Sanctions in directed networks

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Abstract

This paper provides an understanding of trade sanctions (import or export) using a directed network model. Sanctions are common punitive measures taken by a player (sender) to discipline another player (target) in a trade network. Sanctions not only lead to welfare loss, but also, in most cases, fail to discipline the target (ineffective sanction) which aggravates the welfare loss problem. Empirical evidences in the realm of international trade show differences in the effectiveness between import and export sanctions. This paper shows that such differences can be explained by one specific centrality features of the underlying trading network - betweenness-centrality. This measure distinguishes which sanction - import or export - will be effective in imposing more harm to the target and minimum harm to the sender. This centrality measure reflects a player's strategic location and provides intuitive appeal to our results especially in the context of trade. Using this measure, I also discuss the conditions under which geopolitical tensions between sender and target aggravate the problem of sanction ineffectiveness.

Key words: sanctions, directed networks, betweenness centrality
JEL codes: C7,F13,L14

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[†]I would like to thank Sudipta Sarangi, Sunjin Kim for their support and valuable comments.

1 Introduction

Sanctions are imposed as a punitive measure to discipline any agent (target) (individual or nation) when it has engaged in actions not approved by the sanctioning agent (sender). These actions can include the target violating trade norms, involving in weapons trade, inciting domestic warfare, disobeying environmental norms, building nuclear plants and many more. Ex-post sanction, if the target (T) complies to the demands of the sender (S), then the sanction is deemed to be effective, else ineffective. Sanctions has been used throught history. Greif (1993) recounts one of the first instances of sanctions in the eleventh century where Maghiri tribes set up trade-laws to strengthen contractual relations between merchants and their agents who travelled overseas. On the occasion of any code violation, coalition members could take disciplinary actions like sanctions thereby barring the convicted agent from conducting future trade.

The existing literature focuses on sanctions in a trade network where there is a possibility of a two-way exchange of goods or services. Interestingly, effective trade sanctions are rare events (roughly 30% of all sanctions are effective (Felbermayr et al. (2020))). Hufbauer, Schott and Elliott (1990) suggests that most sanctions which have been proven to be a success are due to the presence of contemporaneous forces like military pursuits which had coerced the target to comply to the sender's demand. However, if one looks at sanction effectiveness, then it is seen that effectiveness of sanctions differs across its types - import, export or bilateral. [1]

This paper analyses the merits of export and import sanctions from a directed network perspective. I argue that if the strategic position of sender and target are taken into account before the sender imposes sanctions, then sanction effectiveness can improve. In other words, target will comply to sender's demand with higher probability. In some instances, as I further show, this outcome of complaiance can also be a Pareto improvement when the cost of sanction is smaller for the sender. Strategic location within the network, which is measured by betweenness centrality (henceforth b-centrality) gains instrumental value in the realm of trade networks where it reflects how important a role does a node or agent plays as an intermediary to facilitate trade gains for the entire network. The importance of trade intermediaries has grown with increased globalization where countries find it less costly to outsource manufacturing stages to other nations before buying the final product back. Globalization has also made the OECD increase trade in export of goods which are mainly imported from other nations. This has made the domestic value added to goods exchanged between any two countries increasingly small (see Ali and Dadush (2011),Breda, Cappariello and Zizza (2009)). In essence the role of intermediaries have gained much force. This strengthens the intuition of using b-centrality measure where a higher value suggests more important strategic location of a particular country or player.

[1] Caruso (2003) highlights the superiority of import over export sanctions.

Given the role of intermediaries in modern day trade, severing links (through sanctions) can severely compromise the value of strategic position for both sender and target alongside declining overall gains from trade leading to price-rise and unemployment. Now the question of interest is what type of sanctions (import or export) should S impose such that it inflicts minimum harm on itself but maximum harm on T? The answer lies in the underlying network architecture. Since, strategic location is directly dependent on network architecture, understanding it through b-centrality will give insights in forming a general rule of thumb conditions which will (roughly) determine which sanctions must be imposed.

Previous network-theoretic models have mainly focused on the components model of networks. The standard thought supported by these models is that effective sanctions can be imposed if S has more number of links than the T. More links imply power which can be weiled against T. Alternatively, if the target has more number of links, then sanctions are unlikely to be effective as more number of links generate greater resistance in T to endure sanctions without complying to the demands of sender. Works of Caruso (2003) have shown the outright failure of sanctions by shedding light on how target nations escape the effect of sanctions by forming ties with other nations. For instance, following sanctions by the Arab nations, Qatar formed renewed ties with Iran and China. Effectiveness of sanctions is variant across good-type. Kaempfer and Lowenberg (1999) cites the example the effectiveness of US's export sanction of computers against South Africa due to the limited number of computer suppliers in the world market during the 1980's. However, the ban on coal imports from South Africa was weakened by the fact that South Africa soon found new buyers.

The current paper is novel in two respects. First, it studies sanction efficacy under directed network which allows to differentiate between import and export links. The directed nature gains bite in the ambit of trade where direction of links suggest import or export or the flow in a global supply chain (GSC). Second, payoffs of players do not depend on the number of links, but on strategic location which is quantified by the betweenness-centrality measure. This measure helps to quantify collateral damages on both S and T and proves as a reasonable starting point to understand which sanction -export or import - will be optimal given a specific network. S makes a judgement of how its strategic position will be weakened with respect to T if S imposes an import sanction vis-a-vis an export sanction. A further insight (future work) which this directed network provides is the aspect of T seeking to form links with third parties. If an import sanction is levied on T, it needs to find an alternative market to siphon off its production whereas an export sanction will compel it to either produce domestically or find an alternative partner to buy the good. In essence, efficacy of csanctions are governed by the number of export and import links emanating from or converging to a nation respectively.

Modern world sanctions have evolved in many forms (trade, financial, travel) and methods of

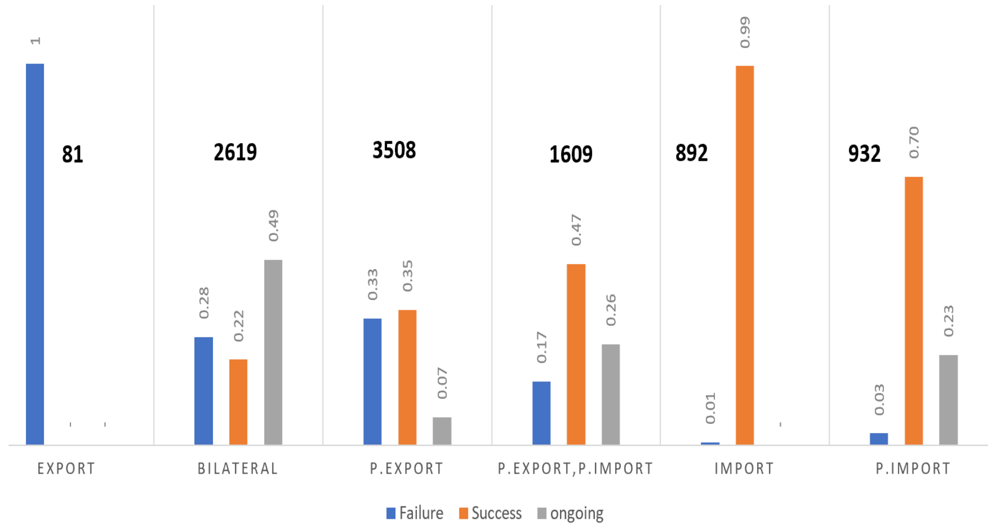


Figure 1: This illustrates the main outcomes of different types of trade sanctions. The numbers are represented as a proportion of the frequency of each sanction type (numbers indicated in bold). Other outcomes apart from these three outcomes include 'partial success' and 'negotiation by settlement' which I have removed for the sake of what this paper is trying to focus.

punishing the guilty (severing trade ties, blocking investments or preventing travel) which in most cases inflicts collateral damage on the sanctioning agent and leads to welfare loss. Felbermayr et al. (2020) shows that complete export sanction has lead to a 76% decrease in trade, an outcome equivalent to an approximate 43% rise in tariffs. While, import sanctions has caused a loss in trade of 52% (an outcome when tariffs rise by 20%). This capacity of sanctions to cast economic and social damage creates avenues for research, like the current paper which presents conditions where more effective sanctions can be imposed such that the welfare loss is minimum.

Previous theoretical work in the same vein consists of Joshi and Mahmud (2016),Joshi and Mahmud (2018),Joshi and Mahmud (2020). The first of the three papers studies how the network structure helps the sender to form a coalition such that it can sanction the target with greater force and whether T would comply to S's demands more readily. They introduce the concept of effective sanctions through the concept of spanning trees and analyse sanctions in short and long run. In essence, this strategy precipitates to relegating T to the smallest possible component to make T succumb. Joshi and Mahmud (2018) extends the previous idea and examines the role of linear, concave or convex utility function of sender in sustaining a larger coalition among sanctioning players. Joshi and Mahmud (2020) further augments their existing work of understanding sanction efficacy under different network specification through strategic complementarities and externalities. For instance,

There are multiple outcomes of how centrality of the target and sender are affected if one considers the initial centrality values prior to sanctions. I define two broad sanction types - strong and weak, each of which further considers multiple sub-types, based on how centrality of sender and target have fared before and after sanctions.

The paper proceeds in the following stages. First, I define model prelimenaries including two broad types of sanctions - strong and weak. Strong sanctions are those where the b-centrality of T decreases while that of S reimains unchanged. The weaker form of sanctions are those where both S and T suffers a loss of b-centrality post-sanctions to varied degrees. I then present a baseline network with 3 players. Henceforth, I propose the insights from the 3-player case which also hold for larger networks. I find cyclicity to be a major factor behind the efficacy of both import and export sanction. I also find that export sanction inflicts more harm on the sanctioned state if there exists a cycle which originates from the target and connects the sender through the other players. Similarly import sanction is more damaging to the target if the cycle is directed in the opposite way.

This paper's aim is to primarily lay out conditions on network structure on the success of mainly two forms of trade sanctions- import and export. In the benchmark model I exclude heterogeneity among the trading players arising from cost of trade, production functions, geopolitical factors to highlight the impact of networks on sanction efficacy.

2 Network model

The graph or network comprising of a set of nodes $\mathbb{N} = \{1, 2, 3, .., n\}$ (denoting players) is denoted by g. g is directed which essentially means that any node can have a link which originates from it or converges to it from another node. This is done to account for export links, import links or bilateral links between two players. The number of links emanating from a player is its out-degree while the number of links converging to a player is its in-degree. I now provide the definitions of some network structures which will be used in this paper.

A path in g connecting i and j is a set of distinct links $il_1, l_2l_n, .., l_kj$. A cycle is a path such that the initial and terminal players are the same. A complete cycle connects all players in the network. A sub-cycle is a cycle connecting n' nodes in a network g which has n nodes where $3 \leq n' < n$. A geodesic σ_{ij} denotes the shortest path from i to j. A network is connected if there exists at least one directed link between a pair of players. A connected network is complete if all players are connected to each other by links in both directions. An incomplete network is one which is not complete. i.e. at least one pair of players are connected by a link directing in one way.

Sanction is imposed by player S (sender) against player T (target) where $S, T \in g$. g' to denote the ex-post sanction network such that $g' = g - l$ where $l \in \{ST, TS\}$. ST denotes export sanction (severing link which directs from S to T). Analogous explanation holds for import sanction TS. This terminology is used to account for the directed nature of the graph g and help differentiate between import and export sanctions. Bilateral sanction is excluded in this paper and focus is given to only import and export.

The traditional network literature cites a number of measures of centrality - eigen-vector centrality, betweenness centrality, degree centrality and closeness centrality. Since, the paper focuses on directed networks, eigen-vector centrality is not a good enough measure. The other two measures works well in trade setting, but cannot capture the strategic value of a player inside a network. The closeness measure is not robust given that sanctioning countries target to sanction countries which are both close and far away. The degree centrality too is not very interesting as nations with smaller value of degree-centrality can impose sanctions against nations with larger values of degree-centrality. I define b-centrality in directed networks as follows:

Definition 1. *Betweenness (b)- Centrality: Let $d(kj)$ denotes the number of geodesics (shortest paths) between k and j in directed network g. Analogously, we denote by $d_i(kj)$ as the number of these geodesics which pass through node i. Then the betweenness centrality of player i is*

$$\beta_i(g) = \sum_{k \neq j : i \notin \{k,j\}} \frac{d_i(kj)/d(kj)}{(n-1)(n-2)}$$

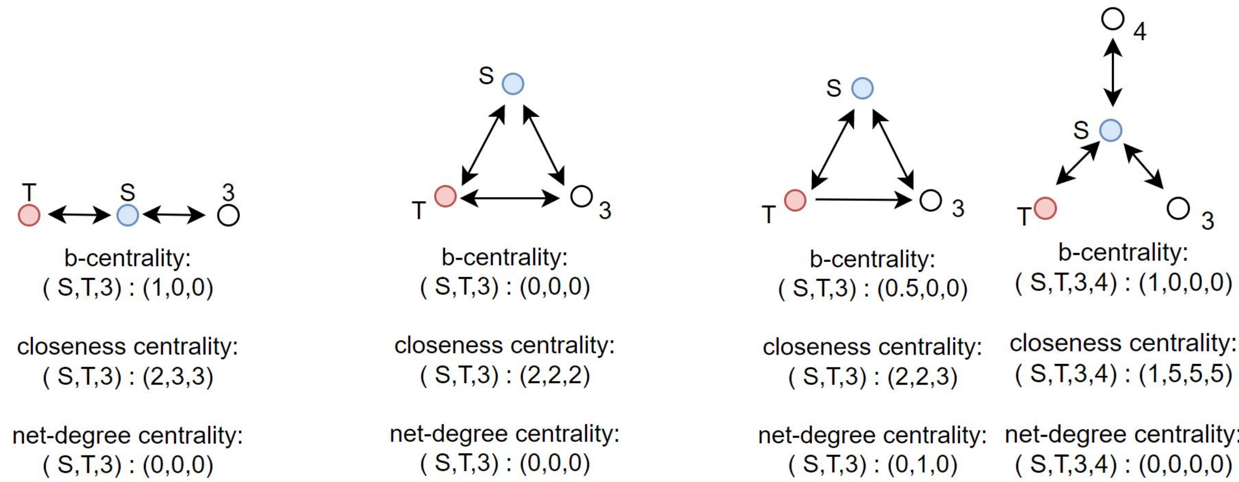
2.1 b-centrality

The usage of this centrality measure in this model of sanctions becomes central because it measures the relative strategic location of each player in the network. The other measures of centrality like closeness and net-degree centrality [2] are more direct measures, while the eigen-value centrality is less robust in a directed network setup. These measures are also unable to capture the trade spill overs from sanctions inside a network by accounting for the change in strategic positions of the sender and target following sanctions. Amidst this, the b-centrality measure provides a simple way (also a reduced way) to show how trade is impacted due to change of strategic position of the sender and target. I have discussed in figure 2, how the b-centrality is distinguishable from the other forms of centrality in some known networks. The figure primarily illustrates the relative strategic position of the players in any given network. I also impose the following assumptions

Assumption 1. *Sender sanctions a target with only direct trade links.*

The direct trade link binds the sender's incentive to participate in sanctioning the target. Without direct trade, sanctions are more likely to be violated by the target. Without direct trade link, the loss from trade is not high enough to make the target obey the sender.

[2] In a directed network, this measure calculates the total out links net of in-links of a node.



(a) Line (b) Complete (c) Incomplete (d) Star

Figure 2: Differences in centrality measures across some standard network structures.

3 Game

I construct a game of sanctions which is represent by $\Gamma = \{g, S, T, A_S, A_T, U_S, U_T\}$. The term g represents the network, the players directly involved are the sender S and target T [3]. Action set of S is $A_S = \{import, export\}$ which signifies whether to impose import or export sanction. Analogously action set A_T comprises of two actions $\{comply, violate\}$. The payoffs of S and T in g are represented by $U_S = U_S(g) = \beta_S(g)$ and $U_T = U_T(g) = \beta_T(g)$ which denotes the value of their b-centrality in g.

4 Partial analysis

I begin my study by first considering the decision making of the S against T. This being a partial analysis, will only focus on the action of the sender. The imposition of sanction generates different outcomes (change in values of b-centrality of S and T) which I classify under two broad classes - strong and weak sanctions. I further elaborate on all possible sub-classes of sanction under these strong and weak sanctions.

The below assumption establishes the incentive compatibility conditions of T in engaging in the restricted activity and of S in imposing the sanctions in the first place.[4]

Definition 2. *T receives a payoff of $P_0 > 0$ by engaging in an action a_0, deemed unlawful by S. S pays a price of $P_s > 0$ from T's action.*

[3]Here I focus on unilateral sanctions and hence do not consider other players in g.

[4]I would discuss this in the general equilibrium section where T's action will enter into the payoff of S.

Assumption 2. ***Participation constraint (PC)*** *is satisfied, such that if T takes a_0, S is better off by sanctioning T.*

$$U(\beta_s(g)) - P_s < U(\beta_s(g'))$$

The inequality states that the utility of S in g from enduring cost P_s (from T's action) is strictly less than the utility S will experience in post-sanction network g'. It implies that the target's activity is causes enough harm to the sender such that the latter is better off by sanctioning.

Definition 3. ***Strong Sanction (S)****: A sanction is strong if the utility of S and T before and after sanction satisfy the following*

$$U_T(g') < U_T(g) \quad and \quad U_S(g') = U_S(g) \tag{1}$$

This entails that after sanction is imposed, the b-centrality of the T decreases while b-centrality of S remains unchanged. This class of sanction can be dissected into five types (Table 1) by combining terminal and initial conditions. Further explanations are given in the appendix. From the table below, it is evident that S5 is the strongest within the class of strong sanctions. The cells with no values does not represent strong sanction.

	Utility in g´		
Utility in g	$U_S(g') > U_T(g')$	$U_S(g') = U_T(g')$	$U_S(g') < U_T(g')$
$U_S(g) > U_T(g)$	**S1**	-	-
$U_S(g) = U_T(g)$	**S2**	-	**S3**
$U_S(g) < U_T(g)$	**S5**	**S4**	-

Table 1: This shows the outcomes of utility of S and T from strong sanctions which preserves the b-centrality of S such that utility of S is unchanged before and after sanctions (in g and g').

Definition 4. ***Weak Sanction (W)****: A sanction is weak if alongside lowering the centrality of T, the centrality of S is weakly decreased, or*

$$U_T(g') \leq U_T(g) \quad , \quad U_S(g') \leq U_S(g) \tag{2}$$

The possible outcomes from weak sanctions which has been grouped in each cell of Table 2. $W5$ is the most effective weak sanction. The appendix expands the conditions for each sanction type under weak sanction.

Definition 5. *(i) A player is net-exporter (net-importer) if its out-degree is strictly greater (lower) than its in-degree.*

	Utility in g´		
Utility in g	$U_S(g') > U_T(g')$	$U_S(g') = U_T(g')$	$U_S(g') < U_T(g')$
$U_S(g) > U_T(g)$	**-**	**W1.2,W1.3**	**W1.1**
$U_S(g) = U_T(g)$	**W2.3**	**W2.2**	**W2.1**
$U_S(g) < U_T(g)$	**W5**	**W4.1,W4.2**	**W3.1,W3.2**

Table 2: This shows the outcomes of utility of S and T from weak sanctions which preserves the b-centrality of S such that utility of S is unchanged before and after sanctions (in g and g‘).

(ii) A player is balanced if its out-degree is equal to its in-degree.

(iii) A player is a complete exporter (importer) if it has only out-links (in-links).

Assumption 3. *All links have equal weight.*

This assumption fits the primary focus of this paper which examines complete sanctions and not partial sanctions. In addition, this unweighted structure provides sufficiency to understand how b-centrality alone can contribute towards explaining the efficacy of the import and export sanctions. A further examination of sanction efficacy under weighted links can be a work of the future.

Assumption 4. *The necessary condition for this analysis of effectiveness of export and import sanction is to have a network which is not complete.*

Proof: If the network is complete, then there exists no geodesic σ between two players k and m ($k \neq m$) such that it passes through a third player i ($i \notin \{k, m\}$). Hence, b-centrality of any i in a complete network g is zero. Following sanctions, in g', geodesic of S and T remains zero.

4.1 3-player network

I consider a network g comprised of the following three players: sender (S), target (T) and a third player denoted by 3. This will help understand the merit of import versus export sanctions within a smaller group of trading nations. The following proposition differentiates the efficacy of export versus import sanctions based on the direction of links with a third player (player 3, who is not directly involved in sanctions).

Proposition 1. *Suppose there exists two bilateral links in g with one bilateral link between S and T. Then depending on whether T or S has the other bilateral link with 3, import and export sanction have the following outcomes.*

- *(i) If T and 3 are bilaterally linked such that T is balanced, then*
  - *If S is a net-importer, then export sanction is S_4, while import sanction is W0.*

- *If S is a net-exporter, then import sanction is S_4, while export sanction is W0.*

- *(ii) If S and 3 are bilaterally linked such that S is balanced, then*
  - *If T is net-importer or net-exporter, then export sanction is W0.2 (ineffective where centrality of T is unchanged but that of S decreases), while import sanction is W0.*

In the above cases, the direction of links take the pivotal role in rendering one sanction weak and the other strong. The position of the third player in this network becomes instrumental. Both the statements can be understood by taking help of figure 3. The first statement corresponds to the top row in figure 3 where the b-centrality of all but T is zero. This entails that S does not lie on the geodesic between T and 3, nor does 3 lie between S and T. Being net-importer, if S severes its import link with T, it turns into a balanced player in g, but the existence of bilateral ties between T and 3 does not affect the b-centrality of S or T. However, severing the export ties with T makes S an importer which does not lowers its b-centrality but lowers that of T. Hence export sanction is $S4$. This network (post export sanction) is not cyclic as S is an importer with only in-links.
The second statement can be analogously explained by the second row in figure 2.
The second sub-part corresponds to the third and fourth row in figure 3. It shows that T has no strategic value (zero value of b-centrality). This is because 3 and S has direct bilateral links, hence do not need T. On the contrary, S's location is strategic as it lies in the geodesic between T to 3. Now, if S deletes its out-link, its strategic value does not change (sanction is ineffective or W0). On the contrary, cutting the in-link from T destroys the geodesic between T and 3 (sanction is W2). So in this case, import sanction is ineffective while export sanction harms the sanctioning player without impacting the target player. Similar reasoning holds for the second statement corresponding to the last row in figure 2.

Corollary 1. *The direction of links between player 3 with S or T has a non-trivial effect on sanction efficacy.*

Proposition 2. *Suppose there is only one bilateral link in g, between S and T.*

(i) If S and T are either both net-exporters or net-importers in g, b-centrality is zero for all three players. Hence import or export sanction is W0.

(ii) If S is net-importer while T is net-exporter in g, then import sanction is W0, while export sanction is W2.

(iii) If S is net-exporter while T is net-importer in g, then export sanction is W2, while import sanction is W0.

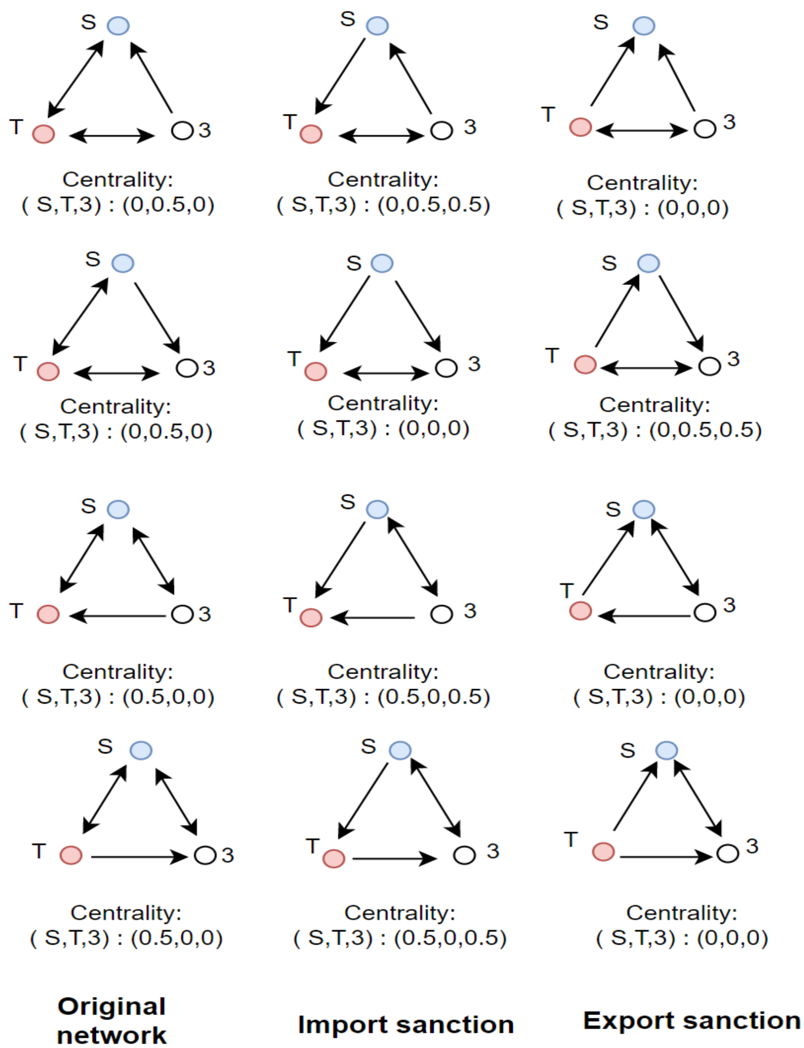


Figure 3: The first two rows illustrates sub-part (i) of proposition 1 and the third and fourth rows illustrates sub-part (i) of proposition 1. The first row exhibits S as net-importer and T is balanced. Import sanction is ineffective as shown.However, the export sanction lowers centrality of T as now it does not lie in the geodesic from S to player 3. Analogous interpretation holds for the rest.

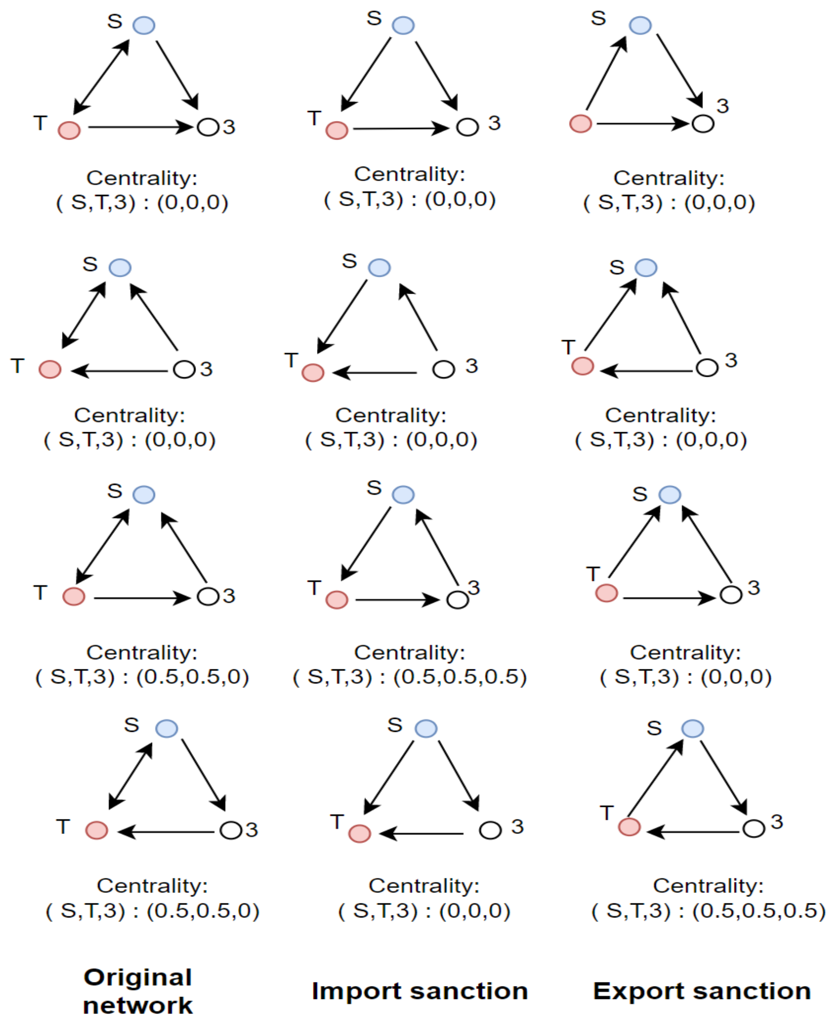


Figure 4: The first two rows illustrates when S and T are net-exporters and net-importers respectively. Row 3 (4) shows the effect of import and export sanctions when S is net-importer (net-exporter) while T is net-exporter (net-importer).

Proof: Figure 4 offers a diagrammatic proof of proposition 2. In the first and second row, S and T are both net-exporters and net-importers respectively. The b-centrality values are zero. Hence, sanctions are $W0$, incapable of eroding centrality any further. In the third and fourth row of figure 4 (depicting statement (ii) and (iii) respectively), the pre-sanction (original) graph is cyclic. As illustrated, the $W2$ sanction type occurs in an acyclic post-sanction network while the ineffective sanction type ($W0$) occurs in a cyclic post-sanction network . In the latter type, trade gains will not to get impacted as all the three players act as intermediaries to the each other.

Corollary 2. *If the betweeness centrality of S or T or both are non-zero in a 3-player network g, then export or import sanction is ineffective when sanctioned network g' is cyclic. On the other hand, sanctions are either of type W2 or S4 when g' is acyclic.*

This statement grabs bite in the world of trade where increasing volume of exported goods from any country are actually made up of intermediate imports made by that country from its trading partners [5]. This result is fairly intuitive and much easier to visualize in a 3-player network. If the post sanction network is cyclic, then each node lies on the geodesic between the other two players which is unlikely to disrupt trade-flow. If on the other hand, the sanctioned network becomes acyclic, then at least one of the 3 players have only out-links or in-links. This entails that the target nation post sanction has been prevented from acting as an essential trading intermediary.
If this player is T, then sanction is S_4 such that betweeness centrality of T falls to zero. However, if S has only out-links and in-links, then sanction is W_2 where S endures loss in centrality.

Remark 1. *If only unilateral links exists among the 3 players in g, then a sanction of type W3 can be imposed iff g is a cycle. If g is acyclic, b-centrality of all 3 players in g remains zero and all sanctions are $W0$.*

When only unilateral links are considered in the 3-player setting, only two situations arise where there exists cycles among the players. In these cases weak sanction of type $W3$ can be imposed. The remaining cases constitute acyclic networks where export or import sanction is $W0$ (ineffective).

4.2 Sanctions within and across trading blocs

This section examines trade with multiple players by constructing trade bloc. Networks with nodes greater than 3 generate the possibility of the remaining forms of strong or weak sanctions. However, with more players in a directed network setting, one runs into the risk of large computational study. Instead, I break these networks into bloc which is more germane to the trade bloc in reality. I give a general insight to the differential impact of import and export sanction on the values of b-centrality of players in such trade bloc.

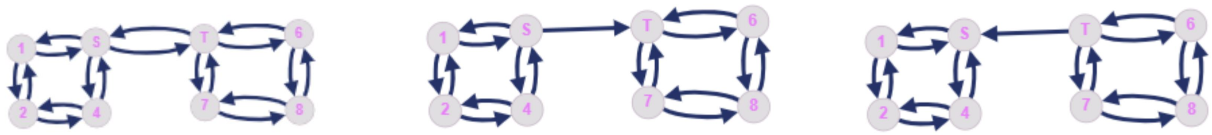
Definition 6. *A component $\mathbb{C}_g$ within network g is a network consisting of $m < n$ players whose cardinality is given by $||\mathbb{C}_g|| = m$. In this trade network, each $||\mathbb{C}_g||$ represents a trading block of a specific size or cardinality.*

Assumption 5. *I assume that g is comprise of two trade bloc $\mathbb{C}_1$ and $\mathbb{C}_2$ with any of the following feature(s).*

(a) S is a connector between $\mathbb{C}_1$ and $\mathbb{C}_2$, but $T \in \mathbb{C}_1$.

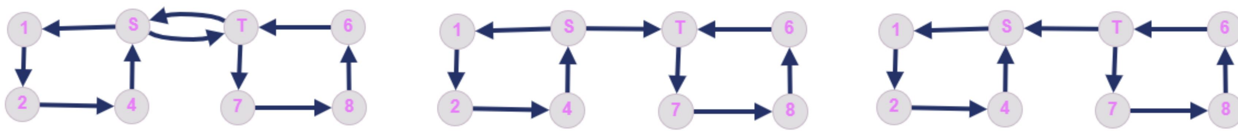
(b) $\mathbb{C}_1 \cap \mathbb{C}_2$ contain multiple players including S, but $T \in \mathbb{C}_1$.

[5] See Ali and Dadush (2011) who shows the increase in this trend in almost all the OECD countries.



(a) Two trade blocs (b) Import sanction by S (c) Export sanction by S

Figure 5: Trading bloc with ST as bridge.



(a) Two trade blocs (b) Import sanction by S (c) Export sanction by S

Figure 6: Trade bloc with ST belonging to same bloc

(c) $\mathbb{C}_1 \cap \mathbb{C}_2$ *contain both* S *and* T*.*

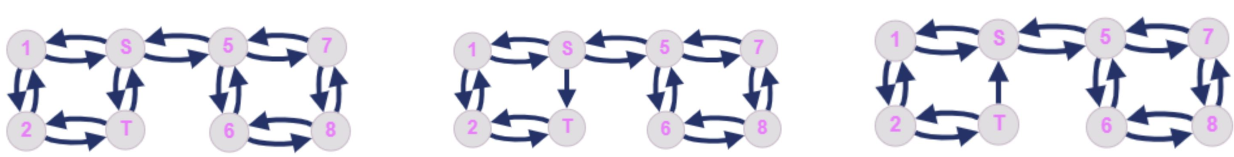
Proposition 3. *(i) Consider assumption 5a is satisfied and suppose both* $\mathbb{C}_1$ *and* $\mathbb{C}_2$ *are cycles comprising of either bilateral or unilateral links between any pair of players (figure 5 and 6). Then, export sanction lowers the b-centrality of* S *and* T *equally.*

(ii) If S *and* T *belong to the same component (figure 7). Then export sanction causes more harm to* T*.*

5 Analysis of a game between S and T

I follow the game described previously between S and T. The normal form game is $\Gamma_G = \{g, S, T, A_S, A_T, \sigma_S, \sigma_T, U_S, U_T\}$. I consider a static game where T's action a_T consists or complying (c) or violating (v) the sanction imposed by S. Action of S (a_S) consists of choosing between import (m) or export (e) sanction. I now lay out the basic premises which will be used for the rest of the paper.

Each player $i \in g$ has intrinsic power α_i which depicts its resistance against any sanction - increase domestic capacity to absorb excess goods in case of import sanction or to initiate its own



(a) Two trade blocs (b) Import sanction by S (c) Export sanction by S

Figure 7: Trade bloc with ST belonging to same bloc

production process in case of export sanction by the sender. There exists a parameter k_i pertaining to each $i \in g$ which denote the number of export links net of import links while N denotes the sum of all links to and from player i. Following definition 5, a player in g is a net exporter (net-importer) if $k_i > 0$ ($k_i < 0$) and it is balanced if $k_i = 0$. k_i' denotes the symmetric meaning of players in g'.

Any form of sanctions (import or export) on the target (weakly) reduces the b-centrality of both S and T. There is further an added cost on the target and sender which is a function of the above parameters, respectively denoted as $C_T(\alpha_T, \frac{k_T}{N}, \frac{k_T'}{N}, a_S, a_T)$ and $C_s(\alpha_S, \frac{k_S}{N}, \frac{k_S'}{N}, a_S, a_T)$. Now, $\frac{\partial C}{\partial \alpha} < 0$ which suggests that greater intrinsic strength reduces cost of sanctions to any player in the network. I now introduce certain properties of the cost function which emerges from this game between S and T in this directed network setup.

Axiom 1. ***Cost of sanction on S:*** $C_S(\alpha', \frac{k_S}{N}, \frac{k_S'}{N}, a_S) < C_S(\alpha', \frac{\bar{k}_S}{N}, \frac{\bar{k}_S'}{N}, a_S)$ *where* $|\bar{k}_S| > |k_S'|$,

The sanctioned network weakly reduces centrality but changes the configuration of S by altering the number of in-links or out-links thereby creating an added cost on trade. In the above axiom, this cost is greater when S has more number of out-links or in-links or absolute value of k_S is near to 1.

Axiom 2. ***Cost of import sanction:*** *Given* $\beta_T > 0$ *and* $k_T' < k_T$,

$$C_T(\alpha, \frac{k_T}{N}, \frac{k_T - 1}{N}, a_S = m, a_T) < C_T(\alpha, \frac{k_T'}{N}, \frac{k_T' - 1}{N}, a_S = m, a_T)$$

The cost of import sanction on the target T is higher when $k_T' < k_T$ and rises as $\frac{k_T}{N} \to -1$. $\frac{k_T}{N} \to -1$ suggests that T mainly imports from other players in g. Hence imposing an import sanction entails that there are lesser out-links originating from T which lowers its strategic power, thereby raising costs of sanctions. Symmetric reasoning applies for the cost on export sanction as stated in axiom 3 below which states that the cost of export sanction is higher when $k' > k$ and rises as $k_T \to 1$.

Axiom 3. ***Cost of export sanction:*** *Given* $\beta_T > 0$ *and* $k_T' > k_T$,

$$C_T(\alpha, \frac{k_T}{N}, \frac{k_T - 1}{N}, a_S = e, a_T) > C_T(\alpha, \frac{k_T'}{N}, \frac{k_T' + 1}{N}, a_S = e, a_T)$$

Axiom 4. *Cost of violating sanction is higher than complying*

$$\frac{C_T(\alpha, \frac{k_T'}{N}, \frac{k_T'+1}{N}, a_S, a_T = v)}{C_T(\alpha, \frac{k_T'}{N}, \frac{k_T'+1}{N}, a_S, a_T = c)} = \mu, \quad \mu > 1$$

	comply (c)	violate (v)
export (e)	$U_S(e,c), U_T(c,e)$	$U_S(e,v), U_T(v,e)$
import (m)	$U_S(m,c), U_T(c,m)$	$U_S(m,v), U_T(v,m)$

Figure 8: Static game

Cost of violating sanction is strictly greater than complying to it. Complying to sanction reflects a momentary costs of link deletion leading to temporary loss in trade gains [6]. However, violation leads to enduring this cost permanently or some added costs of creating new links with other players which raises costs of sanctions on the target.

Lemma 1. *For any player $i \in g$, $\frac{k_i}{N} \in \{-1, 1\}$ implies that $\beta_i(g) = 0$. However the reverse may not hold.*

Proof: The value of $\frac{k_i}{N}$ equals unity (positive or negative) when all m links are either out-links or in-links which entails betweeness centrality is zero. However, if centrality is zero, then i can still have both out-links and in-links such that $\frac{k_i}{N} \neq \{-1, 1\}$. In this case, there exists geodesics between other players which are shorter than the one which includes i.

5.1 Static game

I follow the static game similar to Joshi and Mahmud (2020) where sender decides to sanction or not sanction while target can either comply or violate. However, the current analysis leaves out the action of not sanctioning and dissects the action of sanctioning by considering the sender to choose between import and export sanction. The target's action remains the same - comply or violate. The utility function of S and T in a static game is $U(a_S, a_T)$ and $U(a_T, a_S)$ given other model parameters as previously explained.

Definition 7. *Pure Strategy Nash Equilibrium (PSNE): A pure strategy σ_s^* of the sender in Γ_G is a Nash equilibrium if $U_s(\sigma_S^*, \sigma_T) > U_s(\sigma_S', \sigma_T)$ for any $\sigma_S' \in A_S$ given any strategy σ_T by T and for all $\sigma_s' \in A_S$.*

Successful sanctions are defined when complaince by T sustains as the dominant strategy for T such that the following holds

$$\beta_T(g) - \beta_T(g') \geq 2P_0 \tag{3}$$

[6]This momentary loss from compliance is also present in the strategic study of sanctions in Krustev (2010)

By complying, T loses its benefit $P_0 > 0$ from the activity it was previously engaged in. This means that T will prefer to comply than violate if the loss of centrality is greater than the utility it derives from the activity.

S will wants to make T comply by incurring the least cost. In essence, S imposes the that sanction type which is strong. However, this strategy should erode centrality of T enough such that complying evolves as the dominant strategy for T. Say, given any g, S can impose an strong import sanction or a weak export sanction against T. If T is dovish and always wants to budge before any sanctions then import sanction is the dominant strategy. However, T can be more recalcitrant and complies only if S also endures a high enough costs from sanctions. Then S might want to impose the weaker export sanction. Additionally, either sanction can be successful only if it is P_0 reducing which entails that the by violating sanctions, the loss of centrality will override the gain P_0 from the illegal activity.

Definition 8. *Any sanction by S is P_0-reducing iff $\beta_T(g) - \beta_T(g') = P_0$ and strictly P_0-reducing iff $\beta_T(g) - \beta_T(g') > P_0$.*

The type of sanctions gains force when T complies under export sanction but violates import sanction. Or alternatively, T complies to all import but violate all export sanction. In essence one of the following pair of conditions must hold.

$$U_T(c, e) > U_T(v, e) \tag{4a}$$

$$U_T(v, m) > U_T(c, m) \tag{4b}$$

Now, for the symmetric condition (where T comply to import but violate export sanction), the following must hold

$$U_T(c, m) > U_T(v, m) \tag{5a}$$

$$U_T(v, e) > U_T(c, e) \tag{5b}$$

Proposition 4. *(i) If equation 3 is satisfied then (e, c) or (m, c) is the unique PSNE. If not, then (e, v) or (m, v) is the unique PSNE.*

(ii) If equation 4 is satisfied and if export sanction is stronger than import sanction, then (c, e) is the Pareto efficient outcome. Similarly, if equation 5 is satisfied and import sanction is stronger than export sanction, then (c, m) is the Pareto efficient outcome.

(iii) The equilibrium in mixed strategies is given by

$$q_T^* = \frac{U_S(m,v) - U_S(e,v)}{U_S(m,v) - U_S(e,v) + U_S(e,c) - U_S(m,c)}$$

$$q_S^* = \frac{U_T(v,m) - U_T(c,m)}{U_T(v,m) - U_T(c,m) + U_T(c,e) - U_T(c,m)}$$

If equation 3 is fulfilled, then violating sanctions carries a higher cost which dominates the gain P_0 which T derives from violating the norm. The second statement accounts for scenarios where T surrenders to only one type of sanction, but violates the other type. According to the first sub part, if export is a stronger sanction and T complies to it, then (e, c) is a Pareto efficient outcome. If this sanction was weaker, then certainly it would have led to a Pareto inferior outcome where S loses its own centrality post-sanctions. Analogous reasoning can be applied to the second sub part. The final statement characterizes how S and T will randomize when they derive equal expected payoffs from both their actions.

5.2 Different utility functions

The efficacy of sanctions can be tested under two broad classes of motives of players - pure trade motive and trade combined with geopolitical motive. Geopolitics can act as an impediment to a successful sanction. Nations can well endure losses in terms of trade-gains or strategic power by violating sanctions just to see another nation lose their strategic value.

Example 1. ***Pure trade motive:*** *Here any player in g consider the importance of their strategic power as the singular measure of its well-being. The utility of S and T are respectively $U_S(a_S, a_T|\beta_S)$ and $U_T(a_T, a_S|\beta_T)$.*

The utility of S and T following sanctions are

$$U_S(a_S, a_T = c) = \begin{array}{ll} \beta_S(e) + P_s - C_S^0(.|e), & \textit{if } a_S = e \\ \beta_S(m) + P_s - C_S^0(.|m), & \textit{if } a_S = m \end{array} \tag{6}$$

$$U_S(a_S, a_T = v) = \begin{array}{ll} \beta'_S(e) - P_s - C_S^0(.|e), & \textit{if } a_S = e \\ \beta'_S(m) - P_s - C_S^0(.|m), & \textit{if } a_S = m \end{array} \tag{7}$$

$$U_T(a_T = c, a_S) = \begin{array}{ll} \beta_T(e) - P_0 - C_T^0(.|e), & \textit{if } a_S = e \\ \beta_T(m) - P_0 - C_T^0(.|m) & \textit{if } a_S = m \end{array} \tag{8}$$

$$U_T(a_T = v, a_S) = \begin{array}{ll} \beta'_T(e) + P_0 - C_T^0(.|e), & \textit{if } a_S = e \\ \beta'_T(m) + P_0 - C_T^0(.|m) & \textit{if } a_S = m \end{array} \tag{9}$$

Lets look at T's incentives. The sanction type becomes interesting when T chooses to play (say) c for export sanction while choose v for import. Then 8 and 9 implies that the following two conditions hold simultaneously

$$\beta_T(m) - \beta'_T(m) < 2P_0 + C^0_T(\alpha_T, \frac{k}{N}, \frac{k-1}{N-1}, m, c) - C^0_T(\alpha_T, \frac{k}{N}, \frac{k-1}{N-1}, m, v) \tag{10}$$

$$\beta_T(e) - \beta'_T(e) > 2P_0 + C^0_T(\alpha_T, \frac{k}{N}, \frac{k+1}{N-1}, e, c) - C^0_T(\alpha_T, \frac{k}{N}, \frac{k+1}{N-1}, e, v) \tag{11}$$

The left-hand side of 10 and 11 is the loss in centrality of T under import and export sanction respectively. The inequalities bind conditional on the value of cost differences under both sanction types (final two terms on the right-hand-side of 10 and 11). Chang in cost $C()$ depend on whether T is a balanced ($k = 0$), net-exporter ($k > 0$) or net-importer ($k < 0$) in g and how k changes post sanction.

Given compliance from T, if export sanction is stronger than any import sanction, then (e, c) is the Pareto efficient outcome from the game in figure 4. It can be illustrated how network structure affects the conditions required for this to happen using figure 5. For (e, c) to be a pure strategy equilibrium, export sanction must be stronger than import sanction (cost of export is strictly less than import) which implies from 6 that

$$C^0_S(\alpha, k_S, \frac{k_S - 1}{2}, c, e) < C^0_S(\alpha, k_S, \frac{k_S + 1}{2}, c, m) \tag{12}$$

Now this conditions depend on k_S. If S is a net-importer then an import (export) sanction increases (decreases) the value of k_S by 1 (shown in the top row in figure 5).

$$\begin{cases} C^0_S(\alpha, \frac{-1}{2}, -1, c, e) \leq C^0_S(\alpha, \frac{-1}{2}, 0, c, m) & \text{if} \quad k_s = -1 \qquad (A) \\ C^0_S(\alpha, \frac{1}{2}, 0, c, e) \leq C^0_S(\alpha, \frac{1}{2}, 1, c, m) & \text{if} \quad k_s = 1 \qquad (B) \\ C^0_S(\alpha, 0, \frac{-1}{3}, c, e) \leq C^0_S(\alpha, 0, \frac{1}{3}, c, m) & \text{if} \quad k_s = 0 \qquad (C) \end{cases}$$

The condition A is not possible as S becomes a complete importer following the export sanction. Hence it loses it strategic position completely. In a similar sense, B is always feasible while for C is supposed to hold true because cycle exists as shown in the third network in the last row of figure 6. Therefore if in the initial network g, sender is a balanced agent and target plays *'comply'*, export sanction will be strong (relative to import) iff the following condition holds.

$$\frac{C^0_S(\alpha, 0, \frac{-1}{3}, c, e)}{C^0_S(\alpha, 0, \frac{1}{3}, c, m)} < 1 \tag{13}$$

This fraction is consistent with the last row in figure 6, where export sanction preserves the cyclicity in the sanctioned network while the import renders an acyclic network. Cyclicity allows

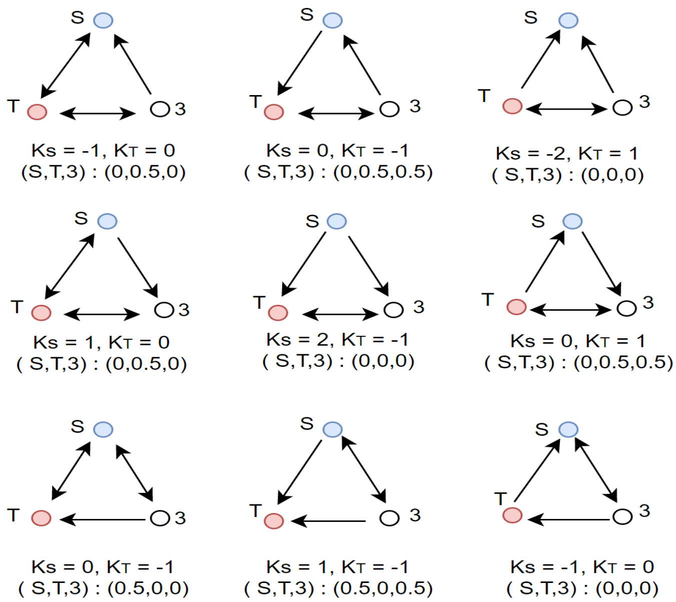


Figure 9:

the possibility of resource diversion for target which weakens the credibility of sanctions.

Example 2. ***Trade and geopolitical motive:*** *The utility functions which accounts for geopolitical motives are formalized by* $U_S((a_S, a_T|\beta_S, \beta_T)$ *and* $U_T(a_T, a_S|\beta_T, \beta_S)$*. post sanction, the utility is* $U_S(a_S, a_T|\beta'_S, \beta'_T)$ *and* $U_T(a_T, a_S|\beta'_T, \beta'_S)$*.*

Imposing sanctions reveal how the strategic position of sender relative to the target can have effects of strategic complemetarity and strategic substitutability to assist or impede the success of a sanction. The utility of S *and* T *following sanctions are*

$$U_S(a_S, a_T = c|\beta_S) = \begin{matrix} x.\beta_S(e) + (1-x).f(\beta_S - \beta_T) + P_s - C_S^0(.|e), & \textit{if } a_S = e \\ x.\beta_S(m) + (1-x).f(\beta_S - \beta_T) + P_s - C_S^0(.|m) & \textit{if } a_S = m \end{matrix} \tag{14}$$

$$U_S(a_S, a_T = v|\beta_S) = \begin{matrix} x.\beta'_S(e) + (1-x).f(\beta'_S(e) - \beta'_T(e)) - P_s - C_S^0(.|e), & \textit{if } a_S = e \\ x.\beta'_S(m) + (1-x).f(\beta'_S(m) - \beta'_T(m)) - P_s - C_S^0(.|m), & \textit{if } a_S = m \end{matrix} \tag{15}$$

$$U_T(a_T = c, a_S|\beta_S) = \begin{matrix} y.\beta_T(e) + (1-y).g(\beta_T - \beta_S) - P_0 - C_T^0(.|e), & \textit{if } a_S = e \\ y.\beta_T(m) + (1-y).g(\beta_T - \beta_S) - P_0 - C_T^0(.|m) & \textit{if } a_S = m \end{matrix} \tag{16}$$

$$U_T(a_T = v, a_S|\beta_S) = \begin{array}{ll} y.\beta'_T(e) + (1-y).g(\beta'_T(e) - \beta'_S(e)) + P_0 - C^0_T(.|e), & \textit{if } a_S = e \\ y.\beta'_T(m) + (1-y).g(\beta'_T(m) - \beta'_S(m)) + P_0 - C^0_T(.|m) & \textit{if } a_S = m \end{array} \tag{17}$$

The above represents that utility of S and T are a convex combination of trade-gains and geopolitical motives. In equation 12, $x \in (0,1)$ is the weight on the trade-gains of S while $(1-x)$ is the weight on geopolitical power. The utility of geopolitical power is given by the function $f(.)$ which increases with the centrality of S relative to T. Analogous explanation holds for the utility of T in equation 16. Following the discussion from the last example, the conditions needed for (c, e) to emerge as the equilibrium. there must exists incentive compatible conditions on both T and S which I keep in the appendix for visual clarity.

The magnitude of geopolitical rivalry is represented by the nature of f and g functions. If the nature is linear, then rivalry is weak. If it is concave, then rivalry increases with the differences in strategic dominance but in a decreasing rate. The existence of aggressive rivalry is given by convex f and g functions.

Corollary 3. *(i) If $4(a)$ holds then (c, e) is the Pareto efficient equilibrium iff the cost of export sanction is weakly stronger than import sanction for S which is ensured if the pair of conditions hold*

$$C^0_S(\alpha, k_S, \frac{k_S - 1}{N}, c, e) \leq C^0_S(\alpha, k_S, \frac{k_S + 1}{N}, c, m) \quad \textit{and} \quad |\frac{k_S - 1}{N}| \neq 1$$

(ii) If $4(b)$ holds then (c, m) is the Pareto efficient equilibrium iff import sanction is stronger than export sanction, or the below pair of conditions hold

$$C^0_S(\alpha, k_S, \frac{k_S - 1}{N}, c, e) > C^0_S(\alpha, k_S, \frac{k_S + 1}{N}, c, m) \quad \textit{and} \quad |\frac{k_S + 1}{N}| \neq 1$$

The fraction $\frac{k}{N}$ carries two features of any node in the network - the net-value of links k (whether T is a net-exporter or net-importer) and the total number of out-links and in-links n. C^0_T increases in the absolute value of $\frac{k}{N}$ which entails that if T has mostly out-links then imposing an export sanction (deletion of in-link) will be more costly than imposing an import sanction (deletion of out-link). Similarly if T has mostly in-links then severing an out-link by S (import sanction) is more costly than export sanction.

Proposition 5. *In presence of relatively lower levels of geopolitical rivalry (g is either linear or concave), strong sanctions be imposed by S which will be obeyed by T[7]. As geopolitical tensions increases, g becomes more convex, only any weak sanction is likely to succeed. The sufficient*

[7] See appendix section 7.3 for the relevant technical sufficiency condition

conditions for a strong sanction to succeed is to make T a complete exporter or complete importer (in limiting sense).

When geopolitical motives are rather weak, T after losing some strategic value might comply even when S suffers no loss or relatively less loss in its own strategic value. However, when T carries rivalry on diplomatic grounds, then T would refuse to budge to the demands of S, unless S suffers a similar or greater loss in terms of centrality value.

6 Reputation concerns by target

The interesting notion of non-compliance to sanctions can be explained via reputation concerns of the target. Compliance to sender's demands can send a signal that the target is vulnerable to sanctions. In a network setup, this action could send a potential signal of the target's vulnerability to the other players in the network. Therefore, present compliance to sanctions can mean future incidence of greater sanctions. So, even though these reputation concerns are futuristic, it acts as a strong deterrent towards sanction compliance in the present date.

The aim of this section is to assess the role of centrality in the target's assessment of reputation. If a target is central enough (say, the position of Ukraine countering Russian sanctions), implying a major share of links pass through it, then sanction is unlikely to alter the target's behavior. Nevertheless, target can alter behavior if the sanction reduces it's centrality to a great extent, in essence, the sanction is strong by. Intuitively the target

7 Conclusion

This discussion has focused on the following question - is the type of sanction levied by a sender against a target a function of the network architecture. Of the various facets of architecture lies the concept of node centrality which measure importance of nodes in different ways. I use the betweenness centrality for this study. Why does this measure matter in examining trade sanctions? Well, a higher value of betweenness centrality of a player means that it holds a key strategic position in bridging trade ties with other nations in the most cost effective way. In simple terms, the player acts as a vital intermediary for majority players in side the network. Hence, issuing sanctions, which consists of link deletion, can lead to heavy compromise of both sender and target. This is also germane to the trade literature which shows strong evidences of how the value added by intermediate trading partners have increasingly surged up over the years.

In essence, the broad question which has been answered here is the following - what network structure supports import sanctions and which ones support export sanctions, if one assumes that the players involved in sanctions take actions based on the value of this measure of centrality. I present certain features in networks which gives force to specific types of sanctions and render other sanction either non-effective or pyrrhic. The consideration of strategic positions while deciding which type of sanction to impose is a new way of examining sanction efficacy. As an application, this analysis lends insights to the strength of sanctions when players are concerned only with trade-gains and when they account for payoffs from geopolitical supremacy alongside trade-gains. This model shows that the presence of such geopolitical motives weakens the power of sanctions and might lead to a Pareto inferior outcome.

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8 Appendix

8.1 Strong sanctions

(i) **Type-S1**: if $U_S(g) > U_T(g)$ and $U_S(g') > U_T(g')$.

This type considers the case when the initial value of b-centrality of S is greater than T in g and g' (both pre-sanction and post sanction).

(ii) **Type-S2**: if $U_S(g) = U_T(g)$ and $U_S(g') > U_T(g')$.

This considers that S and T were equally central (in terms of b-centrality) in g, but following sanction, T loses a portion of its centrality value.

The below 3 types describe a situation where a less central S sanctions T to produce the following three outcomes: T still remains more central than S in the sanctioned network g' (S3), S and T are equally central in g' (S4) and finally S5 which erodes T's centrality such that T becomes less central than S in g'.

(iii) **Type-S3**: if $U_S(g) < U_T(g)$ and $U_S(g') < U_T(g')$.

(iv) **Type-S4**: if $U_S(g) < U_T(g)$ and $U_S(g') = U_T(g')$.

(v) **Type-S5**: if $U_S(g) < U_T(g)$ and $U_S(g') > U_T(g')$.

8.2 Weak sanction

(i) **Type-W0**: if $U_T(g) = U_T(g')$

This defines ineffective sanctions where the centrality of T is not affected.

The below two types sum up situations where a more central sender compared to target by imposing sanctions lowers its own centrality more than the target (W1) or lowers it to a point where in the sanctioned network, S's is as central as T (W2).

(ii) **Type-W1.1**: if $U_S(g) > U_T(g)$ and $U_S(g') < U_T(g')$.

Type-W1.2: if $U_S(g) > U_T(g)$ and $U_S(g') = U_T(g')$ where $U_T(g) = U_T(g')$.

Type-W1.3: if $U_S(g) > U_T(g)$ and $U_S(g') = U_T(g')$ where $U_T(g) > U_T(g')$.

If in pre-sanction network, both S and T are equally central, then the event of a sanction can lead to the below three outcomes: S's centrality is compromised more than T's (W3); S and T remain equally central (W4) and S becomes more central than T post sanctions.

(iii) **Type-W2.1**: if $U_S(g) = U_T(g)$ and $U_S(g') < U_T(g')$.

Type-W2.2: if $U_S(g) = U_T(g)$ and $U_S(g') = U_T(g')$.

Type-W2.3: if $U_S(g) = U_T(g)$ and $U_S(g') > U_T(g')$.

The below three cases depicts a less central S sanctioning a relatively more central T. $W6$ accounts for

(iv) **Type-W3.1**: if $U_S(g) < U_T(g)$ and $U_S(g') < U_T(g')$ and $|U_S(g') - U_S(g)| > |U_T(g') - U_T(g)|$ holds.

Type-W3.2: if $U_S(g) < U_T(g)$ and $U_S(g') < U_T(g')$ and $|U_S(g') - U_S(g)| < U_T(g') - U_T(g)|$ holds.

(v) **Type-W4.1**: if $U_S(g) < U_T(g)$ and $U_S(g') = U_T(g')$ and $|U_S(g') - U_S(g)| > |U_T(g') - U_T(g)|$ holds.

Type-W4.2: if $U_S(g) < U_T(g)$ and $U_S(g') = U_T(g')$ and $|U_S(g') - U_S(g)| < U_T(g') - U_T(g)|$ holds.

(vi) **Type-W5**: if $U_S(g) < U_T(g)$, $U_S(g') > U_T(g')$.

8.3 Proof of Proposition 5

If S is a net-importer then severing an import link still preserves the geodesic between k and m which includes S ($S \notin \{k, m\}$). This keeps the centrality unchanged for both S and T

While, severing an export link makes S a gross importer, thereby implying that no geodesic between k and m can contain S. This leads to a loss in centrality for both S and T.

Some level of complexity enters when we consider acyclic networks. When g is acyclic, betweenness centrality might be lower or higher for players compared to when g is cyclic. If S and T are both net-importers then S will unambiguously gain more by imposing an import link than imposing an export one. Since S is a net-importer, there exists a third player from whom S has an import link. Therefore, acting as a link between any path emanating from k to another player $m \notin \{S\}$ necessitates that S maintains its out-degree or export link with T. This entails that S will not severe its export link with T. If T in also a net importer, then by $A2$, T has an import link with another third player $l \notin \{S\}$ who reaches to some other player in g through T. To create pressure on T, then S must impose an import sanction such that the geodesic from l cannot go beyond T. Then based on S's position and T's position in the network, an import link must be imposed by T

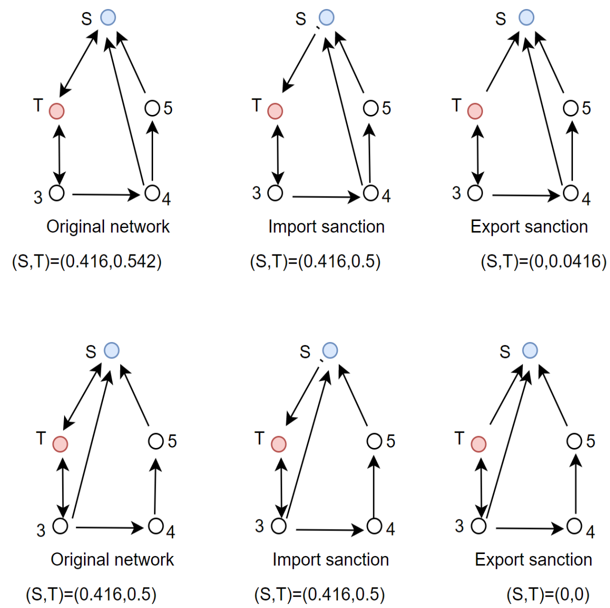

Figure 10: This 5-player network illustrates the effect of sanctions when S is a net-importer while T is balanced (statement (ii) of proposition 3). In the top row, S is part of a cyclic component of size 4 while in the bottom row, the same component is of size 3. In the former, import sanction is $S3$ (centrality of S is unchanged, but remains lower than T even post sanction) while export is $W3.2$ (centrality of T decreases more than S). In the bottem row, import sanction is ineffective or $W0$ (centrality of S and T remains unchanged. Export sanction is $W4.1$.

or T cuts its import link. Hence centrality of S remains unchanged while that of T falls. Hence sanction is $S4$.

8.4 5

For any n player network g, consider two components $\mathbb{C}_1$ and $\mathbb{C}_2$ of g such that $\mathbb{C}_1(g) \cap \mathbb{C}_2(g) = \{S, j\}$ and $\mathbb{C}_1(g) \cup \mathbb{C}_2(g) = g$. Without generality loss, I assume $S \in \mathbb{C}_1$ where $\frac{n}{2} < ||\mathbb{C}_1|| = n_1 < n$ and $||\mathbb{C}_2|| = n_2 < \frac{n}{2}$.

- Consider both $T, S \in \mathbb{C}_1$ and $\mathbb{C}_1$ is a cycle with bilateral links between any pair of players. Then import or export sanction by S against T reduces $\frac{n}{2} - 1$ geodesics that pass within $\mathbb{C}_1$ through S or T. So the value of b-centrality of both S and T are proportionately reduced by $\frac{n}{2} - 1$.

  How does the geodesics from players in $\mathbb{C}_2$ to players in $\mathbb{C}_1$ and vice-versa affect b-centrality of S and T, both of which belong to $\mathbb{C}_1$?

  If there exist a geodesic lk between $l \in \mathbb{C}_2$ to $k \in \mathbb{C}_1$ ($k \neq S$) such that the link kS of geodesic lk precedes the link ST or TS[8]. Then, export sanction by S is relatively stronger than import sanction, given there are initially all bilateral links between players in $\mathbb{C}_1$.

[8]Given the directed nature of the network, the ordering of the links are non-trivial here

8.5 Proof to proposition 7

The following equations assure that T's payoffs are higher when it complies to export sanctions and violates import sanction. I now check how strong binding these conditions are given the loss in initial centrality score ($\beta_T - \beta_T(e)$ and $\beta_T - \beta_T(m)$) following export and import sanctions respectively.

$$\beta_T(m) - \beta'_T(m) < 2P_0 + C^0_T(\alpha_T, \frac{k}{N}, \frac{k-1}{N}, m, c) - C^0_T(\alpha_T, \frac{k}{N}, \frac{k-1}{N}, m, v)$$

or,

$$\beta_T(m) - \beta'_T(m) < 2P_0 - C^0_T(\alpha_T, \frac{k}{N}, \frac{k-1}{N}, m, c)\Big[\mu - 1\Big] \tag{18}$$

$$\beta_T(e) - \beta'_T(e) > 2P_0 + C^0_T(\alpha_T, \frac{k}{N}, \frac{k+1}{N}, e, c) - C^0_T(\alpha_T, \frac{k}{N}, \frac{k+1}{N}, e, v)$$

or,

$$\beta_T(e) - \beta'_T(e) > 2P_0 - C^0_T(\alpha_T, \frac{k}{N}, \frac{k+1}{N}, e, c)\Big[\mu - 1\Big] \tag{19}$$

For larger values of k and n, the cost of sanctions become less convex with the deletion of out-link or in-link. The bindedness of 17 gains force either of the two ways or both- if $\beta_T - \beta'_T(e)$ is large enough positive or if C^0_T is large enough. The latter condition is weakened if T has enough in-links to compensate for the loss. In essence, T can resort to its other partners for importing goods.

The satisfaction of the bindedness condition can be either weakened or strengthened due to the inclusion of the geopolitical factor in the compatibility equations which are shown below.

$$y.\Big[\beta_T - \beta'_T(m)\Big] + (1-y)\Big[g(\beta_T - \beta_S) - g(\beta'_T(m) - \beta'_S(m))\Big] < 2P_0 - C^0_T(\alpha_T, \frac{k}{N}, \frac{k-1}{N}, e, c)\Big[\mu - 1\Big] \tag{20}$$

$$y.\Big[\beta_T(e) - \beta'_T(e)\Big] + (1-y)\Big[g(\beta_T - \beta_S) - g(\beta'_T(e) - \beta'_S(e))\Big] > 2P_0 - C^0_T(\alpha_T, \frac{k}{N}, \frac{k+1}{N}, e, c)\Big[\mu - 1\Big] \tag{21}$$

It is evident that the role of $\frac{k}{N}$ is depleted for $(1-y) > y$, that is when players are focused more on geopolitical gains which might not be aligned to the gains from trade.

8.6 Trade and geopolitical incentives

If players have trade and geopolitical incentives, then the following inequalities ensures that T complies before an export sanction but violates an import sanction. This is similar to equations 6 and 7, but includes the geopolitical motives in its payoff function.

$$\begin{aligned} y[\beta_T(m) - \beta'_T(m)] + (1-y)[g(\beta_T - \beta_S) - g(\beta'_T(m) - \beta'_S(m)] < 2P_0 + \\ C_T^0(\alpha_T, \frac{k}{N}, \frac{k-1}{N}, m, c) - C_T^0(\alpha_T, \frac{k}{N}, \frac{k-1}{N}, m, v) \end{aligned} \tag{22}$$

$$\begin{aligned} y[\beta_T(e) - \beta'_T(e)] + (1-y)[g(\beta_T - \beta_S) - g(\beta'_T(e) - \beta'_S(e)] > 2P_0 + \\ C_T^0(\alpha_T, \frac{k}{N}, \frac{k+1}{N}, e, c) - C_T^0(\alpha_T, \frac{k}{N}, \frac{k+1}{N}, e, v) \end{aligned} \tag{23}$$