CS302-Modulation and Simulation

Lab 4

Undertaken By

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Contents

1 Model for the Spread of Malaria

The model for the spread of malaria is done by dividing the human class and the mosquito class in different compartments.

Here, we will see that all humans are susceptible and on getting bitten by an infected mosquito they would get infected. Once infected the humans can become susceptible again or can also become immune to the disease.

The mosquitoes would have only two compartments which are susceptible and infected. Where, they move from susceptible to infected on biting an infected human.

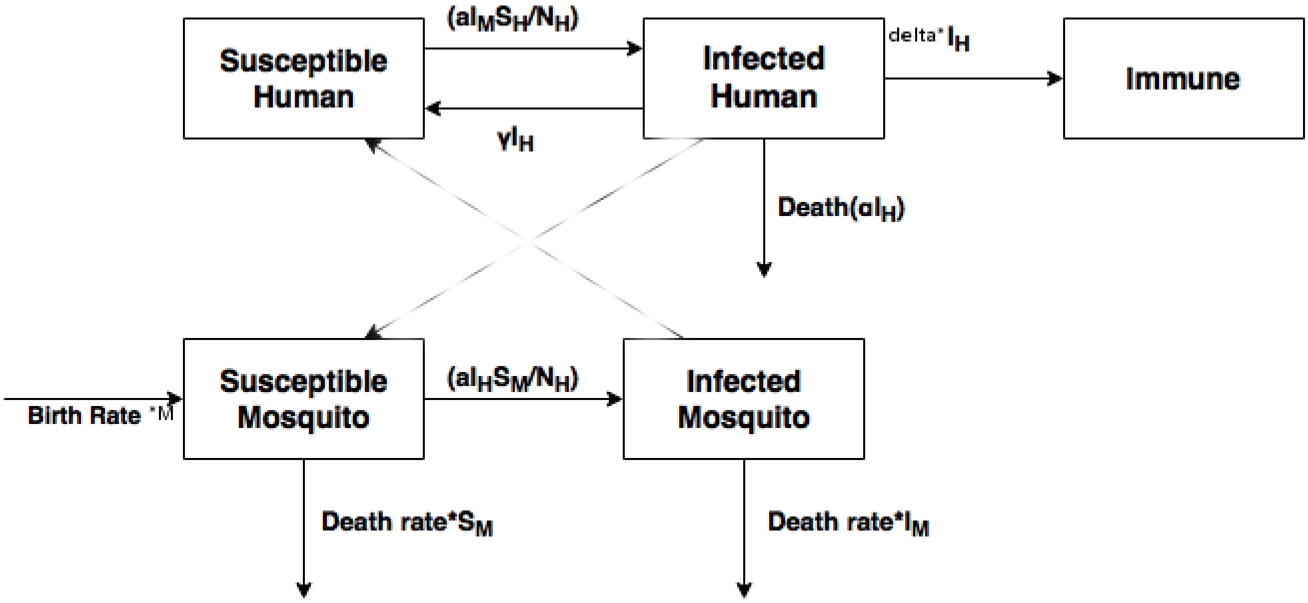

Figure 1: The various compartments of the Malaria Spread Model as discussed above. The dark arrows show the movement from one compartment to the other whereas the light arrows show the interaction between two compartments.

1.1 Assumptions

1. Life span of humans is much longer than the mosquitoes, hence we ignore the natural human birth and death rates, immigration, etc.
2. All the deaths of humans are caused by malaria and no other factor.
3. As soon as an infected mosquito bites a susceptible human, it becomes infected.
4. As soon as a susceptible mosquito bites an infected human it becomes infected.
5. Mosquitoes have a smaller life span and hence we need to take into consideration their birth and death rates.
6. There is homogenous interaction in the population.

1.1.1 Assumptions for Humans

1. Susceptible humans get infected due to mosquito bite from an infected mosquito.
2. Infected humans can recover and become susceptible again.
3. Infected humans can die due to malaria.
4. Infected humans can become immune.

1.1.2 Assumptions for Mosquitoes

1. Susceptible mosquitoes can become infected when they bite an infected human.
2. All mosquitoes are born susceptible.
3. No recovery for infected mosquito.
4. Mosquitoes die naturally and not due to disease infection.

1.2 Mathematical Model

1.2.1 Parameters

$S_h(t)$ =Number of Susceptible Humans at time t
$I_h(t)$ =Number of Infected Humans at time t
$A_h(t)$ =Number of Immune Humans at time t
$S_m(t)$ =Number of Susceptible Mosquitoes at time t
$I_m(t)$ =Number of Infected Mosquitoes at time t
N =Total Population of Humans
M =Total Population of Mosquitoes
a =Probability of a mosquito biting an infected person
α =Malaria Induced Death Rate
γ = Recovery Rate of humans
δ =Immunity Rate
d =Natural Death rate of mosquitoes
b =Natural Birth rate of mosquitoes

1.2.2 Differential Equations

$$S_h + I_h + R_h = N$$

From above given model we can define the equations for the model as follows:

$$\frac{dS_h}{dt} = -\frac{aI_vS_h}{N} + \gamma I_h \tag{1}$$

$$\frac{dI_h}{dt} = \frac{aI_vS_h}{N} - (\alpha + \gamma + \delta)I_h \tag{2}$$

$$\frac{dA_h}{dt} = \delta I_h \tag{3}$$

$$\frac{dS_m}{dt} = bM - \frac{aI_hS_m}{N} - dS_m \tag{4}$$

$$\frac{dI_m}{dt} = \frac{aI_hS_m}{N} - dI_m \tag{5}$$

$$\frac{dD_h}{dt} = \alpha I_h \tag{6}$$

$$\frac{dD_m}{dt} = dS_m + dI_m \tag{7}$$

1.3 Computational Analysis

1.3.1 Analysis of the 5-compartment model

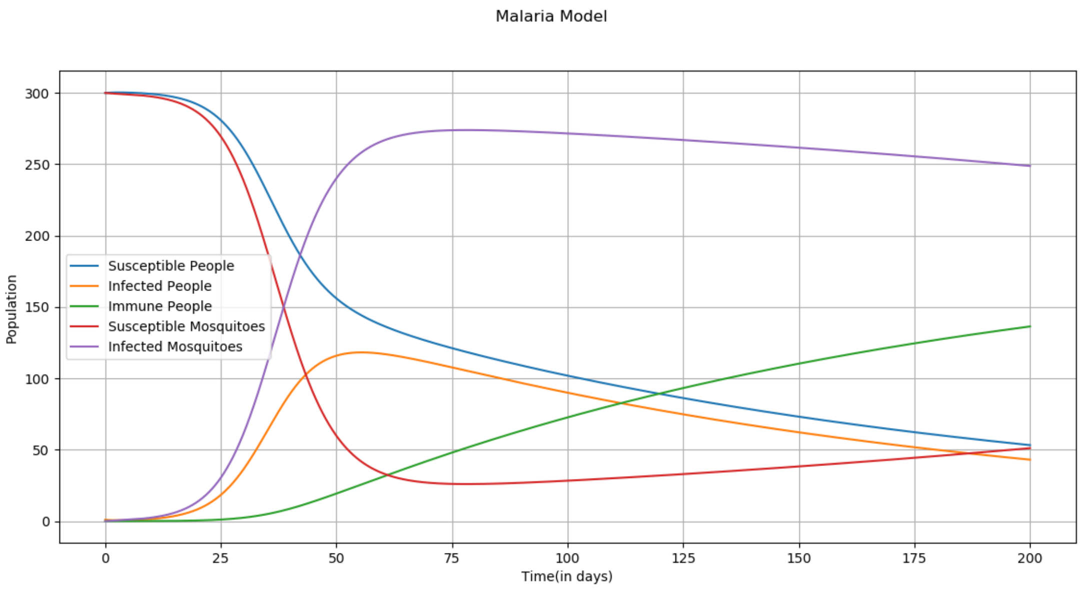

Figure 2: Model for malaria spread taking into account the equations, Eq. (1), Eq. (2), Eq. (3), Eq. (4) and Eq. (5). Here, the quantities are taken as $S_h(0) = N = 300$, $I_h(0) = 1$, $A_h(0) = 0$, $S_m(0) = M = 300$, $I_m(0) = 0$, $\gamma = 0.3$, $\delta = 0.01$, $\alpha = 0.005$, $birthrate_m = 0.01$, $deathrate_m = 0.01$ and $a = 0.3$

As we can see from the above graph, the number of susceptible people decrease initially with time because they would get infected and eventually immune. But some infected people also become susceptible again, hence the susceptible population of humans would increase after a certain time.

The number of immune people would increase at a steady exponential rate and given enough time all the humans would become immune to the disease.

The mosquitoes on the other hand are born at a steady rate and this rate is directly proportional to the original population. Hence, there is an increase in the number of susceptible population of mosquitoes.

Mosquitoes on the other hand can die only of natural deaths and not deaths due to malaria. As their death rates are very low, we can see that the infected population of mosquitoes keeps on increasing. This can result in very serious effects at a later stage because with so many vectors, there is a higher probability of a mosquito biting a human being and infecting him/her with the disease.

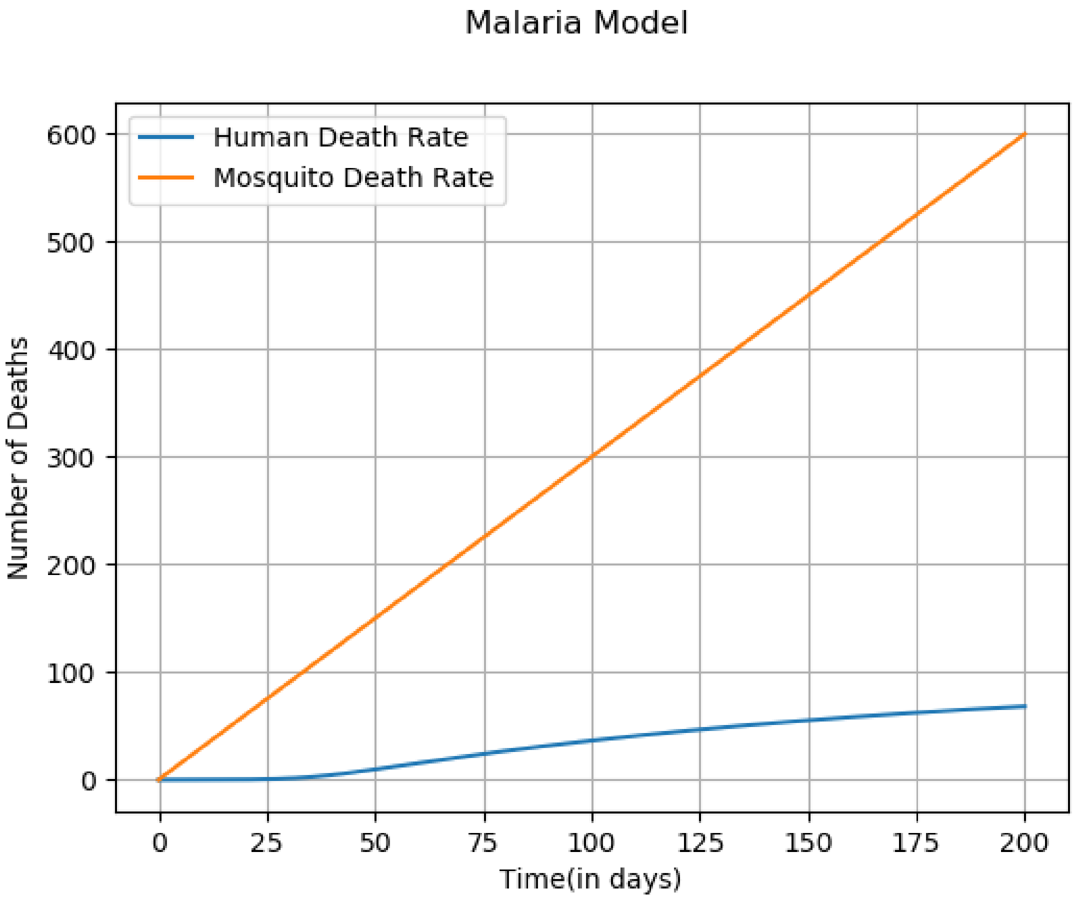

Figure 3: Death Rates of humans and mosquitoes taking ;int in the o account the equations, Eq. (6) and Eq. (7). Here, the quantities are taken as $S_h(0) = N = 300$, $I_h(0) = 1$, $A_h(0) = 0$, $S_m(0) = M = 300$, $I_m(0) = 0$, $\gamma = 0.3$, $\delta = 0.01$, $\alpha = 0.005$, $birthrate_m = 0.01$, $deathrate_m = 0.01$ and $a = 0.3$

The deaths of humans touch an upper value and then get flattened out. This would be because we aren't introducing any humans by natural birth. As a result the total population remains constant for humans.

On the other hand, the mosquitoes are being introduced at a constant rate into the model. As a result we can see that number of deaths for the mosquitoes keep on increasing.

1.3.2 Analysis for fumigation measures

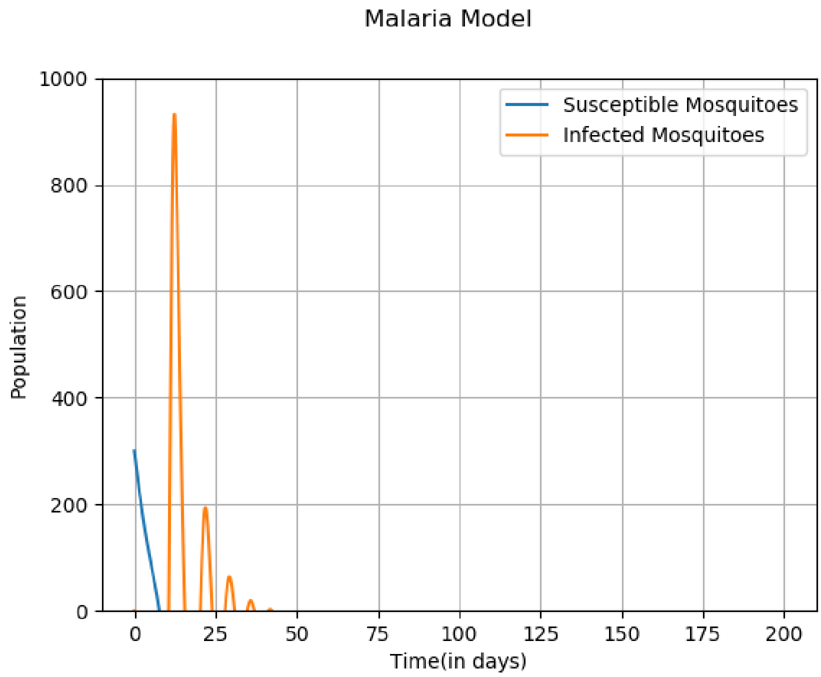

Figure 4: Susceptible and Infected Mosquito Population for constant rates of fumigation

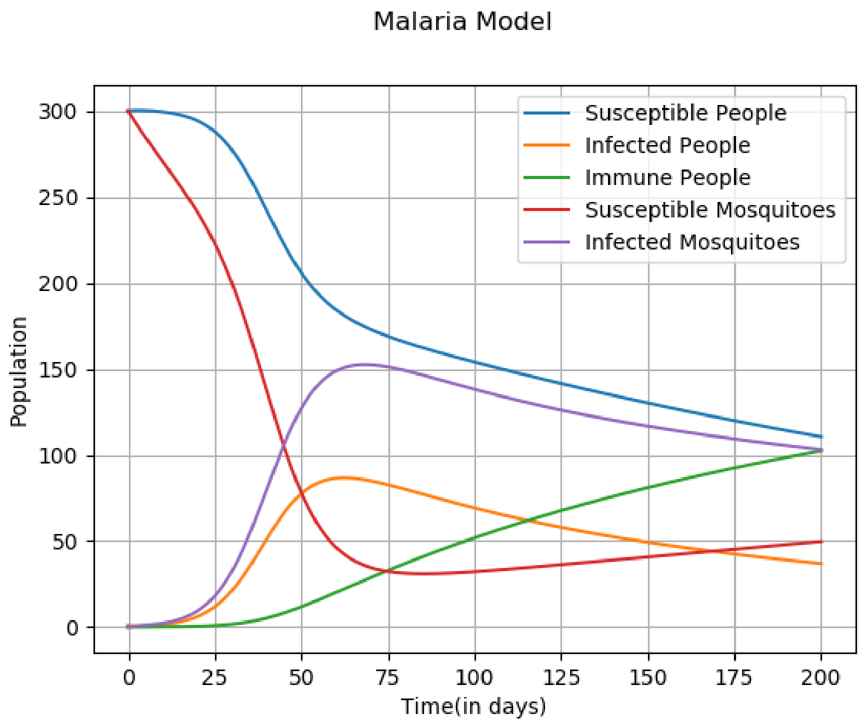

Figure 5: Susceptible and Infected Mosquito Population for variable rates of fumigation. Here the rate of fumigation is taken to be $k = 0.01$

As we can see from the above figures, the number of infected mosquitoes would decrease drastically. For constant fumigation, the number of mosquitoes would slowly decrease to 0 and hence would finish the disease once and for all.

Whereas, for variable fumigation rates i.e. when the number of deaths due to fumigation is dependent on the population size, we see that that infected mosquitoes decrease and go to near 0 but don't get over completely. On the other hand the susceptible mosquitoes would touch towards an upper limit where they would get constant.

We see that it doesn't have much effect on the human population but there are drastic changes in the mosquito population after putting these measures into effect.

Here, we would need continuous fumigation to keep the number of mosquitoes in check.

The graphs of different compartments for different values of fumigation rates are plotted below:

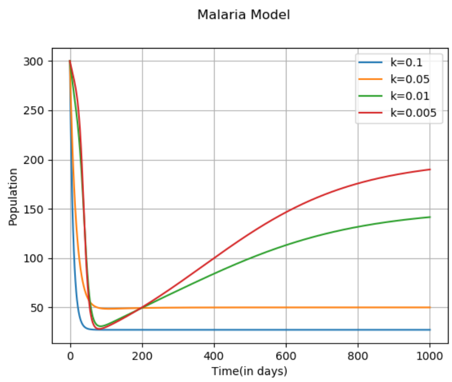

Figure 6: Population of Susceptible Mosquitoes for different rates of fumigation

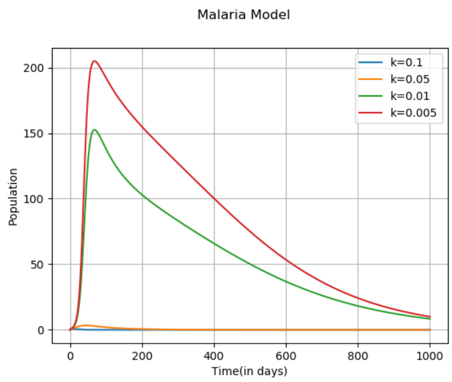

Figure 7: Population of Infected Mosquitoes for different rates of fumigation

1.3.3 Analysis for vaccination measures

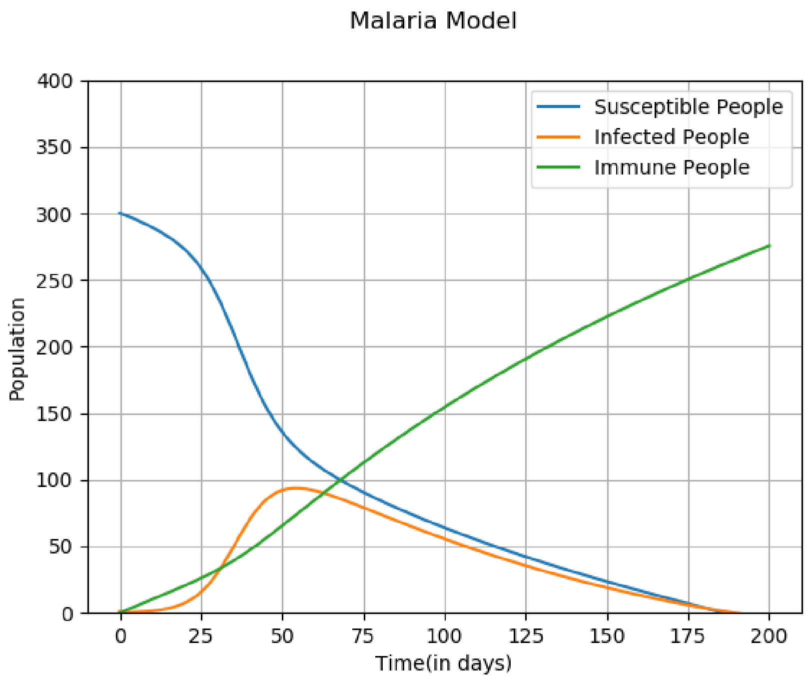

Figure 8: Susceptible, Infected and Immune Human Population for constant rates of vaccination

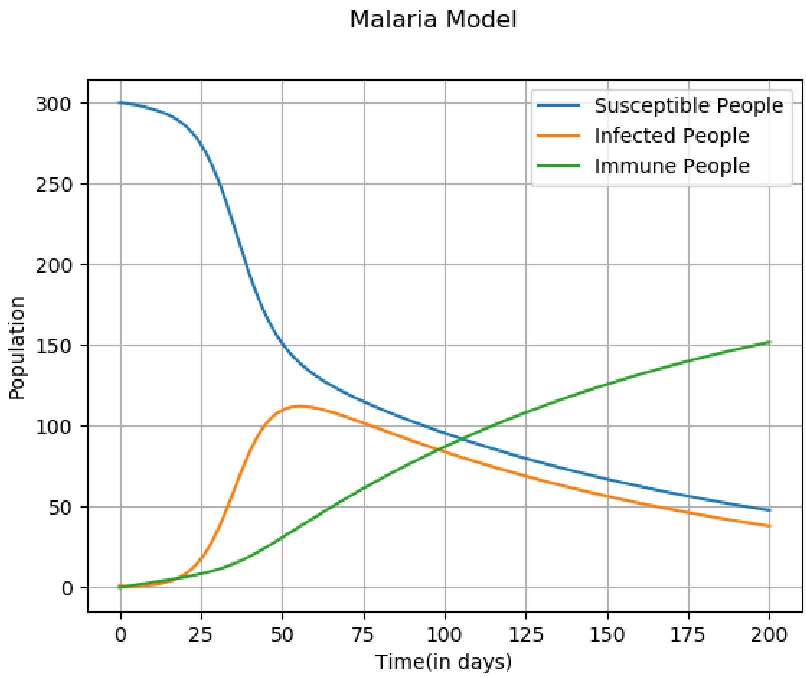

Figure 9: Susceptible, Infected and Immune Human Population for constant rates of vaccination. Here, vaccination rate $k = 0.01$

On vaccination we know that the number of susceptible people would decrease and the number of immune people would increase. With constant vaccination rates there is a sharp decrease in the susceptible population and the immune people increase sharply.

When we have population dependent function instead of constant the immune people and susceptible population reaches its saturation level and then they get stabilized around that value. Because of changes in this, the infected population also decreases since now many people can't contract disease and are safe.

As we change the rate of vaccination, the graphs change accordingly.

- The rate of decrease in the number of susceptible people is directly proportional to the value of k. Higher the k, faster it falls.
- Number of infected people is inversely proportional to the value of k. Hence, with greater values of k it decreases.
- Immune people increase with the increase in the rate of vaccination, hence we can see a direct proportionality there too.

We see that it doesn't have much effect on the human population but there are drastic changes in the mosquito population after putting these measures into effect.

The graphs for different compartments for different values of vaccination rates are plotted below:

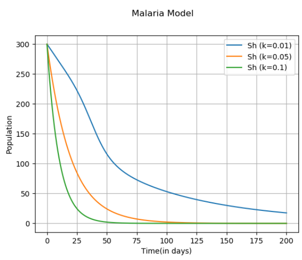

Figure 10: Population of Susceptible Humans for different rates of vaccination

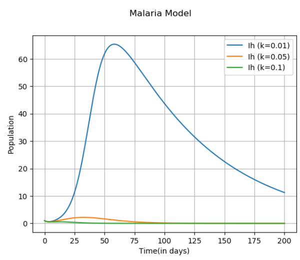

Figure 11: Population of Infected Humans for different rates of vaccination

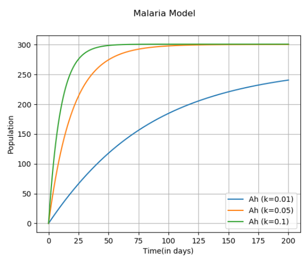

Figure 12: Population of Immune Humans for different rates of vaccination

1.4 Conclusion

- The malaria spread problem can be controlled using the two measures discussed above namely fumigation and vaccination.
- With fumigation we are trying to control the mosquito population whereas with vaccination we are making the people immune from the disease.
- When we use fumigation we are killing other mosquitoes as well. This can have a bad effect on the environment food cycle, so trying to control the population by vaccination is a better option.
- By changing the values of fumigation and vaccination constant we are actually changing R_0 i.e. the basic reproduction number which would in turn change the working of the whole model.

1.5 References

- Using Mathematical Model to Illusrate Spread of Malaria - KS Kipchirchir
- Disease Modeling Institute