Optimization Techniques

→ Interdisplinary team approach

→ operational research is mostly done in digital computer

3 Phases

→ Judgement Phase: → we set a goal, and it is a short phase.

→ Research Phase: → we understand what problem, formulating hypothesis

→ Analysis observation and verifying same, we predict the results, we define alternative method

→ Action Phase:

We act it in particular industry.

→ Modelling in Operation Research.

→ It can't represent real objects.

↳ Account model,

- Model by degree of abstraction

  It is based on past data and model

  eg: language model, case studies.

- Model by function

  Consist of descriptive model, predictive and normative model.

  ~~description~~.

- Model by structure

  → Iconic: Physical model, pictorial representation

  → Analog model:

  → symbolic model:

- Model by nature of environment:

  ↳ Deterministic model

  ↳ Probabilistic model.

→ Model by extent of generality:

General solution for OR

Steps

1. Formulating the problem
2. Construct the mathematical model.
3. Derive the solution from model.
4.

Linear programming problems

→ Decision variables
→ objective.
→ constraints.

Basic assumptions

a) certainity → Every resources should be fixed.

b) Divisibility → non negative values, fractional values

c) Proportionality → Available resources and values should be proportional

d) Additivity →

Major steps in LPP's

1. Key decision.
2. Identify the variables given in the problem.
3. State feasible alternatives which are generating $x_j \geq 0$.
4. Identify the constraints in the problem and express it in as linear inequality or equation.
   LHS → linear function of decision variables

Q) A company has 3 operational departments, weaving, processing and packaging with capacity to produce 3 different types of clothes namely suiting, shirting and woolens. Yielding a profit of Rs 2, Rs 4, Rs 3 per meter respectively. 1m of suiting requires 3min in weaving, 2 min in processing and 1 min in packing. Similarly 1m of shirting requires 4 min of weaving, 1 min of processing and 3 min in packing. 1m of wooling requires 3 mins in each department. In a week, total run time of each dep is 60, 40, 80 hrs for weaving, processing and packing. Formulate LPP problem to find the product to maximize the problem.

	weaving	processing	packing	profit
x_1 suiting	3	2	1	2
x_2 shirting	4	1	3	4
x_3 woolens	3	3	3.	3.
Available	60X60	40X60	80X60	

$$3x_1 + 4x_2 + 3x_3 \leq 36000 \longrightarrow ①$$

$$2x_1 + x_2 + 3x_3 \leq 24000$$

$$x_1 + 3x_2 + 3x_3 \leq 48000$$

$$Z = 2x_1 + 4x_2 + 3x_3.$$

Q) Consider the foll problem faced by production planner in a soft drink plan. He has 2 machines A and B. A is ~~deprived~~ designed for ~~ounce~~ ounds bottle and B for 16 ~~ounce~~ ounds bottles. Each can be used for both types. Following data avail.

~~machine~~ ~~8 ounds~~ ~~16 ounds~~ ~~mach 8 B~~

machine	8 ounce	16 ounce
A	100 per min	40/min
B	60 per min	75/min.

Each machine can run 8hrs per day 5 days per week. Profit on a 8 ounce bottle is 25 paise and 16 ounce bottle is 35 paise. Weekly production of the drink cannot exceed 3 lakh ounce and market can absorb 25000 8 ounce bottles and 7000 16 ounce bottles per week. Planner wishes to maximize profit subject of all the production and marketing restriction. Formulate this as a linear program problem.

SIMPLEX METHOD.

→ Maximize / minimize. Let z be a objective fn.,

Minimum z = - Maximum (-z)

eg: $z = 6x_1 + 7x_2$

$= -(-6x_1 - 7x_2)$ convert to.

→ Inequalities convert to eqn.

$2x_1 + 3x_2 \leq 2$
$4x_1 + 3x_2 \geq 3$ → To convert to eqn we add 2 variables i.e. slack variables and surplus variables.

for $\leq$ ⇒ we add some variable to the inequality i.e for eg $2x_1 + 3x_2 + s_1 = 2$

$\geq$ ⇒ we subtract some variable from the inequality. $4x_1 + 3x_2 - s_1 = 3$

Q) $z = 6x_1 + 4x_2$

$-2x_1 + x_2 \leq 2$
$x_1 - x_2 \leq 2$
$3x_1 + 2x_2 \leq 9$
$x_1, x_2 \geq 0$

⇓

① $-2x_1 + x_2 + s_1 = 2$
$x_1 - x_2 + s_2 = 2$
$3x_1 + 2x_2 + s_3 = 9$

Maximize $z - 6x_1 + 4x_2$ be the objective fn.

B	C_B	X_B	x_1	x_2	s_1	s_2	s_3
s_1	0	2	-2	1	1	0	0
s_2	0	2	$\boxed{1}$	-1	0	1	0
s_3	0	9	3	2	0	0	1
		$z_j = 0$	0	0	0	0	0
		c_j	6	4	0	0	0
		Δ_j	-6	-4	0	0	0

→ Add the variables to objective fn

$z = 6x_1 + 4x_2 + s_1 + s_2 + s_3$

$z_j = \sum C_B \times x_1$ $\Delta_j = z_j - c_j$

$= 0 \times -2 + 0 \times 1 + 0 \times 3 = 0$

$= 0 \times 1 + 0 \times -1 + 0 \times 2 = 0$

c_j $\Delta_j = z_j - c_j$

Since the values of Δ_j is negative we want to select the highest negative row and draw next column θ.

θ_B X_B/x_1
-1
2
3

We consider the lowest +ve value from θ_B.

We select the the position where x_1 and lowest positive intersecting position we call it as key element

B	C_B	X_B	x_1	x_2	s_1	s_2	s_3	$\theta = \frac{X_B}{x_1}$
s_1	0	6	~~0~~ $-\frac{1}{2}$	-1 ~~0~~ ~~$-\frac{1}{2}$~~	1	2	0	-6
$x_1 \leftarrow s_2$	~~0~~ 6	2	$\boxed{1}$	-1 key element	0	1	0	-2
s_3 as per objective fn.	0	3	0	$\boxed{5}$	0	-3	1	3/5
		z_j	6	-6	0	6	0	0
			6	4	0	0	0	
			0	-10	0	6	0	

by row redn $R_1 \Rightarrow R_1 + 2R_2$ 2) $R_3 \to R_3 - 3R_2$

$z_j = 6+6+0$
$= 6$

$z_j = 0-6+0$
$= -6$ //

$c_j = 6 \quad 4 \quad 0 \quad 0 \quad 0$

$\Delta_j = z_j - c_j$
$=$ ~~0 -2j 4 6jk~~

B	C_B	X_B	x_1	x_2	s_1	s_2	s_3	$\theta = X_B/x_3$
s_1	0	~~200~~	33/5	0	1	7/5	1/5	
x_1	6	13/5	1	0	0	2/5	1/5	
x_2	4	3/5	0	$\boxed{1}$	0	-3/5	1/5	3/5
					0	0	0 + 13/5 + 3/5	0 - 3/5

$z_j = 0+6+4 \quad 0+0+4$

$R_1 \xrightarrow{10} R_1 + \frac{1}{5} R_3$ $\quad R_2$ $\quad$ 30 - 3/5

$R_2 \longrightarrow R_2 + R_3$.

6 $\quad$ 4 $\quad$ 0 $\quad$ 0 $\quad$ 0

$\Delta_j = z_j - c_j$

~~10~~ 4 . $\quad$ 2 $\quad$ 0 $\quad$ 0 $\quad$ 0 .

<u>for soln</u>

B	X_B
s_1	33/5
x_1	13/5
x_2	3/5

$z = 6x_1 + 4x_2 + 0$
$= 6 \times 13/5 + 4 \times 3/5$
$= 78/5 + 12/5 = \frac{90}{5} = 18$ //.

$5 \overline{)90}$ = 18

minimum z

$\leq \rightarrow +$ slack variable

$\geq \rightarrow -$ surplus + artificial $+M$

$= +$ artificial variable $-M$

→ if the equation is greater than or equal to the we add artificial variables and surplus variables.

Q) minimize $z = 5x_1 + 6x_2$ using $2x_1 + 5x_2 \geq 1500$, $3x_1 + x_2 \geq 1200$, $x_1 \geq 0$, $x_2 \geq 0$.

$2x_1 + 5x_2 - S_1 + A_1 = 0$

$3x_1 + x_2 - S_2 + A_2 = 0$

$\min z = 5x_1 + 6x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$

$\max = -5x_1 - 6x_2 - 0S_1 - 0S_2 - MA_1 - MA_2$

Key element

B	CB	XB	X1	X2	S1	S2	A1	A2	θ = XB/X2
A1	-M	1500	2	$\boxed{5}$	-1	0	1	0	300 ←
A2	-M	1200	3	1	0	-1	0	1	1200
	Zj		-5M	-6M	M	M	-M	-M	
	cj		-5	-6	0	0	-M	-M	
	$\Delta_j = z_j - c_j$		-5M+5	-6M+6	M	M	0	0	

↑

B	CB	XB	X1	X2	S1	S2	A1	A2	θ = XB/
X2	-6	300	2/5	1	-1/5	0	1/5	0	
A2	-M	1200	3	1	0	-1	0	1	
		900	13/5	0	1/5	-1	-1/5	1	

$R_2 \to R_2 - R_1$

~~$z_j = -6 \times 2/5 + 13/5 \times M$~~ $-12/5 + 13/5 M$ -6

$c_j =$

$-6 \times 2/5 - 13/5 M$ $\quad -6+0 \quad +6/5 - M/5 \quad +M \quad -M$

$= -\frac{12}{5} - \frac{13}{5}M$

$-5 \quad 6 \quad 0 \quad 0 \quad -M \quad -M$

c_j.

$\Delta_j = z_j - c_j = -\frac{13M}{5} + \frac{13}{5} \quad 0 \quad 6/5 - M/5 \quad + M/5 \quad 0 \quad 0$

$\frac{X_B}{X_1} = ?$

$\cancel{300}\,60 \times 2/\cancel{5} = 120$.

Formulate the duality in LPP.

$z = BW$

$A^T W \geq C$

$z = [5 \ 10 \ 15] \ [w_1 \ w_2 \ w_2]$

$z' = 5w_1 + 10w_2 + 15w_2$

$w_1 + 2w_1 + 0w_2 \geq 3$ $\qquad w_1, w_2, w_3 \geq 0$.

$w_1 + 0w_2 + w_3 \geq 1$.

$w_1 + w_2 + 3w_3 \geq 2$

Convert to Dual.

Maximize $z = 8x_1 + 4x_2$

$y_1 \leftarrow 4x_1 + 2x_2 \leq 30$.

$y_2 \leftarrow 2x_1 + 4x_2 \leq 24$

$x_1, x_2 \geq 0$.

$\longrightarrow$ minimize $z = 30y_1 + 24y_2$

$4y_1 + 2y_2 \geq 8$

$2y_1 + 4y_2 \geq 4$

$AX = B$

$Z = CX$

$AX = B \implies z^* = By$

$A^T X = C$

① minimize

$Z = 4x_1 + 2x_2 + x_3$ $\implies$ step 1

$x_1 + x_2 \leq 10$ $\longrightarrow$ ① $-(x_1 + x_2) \leq -10$.

$3x_1 + x_2 + x_3 \geq 23 \longrightarrow$ ② $\qquad -x_1 - x_2 \geq -10 \longrightarrow$ ①

$7x_1 - x_2 = 6$

2nd eqn is greater than ae equal to -

$7x_1 - x_3 \leq 6$

$7x_1 - x_3 \geq 6 \longrightarrow$ ③

$-7x_1 + x_3 \geq 6 \longrightarrow$ ④.

$z =$

Minimize $\implies$ minimize

$z = 4x_1 + 2x_2 + x_3$ $\qquad z = -10y_1 + 23y_2 + 6y_3 + 6y_4$

$-x_1 - x_2 \geq 10 \longrightarrow$ ① $\longrightarrow y_1$

$3x_1 + x_2 + x_3 \geq 23 \longrightarrow$ ② $\longrightarrow y_2$

$-7x_1 + x_3 \geq 6 \longrightarrow$ ③ $\longrightarrow y_3$

$7x_1 - x_3 \geq 6 \longrightarrow$ ④ $\longrightarrow y_4$.

$-y_1 + 3y_2 + 7y_3 + 7y_4 \leq 4$ $\qquad 0y_1 + y_2 + 0y_3 - y_4 \leq 1$

$-y_1 + y_2 + 0y_3 + 0y_4 \leq 2$

Unbounded soln

cases when $\theta = x_B/x_1$ gives only -ve ~~sol~~ numbers or ~~no~~ repeated lowest values are called unbounded solution.

→ Infeasibility soln :

→ Multiple optimal soln

Non-basic variable → s_1, s_2, A_1, A_2

basic → x_1, x_2

Δj values we get 0.

→ Degenerate soln

(Refer)

C_B	X_B	solution value	x_1	x_2	x_3	S_1	S_2	S_3	S_4
0	S_1	10	$-1/2$	0	0	1	0	$1/2$	-4
6	S_2	60	2	0	0	0	1	1	-5
45	x_3	10	$1/2$	0	1	0	0	$1/2$	-1
35	x_2	30	$1/2$	1	0	0	0	$-1/2$	2
	$Z_j - C_j$	1500	8	0	0	0	0	5	25

Manufacturer produce 3 products daily x, y, z. The three products are processed through 3 production operation with time constraints and then stored. Problem has been formulated as $Z = -30x_1 + 35x_1 + 45x_3$

st $2x_1 + 3x_2 + 2x_3 \leq 120$

$4x_1 + 3x_2 + 4x_3 \leq 100$

$x_1 + x_2 + x_3 \leq 40$

$x_1, x_2, x_3 \geq 0$.

1) Explain is the above is feasible

2) is the optimal solution? If yes what is the

3) is above soln unbounded?

4) Is the above soln degenerate? Does it have multiple solutions.