CS302-Modulation and Simulation

Lab 3

Undertaken By

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Contents

1 SIR Model for Modeling Influenza

The SIR Model is used to model simple spread of diseases like influenza or flu. Here, we are looking at diseases which do not kill. At any given time, we need to know the number of infected, susceptible or recovered people.

Also, we would like to see the effect of vaccines on the people. Now, for understanding the following model, we convert it into three models which are:

1. Infected (I(t))
2. Susceptible (S(t))
3. Recovered (R(t))

Figure 1: The compartments of the SIR Model for modeling the Influenza spread

1.1 Assumptions

1. The sum of number of susceptible, infected, vaccinated and recovered people remains constant which is equal to the total population. The population size is large and constant.
2. The number of susceptible remains constant, because we are taking a time scale such that the rate of normal birth and death can be ignored.
3. Homogeneous mixing i.e. each pair of individuals has equal probability of coming into contact with one another.
4. The only way susceptible person can leave the compartment is by becoming infected or by getting vaccinated.
5. The number of people susceptible depends on the current number of susceptible people and also on the contact he/she has with the infected people.
6. The people who have recovered or gotten vaccinated neither get susceptible again nor die.
7. A fixed number of people would be getting better each day. We will later see that if the average time of infection is α, then number of people recovering each day is $\frac{1}{\alpha}$

1.2 SIR Model without vaccination

p =Probability of transmission per infected
k =Number of contacts per individual per unit time
S =Number of Susceptible
I =Number of Infected
R =Number of Recovered
N =Total Population
So, kS/N = Number of contacts (with Infected or Susceptible) per individual per susceptible per unit time

$$S + I + R = N$$

Susceptible Equation

From above details we can define the rate at which susceptible changes,

$$\frac{dS}{dt} = -\frac{pkSI}{N} \tag{1}$$

$$\frac{dS}{dt} = -\beta SI \tag{2}$$

Where, $\beta = \frac{\beta_f}{N}$; $\beta_f = pk$

Infected Equation

$$\frac{dI}{dt} = \beta SI - \alpha I \tag{3}$$

Recovered Equation

$$\frac{dR}{dt} = \alpha I \tag{4}$$

1.2.1 Computational Analysis

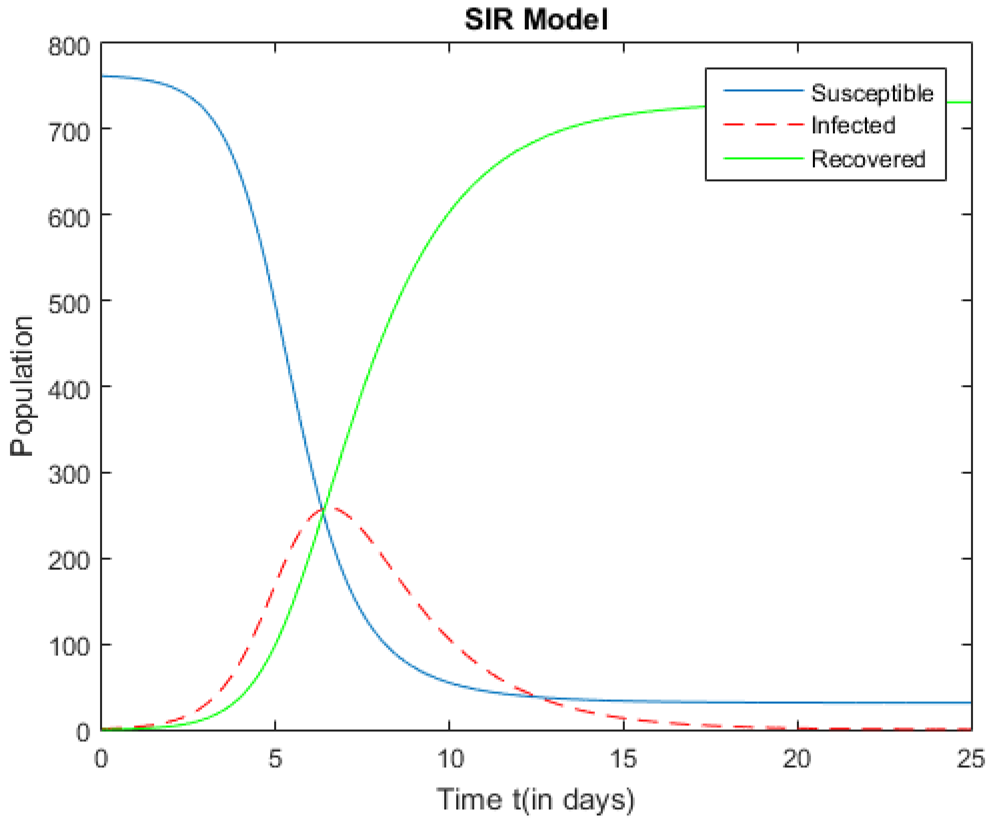


Figure 2: Normal SIR Model without vaccination as seen in Eq. (2), Eq. (3) and Eq. (4)

The above figure shows the rate at which the susceptible, infected and recovered change in the spread of Influenza. Initially the whole population i.e. N is susceptible except a very small number of people who are infected. This number can be neglected as compared to the total population but it is enough to start the cycles for the spread of the disease.

We see that as time passes the number of people who are susceptible get converted to infected at a rate of β as shown in Eq. (2) and hence the number of infected increases. Now, as the infected increases the number of people recovering changes according to Eq. (3). This rate i.e. α is greater than β and so the number of infected people starts dropping until all have recovered.

Since, the recovered people can't get susceptible again the graph of the recovered population saturates to the final population and the infected population goes to zero.

1.3 SIR Model with vaccination

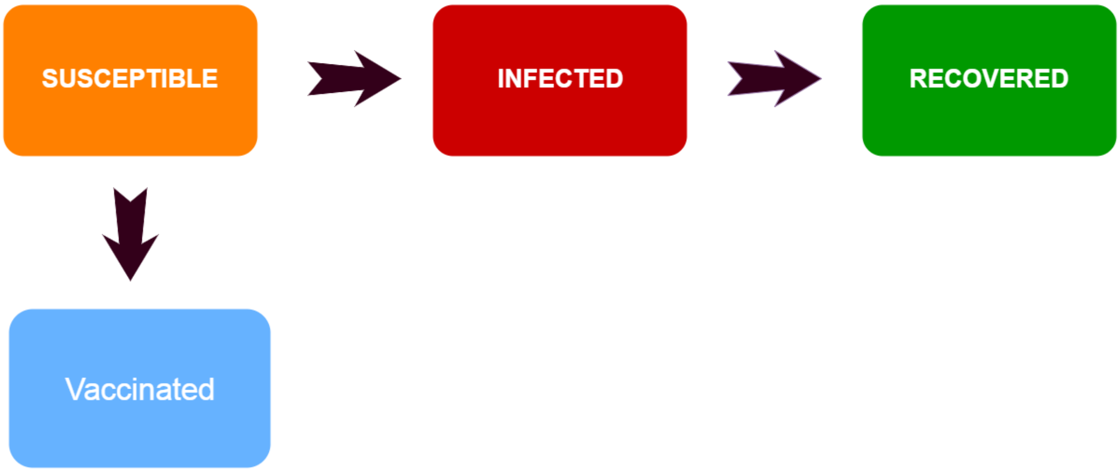


Figure 3: The SIR model for the spread of influenza considering the vaccination of the susceptible people

All other parameters remain the same. Here, μ is the vaccination rate which is taken to be 0.15 in our case. This means that daily 15 % of the susceptible population is vaccinated.

$$S + I + R + V = N$$

Susceptible Equation

$$\frac{dS}{dt} = -\beta SI - \mu S \tag{5}$$

Infected Equation

$$\frac{dI}{dt} = \beta SI - \alpha I \tag{6}$$

Recovered Equation

$$\frac{dR}{dt} = \alpha I \tag{7}$$

Vaccinated Equation

$$\frac{dV}{dt} = \mu S \tag{8}$$

1.3.1 Computational Analysis

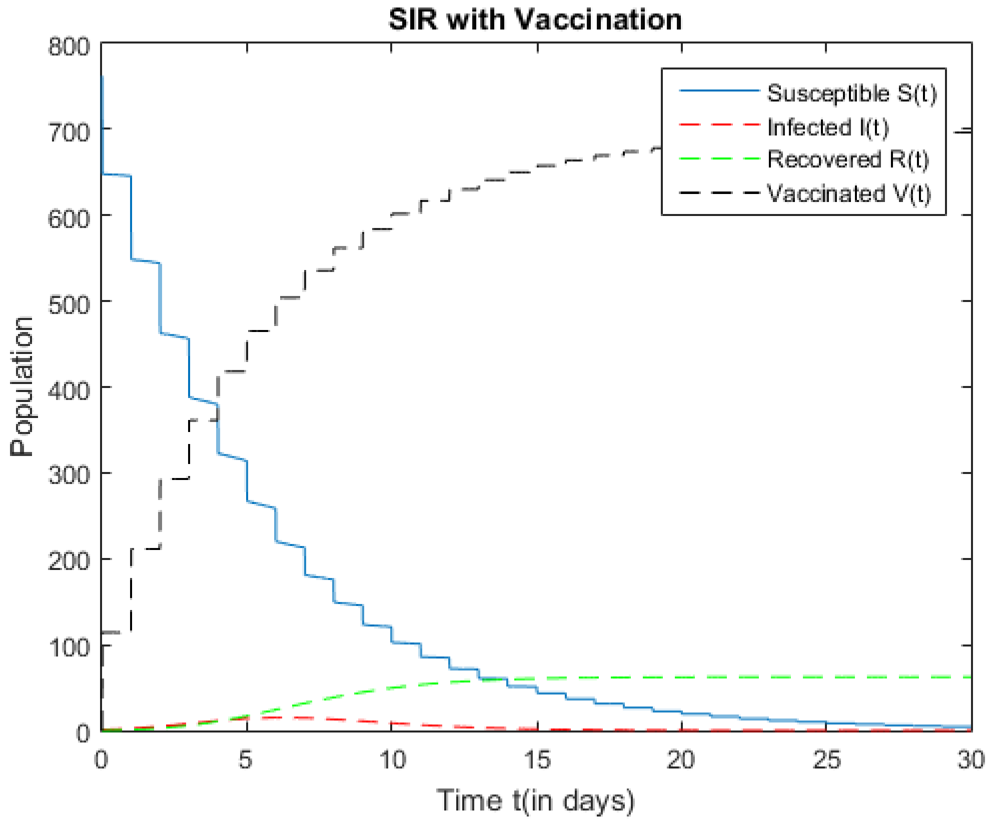
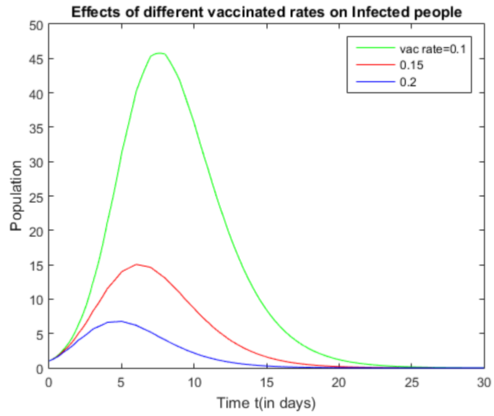


Figure 4: SIR Model with a certain vaccination rate as seen in Eq. (5), Eq. (6), Eq. (7) and Eq.(8)

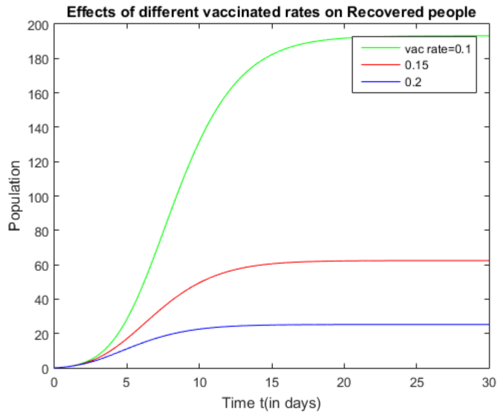
In this case, we have taken into consideration the vaccination of certain people from the susceptible category. The vaccination is done on a daily basis at a certain fixed rate (μ). Here, we have taken that rate to be 15 percent. Now in this case, we assume that the effect of the vaccination is immediate which means that as soon as the person is vaccinated he becomes immune to the disease and hence can be transferred to the vaccinated compartment.

So, here we will need to remove the number of people who are vaccinated from the susceptible compartment and add them to the vaccinated compartment. This can be seen from Eq. (5) and Eq. (7).

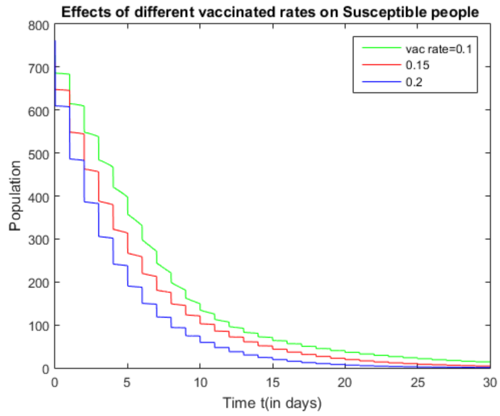
We see that since in this case the number of vaccinated will increase the rate of infected people which depends on the number of susceptible people will decrease drastically, hence it will improve the chances of subduing our infectious disease.



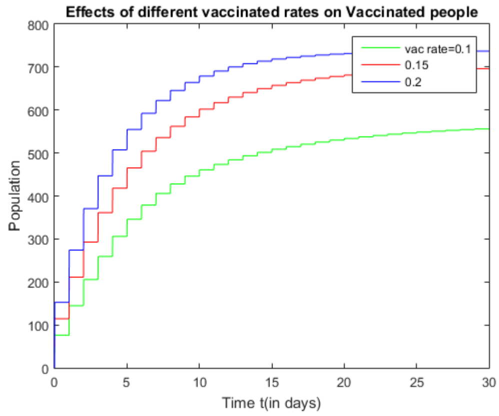
(a) Infected Population



(b) Recovered Population



(c) Susceptible Population



(d) Vaccinated Population

For different values we have plotted the susceptible, infected, vaccinated and recovered populations.

The number of vaccinated people depends on the number of susceptible people according to Eq.(8). Hence, as the rate of vaccination increases the number of people moving from the susceptible compartment to the vaccinated compartment increases.

As more and more people get vaccinated they become immune to the disease. As a result the people becoming infected decreases according to Eq. (6).

The number of recovered people will depend on the number of infected people. Now, as the rate of vaccination increases the number of infected people would decrease and with it would decrease the number of recovered people.

The number of vaccinated people would increase exponentially with the increase in the rate of vaccination.

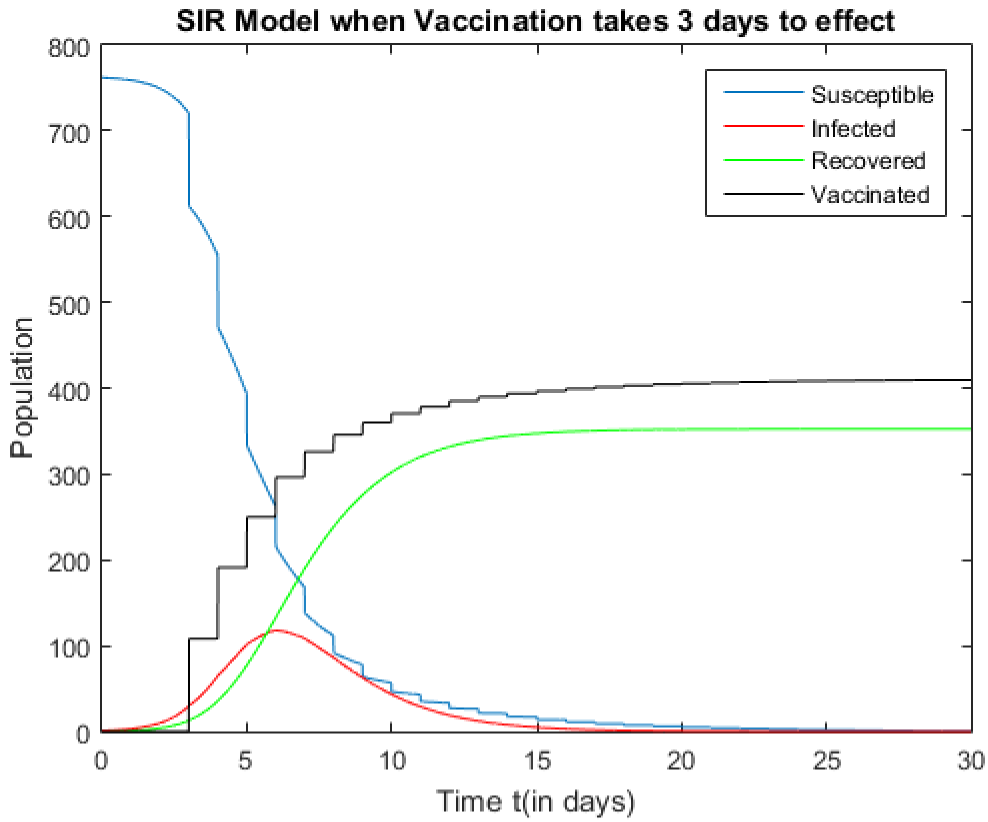
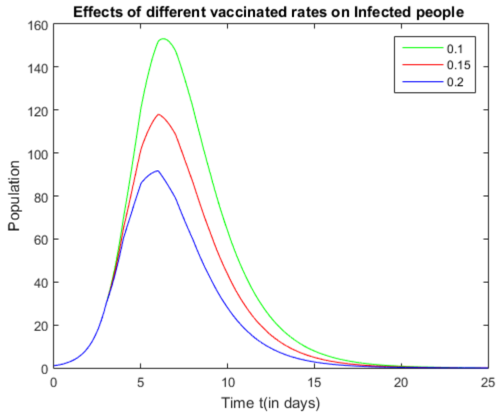


Figure 5: The SIR model for the spread of influenza considering the vaccination (non-instantaneous) of the susceptible people

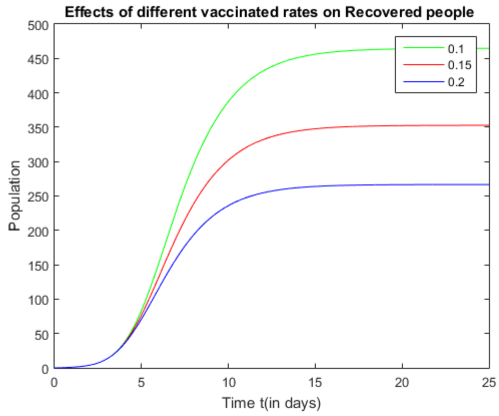
In this case, we have taken into consideration the case where the effect of the vaccination isn't instantaneous but is delayed by 3 days. Hence, the people ho are vaccinated won't be immediately shifted to the vaccinated compartment but will stay in the susceptible compartment for 3 days. During those three days they have a chance to contract the disease which we can see from the figure.

Here, we see that the saturation level of the vaccinated population size is very low. And the recovered and the infected are also high as compared to the scene where there was immediate vaccination.

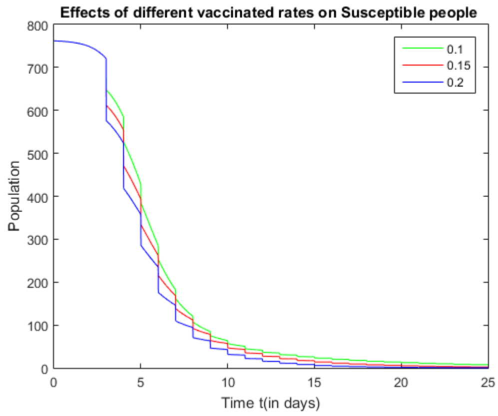
The susceptible, infected, vaccinated and recovered population for different rates of vaccination are shown below. They show the same trends as before.



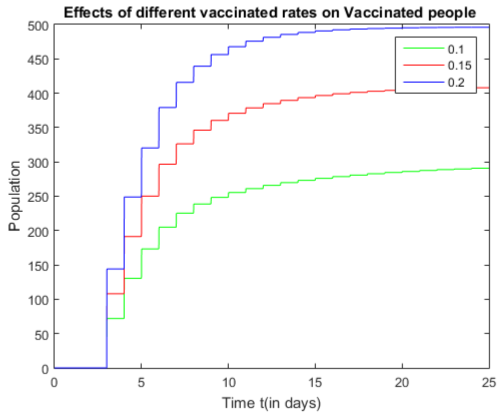
(a) Infected Population



(b) Recovered Population



(c) Susceptible Population



(d) Vaccinated Population

From the above graph we can say that when vaccination takes 3 days to affect,the susceptible and vaccinated population follow step-wise change.Initially when there is no effect of vaccination the susceptible graph follows the natural decrease same as SIR model but when the effect of vaccination takes place,it decreases step-wise.Same thing happens with vaccination but it increases step-wise rather than decreases. Infected population and recovered population graph remain same.

2 SARS Model

The model for the spread of Severe Acute Respiratory Problem (SARS), developed by Lipsitch. It helps us to make predictions on the impact of public health efforts to reduce the spread of the disease. These efforts include quarantine and isolation of the individuals which would help reduce the further spreading of the diseases. The SARS Model is an extension of the SEIR Model (Susceptible, Infected, Exposed, Recovered) which itself is extended from the SIR Model discussed above. The SARS Model adds the compartments, quarantined, isolated and death to the SEIR Model.

2.1 Assumptions

1. The time frame is such that we can ignore natural deaths and births.
2. All the deaths that may occur are due to SARS only and natural death rate can be ignored.
3. The contact of an infected person with rest of the population, mainly the susceptible people is constant and does not depend on the population density.
4. The person who may have the disease but is quarantined or isolated would not be able to spread the disease.
5. Person who is susceptible but not quarantined, would not contract the disease.

2.2 Compartments for the Model

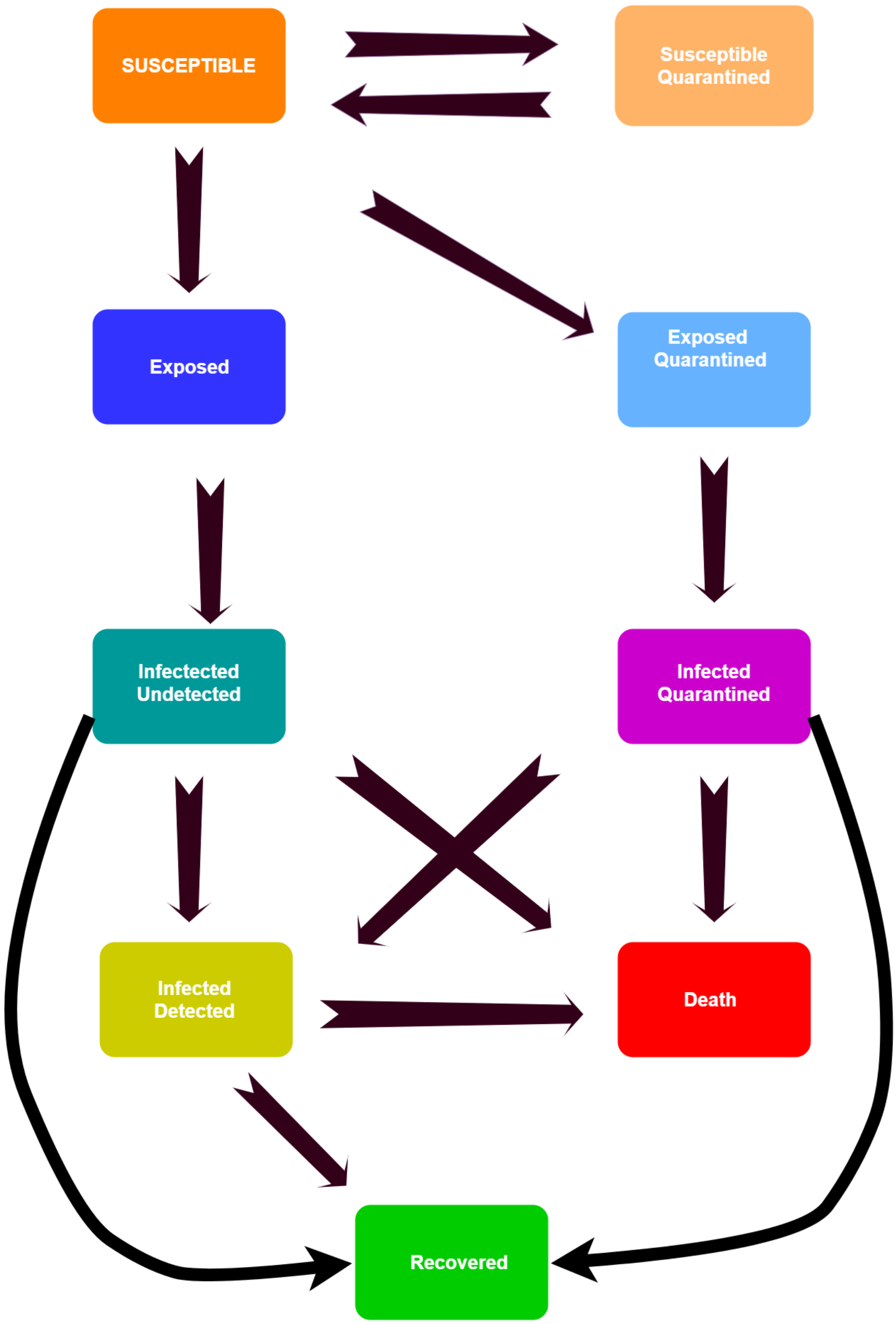


Figure 6: SARS 9 compartment model

- Susceptible (S): They are the people who are at risk from contracting the disease. They can contract it by coming in contact with the infected people.
- Susceptible Quarantined (S_Q): They do not have SARS and are kept in quarantine and hence they can't catch SARS.
- Exposed (E): They have come in contact with the infected people but haven't yet contracted the disease. But at some later time they would get infected ad also show the symptoms.
- Exposed Quarantined (E_Q): They have been exposed but been advised to stay in isolation so that they do not spread it further.
- Infectious Undetected (I_U): Have the infection, but haven't yet been detected.
- Infectious Quarantined (I_Q) : Infected and advised to stay away from the people in order to reduce the chances of spread of disease.
- Infectious Isolated (I_D) : The people who have been detected and are forced to stay in isolation, hence removing the possibility of them coming in contact with anyone else and spreading the disease.
- Death (D) : They have died due to SARS
- Recovered (R) : They have recovered from SARS and are immune from it.

2.3 Differential Equations for the SARS Model

$$\frac{dS}{dt} = uS_q - kq(1-b)I_u\frac{S}{N} - kqbI_u\frac{S}{N} - k(1-q)bI_u\frac{S}{N} \tag{9}$$

$$\frac{dS_q}{dt} = kq(1-b)I_u\frac{S}{N} - uS_q \tag{10}$$

$$\frac{dE}{dt} = k(1-q)bI_u\frac{S}{N} - pE \tag{11}$$

$$\frac{dE_q}{dt} = kqbI_u\frac{S}{N} - pE_q \tag{12}$$

$$\frac{dI_u}{dt} = pE - wI_u - vI_u - mI_u \tag{13}$$

$$\frac{dI_q}{dt} = pE_q - wI_q - vI_q - mI_q \tag{14}$$

$$\frac{dI_d}{dt} = wI_u + wI_q - vI_d - mI_d \tag{15}$$

$$\frac{dD}{dt} = m(I_u + I_q + I_d) \tag{16}$$

$$\frac{dR}{dt} = v(I_u + I_q + I_d) \tag{17}$$

$$\frac{dN}{dt} = -\frac{dD}{dt} \tag{18}$$

2.4 Parameters

All the probabilities and fraction of transfer of people from one compartment to another below are on a daily basis i.e. they are calculated for 24 hour day.

- b = Probability that the contact between an infectious undetected person and a susceptible person results in a successful transmission of SARS
- k = Average number of contacts between undetected infectious people and the susceptible people
- q = Fraction of people in Susceptible which go to Exposed Quarantined or Susceptible Quarantined
- u = Fraction of individuals who move from Susceptible Quarantined to Susceptible
- p = Fraction individuals who become infectious in the following ways:
  Transitions from Exposed to Infectious undetected
  Exposed Quarantined to Infectious Quarantined
- w = Fraction of individuals who move to Infectious Isolated from Infectious Undetected or Infectious Quarantined
- m = Per capita Death Rate
- v = Per capita Recovery Rate

2.5 Computational Model

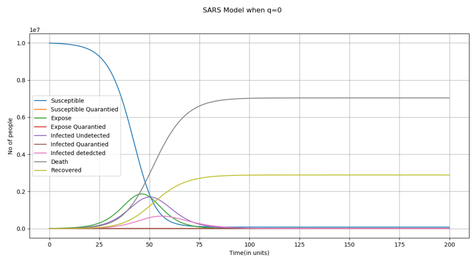


Figure 7: SARS Model for q=0

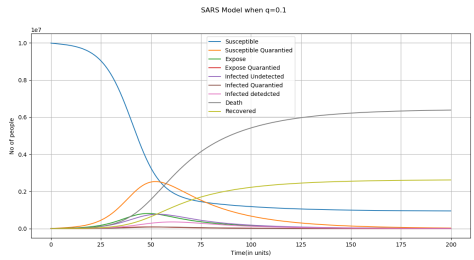


Figure 8: SARS Model for q=0.1

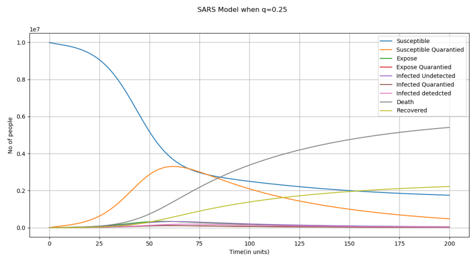


Figure 9: SARS Model for q=0.25

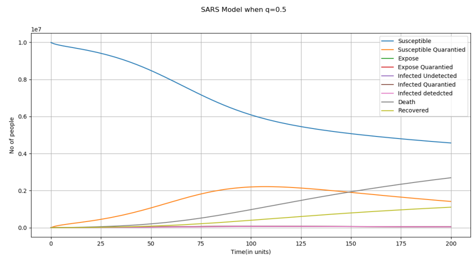


Figure 10: SARS Model for q=0.5

q is the probability of people who go from susceptible to exposed quarantined or susceptible quarantined daily. In the first case we have taken $q = 0$ and hence there is no possibility of the people going into this state. Apart from the quarantined compartments all the other compartments act according to the equations of the model.

As q increases we increase the chances of a person being advised to go into isolation, mainly for two reasons which are:

1) Susceptible Quarantined, so that he doesn't contract the disease
2) Exposed Quarantined, which means that the person doesn't spread it further

As the number of quarantined people increase, the number of susceptible people decrease and hence the chances of a person getting exposed to the disease also decrease. When the number of exposed people who are quarantined increase, they in turn would decrease the number of infected people. With the exposed people decreasing, the number of infected decrease and hence the number of deaths would also decrease.

The susceptible people become 0 in case of $q = 0$. As q increase the susceptible people saturate to a value higher than 0 and this saturation level increases with the increase in q.

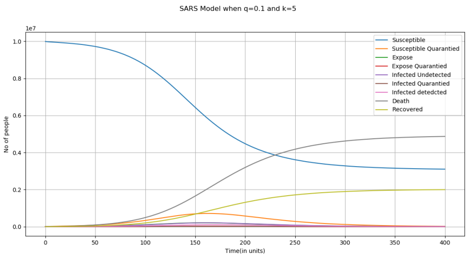


Figure 11: SARS Model for k=5

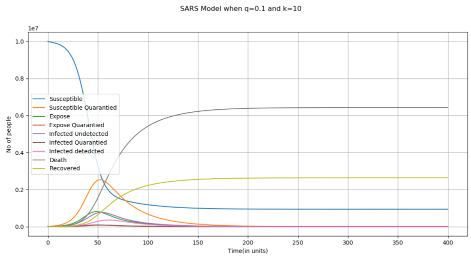


Figure 12: SARS Model for k=10

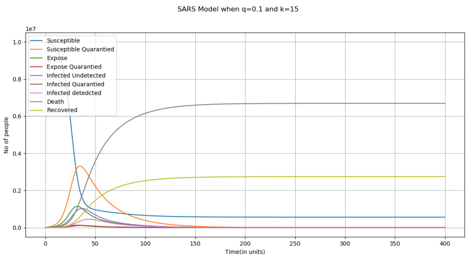


Figure 13: SARS Model for k=15

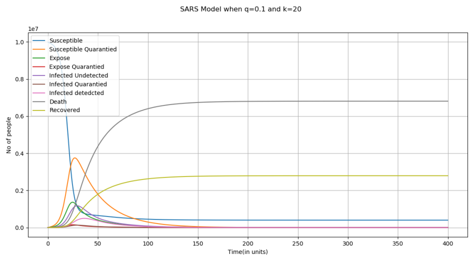


Figure 14: SARS Model for k=20

k is the average number of contacts between the infected and susceptible people on a daily basis. Here, as the number of contact increase we see that the number of people getting exposed increases accordingly. Also with the increase in the exposed people, the people who get infected also increase.

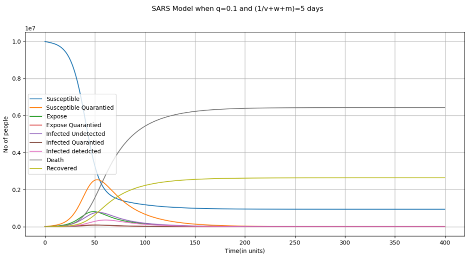


Figure 15: SARS Model for 1/(v+m+w) = 5 days

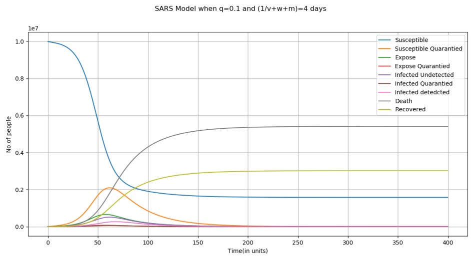


Figure 16: SARS Model for 1/(v+m+w) = 4 days

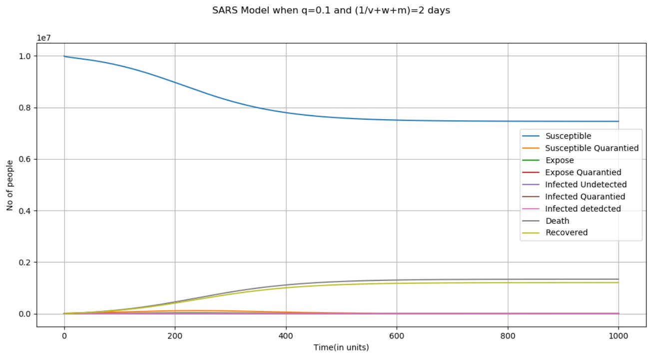


Figure 17: SARS Model for 1/(v+m+w) = 2 days

$v + w + m$ is the summation of the rate at which people move from infected undetected to infected isolated and infected quarantined, recovery rate and the mortality rate. $\frac{1}{v+w+m}$ is the average duration which a person would take to become infected from susceptible. As we increase the value of these three values, the number of infected people increases. As we increase this we will increase the value of R i.e. the recovery rate.

With the increase in this value, the value at which the susceptible population gets saturated at a lower value.

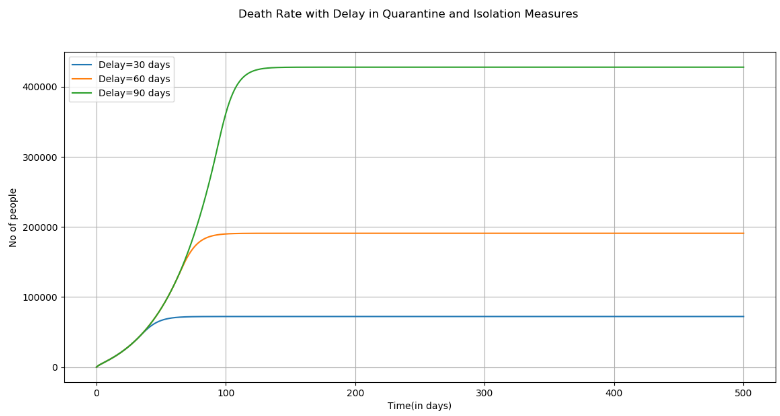


Figure 18: Death Rate for Delayed Measures

Here, we have seen the effect of delayed measures during the spread of SARS. As we delay the effective measures we see that the number of deaths increase i.e. the death saturates at a higher value. So, we need to put these measures at an early stage to decrease the rate of deaths.

2.6 Conclusion

- To reduce the death rate, we should increase q and decrease k and $\frac{1}{v+m+w}$.
- To decrease the death rate we should put the healthcare measures in place at an early stage itself.

2.7 References

- https://www.maa.org/press/periodicals/loci/joma/the-sir-model-for
- mds.marshall.edu/cgi/viewcontent.cgi?article=1737&context=etd