their difference is less than the error).

Also for the Von-Neumann method we see that the time taken is less than that of the Moore method. Reason being, the Von-Neumann would see only the four neighbouring neighbours whereas Moore looks at all the neighbours.

The accuracy of the Von-Neumann model will be lesser as compared with the Moore model because it takes lesser number of neighbours into consideration.

Diffusion Rate	Moore (No. of iterations)	Von-Neumann (No. of iterations)
0.001	1213	461
0.005	757	324
0.05	183	135