Sanctions in directed networks

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Abstract

Sanctions are common policy measures taken by an agent (sender) to discipline another agent (target) when the latter has violated any norm. Empirically, it has been observed that import sanctions are not only more prevalent than export sanctions but also more effective in disciplining the target. I examine that such differences in efficacy of import and export sanctions can be explained by the underlying network architecture which is captured by one special measure of centrality (betweenness centrality) which captures the value of strategic location of any agent in a network. I show that if agents' payoffs directly depend on this measure, then sanctions of certain types can only be effective under specific network features. I also highlight conditions of sanction efficacy under situations of geopolitical tensions between sender and target.

Key words: sanctions, directed networks, betweenness centrality

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1   Introduction

Sanctions are imposed by a sender nation (S) against a target (T) nation as a punitive measure to discipline the latter when it has engaged in actions not approved by the sender. These actions can include the target nation violating trade norms, involving in weapons trade, inciting domestic warfare, disobeying environmental norms, building nuclear plants and many more. Ex-post sanction, if the target complies to the demands of the sender, then the sanction is deemed to be effective or successful. Greif (1993) recounts one of the first instances of sanctions in the eleventh century where Maghiri tribes set up trade-laws to strengthen contractual relations between merchants and their agents who travelled overseas. On the occasion of any code violation, coalition members could take disciplinary actions like sanctions thereby barring the convicted agent from conducting future trade.

The existing literature focuses on sanctions in a trade network where there is a possibility of a two-way exchange of goods or services. Interestingly, effective trade sanctions are rare events (roughly 30% of all sanctions are effective (Felbermayr et al. (2020))). Hufbauer, Schott and Elliott (1990) suggests that most sanctions which have been proven to be a success are due to the presence of contemporaneous forces like military pursuits which had coerced the target to comply to the sender's demand. However, if one looks at sanction effectiveness across its various types, then it is seen that import sanctions are not only more frequently used than export sanctions, but also are more effective [1].

This paper analyses the merits of export and import sanctions from a network perspective. The players payoffs are a function of their strategic location within the network, which is measured by betweenness centrality (henceforth centrality). This measure of centrality gains instrumental value as it directly relates to trade with intermediaries which in modern day trade has gained considerable force. This trend has grown with increased globalization where countries find it less costly to outsource manufacturing stages to other nations before buying the final product back or have made the OECD to increase trade in export of goods which are mainly imported from other nations. This has made the domestic value added to goods exchanged between any two countries increasingly small (see Ali and Dadush (2011),Breda, Cappariello and Zizza (2009)).

In this context, higher value of strategic location of a particular country or player entails that it acts as a major intermediate to most trading countries inside the trade network. The measure of betweenness centrality of say country A accounts for the number geodesics or the smallest trade path between two countries which includes A.

Hence, severing links during sanctions can severely compromise the value of strategic position for both sender and target alongside declining their gains from trade. Given this, a sender will

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[1]Caruso (2003) also highlights the importance of import over export sanctions.

impose import sanction over export sanction if the former inflicts more cost on the target. At the same time, import sanction can also be more costly on the sender itself if ex-post sanction, sender loses its strategic position considerably. Since, strategic location is directly dependent on network architecture, understanding the same can give insights in forming a general rule of thumb conditions which will determine which sanctions must be imposed.

Previous network-theoretic models have mainly focused on the components model of networks. The standard thought supported by these models is that effective sanctions can be imposed if sender has more number of links than the target. Alternatively, if the target has more number of links, then sanctions are unlikely to be effective as more number of links generate greater resistance in T to endure sanctions without complying to the demands of sender. Kaempfer and Lowenberg (1999) cites the example the effectiveness of US's export sanction of computers against South Africa due to the limited number of computer suppliers in the world market during the 1980's. However, the ban on coal imports from South Africa was weakened by the fact that South Africa soon found new buyers.

The current paper takes a detour in two ways. First, I study sanction efficacy under directed network which allows to differentiate between import and export links. Second, payoffs of players do not depend on the number of links, but on strategic location which is quantified by the betweenness-centrality measure. The current paper contributes to this particular area. Other related works on strategic decision making (involving creation or deletion of links) in more general environments introduce refinements of stability - like pairwise stability and Nash stability. These works mainly work with undirected networks while understanding the efficacy of each sanction type requires the need for directed network.

Modern world sanctions have evolved since then in forms (trade,financial or travel) and methods of punishing the guilty (severing trade ties, blocking investments or preventing travel) which in most cases inflicts collateral damage on the sanctioning agent and leads to welfare loss. Felbermayr et al. (2020) shows that complete export sanction has lead to a 76% decrease in trade, an outcome equivalent to an approximate 43% rise in tariffs. While, import sanctions has caused a loss in trade of 52% (an outcome when tariffs rise by 20%). This capacity of sanctions to impose economic and social damage creates avenues for research, like the current paper which presents conditions where it can be more effective and when can welfare loss be minimum.

Works of Caruso (2003) have shown the outright failure of sanctions by shedding light on how target nations escape the effect of sanctions by forming ties with other nations. For instance, following sanctions by the Arab nations, Qatar formed renewed ties with Iran and China.

Previous theoretical work in the same vein consists of Joshi and Mahmud (2016),Joshi and Mahmud (2018),Joshi and Mahmud (2020). The first of the three papers studies how the network structure helps the sender to form a coalition such that it can sanction the target with greater force
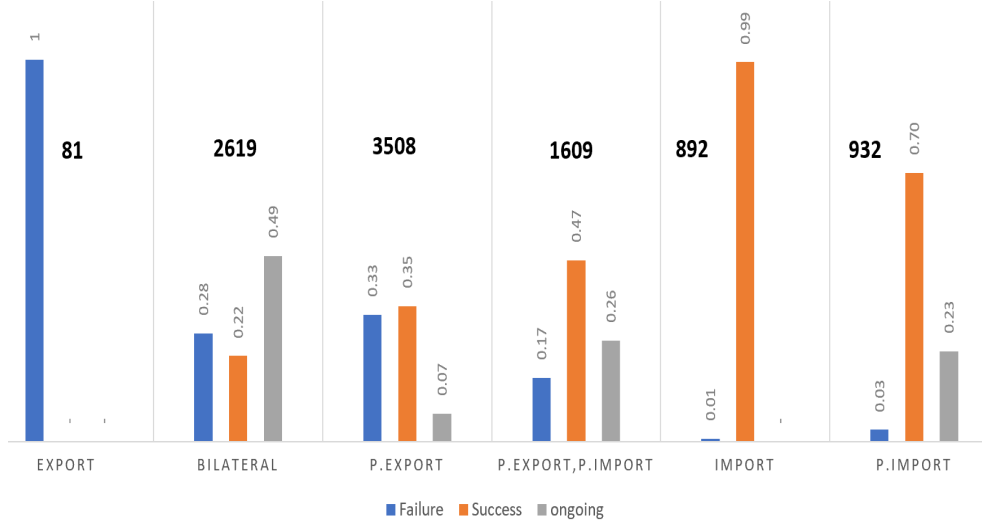
Figure 1: Main outcomes of different types of trade sanctions. The numbers are represented as a proportion of the frequency of each sanction type (numbers indicated in bold). The remaining outcomes include 'partial success' and 'negotiation by settlement' which I have removed after keeping in mind the focus of the current paper.

and whether T would comply to S's demands more readily. They introduce the concept of effective sanctions through the concept of spanning trees and analyse sanctions in short and long run. In essence, this strategy precipitates to relegating T to the smallest possible component to make T succumb. Joshi and Mahmud (2018) extends the previous idea and examines the role of linear, concave or convex utility function of sender in sustaining a larger coalition among sanctioning players. Joshi and Mahmud (2020) further augments their existing work of understanding sanction efficacy under different network specification through strategic complementarities and externalities. For instance,

In the current setup, I consider node centrality (specifically betweenness centrality) as the main factor in determining sanction imposition. Nations who want to impose sanctions make a judgement of how their strategic position will be weakened with respect to the target if they impose an import sanction vis-a-vis an export sanction. The directed nature gains bite in the ambit of trade where direction of links suggest import or export or the flow in a global supply chain (GSC). As discussed in the model, a further insight which this directed network sets is the aspect of the target seeking to form links with third parties. If an import sanction is levied on the target, it needs to find an alternative market to siphon off its production whereas an export sanction will compel it to either produce domestically or find an alternative partner to buy the concerned good. In essence, efficacy of certain sanction types are governed by the number of export and import links emanating from or converging to a nation respectively.

There are multiple outcomes of how centrality of the target and sender are affected if one considers the initial centrality values prior to sanctions. I define two broad sanction types - strong

and weak, each of which further considers multiple sub-types, based on how centrality of sender and target have fared before and after sanctions.

The baseline mode studies sanctions is a network with 3 players. Then I expand it to the more general model with n players. I find cyclicity to be a major factor behind the efficacy of both import and export sanction. I also find that export sanction inflicts more harm on the sanctioned state if there exists a cycle which originates from the target and connects the sender through the other players. Similarly import sanction is more damaging to the target if the cycle is directed in the opposite way.

This paper's aim is to primarily lay out conditions on network structure on the success of various forms of trade sanctions- import or export. I exclude heterogeneity among the trading players arising from cost of trade, production functions, geopolitical factors which could have an effect on sanction efficacy. This is done to highlight the underlying effect of network architecture on sanctions.

The paper is structured as follows. Section 2 discusses the model preliminaries; section 3 discusses a static game between the sender and the target; the partial equilibrium analysis is laid out in section 4 while section 5 presents the general equilibrium analysis.

2   Network model

The set of players or nodes is the set $\mathbb{N} = \{1, 2, 3, .., n\}$. The graph or network comprising these set of players is g. g is directed which essentially means that any node can have a link which originates form it, or which converges to it from another node. This is done to account for export links, import links or bilateral links. The number of links emanating from a player is its out-degree while the number of links converging to a player is its in-degree. A path in g connecting i and j is a set of distinct links $il_1, l_2l_n, .., l_kj$. A cycle is a path such that the initial and terminal players are the same. A complete cycle connects all players in the network. A sub-cycle is a cycle connecting n' nodes in a network g which has n nodes where $3 \leq n' < n$.

A geodesic σ_{ij} denotes the shortest path from i to j. A network is connected if there exists a least one directed link between a pair of players. A connected network is complete if all players are connected to each other by links in both directions. An incomplete network is one which is not complete. i.e. at least one pair of players are connected by a link directing in one way.

The player imposing sanction is termed as sender S while the one being sanctioned is the target T. I use g' to denote the ex-post sanction network such that $g' = g - l$ where $l \in \{ST, TS\}$. This terminology is used to account for the directed nature of the graph g and help differentiate between import and export sanctions.

As discussed before, I assume the payoffs to any player is the value of their strategic location

in g. This value is measured by betweenness centrality. So the sender which decides to sanction the target considers the values of its strategic position, the target's strategic position and how these would be affected if an import or an export sanction is levied.

The traditional network literature cites a number of measures of centrality - eigen-vector centrality, betweenness centrality, degree centrality and closeness centrality. Since, the paper focuses on trade sanctions and eigen-vector centrality does not work too well in a directed network setting, I resort to betweenness centrality. This centrality measures how well situated a node or nation is inside a network. The closeness measure loses its bite in this context as sanctioning nations target nations which are both close and far away. The degree centrality too is not very interesting as nations will small number of nodes impose sanctions against nations with larger values of degree-centrality. I aim to use this measure exploits the entire network architecture strongly enough to posit which sanction type must be imposed given any particular network.

Definition 1. *Betweenness Centrality: Let $d(kj)$ denotes the number of geodesics (shortest paths) between k and j in directed network g. Analogously, we denote by $d_i(kj)$ as the number of these geodesics which pass through node i. Then the betweenness centrality of player i is*

$$\beta_i(g) = \sum_{k \neq j : i \notin \{k,j\}} \frac{d_i(kj)/d(kj)}{(n-1)(n-2)}$$

3   Game

I construct a game of sanctions which is represent by $\Gamma = \{g, S, T, A_S, A_T, U_S, U_T\}$. The term g represents the network, the players directly involved are the sender S and target T [2]. The payoffs of S and T are represented by $U_S = U_S(g)$ and $U_T = U_T(g)$ which denotes the value of their respective strategic position in g.

S' action set $A_S = \{import, export\}$ which signifies whether to impose import or export sanction. Analogously action set A_T comprises of two actions $\{comply, sanction\}$.

4   Partial analysis

I begin my study by first considering the decision making of the sender against the target. This being a partial analysis, will only focus on the action of the sender. The imposition of sanction generates different outcomes which I classify under two broad class - strong and weak sanctions. I further elaborate on all possible sub-classes of sanction under these two classes. Each of these

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[2]Here I focus on unilateral sanctions and hence do not consider other players in g.

sanction types can be differentiated by the change in centrality values of the sender and target ex-post sanctions.

The below assumption establishes the incentive compatibility conditions of T in engaging in the restricted activity and of S in imposing the sanctions in the first place.[3]

Assumption 1. *Player T receives a payoff of $P_0 > 0$ by engaging in some practice which is deemed as unlawful or unfair by sender S. S pays a price of $P_s > 0$ from T's action such that $U_s(g) = \beta_s(g) - P_s < \beta_s(g') = U_s(g')$.*

Definition 2. *Strong Sanction (S): A sanction is strong if the following terminal conditions hold*

$$U_T(g') < U_T(g) \quad , \quad U_S(g') = U_S(g) \tag{1}$$

This entails that after sanction is imposed, the centrality of the T decreases while centrality of S remains unchanged. This class of sanction can be dissected into the following five types by combining terminal and initial conditions:

(i) **Type-S1**: *if $U_S(g) > U_T(g)$ and $U_S(g') > U_T(g')$.*

 This type considers that initial value of S is greater than T in g as well as in g'.

(ii) **Type-S2**: *if $U_S(g) = U_T(g)$ and $U_S(g') > U_T(g')$.*

 This considers that S and T were equally central in g, but following sanction, T loses a portion of its centrality value.

 This below 3 types describe a situation where a less central S sanctions T to produce the following three outcomes: T still remains more central than S in the sanctioned network g' (S3), makes S and T equally central in g' (S4) and finally S5 which erodes T's centrality such that T becomes less central than S in g'.

(iii) **Type-3-S3**: *if $U_S(g) < U_T(g)$ and $U_S(g') < U_T(g')$.*

(iv) **Type-S4**: *if $U_S(g) < U_T(g)$ and $U_S(g') = U_T(g')$.*

(v) **Type-S5**: *if $U_S(g) < U_T(g)$ and $U_S(g') > U_T(g')$.*

 From the above, S5 is the strongest subclass of sanction within the class of strong sanctions.

Definition 3. *Weak Sanction (W): A sanction is weak if alongside lowering the centrality of T, the centrality of S is weakly decreased which is denoted by the below condition.*

$$U_T(g') \leq U_T(g) \quad , \quad U_S(g') \leq U_S(g) \tag{2}$$

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[3]I would discuss this in the general equilibrium section where T's action will enter into the payoff of S.

This type can be of the following six types based on initial and terminal conditions:

(i) **Type-W0**: *if* $U_T(g) = U_T(g')$

   This defines ineffective sanctions where the centrality of T is not affected.

   The below two types sum up situations where a more central sender compared to target by imposing sanctions lowers its own centrality more than the target (W1) or lowers it to a point where in the sanctioned network, S's is as central as T (W2).

(ii) **Type-W1.1**: *if $U_S(g) > U_T(g)$ and $U_S(g') < U_T(g')$.*

   Type-W1.2: *if $U_S(g) > U_T(g)$ and $U_S(g') = U_T(g')$ where $U_T(g) = U_T(g')$.*

   Type-W1.3: *if $U_S(g) > U_T(g)$ and $U_S(g') = U_T(g')$ where $U_T(g) > U_T(g')$.*

   If in pre-sanction network, both S and T are equally central, then the event of a sanction can lead to the below three outcomes: S's centrality is compromised more than T's (W3); S and T remain equally central (W4) and S becomes more central than T post sanctions.

(iii) **Type-W2.1**: *if $U_S(g) = U_T(g)$ and $U_S(g') < U_T(g')$.*

   Type-W2.2: *if $U_S(g) = U_T(g)$ and $U_S(g') = U_T(g')$.*

   Type-W2.3: *if $U_S(g) = U_T(g)$ and $U_S(g') > U_T(g')$.*

   The below three cases depicts a less central S sanctioning a relatively more central T. $W6$ accounts for

(iv) **Type-W3.1**: *if $U_S(g) < U_T(g)$ and $U_S(g') < U_T(g')$ and $|U_S(g')-U_S(g)| > |U_T(g')-U_T(g)|$ holds.*

   Type-W3.2: *if $U_S(g) < U_T(g)$ and $U_S(g') < U_T(g')$ and $|U_S(g')-U_S(g)| < U_T(g')-U_T(g)|$ holds.*

(v) **Type-W4.1**: *if $U_S(g) < U_T(g)$ and $U_S(g') = U_T(g')$ and $U_S(g') < U_T(g')$ and $|U_S(g') - U_S(g)| > |U_T(g') - U_T(g)|$ holds.*

   Type-W4.2: *if $U_S(g) < U_T(g)$ and $U_S(g') < U_T(g')$ and $|U_S(g')-U_S(g)| < U_T(g')-U_T(g)|$ holds.*

(vi) **Type-W5**: *if $U_S(g) < U_T(g)$, $U_S(g') > U_T(g')$.*

   The above sanctions are ordered in an ascending manner such that $W0$ is the ineffective sanction while $W5$ is the most effective weak sanction.

Definition 4. *(i) A player is net-exporter (net-importer) if its out-degree is strictly greater (lower) than its in-degree.*

(ii) A player is balanced if its out-degree is equal to its in-degree.

(iii) A player is a complete exporter (importer) if it has only out-links (in-links)

Remark 1. *The necessary condition for the same requires the network to be not entirely complete.*

Proof: If the network is complete, then there exists no geodesic σ between two players k and m ($k \neq m$) such that it passes through a third player i ($i \notin \{k, m\}$). Hence, BC of any $i \in g$ is zero. Following sanctions, in g', geodesic of S and T remains zero.

4.1 3-player network

I consider a network g comprised of the following three players: sender (S), target (T) and a third player denoted by 3. This will help us understand the merit of sanctions within nations forming small trade ties with its neighbours. [4]

Proposition 1. *If there exists two bilateral links in g, one between S and T and the other between T and player 3, such that T is balanced, then the following holds.*

(i) If S is a net-importer, then import sanction is S_4, while import sanction is W0.

(ii) If S is a net-exporter, then export sanction is S_4, while export sanction is W0.

In the above cases, the direction of links take the pivotal role in rendering one sanction weak and the other strong. The position of the third player in this network also becomes instrumental. Both the statements can be understood by similar reasoning and by taking help of figure. The first statement corresponds to the top row in figure where the centrality of all but T is zero which entails that S does not lie on the geodesic between T and player 3 nor does 3 lie between S and T. Being net-importer, if S severes its import link with T, it turns into a balanced player in g, but the existence of bilateral ties between T and 3 does not change its centrality, nor is the centrality of T affected as it lies on the geodesic between S and 3. However, severing the export ties with T makes S an importer which does not lowers its centrality but lowers the same of T. This network is not cyclic as S is an importer with only in-links.

Proposition 2. *If there exists two bilateral links in g, one between S and T and the other between S and player 3, then*

(i) If S is balanced and T is net-importer, then export sanction is W2, while import sanction is W0.

(ii) If S is balanced and T is net-exporter, then export sanction is W2, while export sanction is W0.

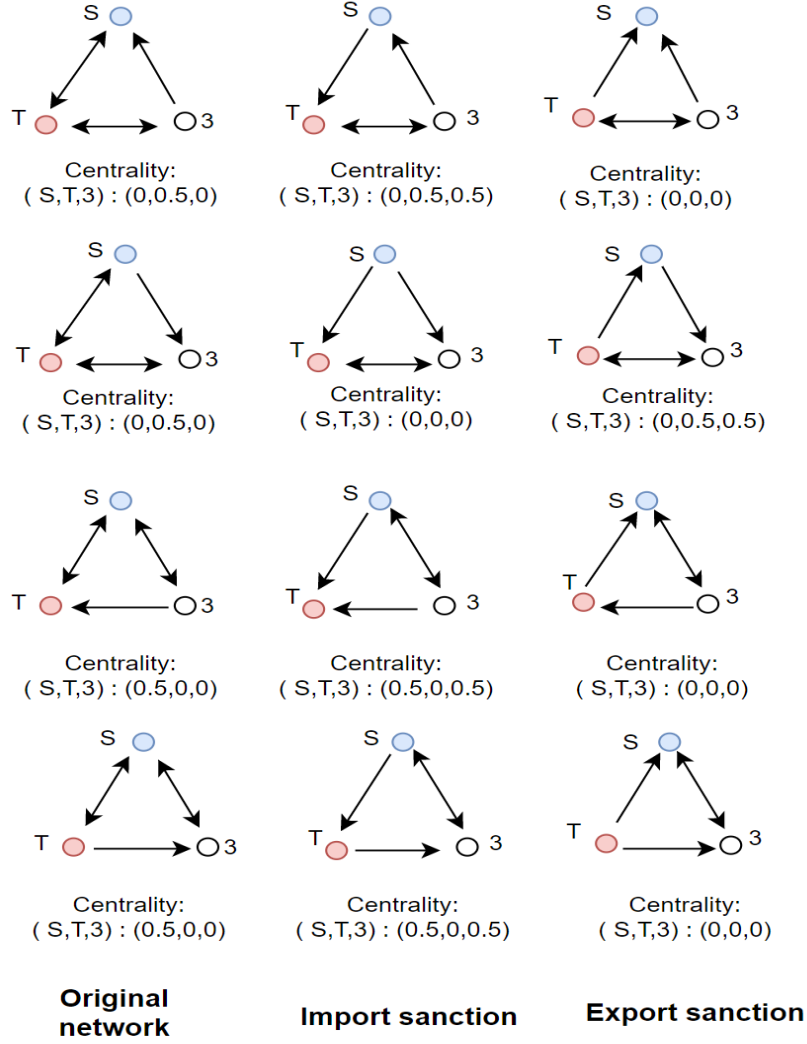[4]The formal proofs for this 3-player will be presented as a diagrammatic form.

Figure 2: The first two rows illustrates proposition 1 and the following two rows illustrates proposition 2. The first row exhibits S as net-importer and T is balanced. Import sanction is ineffective as shown. However, the export sanction lowers centrality of 2 as now it does not lie in the geodesic from S to player 3.

As per the assumption, if S is balanced and T is a net-importer, it entails that T has an in-link from player 3. This structure states that T has no strategic value given its location (3 and 1 has direct bilateral links, hence do not need 2). However, S carries strategic location value as it lies in the geodesic between 2 to 3. Now, if S deletes its out-link, its strategic value does not depreciates (sanction is ineffective or W0) while cutting the in-link from T destroys the geodesic between T and 3 (sanction is W2). Similar reasoning holds for the second statement.

The insight that one gains from the above two propositions is the non-trivial effect of link direction between the third player and S or T. When T carries bilateral link with 3, then strong sanctions can indeed be imposed by S. However, this outcome ceases to exist when S has a bilateral link with 3.

Proposition 3. *If there exists only one bilateral link in g,between S and T, then*

(i) If S and T are either both net-exporters or net-importers, then both betweenness-centrality is zero for all three players and import and exports sanctions are W0.

(ii) If S is net-importer while T is net-exporter, then import sanction is W2, while export sanction is W0.

(iii) If S is net-exporter while T is net-importer, then export sanction is W2, while import sanction is W0.

Proof: Figure 3 offers a diagrammatic proof in the 3-player case. In the first and second row, S and T are both net-exporters and net-importers respectively. In no player carry any importance in regard to their location. Hence, sanctions are incapable of eroding centrality any further. As illustrated, the initial graph g is acyclic and represent networks comprising of unequal agents (at least one player conducts one-way exchanges, ie. either only exports or only imports) engaged in exchanging good or services such that none of the players serve as an important intermediary.

When S and T carry separate identities unlike in sub part (i), there exists a cyclic graphs as seen in rows 3 and 4 of figure 3. Any sanction which renders the graph to a acyclic one is of type $W2$ (where centrality of both S and T decreases but are equal in the post-sanction network). Here, centrality of S and T fall to zero.

Remark 2. *If the betweeness centrality of S or T or both is non-zero in g, then export or import sanction is ineffective when sanctioned network g' is cyclic. On the other hand, sanctions are either of type W2 or S4 when g' is acyclic.*

This grabs bite in the world of trade where increasing volume of exported goods from any country are actually made up of intermediate imports made by that country from its trading partners [5]. This result is fairly intuitive and much easier to visualize in a 3-player network. If the post sanction network is cyclic, then each node lies on the geodesic between the other two players. Thereby betweenness centrality remains unchanged. If on the other hand, the sanctioned network becomes acyclic, then at least one of the 3 players have only out-links or in-links. In the language of trade, this means that the target nation post sanction has been prevented from acting as an essential trading intermediary.

If this player is T, then sanction is S_4 such that betweeness centrality of T falls to zero. However, if S has only out-links and in-links, then sanction is W_2 where S endures loss in centrality.

Proposition 4. *If only unilateral links exists between the three players in g, then a sanction of type W3 can be imposed by S iff g is a cycle. Betweenness centrality of g remains zero when it is acyclic and all sanctions are $W0$*

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[5]See Ali and Dadush (2011) who shows the increase in this trend in almost all the OECD countries.
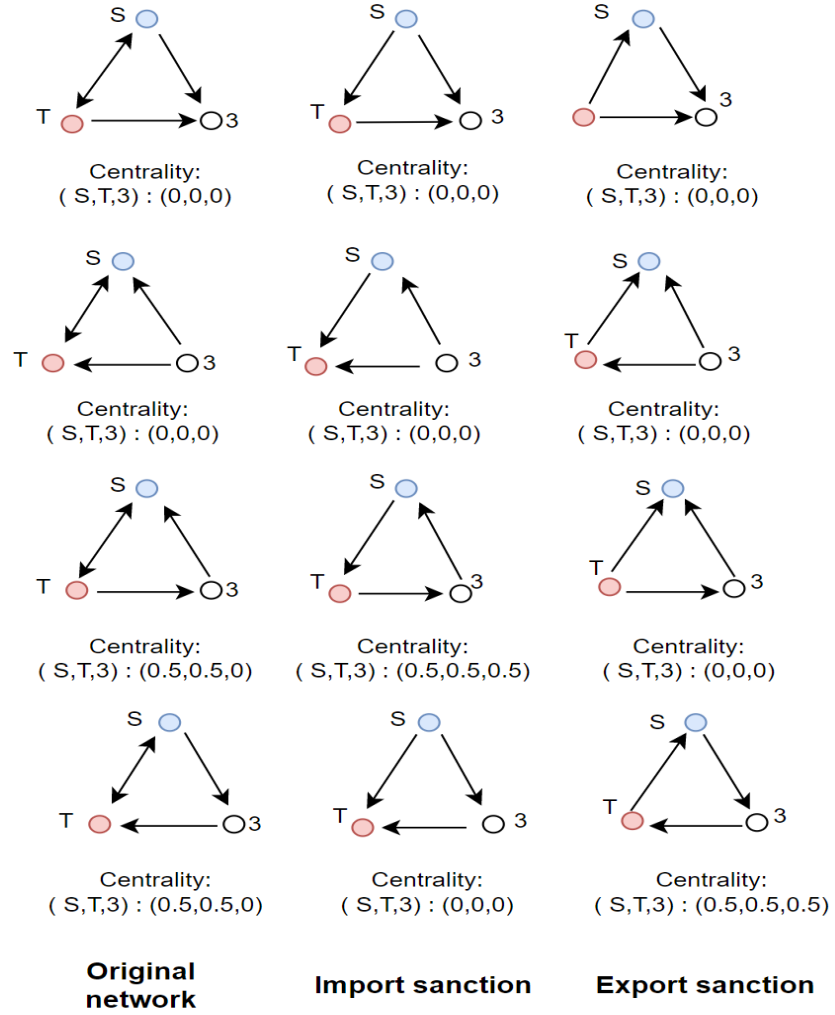
Figure 3: The first two rows illustrates when S and T are net-exporters and net-importers respectively. Row 3 (4) shows the effect of import and export sanctions when S is net-importer (net-exporter) while T is net-exporter (net-importer).

In the 3-player set-up only two situations arise where there exists cycles among the players in which case weak sanction of type $W3$ can be imposed. The remaining four cases constitute acyclic networks where sanction is $W0$ or ineffective.

4.2 Multiple players

Networks with nodes greater than 3 generates the possibility of the remaining forms of strong or weak sanctions. This examination is more robust towards different network architecture and more germane to the complex trade networks in reality. As is seen, the first upgrade that multiple players of size 4 or above bring is the possibility of strong sanctions.

Definition 5. *A component $\mathbb{C}_g$ within g is a graph consists of $m < n$ players whose cardinality is given by $||\mathbb{C}_g||$.*

Proposition 5. *Suppose there is a bilateral link between S and T.*

(i) If S is a net-exporter and T is balanced, then export sanction sanction is $S3$ or $W0$ if S belongs to $\mathbb{C}_g$ such that $||\mathbb{C}_g|| > \frac{n}{2}$. In this case, import sanction is $W3.2$. If $||\mathbb{C}_g|| \leq \frac{n}{2}$, export (import) sanction is $W0$ ($W4.1$).

(ii) If S is a net-importer and T is balanced, then import sanction is $S3$ or $W0$ if S belongs to $\mathbb{C}_g(\frac{n}{2})$. In this situation, export sanction is $W3.2$. If the component size is less than $\frac{n}{2}$, import (export) sanction is $W0$ ($W4.1$).

(iii) If S is balanced and T is net-exporter, import sanction is $W0$, $S3$ or $W2$. Export sanction is $W1$ or

(iv) If S is balanced and T is net-importer, then import sanction is $S3$ or $W0$, while export sanction is $W6.2$.

(v) If S and T are balanced, then export or import sanctions can be either W2,W3 or W6.

A more technical proof has been provide in the appendix.

The essence of the first statement is that when S is a net-exporter, then imposing an export sanction on T still helps maintain its strategic location as in g whereas severing import link will lower its strategic power. Analogous reasoning explains the second statement.

Corollary 1. *If the sanctions are W5 (S4), then the post-sanction network is acyclic (cyclic).*

5 General Analysis

The game in the general equilibrium setting is denoted by $\Gamma_G = \{g, S, T, A_S, A_T, \sigma_S, \sigma_T, U_S, U_T\}$. The resemblance of the terms remains as it were in the partial equilibrium setting. I consider two
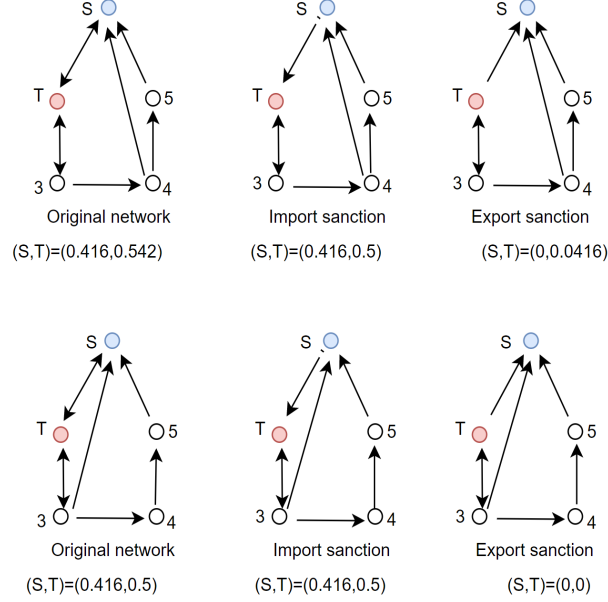
Figure 4: This 5-player network reflects the effect of sanctions when S is a net-importer while T is balanced (statement (ii) of proposition 5). In the top row, S is part of a cyclic component of size 4 while in the bottom row, the same component is of size 3. In the former, import sanction is $S3$ (centrality of S is unchanged, but remains lower than T even post sanction) while export is $W3.2$ (centrality of T decreases more than S). In the bottom row, import sanction is ineffective or $W0$ (centrality of S and T remains unchanged. Export sanction is $W4.1$.

games static and dynamic. In the static game, T's action consists or complying or violating the sanction imposed by S. In the dynamic version, T has a broader set of action if it chooses to violate the sanction, i.e to bargain or negotiate, retaliate by sanctioning S and form new link with player z in network h. I now lay out the basic premises which will be used for the rest of the paper.

Each player $i \in g$ has intrinsic power which depicts its resistance against any sanction and the parameter α_i its domestic capacity to absorb excess goods in case of import sanction or to initiate its own production process in case of export sanction by the sender. The parameter k_i denotes the number of export links net of import links while N denotes the sum of all links to and from player i. Following definition 5, a player in g is a net exporter (net-importer) if $k_i > 0$ ($k_i < 0$) and it is balanced if $k_i = 0$. k_i' denotes the symmetric meaning of players in g'. Forming new links with an external player $z \notin g$ depends on its value of k_z. Sanction can be of two types here export or import which is parameterized by $a_S \in \{e, m\}$.

Any form of sanctions (import or export) on the target (weakly) reduces the centrality of the sender as well as the target. There is further an added cost on the target and sender which is a function of the above parameters, respectively denoted as $C_T^0(\alpha_T, \frac{k_T}{N}, \frac{k_T'}{N}, a_S, a_T)$ and
$C_s^0(\alpha_S, \frac{k_S}{N}, \frac{k_S'}{N}, a_S, a_T)$.

The properties of the cost function are as follows:

- $\frac{\partial C}{\partial \alpha} < 0$, greater intrinsic strength reduces cost of sanctions.

Axiom 1. $C(\alpha', \frac{k}{N}, \frac{k'}{N}, a_S) < C(\alpha', \frac{k'}{N}, \frac{\bar{k}}{N}, a_S)$ *where* $\bar{k} \neq k$,

the sanctioned network weakly reduces centrality but changes the configuration of a player by altering the number of in-links or out-links thereby creating an added cost on trade.

Axiom 2. *Given* $\beta_T > 0$, $C_T(\alpha, \frac{k}{N}, \frac{k-1}{N}, a_S = m, a_T) < C_T(\alpha, \frac{k'}{N}, \frac{k'-1}{N}, a_S = m, a_T)$

The cost of import sanction is higher when $k' < k$ and rises as $k_T \to -1$. [6]

Axiom 3. *Given* $\beta_T > 0$, $C_T(\alpha, \frac{k}{N}, \frac{k-1}{N}, a_S = e, a_T) > C_T(\alpha, \frac{k'}{N}, \frac{k'+1}{N}, a_S = e, a_T)$.

The cost of export sanction is higher when $k' > k$ and rises as $k_T \to 1$.

Axiom 4. *For any given parameter values,*

$$\frac{C_T(\alpha, \frac{k'}{N}, \frac{k'+1}{N}, a_S, a_T = v)}{C_T(\alpha, \frac{k'}{N}, \frac{k'+1}{N}, a_S, a_T = c)} = \mu, \quad \mu > 1$$

Cost of violating sanction is strictly greater than complying to it. Complying to sanction reflects a momentary costs of link deletion leading to temporary loss in trade gains. However, violation leads to enduring this costs permanently or some added costs of creating new links with other players. In essence, costs from violation is always strictly higher.

Lemma 1. $\frac{k_i}{N} \in \{-1, 1\}$ *for any player* $i \in g$ *implies that* $\beta_i(g) = 0$. *However the reverse may not hold.*

Proof: The value of $\frac{k_i}{N}$ equals unity (positive or negative) when all m links are either out-links or in-links which entails betweeness centrality is zero. However, if centrality is zero, then i can still have both out-links and in-links such that $\frac{k_i}{N} \neq \{-1, 1\}$. In this case, there exists geodesics between other players which are shorter than the one which includes i.

5.1 Static game

I follow the static game similar to Joshi and Mahmud (2020) where sender decides to sanction or not sanction while target can either comply or violate. However, the current analysis leaves out the action of not sanctioning and dissects the action of sanctioning by considering the sender to choose between import and export sanction. The target's action remains the same - comply or violate. The utility function of S and T in a static game is $U(a_S, a_T)$ and $U(a_T, a_S)$ given other model parameters as previously explained.

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[6] $k \to -1$ suggests that T mainly imports from other players in g. Hence imposing an import sanction entails that there are lesser out-links originating from T which lowers its strategic power, thereby raising costs of sanctions. Symmetric reasoning applies for the cost on export sanction as stated in axiom 3.

	comply (c)	violate (v)
export (e)	$U_S(e,c), U_T(c,e)$	$U_S(e,v), U_T(v,e)$
import (m)	$U_S(m,c), U_T(c,m)$	$U_S(m,v), U_T(v,m)$

Figure 5: Static game

Definition 6. *Pure Strategy Nash Equilibrium (PSNE): A pure strategy σ_s^* of the sender in Γ_G is a Nash equilibrium if $U_s(\sigma_S^*, \sigma_T) > U_s(\sigma_S', \sigma_T)$ for any $\sigma_S' \in A_S$ given any strategy σ_T by T and for all $\sigma_s' \in A_S$.*

Successful sanctions are defined when complaince by T sustains as the dominant strategy for T such that the following holds

$$\beta_T(g) - \beta_T(g') \geq 2P_0 \tag{3}$$

By complying, T loses its benefit $P_0 > 0$ from the activity it was previously engaged in. This means that T will prefer to comply than violate if the loss of centrality is greater than the utility it derives from the activity.

S will wants to make T comply by incurring the least cost. In essence, S imposes the that sanction type which is strong. However, this strategy should erode centrality of T enough such that complying evolves as the dominant strategy for T. Say, given any g, S can impose an strong import sanction or a weak export sanction against T. If T is dovish and always wants to budge before any sanctions then import sanction is the dominant strategy. However, T can be more recalcitrant and complies only if S also endures a high enough costs from sanctions. Then S might want to impose the weaker export sanction. Additionally, either sanction can be successful only if it is P_0 reducing which entails that the by violating sanctions, the loss of centrality will override the gain P_0 from the illegal activity.

Definition 7. *Any sanction by S is P_0-reducing iff $\beta_T(g) - \beta_T(g') = P_0$ and strictly P_0-reducing iff $\beta_T(g) - \beta_T(g') > P_0$.*

The type of sanctions gains force when T complies under export sanction but violates import sanction. Or alternatively, T complies to all import but violate all export sanction. In essence one of the following pair of conditions must hold.

$$U_T(c, e) > U_T(v, e) \tag{4a}$$

$$U_T(v, m) > U_T(c, m) \tag{4b}$$

Now, for the symmetric condition (where T comply to import but violate export sanction), the following must hold

$$U_T(c, m) > U_T(v, m) \tag{5a}$$

$$U_T(v, e) > U_T(c, e) \tag{5b}$$

Proposition 6. *(i) If equation 3 is satisfied then (e, c) or (m, c) is the unique PSNE. If not, then (e, v) or (m, v) is the unique PSNE.*

(ii) If equation 4 is satisfied and if export sanction is stronger than import sanction, then (c, e) is the Pareto efficient outcome. Similarly, if equation 5 is satisfied and import sanction is stronger than export sanction, then (c, m) is the Pareto efficient outcome.

(iii) The equilibrium in mixed strategies is given by

$$q_T^* = \frac{U_S(m, v) - U_S(e, v)}{U_S(m, v) - U_S(e, v) + U_S(e, c) - U_S(m, c)}$$

$$q_S^* = \frac{U_T(v, m) - U_T(c, m)}{U_T(v, m) - U_T(c, m) + U_T(c, e) - U_T(c, m)}$$

If equation 3 is fulfilled, then violating sanctions carries a higher cost which dominates the gain P_0 which T derives from violating the norm. The second statement accounts for scenarios where T surrenders to only one type of sanction, but violates the other type. According to the first sub part, if export is a stronger sanction and T complies to it, then (e, c) is a Pareto efficient outcome. If this sanction was weaker, then certainly it would have led to a Pareto inferior outcome where S loses its own centrality post-sanctions. Analogous reasoning can be applied to the second sub part. The final statement characterizes how S and T will randomize when they derive equal expected payoffs from both their actions.

5.2   Different utility functions

The efficacy of sanctions can be tested under two broad classes of motives of players - pure trade motive and trade combined with geopolitical motive. Geopolitics can act as an impediment to a successful sanction. Nations can well endure losses in terms of trade-gains or strategic power by violating sanctions just to see another nation lose their strategic value.

Example 1. *Pure trade motive: Here any player in g consider the importance of their strategic power as the singular measure of its well-being. The utility of S and T are respectively $U_S(a_S, a_T | \beta_S)$ and $U_T(a_T, a_S | \beta_T)$.*

The utility of S and T following sanctions are

$$U_S(a_S, a_T = c) = \begin{array}{ll} \beta_S(e) + P_s - C_S^0(.|e), & \textit{if } a_S = e \\ \beta_S(m) + P_s - C_S^0(.|m), & \textit{if } a_S = m \end{array} \tag{6}$$

$$U_S(a_S, a_T = v) = \begin{array}{ll} \beta_S'(e) - P_s - C_S^0(.|e), & \textit{if } a_S = e \\ \beta_S'(m) - P_s - C_S^0(.|m), & \textit{if } a_S = m \end{array} \tag{7}$$

$$U_T(a_T = c, a_S) = \begin{array}{ll} \beta_T(e) - P_0 - C_T^0(.|e), & \textit{if } a_S = e \\ \beta_T(m) - P_0 - C_T^0(.|m) & \textit{if } a_S = m \end{array} \tag{8}$$

$$U_T(a_T = v, a_S) = \begin{array}{ll} \beta_T'(e) + P_0 - C_T^0(.|e), & \textit{if } a_S = e \\ \beta_T'(m) + P_0 - C_T^0(.|m) & \textit{if } a_S = m \end{array} \tag{9}$$

Lets look at T's incentives. The sanction type becomes interesting when T chooses to play (say) c for export sanction and chooses to play v in presence of import. Then 6 and 7 implies that the following two conditions hold simulateneously

$$\beta_T(m) - \beta_T'(m) < 2P_0 + C_T^0(\alpha_T, \frac{k}{N}, \frac{k-1}{N}, m, c) - C_T^0(\alpha_T, \frac{k}{N}, \frac{k-1}{N}, m, v) \tag{10}$$

$$\beta_T(e) - \beta_T'(e) > 2P_0 + C_T^0(\alpha_T, \frac{k}{N}, \frac{k+1}{N}, e, c) - C_T^0(\alpha_T, \frac{k}{N}, \frac{k+1}{N}, e, v) \tag{11}$$

The left-hand side of 8 and 9 is the loss in centrality of T and S. The inequalities are binding given the value of the terms on the left-hand-side and the nature of T, i.e whether T is a balanced ($k = 0$), net-exporter ($k > 0$) or net-importer ($k < 0$) in g.

Given compliance from T, if export sanction is stronger than any import sanction, then (e, c) is the Pareto efficient outcome from the game in figure 4. It can be illustrated how network structure affects the conditions required for this to happen using figure 5. For (e, c) to be a pure strategy equilibrium, export sanction must be stronger than import sanction which implies from 5 and 6 that

$$C_S^0(\alpha, k_S, \frac{k_S - 1}{2}, c, e) \le C_S^0(\alpha, k_S, \frac{k_S + 1}{2}, c, m) \tag{12}$$

Now this conditions depend on k. If S is a net-importer then an import (export) sanction increases (decreases) the value of k by 1 (shown in the top row in figure 5). If $k_S = -1$, then the following must be satisfied

$$C_S^0(\alpha, k_S, \frac{k_S - 1}{2}, c, e) \le C_S^0(\alpha, k_S, \frac{k_S + 1}{2}, c, m)$$

or,

$$C_S^0(\alpha, \frac{-1}{2}, -1, c, e) \le C_S^0(\alpha, \frac{-1}{2}, 0, c, m) \qquad (A)$$
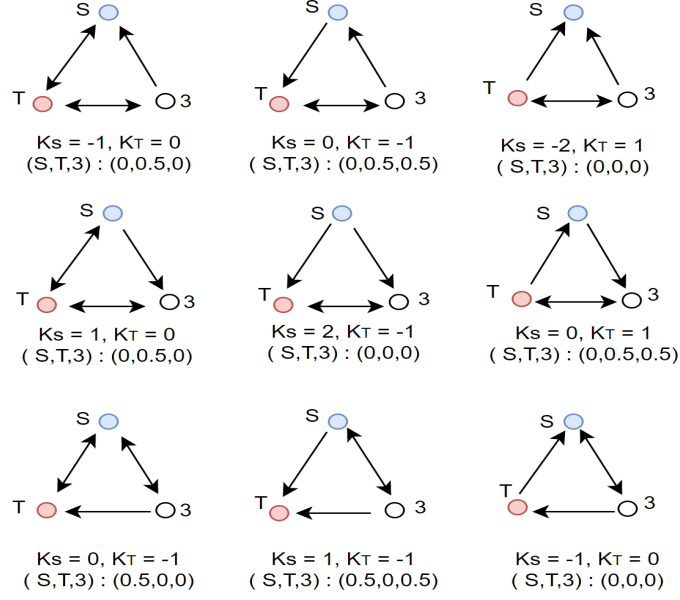
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Figure 6:

If $k = 1$, then

$$C_S^0(\alpha, k_S, \frac{k_S - 1}{2}, c, e) \leq C_S^0(\alpha, k_S, \frac{k_S + 1}{2}, c, m)$$

or,

$$C_S^0(\alpha, \frac{1}{2}, 0, c, e) \leq C_S^0(\alpha, \frac{1}{2}, 1, c, m) \qquad (B)$$

If $k = 0$, then

$$C_S^0(\alpha, k_S, \frac{k_S - 1}{2}, c, e) \leq C_S^0(\alpha, k_S, \frac{k_S + 1}{2}, c, m)$$

or,

$$C_S^0(\alpha, 0, \frac{-1}{2}, c, e) \leq C_S^0(\alpha, 0, \frac{1}{2}, c, m) \qquad (C)$$

The condition A is not feasible as S becomes a complete importer following the export sanction. Hence it loses it strategic position completely. This loss is relatively higher when compared to import sanctions. In similar sense, B is always feasible while C holds because cycle exists as shown in the third network in the last row of figure 5.

Example 2. *Trade and geopolitical motive: The utility functions which accounts for geopolitical motives are formalized by $U_S((a_S, a_T | \beta_S, \beta_T)$ and $U_T(a_T, a_S | \beta_T, \beta_S)$. post sanction, the utility is $U_S(a_S, a_T | \beta_S', \beta_T')$ and $U_T(a_T, a_S | \beta_T', \beta_S')$.*

Imposing sanctions reveal how the strategic position of sender relative to the target can have effects of strategic complemetarity and strategic substitutability to assist or impede the success of

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a sanction. The utility of S and T following sanctions are

$$U_S(a_S, a_T = c|\beta_S) = \begin{array}{ll} x.\beta_S(e) + (1-x).f(\beta_S - \beta_T) + P_s - C_S^0(.|e), & \text{if } a_S = e \\ x.\beta_S(m) + (1-x).f(\beta_S - \beta_T) + P_s - C_S^0(.|m) & \text{if } a_S = m \end{array} \quad (13)$$

$$U_S(a_S, a_T = v|\beta_S) = \begin{array}{ll} x.\beta_S'(e) + (1-x).f(\beta_S'(e) - \beta_T'(e)) - P_s - C_S^0(.|e), & \text{if } a_S = e \\ x.\beta_S'(m) + (1-x).f(\beta_S'(m) - \beta_T'(m)) - P_s - C_S^0(.|m), & \text{if } a_S = m \end{array} \quad (14)$$

$$U_T(a_T = c, a_S|\beta_S) = \begin{array}{ll} y.\beta_T(e) + (1-y).g(\beta_T - \beta_S) - P_0 - C_T^0(.|e), & \text{if } a_S = e \\ y.\beta_T(m) + (1-y).g(\beta_T - \beta_S) - P_0 - C_T^0(.|m) & \text{if } a_S = m \end{array} \quad (15)$$

$$U_T(a_T = v, a_S|\beta_S) = \begin{array}{ll} y.\beta_T'(e) + (1-y).g(\beta_T'(e) - \beta_S'(e)) + P_0 - C_T^0(.|e), & \text{if } a_S = e \\ y.\beta_T'(m) + (1-y).g(\beta_T'(m) - \beta_S'(m)) + P_0 - C_T^0(.|m) & \text{if } a_S = m \end{array} \quad (16)$$

The above represents that utility of S and T are a convex combination of trade-gains and geopolitical motives. In equation 12, $x \in (0,1)$ is the weight on the trade-gains of S while $(1-x)$ is the weight on geopolitical power. The utility of geopolitical power is given by the function $f(.)$ which increases with the centrality of S relative to T. Analogous explanation holds for the utility of T in equation 14. Following the discussion from the last example, the conditions needed for (c, e) to emerge as the equilibrium. there must exists incentive compatible conditions on both T and S which I keep in the appendix for visual clarity.

The magnitude of geopolitical rivalry is represented by the nature of f and g functions. If the nature is linear, then rivalry is weak. If it is concave, then rivalry increases with the differences in strategic dominance but in a decreasing rate. The existence of aggressive rivalry is given by convex f and g functions.

Corollary 2. *(i) If $4(1)$ holds then (c, e) is the Pareto efficient equilibrium iff the cost of export sanction is weakly stronger than import sanction for S which is ensured if the pair of conditions hold*

$$C_S^0(\alpha, k_S, \frac{k_S - 1}{N}, c, e) \leq C_S^0(\alpha, k_S, \frac{k_S + 1}{N}, c, m) \quad and \quad |\frac{k_S - 1}{N}| \neq 1$$

(ii) If $4(2)$ holds then (c, m) is the Pareto efficient equilibrium iff import sanction is stronger than

export sanction, or the below pair of conditions hold

$$C_S^0(\alpha, k_S, \frac{k_S - 1}{N}, c, e) > C_S^0(\alpha, k_S, \frac{k_S + 1}{N}, c, m) \quad and \quad |\frac{k_S + 1}{N}| \neq 1$$

The fraction $\frac{k}{N}$ gives two insights, the net-value of links k (whether T is a net-exporter or net-importer) and the total number of out-links and in-links n. C_T^0 increases in the absolute value of $\frac{k}{N}$ which entails that if T has mostly out-links then imposing an export sanction (deletion of in-link) will be more costly than imposing an import sanction (deletion of out-link). Similarly if T has mostly in-links then severing an out-link by S (import sanction) is more costly than export sanction.

Proposition 7. *In presence of relatively lower levels of geopolitical rivalry (g is either linear or concave), strong sanctions be imposed by S which will be obeyed by T[7]. As geopolitical tensions increases, g becomes more convex, only any weak sanction is likely to succeed. The sufficient conditions for a strong sanction to succeed is to make T a complete exporter or complete importer (in limiting sense).*

When geopolitical motives are rather weak, T after losing some strategic value might comply even when S suffers no loss or relatively less loss in its own strategic value. However, when T carries rivalry on diplomatic grounds, then T would refuse to budge to the demands of S, unless S suffers a similar or greater loss in terms of centrality value.

6   Conclusion

This discussion has focused on the following question - is the type of sanction levied by a sender against a target a function of the network architecture. Of the various facets of architecture lies the concept of node centrality which measure importance of nodes in different ways. I use the betweenness centrality for my study. Why does this measure matter in examining trade sanctions? This is because, a higher value of betweenness centrality of a player means that it holds a key strategic position in bridging trade ties with other nations in the most cost effective way. In simple terms, the player acts as a vital intermediary for majority players in side the network. Hence, issuing sanctions, which consists of link deletion, can lead to heavy compromise of both sender and target. This is also germane to the trade literature which shows strong evidences of how the value added by intermediate trading partners have increasingly surged up over the years.

In essence, the broad question which has been answered here is the following - what network structure supports import sanctions and which ones support export sanctions, if one assumes that

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[7]See appendix section 7.3 for the relevant technical sufficiency condition

the players involved in sanctions take actions based on the value of this measure of centrality. I present certain features in networks which gives force to specific types of sanctions and render other sanction either non-effective or pyrrhic. The consideration of strategic positions while deciding which type of sanction to impose is a new way of examining sanction efficacy. As an application, this analysis lends insights to the strength of sanctions when players are concerned only with trade-gains and when they account for payoffs from geopolitical supremacy alongside trade-gains. This model shows that the presence of such geopolitical motives weakens the power of sanctions and might lead to a Pareto inferior outcome.

References

Ali, Shimelse, and Uri Dadush. 2011. "Trade in intermediates and economic policy." *VoxEU. org*, 9.

Breda, Emanuele, Rita Cappariello, and Roberta Zizza. 2009. "Vertical specialisation in Europe: evidence from the import content of exports." In *Firms' Objectives and Internal Organisation in a Global Economy*. 189–212. Springer.

Caruso, Raul. 2003. "The impact of international economic sanctions on trade: An empirical analysis." *Peace Economics, Peace Science and Public Policy*, 9(2).

Felbermayr, Gabriel, Aleksandra Kirilakha, Constantinos Syropoulos, Erdal Yalcin, and Yoto V Yotov. 2020. "The global sanctions data base." *European Economic Review*, 129: 103561.

Greif, Avner. 1993. "Contract enforceability and economic institutions in early trade: The Maghribi traders' coalition." *The American economic review*, 525–548.

Hufbauer, Gary Clyde, Jeffrey J Schott, and Kimberly Ann Elliott. 1990. *Economic sanctions reconsidered: History and current policy.* Vol. 1, Peterson Institute.

Joshi, Sumit, and Ahmed Saber Mahmud. 2016. "Sanctions in networks:"the most unkindest cut of all"." *Games and Economic Behavior*, 97: 44–53.

Joshi, Sumit, and Ahmed Saber Mahmud. 2018. "Unilateral and multilateral sanctions: A network approach." *Journal of economic behavior & organization*, 145: 52–65.

Joshi, Sumit, and Ahmed Saber Mahmud. 2020. "Sanctions in networks." *European Economic Review*, 130: 103606.

Kaempfer, William H, and Anton D Lowenberg. 1999. "Unilateral versus multilateral international sanctions: A public choice perspective." *International Studies Quarterly*, 43(1): 37–58.

7 Appendix

7.1 Proof of Proposition 5

If S is a net-importer then severing an import link still preserves the geodesic between k and m which includes S ($S \notin \{k, m\}$). This keeps the centrality unchanged for both S and T

While, severing an export link makes S a gross importer, thereby implying that no geodesic between k and m can contain S. This leads to a loss in centrality for both S and T.

Some level of complexity enters when we consider acyclic networks. When g is acyclic, betweenness centrality might be lower or higher for players compared to when g is cyclic. If S and T are both net-importers then S will unambiguously gain more by imposing an import link than imposing an export one. Since S is a net-importer, there exists a third player from whom S has an import link. Therefore, acting as a link between any path emanating from k to another player $m \notin \{S\}$ necessitates that S maintains its out-degree or export link with T. This entails that S will not severe its export link with T. If T in also a net importer, then by $A2$, T has an import link with another third player $l \notin \{S\}$ who reaches to some other player in g through T. To create pressure on T, then S must impose an import sanction such that the geodesic from l cannot go beyond T. Then based on S's position and T's position in the network, an import link must be imposed by T or T cuts its import link. Hence centrality of S remains unchanged while that of T falls. Hence sanction is $S4$.

7.2 Proof to proposition 7

The following equations assure that T's payoffs are higher when it complies to export sanctions and violates import sanction. I now check how strong binding these conditions are given the loss in initial centrality score ($\beta_T - \beta_T(e)$ and $\beta_T - \beta_T(m)$) following export and import sanctions respectively.

$$\beta_T(m) - \beta'_T(m) < 2P_0 + C_T^0(\alpha_T, \frac{k}{N}, \frac{k-1}{N}, m, c) - C_T^0(\alpha_T, \frac{k}{N}, \frac{k-1}{N}, m, v)$$

or,

$$\beta_T(m) - \beta'_T(m) < 2P_0 - C_T^0(\alpha_T, \frac{k}{N}, \frac{k-1}{N}, m, c)\left[\mu - 1\right] \tag{17}$$

$$\beta_T(e) - \beta'_T(e) > 2P_0 + C_T^0(\alpha_T, \frac{k}{N}, \frac{k+1}{N}, e, c) - C_T^0(\alpha_T, \frac{k}{N}, \frac{k+1}{N}, e, v)$$

or,

$$\beta_T(e) - \beta'_T(e) > 2P_0 - C_T^0(\alpha_T, \frac{k}{N}, \frac{k+1}{N}, e, c)\left[\mu - 1\right] \tag{18}$$

For larger values of k and n, the cost of sanctions become less convex with the deletion of out-link or in-link. The bindedness of 17 gains force either of the two ways or both- if $\beta_T - \beta'_T(e)$

is large enough positive or if C_T^0 is large enough. The latter condition is weakened if T has enough in-links to compensate for the loss. In essence, T can resort to its other partners for importing goods.

The satisfaction of the bindedness condition can be either weakened or strengthened due to the inclusion of the geopolitical factor in the compatibility equations which are shown below.

$$y.\left[\beta_T - \beta_T'(m)\right] + (1-y)\left[g(\beta_T - \beta_S) - g(\beta_T'(m) - \beta_S'(m))\right] < 2P_0 - C_T^0(\alpha_T, \frac{k}{N}, \frac{k-1}{N}, e, c)\left[\mu - 1\right] \tag{19}$$

$$y.\left[\beta_T(e) - \beta_T'(e)\right] + (1-y)\left[g(\beta_T - \beta_S) - g(\beta_T'(e) - \beta_S'(e))\right] > 2P_0 - C_T^0(\alpha_T, \frac{k}{N}, \frac{k+1}{N}, e, c)\left[\mu - 1\right] \tag{20}$$

It is evident that the role of $\frac{k}{N}$ is depleted for $(1-y) > y$, that is when players are focused more on geopolitical gains which might not be aligned to the gains from trade.

7.3 Trade and geopolitical incentives

If players have trade and geopolitical incentives, then the following inequalities ensures that T complies before an export sanction but violates an import sanction. This is similar to equations 6 and 7, but includes the geopolitical motives in its payoff function.

$$y[\beta_T(m) - \beta_T'(m)] + (1-y)[g(\beta_T - \beta_S) - g(\beta_T'(m) - \beta_S'(m)] < 2P_0 +$$
$$C_T^0(\alpha_T, \frac{k}{N}, \frac{k-1}{N}, m, c) - C_T^0(\alpha_T, \frac{k}{N}, \frac{k-1}{N}, m, v) \tag{21}$$

$$y[\beta_T(e) - \beta_T'(e)] + (1-y)[g(\beta_T - \beta_S) - g(\beta_T'(e) - \beta_S'(e)] > 2P_0 +$$
$$C_T^0(\alpha_T, \frac{k}{N}, \frac{k+1}{N}, e, c) - C_T^0(\alpha_T, \frac{k}{N}, \frac{k+1}{N}, e, v) \tag{22}$$