

Media bias in the best and worst of times

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Abstract

I examine the influence of supply and demand forces on the reporting choices of a left and right leaning media firm inside a news market. The supply (demand) forces refer to the influence of the ideology preference of media (reader) on news report about a given issue. The issue can be either in favor of the left or right ideology. Media firms strategically locate on an ideology axis to report this issue. This axis also represents the entire ideology spectrum of readers, ranging from leftist to rightist. The model provides a characterization of reader's response to news which helps to show when leftist readers would prefer news of a rightist media and vice versa. I show that the relative strength of the supply and demand forces on each media firm governs to what degree can each differentiate their news reports in terms of ideology. As the strength of the supply shifts towards demand, media profits can change either monotonically or non-monotonically, depending on whether the issue supports the media. Specific sufficiency conditions are identified where both media's reporting choices can either be complements or substitutes. When one moves away from the benchmark model to a monopoly news market, news bias increases unambiguously while introducing more media firms have ambiguous effects on the level of news bias.

Key words: Hotelling's model, news bias, partisan media, strategic complementarity, strategic substitutability

JEL codes: C7,D72,L12,L13,L82

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1 Introduction

Mainstream media is a vital institution in any democracy which gathers and disseminates information from all spheres of social and political life to the public¹. Democracies sustain the course of various negative and positive events like higher unemployment or higher growth respectively by depending significantly on how media outlets describe these events. Media firms bias news in various forms and exercise great power in not only establishing public opinion about the event, but also impedes its readers from construing the event in alternative ways Ansolabehere, Behr and Iyengar (1993).

This paper pays attention to the two main channels which generate news bias -supply and demand - and how these shape up the news diversity in any news economy. The supply channel implies that the bias in news arises from partisan interests of media firms while the demand channel ideologically biased readers. The compounding effects of both these channels are responsible for the commercial makeup of media news (Hamilton, 2011), which provides a good reason to understand its underlying industrial organizational aspect.

This paper studies a linear city model of media competition between two ideologically opposite media firms to study the effects of the supply and demand channels of news bias. Any issue or event like inflation, healthcare, immigration carries a ideologic symbol which triggers associations within a reader based on his ideology or prior knowledge (Rabinowitz and Macdonald (1989)). When a reader reads about these issues from partisan media, these associations alongside media's stance on these issues generate reader evaluation of news. Therefore, the effect of supply or demand channels on the level of news differentiation across both media outlets and their profit levels is driven by the nature of issue and whether readers assess news based on only ideology or prior information or both.

Bias in news have gathered reader criticisms who feel news reports are biased against their beliefs. Liberals complain of conservative bias in news while conservative readers feel the opposite². The past literature of Behavioral Economics and Communication Theory deem such differences to be the manifestations of subjective perceptions ³ of the reader. However, there also exist readers who complain about the compromise in objectivity in news reports with reference to factual information. For instance, not all readers have prior knowledge of the current unemployment statistics from reports by the Bureau of Labor Statistics (BLS). Therefore, there exists differences in news

¹The survey of Smith and Lichter (1997) shows 82% of the participants believed that media must be the foremost news provider. In addition, 75% strongly assert media to take the role of watchdogs on public officials to curb their intentions to abuse power.

²See Gentzkow and Shapiro (2010); Beder (2004); Anand, Di Tella and Galetovic (2007)

³See Bénabou and Tirole (2016) who explains this behavior though motivated reasoning. Vallone, Ross and Lepper (1985), Jerit and Barabas (2012), Ho et al. (2011) provides experimental or empirical evidences of such biased perception of information about events spanning from conflicts, inflation and even stem cell research.

evaluation between a reader who knows the BLS numbers and someone who does not. This paper emphasizes these differences and examines how media tries to account for all reader types alongside satisfying its own partisan interests while it covers any news story.

I contribute to the existing literature in primarily two ways. The first relates to modelling reader perception of news. I develop a simple measure of news evaluation which breaks down readers evaluation of news based on spatial and directional aspects. News perception proceeds in two stages. In the first stage, a reader directionally compares his ideology with the ideological tone of the event. For instance, a liberal and a conservative reader will have opposite mental association towards a pro-immigration policy. In the next stage, readers assess the news of this policy based on how proximate (spatially) he is from the media firms' location choice. In this model, a fraction of readers have access to factual information before they read from partisan media. This implies their news evaluation news will be based on both spatial and directional forces. On the other hand, readers who do not have access to the factual information evaluates news based on how close the news is to his ideology. This initiates different tradeoff between accuracy and desirability between these two reader types⁴. This asymmetry in information also governs policy outcomes when readers use media news to make decisions regarding policy choices. Secondly, this model throws light on how the news economy is impacted by the demand and supply channels of bias. The benchmark model considers two ideologically opposite media channel who vary exogenously on two fronts-their ideology and the relative weights assigned to the demand and supply channels of news bias. Hence, the degree of news bias can be compared by varying the weight distribution on the two channels. This exercise not only explains the condition when news will be more differentiated across the two outlets, but also how it affects the strategic substitutability and complementarity between the competing media outlets.

The present model begins with an honest information source which provides the reader populace with a factual report about a topic. Examples of such sources include press releases of the Federal Reserve, Bureau of Labor Statistics (BLS), Reuters who are known to ‘.. represent the essence of objective news coverage, as they self-consciously avoid politically based editorial judgments in their news content’, Baum and Groeling (2008). A fraction of the reader populace learns about this factual report. Let us assume that the Federal Reserve has released a 1% hike in interest rates which is received by a fraction of the public. Following this, two ideologically opposite media firms (left and right) locate themselves on a linear ideology axis to report their stance on the topic. Media's profit is a convex combination of payoffs from ideology and reader evaluation with a particular weight⁵. The left biased media can defend this hike by highlighting the broader

⁴See Bénabou and Tirole (2016)

⁵If the weight on ideology is greater than reader evaluation, then media is more concerned about its own partisan interests and bias originates from the supply side. Analogously, when the weight on reader payoffs is relatively higher than ideology payoffs, bias is more demand oriented.

goal of the Fed in checking inflation. The right biased media can attack this hike in rates by relating it to lower forecasts of economic growth.⁶ Readers are ideologically heterogeneous and will evaluate both these news stories based on two aspects - how strongly they support or oppose the Fed's function of checking inflation by increasing interest rates (directional aspect) and how close are the location choices of both media from his ideology (spatial aspect).

The insights I get from the model is as follows. First, the relative profit motives of the two firms govern the strategic interaction between the two media outlets. This entails that if news bias emerges from supply side from one firm (where the firm places greater weight on ideology payoffs) then it will locate differently while competing with a media which generates news bias through the demand side (greater weight on reader evaluation of news) and supply side. Second, for particular topics and certain parameter values, not biasing news leads to lower profits. Third, for any event (ideologically unfavorable or favourable), media's profits are lower when it generate bias through both the supply and demand channels (for instance when ideology and reader evaluation receive equal weights). For ideologically unfavourable issues, comparative statics show that the equilibrium profits is non-monotonic when weights shift from ideology payoffs towards payoffs from reader evaluation. Fourth, a novel measure of reader-satisfaction is provided which reflects that readers can gain relatively more utility from news whose location is farther away from their ideology than one which lies closer. Fifth, the weights on the supply and demand channels also govern the degree of strategic complementarity and substitutability between the two firms. Sixth, media enjoys greater leeway to bias news in its favor when the audience is more unsophisticated, who are less educated and tolerates bias⁷. However, the impact on bias from higher reader unsophistication gathers force when both the media firms are more focused towards ideology gains than gains from reader-assessment. Sixth, welfare is not necessarily enhanced in presence of media firms which care more for reader-assessment. Welfare is dependent on the number of readers in the economy and how they are spread across the ideology spectrum.

Before proceeding with my model, I briefly layout the main forms of media-bias. Following Puglisi and Snyder Jr (2011), news bias by a partisan media mainly occurs in three forms - selective reporting (reporting on strongly partisan topics); issue framing (how an event is portrayed by reporters)⁸ and 'agenda setting' (determined by amount of coverage on each incident). In the present setup, bias takes the form of issue-framing and is generated by both demand (reader-assessment) and supply (ideology of media firm) factors. Since competition between media is spatial, the placement of news is a single point on the ideology spectrum which represents the bottom line or a

⁶Groseclose and Milyo (2005) discusses how partisan media bias inflation statistics based on which party is the incumbent.

⁷Prat (2018) discusses a measure of media power which gains force as reader sophistication tends to zero.

⁸ (Mullainathan and Shleifer, 2002) dissects these two as 'bias' and 'spin', the former being in context of traditional left-right ideology while the latter helps to create a memorable story

condensed form of the event. The location of this point then signifies how close a particular media has chosen to be to the left or right ideology.

In later sections, the results of the benchmark model of duopoly competition is compared with respect to three settings - monopoly media (absence of competition), more polarized reader distribution (for instance, when majority readers are biased to the left or to the right) and a market with three firms. I find equilibrium bias increases in the monopoly setup, due to absence of any competition. In the three-media case, if the event to be reported is has no ideology bearings, then equilibrium media bias rises above the duopoly bias level only if the third firm is ideologically biased. I then analyse welfare by aggregating reader utility and media firms payoffs. I find welfare to depend crucially on the nature of topic to be reported, the weights assigned by a media to its twin motive - ideology and reader-assessment and reader polarization.

The remaining of the paper proceeds in the following manner: section 2 the related theoretical and empirical literature. Section 3 introduces the model preliminaries; the game timeline has been laid out in section 4; section 5 examines the duopoly competition; section 6 analyses media bias in presence of a more polarized reader pool; section 7 provides a brief insight into the outcomes when a third media enters the duopoly market and section 8 presents the welfare analysis.

2 Related Literature

This paper fits in the literature of industrial organization of news market where biasing information to report about the real world is viewed as a model of product differentiation. In a spatial model, this differentiation is basically a product placement problem. Contingent on the ideological tone of an issues, firms either locate close or far away from each other which is governed by the forces of the supply and the demand channels. The particular attention paid to the role of supply and demand channels has lead to insights which complement or add to the existing theoretical literature surrounding media bias.

The existing theoretical literature which closely relates to our model consists of Chan and Suen (2008), Bernhardt, Krasa and Polborn (2008) (BKP) and Duggan and Martinelli (2011)). Chan and Suen (2008) studies candidate endorsements by newspapers; Bernhardt, Krasa and Polborn (2008) discusses news suppression by media firms which lead to electoral mistakes; Duggan and Martinelli (2011) studies a novel technology of slant by media firms where citizens use media information to learn about others actions before they cast their vote for an incumbent or a challenger. Like the current paper, media bias is endogenous in BKP who studies the incidence of electoral mistakes by the citizens whose chances increases with asymmetric distribution of ideologies. In my model of news economy, I show that electoral mistakes can be committed based on the information asymmetries and how readers evaluate news. BKP also shows that media firms find it

profitable to bias news amidst strong competition. I show that the profit from biasing is dependent on the ideology tone of the issue and the relative weights on the supply and demand channels of each media. For instance, I find that for ideologically neutral events, biasing news become more profitable when the weight shifts from the demand to the supply side.

Models which studies media bias through supply-side factors includes the seminal paper by (Gentzkow and Shapiro, 2006). However, here media reputation is the driver of editorial choices. However, in their model, there exists chances of truth revelation which disciplines media and prevents it from biasing news. Mullainathan and Shleifer (2005) studies the demand-side sources of news bias where media caters to the ideological predispositions of readers who prefer confirmatory news.

The structural aspects of Mullainathan and Shleifer (2002) closely resonates with this model where readers learn about the issue before reading media reports. However, both differ in other underlying model assumptions and the nature of news provision. A finding common to both is the information slant by media about reports on events with no ideology. They argue that the bias exist through the channel of ‘spin’ which creates a memorable story whereas my argument depends on a result following Hotelling’s lemma, where each partisan media outlet segment the economy and bias the ideology-free event to cater to their like-minded readers. In addition, this model offers an added insight which can be explained through the following example. Consider two scenarios - *A*, where both media firms refuse to compromise with their partisan interests and *B* where both are relatively flexible about adjusting partisan priorities to satisfy readers. Then while reporting a neutral incident, media firms in *A* will earn relatively higher equilibrium profits than the ones in *B*. Apart from this, I add to the literature, by providing a formalized way to detect when media firms will speak indifferently and when they will not.

Nevertheless, among these papers we find support of the model premises. The manner in which we define bias matches with Duggan and Martinelli (2011), D’Alessio and Allen (2000) or Mullainathan and Shleifer (2002). This definition directs us to a specific strand of works within the spatial product-placement literature of Anderson and McLaren (2012), Chan and Suen (2008) and Bernhardt, Krasa and Polborn (2008). These papers however work at the conjunction of media bias and its extensions in various political and electoral environments.

2.1 Empirical Implications

The theoretical settings as well as some findings are consistent with empirical papers analysing news bias through secondary data, surveys and experimental findings. But firstly, it is important to gain empirical support of the model premises. Firstly, I assume media’s profit as a function of partisan gains and gains from reader assessments or evaluations. Such design of media payoffs

find support in the supply-side estimation of Gentzkow and Shapiro (2010) where media's slant responds to customer ideology and their owner's type. Further empirical support behind partisan motivations of media are established in Budak, Goel and Rao (2016). The demand-side estimations from Gentzkow and Shapiro (2010) strengthens the importance of reader evaluations where they find that consumers try to match their own ideology with the media's slant. The latter assumption finds further support in (Iyengar et al., 1984) whose experimental findings suggest that reader evaluations based on media news are indeed instrumental in the ambit of political consequences. The point where we deviate from the above works is the assumption that readers and media already know the reality through fact-based reporting. Since readers are ideological and even deny facts which contradict their own beliefs, it creates an additional market for partisan media to send their version of reality. Surveys by *Laura Silver* (2021) shows how public are averse towards undesirable facts and become resistant towards accepting any information which advocates it. Similarly *Mitchellet al.* (2019) shows the prevalent lack of trust of news readers towards media belonging to opposite ideology groups.

Empirical evidences of these are found in Iyengar et al. (1984) and Higgins, Rholes and Jones (1977). These papers state that the presence of such coverage not only provokes readers to recollect memories on a previous event, but also plants an initial comprehension of the same in the reader's mind, based on which he forms his evaluations regarding the current news. The prior information in essence leads the reader to judge information differently in context to his initial comprehension.

The structure of media bias as a product differentiation model is found in Hamilton (2011) who states that readers and media firms can be mapped on an ideology spectrum and readers deem a media as biased depending on how far it is from his ideology. However, the current paper adds the role of a factual report which makes the reader match a news story with his own ideology as well as with the factual report. The concept of reader-assessments of news is also found in Hamilton (2011) (p. 74) where it is stated that an economy comprised of mostly liberal public will deem the news of more liberal-oriented news as not biased. This is one of the results in section 6 which analyses the behavior of partisan media in a biased reader pool. The effect of biased reader pool on the degree of media slant is also found in Gentzkow and Shapiro (2010) who shows a statistically significant rise in slant in presence of like-minded public. However, my analysis also provides the way readers perceive such news which ultimately affects media profits. We show that strongly idealistic readers may deem a news story of their like-minded news channel as unsatisfactory which might be due to the absence of enough ideology slant.

The model also sheds light on the tendency of media to take indifferent stances on a range of issues has been supported by anecdotal evidences which has been cited in proposition 3. The theoretical observation that under certain conditions, a reader may prefer news from an ideologically-opposite media channel only if the incident supports their ideology is found in the experimental

evidences of (Kuklinski and Hurley, 1994). If the incident seems harmful towards their beliefs, then readers prefer their like-minded channel.

3 Model

This fundamental model is akin to the linear city model where partisan media at th two extreme ends provide new to uniformly distributed readers by choosing their respective locations. In the baseline case, there are two partisan media firms $j \in \{L, R\}$ at opposite ends which signifies their ideological rivalry. The entire game spans across three periods. In the first period, readers receive an exogenous factual report by a honest media E about a particular event ω belonging to the universe $\Omega = [-1, 1]$. The probability of reception of this factual report by any reader i is $p > 0.5$. The point -1 represents an incident which aligns to the extreme left ideology while 1 denotes an event with extreme rightist ideology. Any intermediate points are relatively moderate and the midpoint 0 is absolutely neutral.

The above interval also represents the linear city where $N \in \mathbb{N}$ readers are uniformly ⁹ placed and their location denotes their subjective ideological leanings. A reader i 's position is denoted as x_i on $[-1, 1]$. The neutral (or moderate) reader is positioned at 0 while the extreme leftist (rightist) reader is placed at -1 (1) as shown in Figure 1. Readers are rational and are aware of the partisan interests of media.

The utility of a reader i is additively separable ¹⁰ across news of media $j \in \{L, R\}$. The utility of a reader if he receives the factual report is

$$U_{ij} = -(\alpha_{ij}\theta_j - x_i)^2 - (\alpha_{ij}\theta_j - \theta_E)^2 \quad (1)$$

If he does not receive the factual report,

$$U_{ij} = -(\alpha_{ij}\theta_j - x_i)^2 \quad (2)$$

The action of i is to choose $\alpha_{ij} \in \mathbb{R}$ which denotes his assessment or weight of the news story by media j . Intuitively, this is a measure of the degree of satisfaction from a news story. This assessment is therefore a mapping $\alpha_{ij} : x_i \times \theta_E \times \theta_j \rightarrow \mathbb{R}$ when i has knowledge of θ_E . When he has no knowledge of θ_E , $\alpha_{ij} : x_i \times \theta_j \rightarrow \mathbb{R}$.

Media firms are located at the extremes. This is akin to the concept of bliss point or where the partisan media firms ideally want to be. We parameterize this location by $\bar{\theta}_j \in [-1, 1]$. In the

⁹Cumulative mass function F and probability mass function f .

¹⁰This convention has been used in Gentzkow and Shapiro (2010) with a more ordinal utility form, where a household's utility is additive in the number of newspapers chosen among the ones available within its zip code.

baseline model, $\bar{\theta}_L = -1$ and $\bar{\theta}_R = 1$. So my aim is to understand how information bias percolates into an economy when its news suppliers are inherently extreme partisans. One can do the same for other moderate values of $\bar{\theta}_j$ and examine the levels of information slant.

The payoff function of media j accounts for the action of its rival firm ($-j$), given the report of the honest media as shown below.

$$\Pi_j(\theta_j, \theta_{-j} | \theta_E) = -\lambda_j \cdot \left[\mathbb{E}(\alpha_j^*) - 1 \right]^2 - (1 - \lambda_j)(\theta_j - \bar{\theta}_j)^2 - C(\theta_j, \theta_E, b, \theta_{-j})$$

In the benchmark model, I consider media to report with perfect precision or zero variance which leads us to the following form of utility. This exercise is conducted to throw light on the marginal effects of noise on the equilibrium level of bias. This method as will be discussed gives a novel way of measuring demand of media reports based on its qualitative features.

$$\Pi_j(\theta_j, \theta_{-j} | \theta_E) = -\lambda_j \cdot (\alpha_j^* - 1)^2 - (1 - \lambda_j)(\theta_j - \bar{\theta}_j)^2 - c \frac{(\theta_j - \theta_E)^2}{b + (\theta_{-j} - \theta_E)^2} \quad (3)$$

The action of media j is a mapping θ_j where $\theta_j : \theta_{-j} \times \theta_E \rightarrow \mathbb{R}$ where θ_{-j} denotes strategy of the rival outlet. The first two terms depicts the trade-off between accuracy and ideology to media j respectively. Hence, media's payoff is a convex combination of these two factors with respective weights λ_j and $(1 - \lambda_j)$ where $\lambda_j \in (0, 1)$. The first term implies gaining better reader assessment¹¹ while the second term denotes the gain in ideology payoff by locating closer to its ideology bliss point $\bar{\theta}_j$. If λ_j is very closer to 1 then j places greater weight on reader satisfaction. On the contrary, when λ_j is closer to 0, media j weighs ideological gains more than reader satisfaction.

The final term denotes the cost function $C(\cdot)$ of j from biasing news which is basically the deviation of θ_j from θ_E . The marginal cost is $c > 1$. The parameter $b \in (0, 1)$ represents the sophistication within the readers. This has a cross-over effect of one firm's bias on its rival. Higher (lower) value of b implies lower (greater) cost of bias, given the level of bias of the other firm. $C(\cdot)$ has the following properties:

- (i). $\frac{dC}{d(\theta_j - \theta_E)^2} > 0$, firm j incur greater cost by biasing news.
- (ii). $\frac{dC}{d(\theta_{-j} - \theta_E)^2} < 0$, firm j faces lower cost from biasing when its rival firm biases news and vice versa.
- (iii). $\frac{dC}{db} < 0$, cost of bias decreases when level of reader un-sophistication increases.

I solve this duopoly game Γ_D of complete information using Subgame Perfect Nash Equilibrium (henceforth SPNE).

¹¹I explain this functional form more clearly using Lemma 1 in section 5.

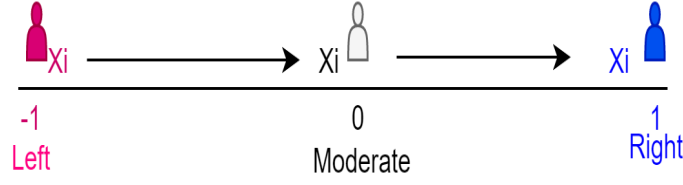


Figure 1: Location of readers

Definition 1. A strategy profile $s = \{\theta_L, \theta_R, < (\alpha_{1L}, \alpha_{1R}), \dots, (\alpha_{NL}, \alpha_{NR}) >\}$ of Γ_D is a subgame perfect Nash equilibrium (SPNE) if s induces a Nash equilibrium in every subgame of Γ_E . Nash Equilibrium of the duopoly game (Γ_D) between the media is a pair (θ_L^*, θ_R^*) of editorial choices for which θ_L^* is a best response to θ_R^* and θ_R^* is a best response to θ_L^* .

4 Timeline of game

Figure 2 illustrates the timeline of this model which begins with a naturally occurring event ω , through some exogenous random process. An unbiased media E sends a factual public report θ_E which becomes available to the partisan media with certainty, but readers learn about it with probability $p > 0$. Mullainathan and Shleifer (2002) refers this as a signal ‘ r ’ which sets a prejudice within a reader before he reads the news. Readers who receive the news process the issue as either sharing their ideology or contradicting it (as referred the directional aspect). The partisan media firms L and R observe the event and θ_E and designs its own report (reflected by its editorial positions θ_L and θ_R respectively) for the readers in the next stage. Readers are heterogeneous and rational and they cannot observe the true event *prima facie* but has access to the news of E . After the partisan media publishes the report, readers assess its report and provides a rating (α_L to L ; α_R to R) which measures the report’s consistency with E ’s report and their subjective ideology. These ratings can act as instruments to measure the unrest or ecstasy among the readers about any particular event. I study the editorial decision of partisan media through a simple backward-induction game in a duopoly media market. Media firms L and R compete over ideology and reader evaluation along a spatial Hotelling’s axis which measures ideology.

5 Duopoly Model

I consider the duopoly media market with firms L and R . Then the corresponding normal form

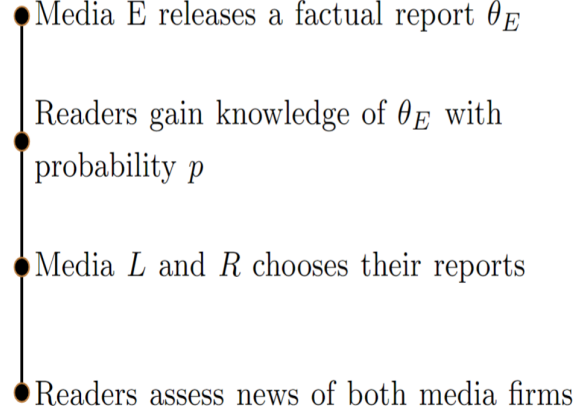


Figure 2: Timeline of the duopoly game

game of this duopoly case is defined as

$$\Gamma_D = [I, \{u_i(\cdot)\}, \{\Pi_L(\cdot)\}, \{\Pi_R(\cdot)\}]$$

I denotes the player set comprising of media L and R and reader $i \in \{1, \dots, n\}$. u_i is the utility of reader i from reading news and Π_L and Π_R denotes the profits of media L and R . Thereby the equilibrium strategy profile constituting the SPNE is characterized as $s^* = (\theta_L^*, \theta_R^*, \alpha_{iL}^*(\theta_L^*), \alpha_{iR}^*(\theta_R^*))$.

5.1 Utility maximization of reader

First order condition following equation 1 and 2 leads to the optimal assessment (weight) given by reader i towards media j 's editorial position θ_j

$$\mathbb{E}(\alpha_{ij}^*) = \frac{2x_i + p(\theta_E - x_i)}{2\theta_j} \quad (4)$$

In presence of no noise in news report and perfect reception of the factual report within the reader populace ($p = 1$), the above expression reduces to the following lemma.

Lemma 1. *The first best evaluation by reader i reading news of media $j \in \{L, R\}$ is achieved when $\theta_j = \theta_E = x_i$, or*

$$\mathbb{E}(\alpha_j^*) = 1$$

The rating of 1 suggests that media j 's editorial position matches both media E 's position θ_E and the ideology x_i on $[-1, 1]$ in tandem. Intuitively, if $\theta_j = \theta_E = x_i$ then not only does i perceive j to be as honest and accurate as E , but also can relate it perfectly with his own ideology x_i . Hence this news is perfectly cohesive with his rational self.

The first term in the profit function of j is a distance function which accounts for the loss of reader satisfaction from a piece of news which cannot be assigned this first-best weight.

Borrowing equation 3, the expected rating from N readers of j is given as

$$E(\alpha_j^*) = \sum_{i=1}^N \frac{2x_i + p(\theta_E - x_i)}{2\theta_j} \cdot f(x_i) = \sum_{i=1}^N \frac{2x_i + p(\theta_E - x_i)}{2\theta_j} \cdot \frac{1}{N} = \frac{p \cdot \theta_E}{2\theta_j} \quad (5)$$

The above result arrives from the assumption that readers are distributed such that mass of leftist and rightist readers are equal, hence they offset each other ($\sum_{i=1}^{i=N} x_i = 0 \quad \forall i \in \{1, \dots, N\}$). $\sum_{i=1}^{i=N} x_i \neq 0$ implies more polarised readers such that the distribution of readers f is such that the mass of leftist readers either greater or lesser than their rightist counterparts. If $\sum_{i=1}^{i=N} x_i \leq 0$, ($\sum_{i=1}^{i=N} x_i \geq 0$) the economy has a leftist (rightist) majority. The impact of such an unbalanced reader base on the editorial positions has been explored in section 6.

5.2 Payoff maximization of media

The optimal action of media j is directed by the below first-order-condition

$$\begin{aligned} \frac{d\Pi_j}{d\theta_j} = & \theta_j^4 \left[(1 - \lambda_j) + \frac{c}{(b + (\theta_{-j} - \theta_E)^2)} \right] - \theta_j^3 \left[(1 - \lambda_j)\bar{\theta}_j - \frac{c\theta_E}{b + (\theta_{-j} - \theta_E)^2} \right] \\ & + 0.5p\lambda_j\theta_E\theta_j - 0.25p^2\lambda_j\theta_E^2 = 0 \end{aligned} \quad (6)$$

This represents the best response function of j to the action of its rival θ_{-j} . The equilibrium editorial choice(s) is attained at the intersection of these functions. To bring out the possible behavior traits of media, I limit the value of b to be above some threshold as stated in Assumption 1. It is only above a cutoff that the effects of media under this setup becomes pronounced enough for a deeper analysis.

Assumption 1. b is above a threshold level $b' \in (0, 1)$.

This threshold value can act as a direct measure of reader un-sophistication and finds support in the experimental findings of (Iyengar et al., 1984) who posits that experts are much less influenced by manipulations by media and have already established their own evaluations about a particular event. On the other hand novices are the vulnerable ones, totally non-immune to information manipulations by media. L faces much higher cost in the event when rival media R does not bias. As b increases, it allows L to bias news and insulates against any negative feedback from the public.



Figure 3: The blue (orange) segment denotes class of events which supports the mixed-strategy equilibrium of L (R) at a particular threshold of λ_L (λ_R). These thresholds are unique for every event $\theta_E \in [\bar{\theta}_E^R, \bar{\theta}_E^L]$. Events outside these area support unique equilibrium of for all values of λ_L or λ_R

This simultaneously weakens competition to publish more accurate information and exacerbates the level of information slant.

Before proceeding into the equilibrium properties, it must first be ensured that the above system of equations have at least one real root within the interval of interest i.e $[-1, 1]$. Given the quartic nature of equation 5, it is close to impossible to postulate an explicit solution for θ_j . However, using *Sturm's Theorem*, it is suggested that two real solutions exists in $[-1, 1]$, as proposed by *Lemma 2*. For any parameter values, each polynomial has two real roots within $(-1, 1)$, one positive and one negative. I provide detailed explanation about this rule in section 8.1 of the appendix.

Lemma 2. *There exists two distinct real roots (one positive, one negative) in $(-1, 1)$ of the best response function of each media.*

The following proposition (refer to figure 3) describes the conditions which support both pure and mixed strategy equilibrium. In equilibrium, the BR functions intersects providing the associated profit levels to each media firm.

Proposition 1. (i) **Unique equilibrium:** *There exists a unique equilibrium pair of strategies (θ_L^*, θ_R^*) any $\theta_E \in [-1, 1]$.*

(ii) **Multiple equilibrium:** *For the class of events $(0, \bar{\theta}_E^L]$ (blue interval in figure 3) supporting the right, there exists a threshold $\bar{\lambda}_L$ where L 's profit is equal by supporting the left or the right. Similarly, for left-favorable events in $[\bar{\theta}_E^R, 0)$ (orange interval in figure 3), equilibrium profit of R at $\lambda_R = \bar{\lambda}_R$ is equal when it either supports the right or the left.*

The first two statements can be understood with more clarity through figure 3. Events to the right of the blue interval support the right ideology strongly enough such that L always locates on the right of 0 for all values of λ_L . This is unique pure strategy equilibrium for both L and R . Symmetric results evolve for event to the left of the blue interval. Compared to this, events in the

$(\lambda_L, \lambda_R) \setminus \theta_E$	-1	0	1
(0.1,0.1)	(-0.986,-0.346)	(-0.417,0.417)	(0.346,0.986)
(0.1,0.5)	(-0.988,-0.509)	(-0.387,0.279)	(0.344,0.908)
(0.1,0.9)	(-0.989,-0.722)	(-0.365,0.07)	(0.331,0.748)
(0.5,0.1)	(-0.907,-0.344)	(-0.279,0.389)	(0.508,0.988)
(0.5,0.5)	(-0.919,-0.507)	(-0.258,0.258)	(0.506,0.92)
(0.5,0.9)	(-0.93,-0.719)	(-0.242,0.064)	(0.497,0.789)
(0.9,0.1)	(-0.748,-0.331)	(-0.071,0.366)	(0.722,0.989)
(0.9,0.5)	(-0.789,-0.497)	(-0.065,0.242)	(0.72,0.93)
(0.9,0.9)	(-0.827,-0.713)	(-0.06,0.06)	(0.712,0.827)

Table 1: The first column shows that when true state totally favors the left, then L speaks closer to the truth than R for all values of λ_R (in blue). Symmetric results hold for R (in red). As λ_L increases, L locates itself closer to the median reader at 0. The only symmetric equilibrium occurs when $\theta_E = 0$ and $\lambda_L = \lambda_R$ (shown in bold).

blue (orange) intervals favor the left (right) relatively with lower magnitude. Then, reporting in favor of the left for events in the blue interval is no longer binding for R unless when λ_R is high enough (greater weight on reader assessment). The model provides a cutoff $\bar{\lambda}_R$ which determines the equilibrium response of R . below which R will still speak in favor of the left. At the cutoff value, R is indifferent between speaking in favor of either ideology, hence leading to a mixed strategy equilibrium.

Media designing a report which extol their own ideology even in the face of a contradicting event follows (Baum and Groeling, 2009). The current model formalizes the sufficiency conditions where events contradicting a media's ideology will bind it to speak closer to the truth. Media j with value of λ_j greater than threshold speaks closer to the true events and does not jeopardize with reader-assessments while the ones below the threshold advocates more towards ideology motive, thereby publishing stories contradicting the true event.

Remark 1. Comparison of magnitude of editorial positions: *The class of events which strictly favors the left, $\theta_E \in [-1, 0)$, L chooses to locate closer to the event than R . Analogously, for events favoring the right, R chooses to locate closer to the event than L .*

This phenomenon is illustrated through table 1. Additionally, L and R locate symmetrically around zero when $\theta_E = 0$ (neutral event) and $\lambda_L = \lambda_R$ holds (shown in bold in table 1).

Remark 2. *Intuitively, the threshold value $\bar{\lambda}_j$ of λ_j is a measure of the extent to which L is willing to champion its ideology in presence of a contradicting reality.*

Proposition 2. *(i) There exists a reader in $[-1, 1]$ with ideology x_i to whom both news of L and R are redundant, and he assigns identical assessments to news of L or R . x_i can be uniquely solved*

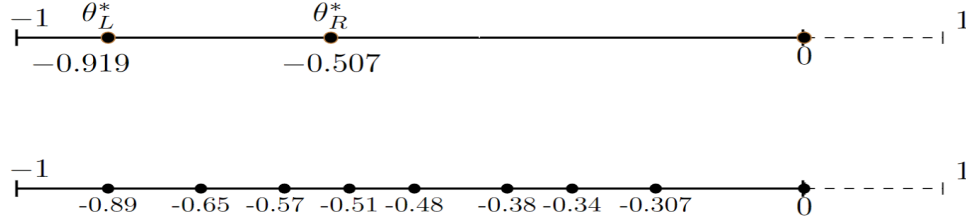


Figure 4: The top figure shows the equilibrium locations θ_L^* and θ_R^* of L and R respectively to report $\theta_E = -1$ when $(\lambda_L, \lambda_R) = (0.5, 0.5)$. The bottom figure segments the leftist readers into intervals. These intervals distinguish leftist readers based on how they evaluate news of L and R , given that a fraction of them have access to the factual report prior to reading the news. So a portion of readers evaluate news based on the factual report and their ideology while the rest evaluate it only based on ideology.

from the below identity when $\theta_L^* \neq -\theta_R^*$ ¹²

$$x_i(2 - p) + p\theta_E = 0$$

(ii) While reading news about an event which is (say) biased to the left ($\theta_E < 0$), there exists sufficient non-uniformity in new evaluation across the leftist readers when R reports accurately of the event (R 's reporting is more demand oriented). Else, if R chooses to report in favor of its own ideology (R 's reporting is more supply oriented), then leftist readers evaluates L 's news above of that of R , implying more uniformity in evaluation.

The first statement resonates the idea of any spatial model where one consumer's location is equidistant from either media outlets. However, here the directional aspect of the issue. If $p = 1$, then $x_i = -\theta_E$ which suggests that $\alpha_{ij} = 0$. For this reader, the issue is exactly directionally opposite to his ideology which makes either of the reports from L and R redundant.

The second statement reflects the idea that if the right media reports a pro-left event without challenging it, then a fraction of leftist readers will evaluate R 's report above L 's. There will also be a fraction of readers who will prefer news of L than R even when L is located farther away from them than R . These behavior will vary across readers depending on their learning about θ_E .

5.3 Choice of reporting neutrally

There occurs two broad scenarios where media j can report neutrally by locating closer to zero. First, when the true event is actually neutral and second, when the event is unfavourable to j 's ideology. The former indicates truthful reporting, while the latter can be termed as 'indifferent reporting', a form of biased news reporting, where the media is reluctant to speak in favor of the

¹²The outcome $\theta_L^* = -\theta_R^*$ is endogenously arrived iff $\lambda_L = \lambda_R = 1$ and are reporting a neutral event ($\theta_E = 0$). In this case, the median reader at 0 is indifferent between either outlets. We can exclude this case as λ_L and λ_R lies between $(0, 1)$.

rival ideology. However, as this model predicts from proposition 1, media does not want to sound indifferent even when faced with an ideologically ‘bad’ event.

For example, when the event favors the left (θ_E^1 in figure 5), then L does not position itself in the blue region. When L is more attached to its ideology ($\lambda_j \leq \hat{\lambda}_j$), it places itself on the left of the blue interval. On the other side of this cutoff, L places itself in the territory of the rightist readers, on the right of the blue interval. In essence, L avoids a more indifferent location (around zero)

Proposition 3. (i) *Segmented Equilibrium:* When $\theta_E = 0$, the equilibrium editorial choice of j is given by

$$\theta_j^* = \frac{(1 - \lambda_j)\bar{\theta}_j}{(1 - \lambda_j) + \frac{c}{b + (\theta_j^*)^2}}$$

(ii) *Non-indifferent reporting:* Given any unfavourable event, media j either supports its own ideology (for $\lambda_j \leq \bar{\lambda}_j$) or the opposite ideology (for $\lambda_j > \bar{\lambda}_j$). However it never locates on a region surrounding zero which implies indifferent reporting.

The technical proof is in the appendix. The first statement is analogous to Osborne and Pitchik (1987) where the optimal choice of location between two firms is not the mid-point of the $[0, 1]$ line but at roughly 0.27 units from either ends of $[0, 1]$. Firms choose this by minimizing the consumer’s transportation cost which in our model reflects the cost of reading a news story which is far away from a reader’s ideology. Locating at the midpoint of the $[-1, 1]$ interval only increases the transportation costs of extreme readers. Therefore when the event is neutral, both firms try to segment the entire reader populace into like-minded and opposite minded readers (in essence readers on either side of 0) and then locate at the mid-point of the like-minded segment. This equilibrium strategy connects to the psychology literature on news perception where more extremely ideology readers misinterpret neutral reporting of media as biased Giner-Sorolla and Chaiken (1994). Hence firms locate away from such neutral reporting.

The implication of the second statement can be derived from figure 5. If there is a rise in inflation during the presidency of the left, then statement *ii* implies that if L is too partisan-motivated, then it will attempt to defend the rise in inflation by reporting that its unemployment reducing monetary policies are targeted to lower unemployment which comes at a cost of higher inflation or raise doubts in readers’ minds about the possibility that the reported numbers are overestimated. Alternatively, if L is more motivated towards reader-assessments, then the coverage can be a fairly accurate report of the high inflation rates.

The above phenomenon was found in the way Fox news covered ICE’s decision of deporting international students during pandemic. The news story did not criticise the decisions but highlighted the dire impact it had on the lives of international students.¹³ What appears is that media will speak

¹³ A report by Fox5 Atlanta on July 8th 2020 titled “International students face uncertain future due to new ICE rule”.

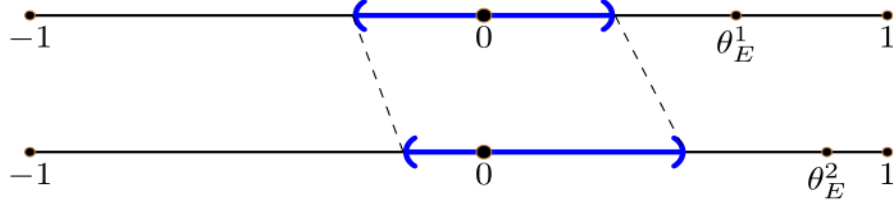


Figure 5: **Deviation from neutral reporting:** Suppose, readers are uniformly distributed and there occurs an event θ_E^1 which favors the right. Then L does not locate on the blue region. It reports on the left of this region (supporting the left) when λ_L lies below the cut-off $\bar{\lambda}_L$ (it is ideologically stronger) and reports on the right for values of λ_L above $\bar{\lambda}_L$ (it is more motivated towards reader-assessment). When the event favors the right more strongly, say θ_E^2 , then this blue region shifts to the right.

(not strongly enough) in favor of its adversary instead of positioning itself near zero, which intuitively leads to a tendency to build better reader-assessment credibility even from opposite-minded readers.

5.4 Comparative Statics

I now consider how the parameters λ_L and λ_R affect the equilibrium choices of L and R respectively. For more clarity of the stated propositions, I study the effects of the equilibrium choices of media L . Analogous explanations will hold for a similar study of R 's equilibrium choice. I also study the cross-over effect the rival media imposes on the equilibrium choices of the media firms (through the parameter b). Applying IFT to (5), I arrive at the following

$$\frac{d\theta_L^*}{d\lambda_L} = \frac{\theta_L^4 + \theta_L^3 + 0.25\theta_E^2 p^2 - 0.5p\theta_E\theta_L}{4\theta_L^3(1 - \lambda_L + \frac{c}{b+(\theta_R - \theta_E)^2}) + 3\theta_L^2(1 - \lambda_L + \frac{c\theta_E}{b+(\theta_R - \theta_E)^2}) + 0.5p\lambda_L\theta_E} \quad (7)$$

Let us take numerical values of exogenous parameters to better understand the comparative statics. I choose $b = 0.7$ and $c = 1.1$ and given $\lambda_L = \lambda_R = 0.1$, I get $(\theta_L^*, \theta_R^*) = (-0.417, 0.417)$ when $\theta_E = 0$. Incorporating in (8), I get,

$$\frac{d\theta_L^*}{d\lambda_L} = 0.27$$

As I will see later that this magnitude is greater than the comparative statics result from the monopoly model in section 6 where $\frac{d\theta_L^*}{d\lambda_L} = 0.2475$. Intuitively, given $\theta_E = 0$, when media L puts more weight on payoffs from readers, then it takes an editorial stance closer to the median reader.

I now conduct a similar comparative statics exercise with parameter b . This will allow us to measure the cross-effects of editorial choice of R on the choices of L and vice versa. Using IFT on equation (5) through parameter b gives us the following equality.

$$\frac{d\theta_L^*}{db} = \frac{\frac{-c\theta_L^3(1-\theta_L)}{(b+(\theta_R-\theta_E)^2)^2}}{4\theta_L^3(1-\lambda_L + \frac{c}{b+(\theta_R-\theta_E)^2}) + 3\theta_L^2(1-\lambda_L + \frac{c\theta_E}{b+(\theta_R-\theta_E)^2}) + 0.5p\lambda_L\theta_E} \quad (8)$$

As the weight on satisfying the average reader increases, both rival partisan media firms tries to place themselves near the median reader. The sign of the derivatives shows that the equilibrium editorial stance of L moves rightward towards 0 while the position of R moves leftward towards 0.

The nature of signs of the change in equilibrium level of slant depends on whether the events are themselves too strongly or too weakly biased. As before, $[\theta_E^L, \bar{\theta}_E^L]$ depicts events which are weakly biased (centered around 0) while its complement within $[-1, 1]$ denote the events which are biased strongly enough to either ideology.

$$\frac{d\theta_L^*}{dp} = \frac{0.5p\lambda_L\theta_E^2 - 0.5\lambda_L\theta_E\theta_L}{4\theta_L^3(1-\lambda_L + \frac{c}{b+(\theta_R-\theta_E)^2}) + 3\theta_L^2(1-\lambda_L + \frac{c\theta_E}{b+(\theta_R-\theta_E)^2}) + 0.5p\lambda_L\theta_E} \quad (9)$$

Proposition 4. (i) *If an event favors j 's ideology, j 's editorial choice moves closer to the median reader at 0 as λ_j increases. In other words, θ_j^* increases with λ_j , ($\frac{d\theta_j^*}{d\lambda_j} > 0$).*

(ii) *For any unfavourable event, $\frac{d|\theta_j^*|}{d\lambda_j} < 0$ for all $\lambda_j \in (0, \bar{\lambda}_j)$ and $\frac{d|\theta_j^*|}{d\lambda_j} > 0$ for all $\lambda_j \in (\bar{\lambda}_j, 1)$. At $\bar{\lambda}_j$, θ_j^* is discontinuous.*

(iii) *The impact of a more sophisticated reader pool reduces bias of L given any nature of event. However, the weights on ideology and reader-assessment of both media weakens or strengthens this impact.*

(a) *When the event has no ideology ($\theta_E = 0$), the impact gains strength in the presence of a media R which is less focused on ideology motive and assigns greater weight on reader assessment.*

(b) *If the event supports the ideology of media L , then the impact is greater in the presence of R whose motive is more driven towards ideology gains than reader-assessment.*

(iv) *Effect of an increase in the share of readers uninformed about θ_E .*

(a). *Suppose the event is favourable to j 's ideology, then media j with almost all weight on ideology ($\lambda_j \rightarrow 0$) locates nearer the median reader at zero ($\frac{d\theta_j^*}{dp} > 0$). On the contrary, when j weighs reader satisfaction ($\lambda_j \rightarrow 1$), it locates farther away from the median reader.*

(b) *When the event is unfavourable to j 's ideology beyond the threshold in figure 3, then $\frac{d\theta_j^*}{dp} \geq 0$ if $p \geq \frac{\theta_L}{\theta_E}$ and $\frac{d\theta_j^*}{dp} < 0$ if $p < \frac{\theta_L}{\theta_E}$.*

Sub-part (i) points out that as j 's attachment towards its own ideology falls, its comparative statics with respect to λ_j naturally segments the event space into two classes - events which are biased enough vis-a-vis the ones which are not. For the first class, media L 's editorial choice

increases (moves towards right on the ideology spectrum) when the weight on reader satisfaction increases. Analogously media R 's response decreases and moves towards the left.

(ii) implies that θ_j^* is piecewise continuous. As stated in statement (ii) of proposition 1, at the advent of a ideologically negative event, media j 's location strategy varies distinctly around a threshold value of λ_j . This variation is clearly suggested by the direction in change of θ_j^* on either side of the threshold. At the threshold, θ_j^* exhibits non-removable discontinuity of the first kind where $\theta_j^*(\bar{\lambda}_j + 0)$ and $\theta_j^*(\bar{\lambda}_j - 0)$ exists but have different values. θ_j^* remains continuous for all other values of λ_j .

Higher value of b , implies lower reader sophistication, thereby a greater leeway to bias in favor of ideology. According to Ansolabehere, Behr and Iyengar (1993), more educated people will generally call upon alternative information before accepting a news story and that increases the likelihood of them posing a stronger counter-argument to a overtly biased news story. This argument augments the third statement. The effect of a more sophisticated reader-base on reducing bias of a particular media is affected by contemporaneous effects of the preference of its rival. When the event has no bearing on ideology, then the presence of a rival which prefers reader-assessment will lead to a reduction in bias. This is because, it would pay the media more to locate towards the median reader by the standard Hotelling argument.

To evaluate the effect of increasing λ_L on the equilibrium payoffs, I use envelope theorem. By the envelope theorem, the effect of any parameter on the maximum value function is entirely the direct effect of the parameter on the maximum value function. The maximum value function V_j is calculated by substituting θ_j^* in the payoff functions of media j .

Proposition 5. Responsiveness of maximum value function

(i) Suppose the event is neutral ($\theta_E = 0$), then the maximum value function decreases as the weight on reader ratings are increased, $\frac{dV_j}{d\lambda_j} < 0$.

(ii) Suppose the event is not neutral ($\theta_E \in [-1, 0) \cup [-1, 1]/\{0\}$), then the maximum value function is U-shaped as λ_j as increased.

The technical proof is in the appendix. What is implied by this is the following. Given the reader pool is balanced and $\theta_E = 0$, the first term of media j 's profit function is zero (see equation 2). Then profit in equilibrium will always be enhanced when $\lambda_L \rightarrow 0$, or media j is more ideology-motivated.

Intuitively, if the event is neutral, then an average reader has zero bias (balanced reader pool) and he will tune in to the partisan channels to learn about potential ideological subtleties. Hence, placing more weight on ideology brings in higher rewards for the media. Alternatively, placing weight of reader assessment and providing a neutral report only leads to worse experience of like-minded readers. This resonates with the location choice model of Osborne and Pitchik (1987)

where firms does not choose the midpoint of the linear city economy (of unit length) but at roughly at points 0.25 and 0.75. These points minimize the consumers transportation costs.

The second statement means that while reporting a story which is not neutral (the story either supports or attacks the ideology of j), higher equilibrium profits are achieved when media either focuses on ideology or on reader ratings. Equilibrium profits are compromised if j wants to produce a report by balance both the factors. Hence, higher profits are realized at the extreme values of λ_j .

By continuity of the maximum value function, then there exists a threshold where the media experiences the lowest equilibrium profit. This is the exact threshold which reflects the desperation of a media to support its ideology even when the true event stands in contradiction. Following proposition 1, this threshold is denoted by $\bar{\lambda}_j$ and $\frac{\partial V_j}{\partial \lambda_j}$ vanishes at $\bar{\lambda}_j$.

Remark 3. *At the threshold value, media experiences greater equilibrium losses which primarily stems from poorer reader ratings.*

Desperately supporting its ideology can besmirch j 's image to a certain mass of readers who will doubt j 's credibility. This also intuitively connects to the experimental findings of Baum and Groeling (2009) where media often engage in such risky editorial decisions. As I will see from the comparative statics section below that this type of reporting comes at a high cost. At this threshold, media actually experiences the highest equilibrium loss.

The strategic interaction between L and R affects each other profit levels contingent on the nature of the topic and through which channels both media introduces bias- supply or demand. These channels then decide whether the firm interaction leads to strategic substitutability and complementarity and their respective magnitudes.

Proposition 6. (i) *When the event is neutral ($\theta_E = 0$), then any action by firm j is always a strategic substitute towards its opponent. The magnitude of strategic substitutability is greater when j is more partisan-oriented.*

(ii) *Suppose the event supports media L ($\theta_E < 0$). Then strategic substitutability persists when L 's reporting is more supply oriented (λ_L is lower than λ'_L) while when L is more demand oriented (λ_L is greater than λ'_L), there exists strategic complementarity.*

$$\frac{d^2 \Pi_L}{d\theta_L d\theta_R} = \begin{cases} < 0, & \text{if } \lambda_L < \lambda'_L \\ > 0, & \text{if } \lambda_L > \lambda'_L \end{cases}$$

(ii) *Suppose the event supports media R ($\theta_E > 0$). Then strategic substitutability persists when R 's reporting is more supply oriented (λ_R is lower than λ'_R). When R is more demand oriented (λ_R is greater than λ'_R), there exists strategic complementarity.*

$$\frac{d^2\Pi_L}{d\theta_L\theta_R} = \begin{cases} < 0, & \text{if } \lambda_R < \lambda'_R \\ > 0, & \text{if } \lambda_R > \lambda'_R \end{cases}$$

6 Equilibrium in a left or a right majority reader pool

Until now, this paper has dealt with the case where the share of leftist and rightist readers in the economy readers. I now study the equilibrium strategies of L and R which publishes reports to a distribution of readers who are either left or right-leaning.

I assume that media has perfect knowledge about the mass of N readers who lie in their territory on $[-1, 1]$.

The aggregated α_{ij} across N readers towards media $j \in \{L, R\}$ in equation (3) is :

$$\alpha_j^* = \sum_{i=1}^N \frac{x_i + \theta_E}{2\theta_j^*} \cdot \frac{1}{N} = \frac{\theta_E}{2\theta_j^*}$$

The political neutrality or balance between the share of leftist and rightist readers was formalized by $\sum_{i=1}^N x_i = 0$, thereby arriving at the above result.

Relaxing the condition in this section entails $\sum_{i=1}^N x_i \neq 0$. Equation (3) then becomes,

$$\alpha_L^* = \sum_{i=1}^N \frac{x_i + \theta_E}{2\theta_L^*} \cdot \frac{1}{N} = \frac{\kappa + \theta_E}{2\theta_L^*}, \kappa \neq 0 \quad (10)$$

I now have two possible scenarios:

1. Majority of readers are rightist: $\frac{1}{N} \sum_{i=1}^N x_i = \kappa$ and $0 < \kappa < 1$.
2. Majority of readers are leftist: $\frac{1}{N} \sum_{i=1}^N x_i = -\kappa$ and $0 < \kappa < 1$

First order condition for media L now becomes:

$$\frac{d\Pi_L}{d\theta_L} = \theta_L^4 \left[(1 - \lambda_L) + \frac{c}{(b + (\theta_R - \theta_E)^2)} \right] + \theta_L^3 \left[(1 - \lambda_L) - \frac{c\theta_E}{b + (\theta_R - \theta_E)^2} \right] \quad (11)$$

$$+ 0.5\lambda_L(\theta_E + \kappa)\theta_L - 0.25(\theta_E + \kappa)^2\lambda_L = 0$$



Figure 6: If there is a leftist majority of κ_L , then L will always support its own ideology for events in the cyan region. For events in the blue region, which are biased more strongly than the ones in cyan, L will support its leftist (rightist) ideology for $\lambda_L < \tilde{\lambda}_L$ ($\lambda_L > \tilde{\lambda}_L$). At the threshold, L is indifferent. One can interpret the orange and red intervals for media R in an analogous fashion.

First order condition for R is

$$\begin{aligned} \frac{d\Pi_R}{d\theta_R} &= \theta_R^4 \left[(1 - \lambda_R) + \frac{c}{(b + (\theta_L - \theta_E)^2)} \right] - \theta_R^3 \left[(1 - \lambda_R) + \frac{c\theta_E}{b + (\theta_L - \theta_E)^2} \right] \\ &+ 0.5\lambda_R(\theta_E + \kappa)\theta_R - 0.25(\theta_E + \kappa)^2\lambda_L = 0 \end{aligned} \quad (12)$$

In equilibrium, the strategy pair (θ_L^*, θ_R^*) solves (9) and (10). The next proposition presents how the presence of a dominant leftist-reader base emancipates media L while restraining R and vice-versa. We draw in comparisons with equilibrium editorial choices with balanced reader base in *Proposition 1* and also combine the comparative statics due to changes in trade-off between ideology and ratings. The comparative statics results follows from applying the implicit function theorem on (9) and (10). This is similar to what we have done before.

Remark 4. Suppose that media j has majority share of like-minded readers equal to κ_j . Then the nature of equilibrium as stated in proposition 1 still holds true. However, now with a favorable share of like-minded voters, media j enjoys leeway to bias events which strongly contradict their ideology.

The threshold which support the multiple equilibrium now is denoted by $\tilde{\lambda}_j$. Intuitively this suggests that for events which support the right, a left-biased media L with λ_L below $\tilde{\lambda}_L$ will locate itself to the left of zero, or bias in favor of L . Comparing this threshold with the one in proposition 1, $\bar{\lambda}_L < \tilde{\lambda}_L$.

This is easier to explain using figure 6. Suppose leftist readers form a majority share of κ_L . Given this, any event lying to the left of zero will be reported by media L with more intensity for all values of λ_L (this is consistent with *proposition 1*). In the current proposition, the dominance of like-minded readers by a magnitude, say κ_L , offer L a leverage to bias events which also lie to the right of zero, (denoted by the cyan interval $(0, \tilde{\theta}_E(\kappa_L))$ in figure 6). However, κ_L does not give

L the liberty to unconditionally bias news for topics lying to the right of the cyan region. So for events in the blue interval, biasing information becomes conditional on λ_L (L will bias in favor of left (right) below a threshold value $\tilde{\lambda}_L$ of λ_L , from proposition 1). For events lying to the right of the blue interval (topics more strongly favoring the right), L speaks in favor of the right for all values of λ_L . Symmetric interpretation with regard to media R 's strategy can be given for events lying in the orange and red intervals.

To elicit some important characteristics of media behavior, we first assume the parameters signifying the majority share and the cost of biasing through cross-over effect to be below some cutoff such that $|\kappa| < \hat{\kappa}$ and $b \geq \hat{b}$. This guarantees that the editorial choice are not shackled too much either by the extent of biased readers or by a very high cost of bias.

Remark 5. (i) *The magnitude of movement of θ_L^* is dictated by the gross effect of the absolute values of $\frac{d\theta_L^*}{d\lambda_L}$ and $-\frac{d\theta_L^*}{d\kappa}$. θ_L^* moves towards (away from) 0 if the net effect is positive (negative).*

(ii) *Analogously, the magnitude of movement of θ_R^* is dictated by the net effect of $\frac{d\theta_R^*}{d\lambda_R}$ and $\frac{d\theta_R^*}{d\kappa}$. θ_R^* moves towards (away from) 0 if the net effect is positive (negative).*

When λ_L and λ_R increases, then media L and R respectively concentrates more on average reader, hence moves towards the mean reader. As κ decreases (readers are majorly left-biased), it gives L more leeway to position itself more extremely towards the left. Symmetrically when κ increases (readers are majorly right-biased), then R has more freedom to bias in favor of the right.

We now examine the features of the maximum value function and draw in comparisons with the statements in proposition 3.

In essence, higher profit is enjoyed by media which are either more ideologically extreme (close to -1 or 1) or mainly care only about reader ratings (close to 0). The sequence of thresholds $\{\bar{\lambda}_L\}$ where equilibrium profit equals zero, originates from a point which is closer to the optimal ideology at -1 than in a situation when reader pool was balanced (Proposition 3). This implies that, in an economy dominated by leftist readers, by compromising with ideology (by increasing λ_L away from some $\epsilon \rightarrow 0$), L gets penalized in terms of media ratings.

7 Model with 3 media outlets

We expand the previous analysis by adding one more firm on the ideology axis. We denote this firm by Q which has an ideological bliss point at $\tilde{q} \in (-1, 1)$. The remaining features of the model comprising the readers and the media outlets L and R carries on unchanged in this section. This exercise is expected to reveal how more competition among the media outlets affect the equilibrium level of bias.

The corresponding normal form game of this three firm model is defined as $\Gamma_T = [I, \{S_i\}, \{S_L\}, \{S_R\}, \{S_Q\}, \{u_i(\cdot)\}, \{\Pi_L(\cdot)\}, \{\Pi_R(\cdot)\}, \{\Pi_Q(\cdot)\}]$. I denotes the player set comprising of media L , R and Q and reader $i \in \{1, \dots, n\}$. u_i is the utility of reader i from reading news and Π_L, Π_R and Π_Q denotes the profits of media L , R and Q . Thereby the strategy profile constituting the SPNE is characterized as $s^* = (\theta_L^*, \theta_R^*, \theta_Q^*, \alpha_{iL}^*(\theta_L^*), \alpha_{iR}^*(\theta_R^*), \alpha_{iQ}^*(\theta_Q^*)) \quad \forall \quad i = \{1, \dots, N\}$.

7.1 Utility Maximization of reader

We will inherit equation (1) with one more media firm Q such that for $j \in \{L, R, Q\}$, utility of any reader i is given by

$$U_i(\alpha_{ij}|\theta_j, \theta_E) = -(\alpha_{ij}\theta_j - x_i)^2 - (\alpha_{ij}\theta_j - \theta_E)^2$$

7.2 Backward Induction by Media

With three firms, the payoff function takes a slightly revised form where the nature of cost function gets updated to account for the bias of the third firm. In the following three equations, we layout the payoffs of media $j \in \{L, R, Q\}$.

$$\Pi_L(\theta_L, \theta_R, \theta_Q) = -\lambda_L \cdot (\alpha_L^* - 1)^2 - (1 - \lambda_L)(\theta_L + 1)^2 - \frac{c(\theta_L - \theta_E)^2}{b + (\theta_R - \theta_E)^2 + (\theta_Q - \theta_E)^2} \quad (13)$$

$$\Pi_R(\theta_R, \theta_L, \theta_Q) = -\lambda_R \cdot (\alpha_R^* - 1)^2 - (1 - \lambda_R)(\theta_R - 1)^2 - \frac{c(\theta_R - \theta_E)^2}{b + (\theta_L - \theta_E)^2 + (\theta_Q - \theta_E)^2} \quad (14)$$

$$\Pi_Q(\theta_Q, \theta_L, \theta_R) = -\lambda_Q \cdot (\alpha_Q^* - 1)^2 - (1 - \lambda_Q)(\theta_Q - \tilde{q})^2 - \frac{c(\theta_Q - \theta_E)^2}{b + (\theta_L - \theta_E)^2 + (\theta_R - \theta_E)^2} \quad (15)$$

The only difference between L 's (R 's) payoff function from previous section lies in the cost function which now takes account for the bias of the the third media house Q .

Definition 2. *Nash Equilibrium of this game Γ_T is a triple $(\theta_L^*, \theta_R^*, \theta_Q^*)$ of editorial choices for which θ_j^* is a best response to θ_{-j}^* where $j \in \{L, R, Q\}$*

The below table shows a numerical depiction of the equilibrium choices of L and R with the entry of a new media with two respective ideology bliss points- -0.5 and -0.75 and for two values

of $\lambda_Q = \{0.1, 0.5\}$.

(λ_L, λ_R)	λ_Q	0.1	0.5
(0.1,0.1)		(-0.436,0.436,-0.23)	(-0.426,0.426,-0.162)
(0.1,0.5)		(-0.4,0.293,-0.218)	(-0.396,0.28,-0.15)
(0.5,0.5)		(-0.27,0.27,-0.20)	(-0.264,0.264,-0.138)

Table 2: Equilibrium editorial position of media L , R and Q ($\theta_L^*, \theta_R^*, \theta_Q^*$) when $\theta_E = 0$ and Q is located at -0.5 . For comparison purposes, we have highlighted $L(R)$'s choices in blue (red) for $\lambda_Q = 0.1$. Editorial choices are more extreme with the new biased media Q from the duopoly model in Table 1.

(λ_L, λ_R)	λ_Q	0.1	0.5
(0.1,0.1)		(-0.46,0.46,-0.36)	(-0.436,0.436,-0.247)
(0.1,0.5)		(-0.426,0.31,-0.33)	(-0.41,0.294,-0.226)
(0.5,0.5)		(-0.285,0.285,-0.31)	(-0.27,0.27,-0.20)

Table 3: Equilibrium editorial position of media L , R and Q ($\theta_L^*, \theta_R^*, \theta_Q^*$) when $\theta_E = 0$ and Q is located at -0.75 . For comparison purposes, we have highlighted $L(R)$'s choices in blue (red) for $\lambda_Q = 0.1$. Editorial choices are not only more extreme from the duopoly model, but also from Table 2 where Q is relatively less biased.

Taking into account of the nature of profit functions in 16, 17 and 18, we have the following corollary.

Remark 6. Given $\theta_E = 0$, the equilibrium editorial choices of L and R become more biased with the entry of a biased third firm Q . If Q is unbiased or positioned at 0, then it has no effect on the equilibrium editorial choices of L and R .

If media is covering a story about a neutral event, then the entry of a new firm which is biased, increases the absolute levels of slants in both L and R . To get more clarity, one can compare the numbers of the editorial choices of L (R) highlighted in blue (red) across Tables 1, 2 and 3. The proof of the second part is straightforward and entails that the presence of the unbiased media is unable to cater to the ideological beliefs of readers along the ideological spectrum- thereby, biased media firms stay persistent in their prior editorial choices and refuses to decrease their slant.

8 Instrumental value of information

I now study the above information economy when media report carries instrumental value such that readers use media report by taking a particular decision of voting for a particular policy. I retain the original framework but now consider that media L and R endorse policies of political party l and r respectively. A policy of party $j' \in \{l, r\}$ which provides an amount y_l and y_r lying within $(0, 1)$ to every reader in the economy. So now, the choice variable of say, media L is to

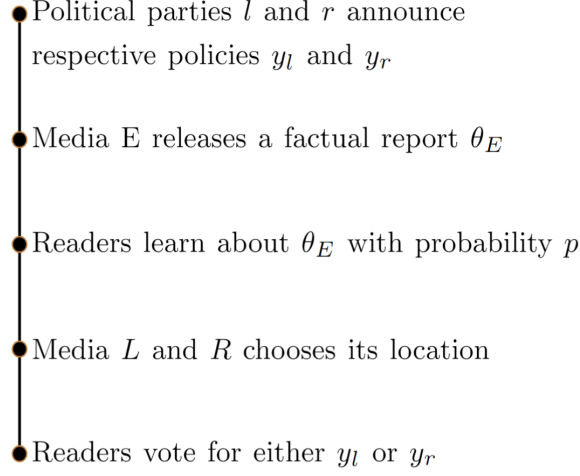


Figure 7: Timeline of game where media's location choice endorses policies of their respective parties.

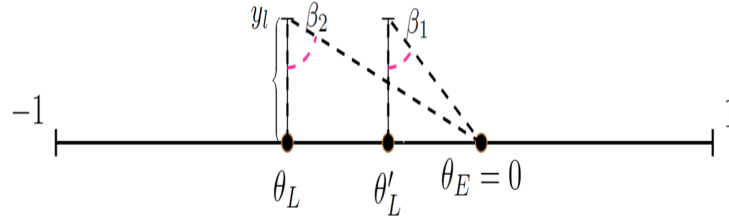


Figure 8: If L chooses to locate at θ'_L while reporting a neutral event ($\theta_E = 0$), the report casts a angle of β_1 with the existing policy y_l . A larger value of β_1 implies a lower level of viability of y_l given θ_E . As seen above, if L biases its report further to θ_L , the angle gets steeper (β_2), thereby weakening the viability of y_l at $\theta_E = 0$.

choose θ_L such that y_l seems most viable given θ_E . Analogous explanation holds for the location choice of media R .

The policies y_l and y_r are assumed to be exogenously chosen by the political parties l and r prior to the occurrence of θ_E . Readers who learn about θ_E form a prior equal to $G(1 - \theta_E)$ which suggests the chances of success of party l 's policy and equals $H(\theta_E)$ for party r 's policy. I assume the cumulative distribution functions L and R to be uniform. Readers who do not have access to θ_E , form a constant prior of $q \in (0, 1)$. After any reader i reads news of media j , he updates his prior based on the weight he assigns to the news report. Hence, if y_l seems to be more viable than y_r , he casts his vote for party l .

The payoff from both policies are common knowledge but readers chooses to vote for the policy which has higher expected payoff given θ_E . This expected payoff is a function of how the news assessment and the projection of media j 's location choice onto the policy space. The angle β_j is formed between the perpendicular line denoting the policy y_l and the line joining θ_E and y_l . This angle resembles the degree of viability of y_l when true event is θ_E .

The utility from policy y_l and y_r when reader i learns about θ_E be denoted as follows.

$$U_i(y_l, \theta_E) = y_l - \theta_E - y_l \cdot x_i \quad U_i(y_r, \theta_E) = y_r + \theta_E + y_r \cdot x_i \quad (16)$$

The prior belief about y_l 's viability given θ_E which I denote as $1 - g(\theta_E)$. Similarly, the prior of y_r 's viability is $h(\theta_E)$. So under uniform prior, if $\theta_E = 0$, then both $g(\theta_E) = h(\theta) = \frac{1}{2}$. If $\theta_E = -0.5$, then viability of y_l is $1 - g(\theta_E) = 1 - l(-0.5) = 0.75$ while that of y_r is $h(-0.5) = 0.25$.

The utility from policy y_l and y_r when reader i has no knowledge of θ_E be denoted as follows.

$$U_i(y_l) = y_l - y_l \cdot x_i \quad U_i(y_r) = y_r + y_r \cdot x_i \quad (17)$$

These readers form a constant prior q about the viability of policy. If a reader is aligned to the left, he assigns probability q_l (q_r) to the viability of left (right) policy and $q_l > q_r$. The reverse holds if the reader is aligned to the right.

After reading news of L and R , reader i maximizes his expected utility from each policy by updating his prior belief such that $\alpha_{ij}^* = \arg\max \mathbb{E}(y_{j'})$. The belief updation is maximized when the reader assess the news based on (1), implying that the optimal valuation will lead to the more accurate updation of his prior regarding the viability y_l or y_r .

Media profits remains a convex combination of payoffs from reader assessment and policy payoffs. The goal of getting its policy elected by majority readers is akin to the ideology payoffs as in (3). Therefore, this payoff is are maximized when L can position itself as close as possible to his ideal ideology location of -1 which for media R is the point 1. As the equilibrium location choice converges to these ideal points, the policy projects an angle β with respect to the location choice. This signals the leverage of policy y_l in becoming viable under θ_E . Lower the value of β , higher is the leverage.

In this section, one needs to focus on the manner of belief updation of reader i after learning θ_j and having knowledge of θ_E about the viability of y_l at θ_E . The Bayesian updating rule is given by

$$\sigma(y_l | \theta_j, \theta_E) = \frac{g(1 - \theta_E) \cdot \pi(\theta_j | y_l, \theta_E)}{g(1 - \theta_E) \cdot \pi(\theta_j | y_l, \theta_E) + g(\theta_E) \cdot \pi(\theta_j | y_l^c, \theta_E)} \quad (18)$$

The prior belief of viability of policy y_l at θ_E is denoted by $1 - L(\theta_E)$. Similarly, the probability of policy y_r at θ_E is denoted by $R(\theta_E)$ in the below equation. $\pi(\cdot)$ denotes the likelihood of θ_j being true, given the knowledge of y_l and θ_E .

$$\sigma(y_r | \theta_j, \theta_E) = \frac{h(\theta_E) \cdot \pi(\theta_j | y_r, \theta_E)}{h(\theta_E) \cdot \pi(\theta_j | y_r, \theta_E) + h(1 - \theta_E) \cdot \pi(\theta_j | y_r^c, \theta_E)} \quad (19)$$

The likelihood is a function of the assessment reader attaches to news of media α_{ij} and the parameter β_j .

The likelihood function $\pi(\cdot)$ measures how likely is the report of L true given that y_l is viable

in $\theta_E = 0$. In reduced form, this can be denoted as $\pi(\hat{\alpha}_{ij}, \beta)$ where $\hat{\alpha}_{ij} = |\alpha_{ij} - 1|$. π is continuously differentiable and strictly decreases with $\hat{\alpha}_{ij}$ or $\frac{\partial \pi}{\partial \hat{\alpha}_{ij}} < 0$ and strictly decreases with β , or $\frac{\partial \pi}{\partial \beta} < 0$. Since $\alpha_{ij} \in \mathbb{R}$, I consider functional forms of π such that the entire real range is mapped on the interval $[0, 1]$. In addition, the assumptions of $y_j \in (0, 1]$ and $\theta_E \in [-1, 1]$ imply that $\beta \in (0, \arctan 0.5]$.

In the absence of any knowledge of θ_E , readers form a constant prior q . belief updation about the viability of y_l at θ_E follows in the following manner where the likelihood is only a function of reader assessment.

$$\sigma(y_l|\theta_j) = \frac{q \cdot \pi(\theta_j|y_l)}{q \cdot \pi(\theta_j|y_l) + (1 - q) \cdot \pi(\theta_j|y_l^c)} \quad (20)$$

$$\sigma(y_r|\theta_j) = \frac{q \cdot \pi(\theta_j|y_r)}{q \cdot \pi(\theta_j|y_r) + (1 - q) \cdot \pi(\theta_j|y_r^c)} \quad (21)$$

Consider one representative reader i who has knowledge about $\theta_E = 0$ prior to listening to L and R . Then given g and h to be uniform, 16 and 17 implies that i will vote for y_l over y_r iff

$$y_l \frac{g(0.5) \cdot \pi(\theta_j|y_l, 0)}{g(0.5) \cdot \pi(\theta_j|y_l, 0) + (0.5) \cdot \pi(\theta_j|y_l^c, 0)} > y_r \frac{h(0.5) \cdot \pi(\theta_j|y_r, 0)}{h(0.5) \cdot \pi(\theta_j|y_r, 0) + h(0.5) \cdot \pi(\theta_j|y_r^c, 0)}$$

Given g and h are both uniform, the above reduces to

$$y_l \frac{\pi(\theta_j|y_l, 0)}{\pi(\theta_j|y_l, 0) + \pi(\theta_j|y_l^c, 0)} > y_r \frac{\pi(\theta_j|y_r, 0)}{\pi(\theta_j|y_r, 0) + \pi(\theta_j|y_r^c, 0)}$$

$$y_l \frac{\pi(\hat{\alpha}_{iL}, \hat{\beta}_L)}{\pi(\hat{\alpha}_{iL}, \hat{\beta}_L) + \pi^c(\hat{\alpha}_{iL}, \hat{\beta}_L)} > y_r \frac{\pi(\hat{\alpha}_{iR}, \hat{\beta}_R)}{\pi(\hat{\alpha}_{iR}, \hat{\beta}_R) + \pi^c(\hat{\alpha}_{iR}, \hat{\beta}_R)}$$

Now, imagine that both $\lambda_L \rightarrow 1$ and $\lambda_R \rightarrow 1$. Then when $\theta_E = 0$ $\theta_L^* \rightarrow 0$ and $\theta_R^* \rightarrow 0$ which suggests $\beta_L, \beta_R \rightarrow \infty$ which suggests $\hat{\beta}_L, \hat{\beta}_R \rightarrow 0$. The convergence to zero suggests that the report from the media L and R does not posit stronger viability of y_l and y_r respectively. Therefore leftist chooses y_L and rightist chooses y_R which leads to welfare maximization. Similarly, welfare will be maximized when $\lambda_L \rightarrow 0$ and $\lambda_R \rightarrow 0$ where both media differentiates their news away from the point of neutral ideology (0).

Proposition 7. *Suppose the event is neutral ($\theta_E = 0$).*

(a) *Then if $\lambda_L, \lambda_R \rightarrow 0$, then welfare is maximized as readers chooses policy aligned towards their ideology without regard to whether $y_l = y_r$ or $y_l \neq y_r$.*

(b) *If $\lambda_L, \lambda_R \rightarrow 1$, and $y_l > y_r$, then a share of rightist readers close to zero will choose y_l iff $\frac{y_l}{y_r} > \delta$.*

Suppose the event is favourable to left ($\theta_E < 0$).

(c) If $\lambda_L \rightarrow 0, \lambda_R \rightarrow 1$, then this can lead to welfare degrading equilibrium such that majority readers vote for y_r when readers surrounding zero have no access to the factual report θ_E .

9 Welfare analysis and policy prescription

In this spatial linear city model of product placement, where firms with different biases compete to serve news to readers who also differ, truth revelation comes with a trade-off. The primary objective of welfare improvement lies not only when media outlets reports accurately, but also when the readers perceive the news in light of the facts underlying the issue and not let their ideology override their judgement of those facts. This trade-off gets worse in presence of reader polarization and reader population as this section as this analysis will show. I first layout how reader surplus varies with more or less polarization, followed by a general description of how media profits and reader surplus varies with reader polarization and the issue to be reported. Finally I suggest policy prescription which will lead towards truth revelation alongside moderating the trade-off of the ideology effect which can dictate a reader's judgement.

9.1 Reader surplus

Reader surplus from a news story is represented by the gap between the value of assessment and 1 (this follows from Lemma 1 which shows the first best assessment value equals 1). This reflects the idea of deriving consumer surplus in by subtracting market price from the reservation price of a consumer. *Lemma 1* therefore entails that the first best assessment is akin to the reservation weight of a reader.

Following from *Equation 2*,

$$\alpha_{ij}^* = \frac{x_i + \theta_E}{2\theta_j^*} = \frac{x_i + \theta_E}{\theta_j^* + \theta_j^*} \quad (22)$$

Utility of i from reading a report of j is given as

$$U_i(\alpha_{ij}^*, \theta_j^* | \theta_E) = -(\alpha_{ij}^* \theta_j^* - x_i)^2 - (\alpha_{ij}^* \theta_j^* - \theta_E)^2$$

Therefore utility loss of i from a news to which i attaches a weight of $\hat{\alpha}_j \neq 1$ is

$$\Delta U_{ij} = U_i |_{\alpha_{ij}^* = \hat{\alpha}_j} - U_i |_{\alpha_{ij}^* = 1} = 4(\theta_j^* - \frac{x_i + \theta_E}{2})^2 \quad (23)$$

Given 15, any reader i who faces zero utility loss is characterized by $x_i = 2\theta_j^* - \theta_E$. The total

utility loss across all N readers due to media j is then calculated by summing the individual utility losses across N readers, denoted as

$$\Delta U_j = \sum_{i=1}^N \Delta U_{ij} = 4N((\theta_j^*)^2 - \theta_E \theta_j^*) + N\theta_E^2 - 2\theta_E \sum_i x_i + \sum_i x_i^2$$

or,

$$\Delta U_j = 4N((\theta_j^*)^2 - \theta_E \theta_j^*) + N\theta_E^2 - 2\theta_E \kappa + \sum_i x_i^2 \quad (24)$$

The above equation is intuitive and brings out the avenues where utility of readers decreases in the economy. The first term within parenthesis resembles the level of loss imposed on readers when media reports the true event. This term increases if media speaks overtly opposite to the truth ($\theta_E \cdot \theta_j < 0$). The third term signifies whether the event is favourable to the majority readers ($\theta_E \cdot \kappa > 0$). It is quite evident that any policy interventions that can be implemented must be focused on taxing media firms to report closer to the truth, hence lowering the first term. Alternatively, policies to enhance readers to weigh the true event more than their ideology affiliations can lead to welfare improvement when both media reports closer to true event. The remaining terms are exogenous and no welfare improving policy can target to mitigate this loss.

I illustrate some simple examples with different reader demography which illustrates that policies will have a bite only relating to the first term. The effect of demography will either augment or impede the goal of any policy. I consider a fixed event and elaborate the loss reader will face from either L or R .

Example 1. *Balanced readership with high polarization:* Consider an economy with 4 readers such that two are located at -0.5 and two at 0.5 . The reader pool is balanced as both readers on either side of zero neutralize each other. However this population has variance of 1 ($\sum_i x_i^2 = 1$). Suppose now there occurs an event totally favorable to the left, $\theta_E = -1$. The loss in reader surplus due to media L and R is given by

$$\Delta U_L = 16((\theta_L^*)^2 + \theta_L^*) + 5$$

$$\Delta U_R = 16((\theta_R^*)^2 + \theta_R^*) + 5$$

Example 2. *Balanced readership with low polarization:* Consider an economy with 4 readers located at $-0.5, -0.25, 0.25$ and 0.5 . The reader pool is still balanced as example 1. However the variance term is now lower, $\sum_i x_i^2 = 0.625$. Suppose there occurs an event totally favorable to the left, $\theta_E = -1$. The loss in reader surplus due to any media L and R is given by

$$\Delta U_L = 16((\theta_L^*)^2 + \theta_L^*) + 4.625$$

$$\Delta U_R = 16((\theta_R^*)^2 + \theta_R^*) + 4.625$$

Example 3. Readership biased towards the event: Now assume that these 4 readers are at -0.5 , -0.30 , -0.1 and 0.5 . The reader pool now is left biased by -0.4 variance of 0.6 . Suppose there occurs an event totally favorable to the left, $\theta_E = -1$. The loss in reader surplus due to any media L and R is given by

$$\Delta U_L = 16((\theta_L^*)^2 + \theta_L^*) + 3.8$$

$$\Delta U_R = 16((\theta_R^*)^2 + \theta_R^*) + 3.8$$

Example 4. Readership biased against the event: Consider now that the 4 readers are at -0.5 , 0.30 , 0.1 and 0.5 . The reader pool now is right biased by 0.4 variance of 0.6 . Suppose there occurs an event totally favorable to the left, $\theta_E = -1$. The loss in reader surplus due to any media L and R is given by

$$\Delta U_L = 16((\theta_L^*)^2 + \theta_L^*) + 5.4$$

$$\Delta U_R = 16((\theta_R^*)^2 + \theta_R^*) + 5.4$$

The constant terms resembles the demography effects on reducing consumer surplus on the entire economy due to media L and R . The exogenous effect of having high polarization comparative to low polarization can be estimated by the difference of the constant terms ($5 - 4.625 = 0.375$) in example 1 and 2. Analogously, the differences in the constant terms in examples 3 and 4 amounting to 1.6 ($5.4 - 3.8$) shows the exogenous effect on reader welfare due when the event stands contradictory to the majority's beliefs.

9.2 Media payoffs

The profit of any media j with ideological bliss point at $\theta_j \in [-1, 1]$ is given by

$$\Pi_j(\theta_j, \theta_{-j}) = -\lambda_j \cdot (\alpha_j^* - 1)^2 - (1 - \lambda_j)(\theta_j - \dot{\theta}_j)^2 - \frac{c(\theta_j - \theta_E)^2}{b + \sum_{-j}(\theta_{-j} - \theta_E)^2}$$

Applying the characterization of α_j^* in Lemma 1, the profit of media j is

$$\Pi_j(\theta_j, \theta_{-j}) = -\lambda_j \cdot \left(\frac{\kappa + \theta_E}{2\theta_j^*} - 1 \right)^2 - (1 - \lambda_j)(\theta_j - \theta_{-j})^2 - \frac{c(\theta_j - \theta_E)^2}{b + \sum_{-j}(\theta_{-j} - \theta_E)^2} \quad (25)$$

Equilibrium welfare (henceforth, welfare) denoted by W comprises of media j 's payoff and the loss in utility faced by N readers. It then becomes evident that absolute number of readers N , their share net of ideology κ and the spread of readers from 0 denoted by $\sum_i x_i^2$ affect welfare, however to different extents.

$$W_j(\theta_j, \theta_{-j} | \theta_E, \kappa, N, c, b) = \Delta U_j(\theta_E, \kappa, N) + \Pi_j(\theta_j, \theta_{-j} | \theta_E, \kappa, c, b)$$

Remark 7. (i) If the number of readers N increases, then welfare changes by $4((\theta_j^*)^2 - \theta_E \theta_j^*) + \theta_E^2$.
(ii) If the share of readers, κ increases, then welfare changes by $-\frac{\lambda_j \kappa}{(\theta_j^*)^2} + \frac{\lambda_j}{\theta_j^*} - 2\theta_E$
(iii) Welfare decreases at a unit rate with the rise in variance of readers on the ideology axis.

The first two statements implies that readers face some utility loss from reading news about an event which goes against their ideological orientation. As the number of readers increases, it gets more challenging to satisfy everyone. Similarly, if the reader pool is skewed to the left or right, then welfare increases or decreases depending on the nature of event.

The final statement is relatively unambiguous about the nature of welfare change. It suggests that as the spread of readers rises, it gets more tough to satisfy them irrespective of the nature of event or the level of heterogeneity among readers.

9.3 Policy recommendation

By expanding the role of the honest media, truth telling can be better sustained such that news consumers start to accept the truth without being clouded by their ideology beliefs. To achieve this, I suggest that government spending can be targeted to develop the honest media in taking up an educative role while releasing the factual report. The aim of such an initiative lies in making readers assign greater weight on the true information than their ideology.

The degree of command of the media in making readers process the factual report is parameterized by $\beta \in (1, 2]$. The utility of readers from reading news reports of media L and R is represented by

$$U_{ij} = -(\hat{\alpha}_{ij}\theta_j - (2 - \beta).x_i)^2 - (\hat{\alpha}_{ij}\theta_j - \beta.\theta_E)^2 \quad (26)$$

As shown above, any reader i while reading news of media j assigns a weight of β to the factual information which is strictly greater than the weight on ideology x_i . The first order condition leads

to

$$\alpha_{ij}^* = \frac{(2 - \beta) \cdot x_i + \beta \cdot \theta_E}{2\hat{\theta}_j^*} \quad (27)$$

Media j 's profit function is given by

$$\Pi_j(\hat{\theta}_j, \theta_{-j}) = -\lambda_j \cdot \left(\alpha_{ij}^* - 1 \right)^2 - (1 - \lambda_j)(\theta_j - \hat{\theta}_j)^2 - \frac{c(\theta_j - \theta_E)^2}{b + \sum_{-j}(\theta_{-j} - \theta_E)^2} \quad (28)$$

The choice variable of $\hat{\theta}_j$ is now a function of β alongside the previous parameter used in the baseline model. To assess the merits of this policy intervention, I discuss the sufficiency conditions when the overall reader welfare in the economy. These conditions depend on how media j chooses to cover a topic considering the nature of reader polarization and given the value of λ_j .

Proposition 8. (i). Suppose the event is neutral ($\theta_E = 0$).

- a. Then overall reader welfare is improved for any $\beta > 0$ if reader pool is balanced.
- a. If the reader pool is biased, then reader utility is enhanced if the following is satisfied

$$\frac{\sum_i x_i^2}{4N} \left(\frac{2 - \beta}{\hat{\theta}_j^*} + \frac{1}{\theta_j^*} \right) < \kappa$$

(ii). Suppose the event supports the left completely¹⁴. Then the following conditions must hold for welfare improvement of readers.

- a. If the reader pool is also polarized completely to the left, then $\frac{(\hat{\theta}_j^*)^2}{(\theta_j^*)^2} > \beta$ must be satisfied.
- b. If the reader pool is polarized completely to the right, then $2 - \beta < \frac{(\hat{\theta}_j^*)^2}{(\theta_j^*)^2} < \beta(2 - \beta)$ must hold.
- c. If the reader pool is balanced, then $2 - \beta < \frac{(\hat{\theta}_j^*)^2}{(\theta_j^*)^2} < \beta$.

See appendix section 10.9 for technical proof. In terms of intuition, when the topic is free of ideology, then strongly ideology oriented media will try to deviate and bias the news away from the point zero. If prior to media E undertaking such educative role, media j takes position θ_j^* , then ex-post E taking such a role media j moves towards zero by taking $\hat{\theta}_j^*$ which implies that $\frac{\hat{\theta}_j^*}{\theta_j^*} < 1$. The policy by media E generates a reporting interval where reader welfare will be enhanced if media j chooses to report within it. For higher values of β , such intervals are wider which suggests higher chances of welfare improvement.

In sub part (ii), when the event is itself favourable to the left, then welfare improvement becomes either redundant (implied by stronger the condition in a, when majority readers also share

¹⁴Exactly symmetric results will hold if the topic supports the right.

similar ideology beliefs with the topic) or challenging (when majority readers themselves hold beliefs opposite to the event, as in b). In b , when the topic is absolutely pro-left, then this condition is satisfied when R 's motive is to maximize ideology payoffs ($\lambda_R \rightarrow 0$)

10 Concluding comments

The model presents a spatial model of news bias where bias in news evolves through the strategic interaction between media outlets, each of which choose to report events through the demand or supply oriented forms of news bias. The demand and supply oriented channels lead media to location close or far away from each other in response to the nature of the event. This paper also provides a measure of news evaluation which possess both spatial and directional features of how readers process information about any event. The spatial aspect measures the ideology difference between the reader and the news story while the directional aspect consider whether the event and the reader are on the same side of the ideology axis. For example, conservative readers would discount the liberal media news about the demerits of ban in abortion not only because there is a difference in liberal conservative ideology, but also due to the fact that abortion is directionally opposite to the conservative ideology.

In this set-up, policy measures should be designed which make factual information more receptive to the entire reader populace. I suggest that the exogenous media can take up an educative role by providing factual information in a manner such that the intrinsic facts get primary attention to the readers, thereby not letting their ideology override their judgement of a topic. Ideology worsens societal divide which raises disagreement regarding societal issues like proscribing abortion, anti-immigration attitudes¹⁵ which according to the present analysis can be assuaged when people's reception of news is guided strongly by the facts of the matter. Simply increasing media competition by introducing less partisan media might lead to more accurate information provision, but its effect on reader perception remains ambiguous.

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11 Appendix

11.1 Example of news bias from mainstream news

As an example of how two opposing media can publish reports which evoke supremacy of their own ideology, I present the stories of CNN and Fox while covering the hike in tariff rates on Chinese goods by the Trump administration.

CNN: The US just raised tariffs on Chinese goods. China says it will hit back: The United States has escalated its trade war with China, hiking tariffs on \$200 billion worth of Chinese exports hours after trade talks held in Washington failed to produce a breakthrough. Tariffs on the targeted exports increased from 10% to 25% at 12:01 a.m. ET on Friday, prompting a swift rebuke from Beijing... "The tariff increase inflicts significant harm on US industry, farmers and consumers," said Jacob Parker, vice president of the US-China Business Council...

Fox News: Trump absolutely right to slap new tariffs on China: President Trump on Sunday announced additional incoming tariffs on China, reminding Beijing that its days of negotiating with weak counterparts are over, at least as far as it concerns the United States. While Trump's move may cause short-term stock market turbulence, it's great news for U.S. national security and our economy over the longer term.

As can be inferred, CNN's report is a blatant criticism of the policy and predicts a backlash from China while Fox News champions Trump for being aggressive with China, and hopes that this will instill renewed resilience on the part of the United States. Likewise, for any other incident, media will bias news bearing in mind its partisan interests and making its reader's happy.

11.2 Existence of roots of best response functions

Strum's theorem allows us to find the number of real distinct roots of each best response (BR) of L and R . This from Worth(2005) and helps us determine the number of real distinct roots within the interval $[-1, 1]$ for any given θ_E , λ_L and λ_R . This exercise allows us to know whether each of these equations have a real zero within $[-1, 1]$. The Nash equilibrium choices of θ_L and θ_R is then determined at the intersection of each of these BR .

We denote BR of L and R below as

$$g(\theta_L) = \frac{d\Pi_L}{d\theta_L} = \theta_L^4 \left[(1 - \lambda_L) + \frac{c}{(b + (\theta_R - \theta_E)^2)} \right] + \theta_L^3 \left[(1 - \lambda_L) - \frac{c\theta_E}{b + (\theta_R - \theta_E)^2} \right] \quad (29)$$

$$+ 0.5\lambda_L\theta_E\theta_L - 0.25\theta_E^2\lambda_L = 0$$

$$g(\theta_R) = \frac{d\Pi_R}{d\theta_R} = \theta_R^4 \left[(1 - \lambda_R) + \frac{c}{(b + (\theta_L - \theta_E)^2)} \right] - \theta_R^3 \left[(1 - \lambda_R) + \frac{c\theta_E}{b + (\theta_L - \theta_E)^2} \right] \quad (30)$$

$$+ 0.5\lambda_R\theta_E\theta_R - 0.25\theta_E^2\lambda_R = 0$$

Definition 3. *Strum's sequence: The Strum sequence for a univariate polynomial $f(x)$, is a sequence f_0, f_1, f_2, \dots such that*

$$f_0 = f$$

$$f_1 = f'$$

$$f_{i+1} = -\text{rem}(f_{i-1}, f_i) \text{ where } \text{rem}(f_{i-1}, f_i) \text{ is the remainder when } f_{i-1} \text{ is divided by } f_i.$$

Definition 4. *Strum's Theorem: - Let $f(x)$ be a polynomial of positive degree with real coefficients and let $\{f_0(x) = f(x), f_1(x) = f'(x), f_2(x), \dots, f_s(x)\}$ be the standard sequence for $f(x)$. Assume $[a, b]$ is an interval such that $f(a) \neq 0 \neq f(b)$. Then the number of distinct real roots of $f(x)$ in (a, b) is $V(a) - V(b)$ where $V(c)$ denotes the number of variations in sign of the Strum's sequence $\{f_0(c), f_1(c), \dots, f_s(c)\}$*

11.3 Proof of statement 2 proposition 2

The below diagram illustrates the non-uniformity in news evaluation of leftist readers for news of L and R . Readers who have access to the factual report evaluate news based on the report as well as their ideology. Their level of dissatisfaction from the news of media j is determined by the distance $-\left(\frac{x_i + \theta_E}{2\theta_j^*} - 1\right)^2$. Those without the factual report evaluate news based on the distance $-\left(\frac{x_i}{\theta_j^*} - 1\right)^2$. In the below diagram, I illustrate an example using the equilibrium one of the location choices from table 1. When $\theta_E = -1$, $(\theta_L^*, \theta_R^*) = (-0.919, -0.507)$ when $(\lambda_L, \lambda_R) = (0.5, 0.5)$. The diagram divides the segment of leftist readers on $[-1, 0)$. Readers within each intervals rank the news of L and R differently depending on their ideology and their knowledge of the factual report. In essence, the values of the following four functions are plotted to calculate the differences in news

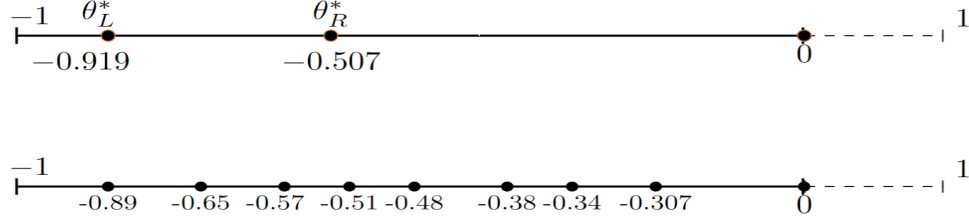


Figure 9: The top figure shows the equilibrium locations θ_L^* and θ_R^* of L and R respectively to report $\theta = -1$ when $(\lambda_L, \lambda_R) = (0.5, 0.5)$. The bottom figure segments the leftist readers into intervals. These intervals distinguish leftist readers based on how they evaluate news of L and R , given that a fraction of them have access to the factual report prior to reading the news. So a portion of readers evaluate news based on the factual report and their ideology while the rest evaluate it only based on ideology.

evaluations of readers in each of these intervals - $\left(\frac{x_i + \theta_E}{2\theta_L^*} - 1\right)^2$, $-\left(\frac{x_i + \theta_E}{2\theta_R^*} - 1\right)^2$, $-\left(\frac{x_i}{\theta_L^*} - 1\right)^2$, $-\left(\frac{x_i}{\theta_R^*} - 1\right)^2$.

For $x_i \in [-1, -0.89)$, $-\left(\frac{x_i + \theta_E}{2\theta_L^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_L^*} - 1\right)^2 > -\left(\frac{x_i + \theta_E}{2\theta_R^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_R^*} - 1\right)^2$.

For $x_i \in [-0.89, -0.65)$, $-\left(\frac{x_i + \theta_E}{2\theta_L^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_L^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_R^*} - 1\right)^2 > -\left(\frac{x_i + \theta_E}{2\theta_R^*} - 1\right)^2$.

For $x_i \in [-0.65, -0.57)$, $-\left(\frac{x_i + \theta_E}{2\theta_L^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_R^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_L^*} - 1\right)^2 > -\left(\frac{x_i + \theta_E}{2\theta_R^*} - 1\right)^2$.

For $x_i \in [-0.57, -0.51)$, $-\left(\frac{x_i}{\theta_R^*} - 1\right)^2 > -\left(\frac{x_i + \theta_E}{2\theta_L^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_L^*} - 1\right)^2 > -\left(\frac{x_i + \theta_E}{2\theta_R^*} - 1\right)^2$.

For $x_i \in [-0.51, -0.48)$, $-\left(\frac{x_i}{\theta_R^*} - 1\right)^2 > -\left(\frac{x_i + \theta_E}{2\theta_L^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_R^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_L^*} - 1\right)^2$.

For $x_i \in [-0.48, -0.38)$, $-\left(\frac{x_i}{\theta_R^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_R^*} - 1\right)^2 > -\left(\frac{x_i + \theta_E}{2\theta_L^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_L^*} - 1\right)^2$.

For $x_i \in [-0.38, -0.34)$, $-\left(\frac{x_i + \theta_E}{2\theta_R^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_R^*} - 1\right)^2 > -\left(\frac{x_i + \theta_E}{2\theta_L^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_L^*} - 1\right)^2$.

For $x_i \in [-0.34, -0.307)$, $-\left(\frac{x_i + \theta_E}{2\theta_R^*} - 1\right)^2 > -\left(\frac{x_i + \theta_E}{2\theta_L^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_R^*} - 1\right)^2 > -\left(\frac{x_i}{\theta_L^*} - 1\right)^2$.

11.4 Proof of proposition 3

For sub part 1, I can simply substitute $\theta_E = 0$ in the above equations 21 and 22 to get

$$g(\theta_L) = \frac{d\Pi_L}{d\theta_L} = \theta_L^4 \left[(1 - \lambda_L) + \frac{c}{(b + (\theta_R)^2)} \right] + \theta_L^3 \left[(1 - \lambda_L) \right] = 0$$

$$g(\theta_R) = \frac{d\Pi_R}{d\theta_R} = \theta_R^4 \left[(1 - \lambda_R) + \frac{c}{(b + (\theta_L)^2)} \right] - \theta_R^3 \left[(1 - \lambda_R) \right] = 0$$

The above system leads to

$$\theta_L^* = \frac{(1 - \lambda_L)}{(1 - \lambda_L) + \frac{c}{b + (\theta_R^*)^2}}$$

$$\theta_R^* = \frac{(1 - \lambda_R)}{(1 - \lambda_R) + \frac{c}{b + (\theta_L^*)^2}}$$

For the second sub part, I again use equations 20 and 21 and substitute $\theta_E = 1$, when the event is extreme pro-right. Symmetric outcomes emerge when $\theta_E = -1$.

$$g(\theta_L) = \frac{d\Pi_L}{d\theta_L} = \theta_L^4 \left[(1 - \lambda_L) + \frac{c}{(b + (\theta_R - 1)^2)} \right] + \theta_L^3 \left[(1 - \lambda_L) - \frac{c}{b + (\theta_R - 1)^2} \right]$$

$$+ 0.5\lambda_L\theta_L - 0.25\lambda_L = 0$$

$$g(\theta_R) = \frac{d\Pi_R}{d\theta_R} = \theta_R^4 \left[(1 - \lambda_R) + \frac{c}{(b + (\theta_L - 1)^2)} \right] - \theta_R^3 \left[(1 - \lambda_R) + \frac{c}{b + (\theta_L - 1)^2} \right]$$

$$+ 0.5\lambda_R\theta_R - 0.25\lambda_R = 0$$

Since it is nearly impossible to derive a closed form solution of (θ_L^*, θ_R^*) , I resort to solutions based on heuristics to give a suggestive solution about where the optimal values will lie. First I provide a closed-form solution of λ_L in presence of $\theta_E = 1$ and $\lambda_L = 0$. This is as follows,

$$\theta_L^* = -\frac{1 - \frac{c}{b + (\theta_R^* - 1)^2}}{1 + \frac{c}{b + (\theta_R^* - 1)^2}}$$

I now need to prove that θ_L^* is sufficiently away from zero and is positive. Now throughout the model, I have assumed $c = 1.1$ and $b = 0.7$. Then θ_L^* is positive iff $(\theta_R^* - 1)^2 < 0.4$. This implies that R locates between $(0.8, 1)$ in equilibrium. If R is motivated towards ideology more strongly, it will report very close to 1 and θ_L^* will be strictly positive. However, when R has almost no ideological motivation, R can place itself a bit away from zero ¹⁶during which θ_L^* will be negative. This happens for $\lambda_L = 0$. So when λ_L is increased beyond zero, then the above inequality becomes less binding and is more easily satisfied.

If one refers to assumption 1 that $b > b'$, then it is reasonable to infer that with a lower value

¹⁶One can refer to table 4 below of the monopoly model to see how any media j reports when λ_j approaches the value 1.

of b , the chances of truthful reporting increases which entails that when L has to report a pro-right event like the one discussed, it will locate farther away from zero towards that event, thereby refraining from indifferent reporting.

A more general way of presenting the conditions when media will refrain from locating near zero is by the following method. I first assume $\lambda_L = 0$ and incorporate it to 20 and 21 to get,

$$g(\theta_L) = \theta_L^4 \left[1 + \frac{c}{(b + (\theta_R - \theta_E)^2)} \right] + \theta_L^3 \left[1 - \frac{c\theta_E}{b + (\theta_R - \theta_E)^2} \right] = 0$$

This gives

$$\theta_L^* = - \frac{1 - \frac{c\theta_E}{b + (\theta_R^* - \theta_E)^2}}{1 + \frac{c}{b + (\theta_R^* - \theta_E)^2}}$$

As θ_E increases beyond zero, it raises the value of $c\theta_E$ which leads to a positive value of θ_L^* . Hence, with a more pro-right topic to cover, L will choose to locate at a point which is farther right away from zero. This holds for $\lambda_L = 0$. Hence for $\lambda_L > 0$ (no matter how small), this shift will be of greater magnitude.

Hence, it is proved that L will refrain from taking an indifferent stance while covering a pro-right event.

11.5 Proof of proposition 4

11.5.1 Change in λ_j

As λ_j is close to 1, it suggests that the representative media j is mostly concerned about payoffs from reader's news evaluation which propels its location choice towards the median reader. Compared this to another media j' such that $\lambda_{j'}$ is close to 0. Then media j' will compromise less on ideology and inject bias in its news which shifts its location towards its own ideology bliss point on the ideology axis. In this example, the bliss points of the two media are at the extreme ends (-1 and 1).

Using *IFT* on the best response function, I arrive at the below equation

$$\frac{d\theta_L^*}{d\lambda_L} = \frac{\theta_L^4 + \theta_L^3 + 0.25\theta_E^2 p^2 - 0.5p\theta_E\theta_L}{4\theta_L^3(1 - \lambda_L + \frac{c}{b + (\theta_R - \theta_E)^2}) + 3\theta_L^2(1 - \lambda_L + \frac{c\theta_E}{b + (\theta_R - \theta_E)^2}) + 0.5p\lambda_L\theta_E} \quad (31)$$

11.5.2 Change in b

$$\frac{d\theta_L^*}{db} = \frac{\frac{-c\theta_L^3(1 - \theta_L)}{(b + (\theta_R - \theta_E)^2)^2}}{4\theta_L^3(1 - \lambda_L + \frac{c}{b + (\theta_R - \theta_E)^2}) + 3\theta_L^2(1 - \lambda_L + \frac{c\theta_E}{b + (\theta_R - \theta_E)^2}) + 0.5p\lambda_L\theta_E} \quad (32)$$

The numerator is positive (negative) when θ_L is negative (positive). θ_L is negative when $\theta_E \leq 0$ and when θ_L lies in the *orange* interval in figure 3 together with the condition that $\lambda < \bar{\lambda}_L$. The terms $1 - \lambda_L + \frac{c}{b+(\theta_R-\theta_E)^2} > 0$ and $1 - \lambda_L + \frac{c\theta_E}{b+(\theta_R-\theta_E)^2} \geq 0$. The latter is negative when $\theta_E < 0$ unless $1 - \lambda_L > \frac{c\theta_E}{b+(\theta_R-\theta_E)^2}$. This is satisfied when L is more ideology oriented or $\lambda_L \rightarrow 0$.

An unambiguous observation about the sign of $\frac{d\theta_L^*}{db}$ can be given whenever θ_L lies to the left of -0.75 where $4\theta_L^3 > 3\theta_L^2$ holds. The denominator becomes negative which implies $\frac{d\theta_L^*}{db} < 0$, given the numerator is negative when $\theta_L^* < 0$. Further whenever $\theta_L^* > 0$, $\frac{d\theta_L^*}{db} < 0$ is satisfied as the denominator stays positive.

11.5.3 Change in p

The equilibrium location choice varies with the fraction of readers who gains access to the factual report. As mentioned earlier, readers with knowledge of factual report have two reference points to evaluate a piece of news - the fact and their subjective ideology. On the other hand, readers without the factual knowledge assess news based on a single reference point - their subjective ideology. Now is it easier for

$$\frac{d\theta_L^*}{dp} = \frac{0.5p\lambda_L\theta_E^2 - 0.5\lambda_L\theta_E\theta_L}{4\theta_L^3(1 - \lambda_L + \frac{c}{b+(\theta_R-\theta_E)^2}) + 3\theta_L^2(1 - \lambda_L + \frac{c\theta_E}{b+(\theta_R-\theta_E)^2}) + 0.5p\lambda_L\theta_E} \quad (33)$$

Let us see what happens when L has to report any event to the right of the *orange* interval in figure 3. In this case $\theta_L^* > 0$ for all values of $\lambda_L \in (0, 1)$ such that $\theta_L < \theta_E$. Then the denominator is positive. The numerator is positive iff $0.5p\lambda_L\theta_E^2 - 0.5\lambda_L\theta_E\theta_L$ or $p > \frac{\theta_L}{\theta_E}$. There the sign of $\frac{d\theta_L^*}{dp}$ is positive (negative) iff $p > \frac{\theta_L}{\theta_E}$ ($p < \frac{\theta_L}{\theta_E}$). If $\frac{d\theta_L^*}{dp} = 0$ if $(p = \frac{\theta_L}{\theta_E})$

Now consider L reports events within the *orange* interval in figure 3. Then $\theta_L^* < 0$ ($\theta_L^* > 0$) for $\lambda_j \leq \bar{\lambda}_L$ ($\lambda_j > \bar{\lambda}_L$). When $\theta_L^* < 0$, then numerator is positive. When $\theta_L^* > 0$, then numerator too is positive as $\theta_L < \theta_E$ for $\lambda_j > \bar{\lambda}_L$. So when $\theta_L^* < 0$, the sign of $\frac{d\theta_L^*}{dp}$ depends on whether the following inequality holds $-4|\theta_L|^3(1 - \lambda_L + \frac{c}{b+(\theta_R-\theta_E)^2}) < 3\theta_L^2(1 - \lambda_L + \frac{c\theta_E}{b+(\theta_R-\theta_E)^2}) + 0.5p\lambda_L\theta_E$

For ideologically neutral events where $\theta_E = 0$, $\frac{d\theta_L^*}{dp} = 0$ as the numerator reduces to zero.

For events biased to the left where $\theta_E < 0$, $sign \frac{d\theta_L^*}{dp}$ depends on λ_L . As $\lambda_L \rightarrow 0$, $|\theta_L| > |\theta_E|$ which makes the numerator a negative number. As $\lambda_L \rightarrow 1$, $|\theta_L| \leq |\theta_E|$ which makes the numerator a positive number iff $p > \frac{\theta_L}{\theta_E}$.

11.6 Proof of proposition 5

By envelope theorem, the effect of a change in the maximum value function is equal to the direct effect of the parameters. We differentiate the profit function of L in equation 4 at equilibrium editorial stance of L . This will also hold true for media R .

$$\frac{dV_L}{d\lambda_L^*} = -(\alpha_L^* - 1)^2 + (\theta_L^* + 1)^2$$

Upon expanding,

$$\frac{dV_L}{d\lambda_L^*} = \frac{\theta_E}{\theta_L} - \left(\frac{\theta_E}{2\theta_L}\right)^2 + (\theta_L^*)^2 + 2\theta_L^*$$

If topic is neutral or $\theta_E = 0$, then $\frac{dV_L}{d\lambda_L^*} = (\theta_L^*)^2 + 2\theta_L^*$. Now, suppose $\theta_E = q \in \mathbb{R}_{++}$ or a pro-right topic but not an extreme one, or $q < 1$. Then $\frac{dV_L}{d\lambda_L^*}$ is U-shaped. As $\lambda_L \rightarrow 0$, the fraction $\frac{\theta_E}{\theta_L}$ is negative. Now as λ_L increases such that $|\theta_L^*|$ decreases, then $\frac{\theta_E}{\theta_L}$ becomes more negative until λ_L increases enough to make L locate on the positive part of the ideology axis. Therefore, for ideologically negative issues, the maximum value function is U-shaped.

At the above threshold, the derivative of the maximum value function with respect to the equilibrium editorial choice vanishes.

11.7 Proof of proposition 6

I examine the effects of strategic substitutability and complementarity with of L with respect to changes in location choices of R . Analogous observations will hold as the model is symmetric. To proceed, I take the derivative of L 's best response with respect to θ_R which gives the following.

$$\frac{d^2\Pi_L}{d\theta_L d\theta_R} = -\theta_L^4 \frac{2c(\theta_R - \theta_E)}{(b + (\theta_R - \theta_E)^2)^2} + \theta_L^3 \frac{2c\theta_E(\theta_R - \theta_E)}{(b + (\theta_R - \theta_E)^2)^2} \quad (34)$$

When $\theta_E = 0$, the above reduces to

$$\frac{d^2\Pi_L}{d\theta_L d\theta_R} = -\theta_L^4 \frac{2c\theta_R}{(b + (\theta_R)^2)^2} < 0$$

The inequality sign holds because $\theta_R > 0$ for all values of λ_R when $\theta_E = 0$. Therefore this suggests that any change in location choice by R will reduce L 's profit. The magnitude of this effect is greater when $|\theta_L|$ farther away from zero, i.e. when $\lambda_L \rightarrow 0$ and in tandem, λ_R is closer to zero i.e. when $\lambda_R \rightarrow 1$. So, it is evident that strategic substitutability gains force when L and R 's primary profit motive is different.

Now, suppose θ_E is positive. Then let's analyse four cases - a. $(\lambda_L, \lambda_R) \rightarrow (1, 1)$ b. $(\lambda_L, \lambda_R) \rightarrow (1, 0)$ c. $(\lambda_L, \lambda_R) \rightarrow (0, 1)$ d. $(\lambda_L, \lambda_R) \rightarrow (0, 0)$. In case a, $\theta_L^* > 0$ and $|\theta_R^*| < |\theta_E|$ which implies the first term in 29 to be positive while the second is negative. However in terms of magnitude, the second term is weakly lesser than the first term which suggests $\frac{d^2\Pi_L}{d\theta_L d\theta_R} \geq 0$. In case b, $\theta_L^* > 0$ and $|\theta_R^*| > |\theta_E|$, and by the previous reasoning, $\frac{d^2\Pi_L}{d\theta_L d\theta_R} \leq 0$. In case c, $\theta_L^* < 0$ while $\theta_R^* < \theta_E$

making both the terms positive in 29. In case d, $\theta_L^* < 0$ and $\theta_R^* \geq \theta_E$. The sign of $\frac{d^2\Pi_L}{d\theta_L d\theta_R} \geq 0$ iff $|\theta_L^3 \cdot \theta_E| > |\theta_L^4|$.

11.8 Comparative statics in unbalanced reader population

$$\frac{d\theta_L^*}{d\lambda_L} = \frac{\theta_L^4 + \theta_L^3 + 0.25(\theta_E + \kappa)^2 - 0.5(\theta_E + \kappa)\theta_L}{4\theta_L^3(1 - \lambda_L + \frac{c}{b+(\theta_R - \theta_E)^2}) + 3\theta_L^2(1 - \lambda_L - \frac{c\theta_E}{b+(\theta_R - \theta_E)^2}) + 0.5\lambda_L(\theta_E + \kappa)} \quad (35)$$

$$\frac{d\theta_L^*}{db} = \frac{\frac{-c\theta_L^3(1-\theta_L)}{(b+(\theta_R - \theta_E)^2)^2}}{4\theta_L^3(1 - \lambda_L + \frac{c}{b+(\theta_R - \theta_E)^2}) + 3\theta_L^2(1 - \lambda_L - \frac{c\theta_E}{b+(\theta_R - \theta_E)^2}) + 0.5\lambda_L(\theta_E + \kappa)} \quad (36)$$

$$\frac{d\theta_L^*}{d\kappa} = \frac{0.5\theta_L(\theta_E + \kappa) - 0.5\lambda_L\theta_L}{4\theta_L^3(1 - \lambda_L + \frac{c}{b+(\theta_R - \theta_E)^2}) + 3\theta_L^2(1 - \lambda_L - \frac{c\theta_E}{b+(\theta_R - \theta_E)^2}) + 0.5\lambda_L(\theta_E + \kappa)} \quad (37)$$

To elicit some important characteristics of media behavior, we first assume the parameters signifying the majority share and the cost of biasing through cross-over effect to be below some cutoff such that $|\kappa| < \hat{\kappa}$ and $b \geq \hat{b}$. This guarantees that the editorial choice are not shackled too much either by the extent of biased readers or by a very high cost of bias.

11.9 Proof of proposition 7

The Bayesian updation of a reader i 's belief about the success of policy y_l at θ_E after learning L 's endorsement position θ_L

$$\sigma(y_l|\theta_L, \theta_E) = \frac{g(1 - \theta_E) \cdot \pi(\theta_L|y_l, \theta_E)}{g(1 - \theta_E) \cdot \pi(\theta_j|y_l, \theta_E) + g(\theta_E) \cdot \pi(\theta_L|y_l^c, \theta_E)} \quad (38)$$

The term $\pi(\theta_L|y_l, \theta_E)$ is the conditional probability that L 's endorsement θ_L of policy y_l is true given that y_l is viable for a given value of θ_E . I formalize this through a reduced form function $\pi(\alpha_{iL}, \beta_l)$. When α_{iL} is a negative transformation of L 's news evaluation of reader i , such that the closer is α_{ij} to 1 (following Lemma 1), higher is α_{iL} which increases π . π is also a function of β_l whose lower value denotes higher leverage of y_l given θ_E .

In the denominator, the term $\pi^c(\theta_L|y_l, \theta_E)$ is the conditional probability that L 's endorsement θ_L of policy y_l is true given that y_l is not viable for a given value of θ_E . The reduced form equation is given by $\pi(\alpha_{iL}, \beta_l)$ such that π^c rises with the distance of α_{ij} from 1 and the value of β .

As a numerical example, let $\theta_L^* = -0.4$ when $\theta_E = 0$. For reader at -0.5 $\alpha_{iL} = 0.125$ such that $\alpha_{iL} = 1 - 0.125 = 0.875$. Then if $y_l = y_r = 1$, $\beta_l = \arctan -2.5 \sim -68.2^\circ$. Then equation

38 becomes

$$\sigma(y_l|\theta_L, \theta_E) = \frac{\pi(0.875, -68.2)}{\pi(0.875, -68.2) + \pi^c(0.875, -68.2)}$$

For an individual at 0.5, whose evaluation $\alpha_{iL} = -0.125$ the belief updation will take the following form

$$\sigma(y_l|\theta_L, \theta_E) = \frac{\pi(1.125, -68.2)}{\pi(1.125, -68.2) + \pi^c(1.125, -68.2)}$$

12 Model of a Monopoly news market

We do the similar analysis with only one partisan media serving the readers. Without loss of generality, let that firm be L situated at -1 on the ideology axis $[-1, 1]$. Symmetric results will hold if R is the monopoly firm.

This exercise is done to separate out the effects of competition for readership (in the absence of R) and learn the magnitude and direction of media slant due to ideology.

12.1 Utility Maximization of any reader i

This strategy of reader i is providing a rating to media L 's report. Rating is a mapping from the ideology space to the real line \mathbb{R} , $\alpha_{iL} : \theta_L \rightarrow \mathbb{R}$.

$$U_i(\alpha_{iL}|\theta_L, \theta_E) = -(\alpha_{iL}\theta_L - x_i)^2 - (\alpha_{iL}\theta_L - \theta_E)^2 \quad (39)$$

The first term is quadratic loss in the distance between i 's ideology x_i and the value (α_{iL}) which i attaches to the editorial position of L . The second term is similarly the distance between the weighted editorial position and the true signal θ_E from media E .

First Order Condition gives

$$\alpha_{iL}^* = \frac{x_i + \theta_E}{2\theta_L} = \frac{x_i + \theta_E}{\theta_L + \theta_L} \quad (40)$$

Second Order Condition for utility maximization is,

$$\frac{d^2U}{d\theta_L^2} = -4\theta_L^2 < 0 \quad (41)$$

12.2 Backward Induction by Media L

The action of the firm L is choosing a optimal editorial position $\theta_L^* \in [-1, 1]$, where

$\lambda_L \setminus \theta_E$	-1	0	1
0.1	-0.987	-0.45	0.296
0.5	-0.915	-0.312	0.453
0.9	-0.778	-0.083	0.635

Table 4: Equilibrium editorial position of media $L(\theta_L^*)$

$$\theta_L^* = \operatorname{argmax}_{\theta_L} \Pi_L(\theta_L)$$

Analogous interpretation holds for the optimal editorial stance θ_R^* of media R .

The payoff function of L is a quadratic loss function as shown below.

$$\Pi_L(\theta_L | \lambda_L, \theta_E) = -\lambda_L \cdot (E(\alpha_L^*) - 1)^2 - (1 - \lambda_L)(\theta_L + 1)^2 - c(\theta_L - \theta_E)^2 \quad (42)$$

Equation (15) shows media L minimizes losses from two sources. θ_L^* minimizes the distance of reader i from attaining his best rating of 1 (given by $(E(\alpha_L^*) - 1)^2$). Simultaneously this choice also determines L 's distance from its preferred ideology position of -1 (given by $(\theta_L + 1)^2$).

The final term $c(\theta_L - \theta_E)^2$ denotes the convex cost of biasing news which is increasing with the distance of θ_L from the unbiased position θ_E . Moving farther away from the true signal θ_E require media to modify information more, thereby they incur higher cost. c denotes the marginal cost parameter with $c > 1$.

First order condition of equation 6 gives:

$$\theta_L^4((1 - \lambda_L) + c) + \theta_L^3((1 - \lambda_L) - c\theta_E) + 0.5\lambda_L\theta_E\theta_L - 0.25\lambda_L(\theta_E)^2 = 0 \quad (43)$$

For clarity and ease of comparison with the previous sections we denote the equilibrium editorial choice of monopoly media L as θ_L^{M*} .¹⁷

Remark 8. Comparing with proposition 1, θ_L^{M*} is always more biased towards the left than θ_L^* . The class of events supporting multiple equilibria is more biased to the right compared to the one in proposition 1.

Table 4 illustrates the unique equilibrium values of monopoly media L which underscores corollary 2.¹⁸

¹⁷ If R was the monopoly media, it would have been θ_R^{M*}

¹⁸To derive the numbers Table 4, we assume $c = 1.1$ and $b = 0.7$.

12.3 Comparative Statics

We devote this section to bring out subtle insights on how the parameter λ_L affects equilibrium strategy θ_L^* . In essence, we express θ_L^* as a function of λ_L .

Applying IFT on equation (7), we get,

$$\frac{\partial \theta_L}{\partial \lambda_L} = \frac{\theta_L^4 + \theta_L^3 + 0.25\theta_E^2 - 0.5\theta_E\theta_L}{4\theta_L^3(1 - \lambda_L + c) + 3\theta_L^2(1 - \lambda_L - c\theta_E) + 0.5\lambda_L\theta_E} \quad (44)$$

The below proposition entails that moving away from the truth does pay the media with higher payoff upto threshold. For higher values of λ_L (media caring more about rating), then it will move closer to the truth.

Remark 9. *There exists a threshold $\lambda_L^* \in (0, 1)$ such that $\frac{\partial \theta_L^*}{\partial \lambda_L} < 0$ for $\lambda_L \in (0, \lambda_L^*)$ and $\frac{\partial \theta_L^*}{\partial \lambda_L} > 0$, for any $\lambda_L > \lambda_L^*$.*

The next proposition gives us a fair understanding of a comparison of the equilibrium profit levels of L due to a change in the values of the exogenous parameter λ_L . Once we express the equilibrium solution to the maximization problem in (6), we have θ_L^* as a function of λ_L and θ_E . If we substitute θ_L^* in the profit function, we obtain the maximum value profit function $V_L(\cdot)$ of media L . This is a function of λ_L given a certain event characterized by θ_E . The variation in V_L due to changes in λ_L is a direct outcome of the envelope theorem.

12.4 Policy recommendation

As suggested by the educative role of media E instills a habit within readers to put greater weight on the true facts of the event which leads to the following characterization of news of media j by reader i .

$$\hat{\alpha}_{ij}^* = \frac{(2 - \beta).x_i + \beta.\theta_E}{2\theta_j^*} \quad (45)$$

Initially, without such such educative role, this characterization is given by

$$\alpha_{ij}^* = \frac{x_i + \theta_E}{2\theta_j^*} \quad (46)$$

Policy recommendation matters when this role of media E increases welfare from reading news which is implied by the following.

$$-\sum_i [\hat{\alpha}_{ij}^* - 1]^2 > -\sum_i [\alpha_{ij}^* - 1]^2 \quad (47)$$

Expanding this leads to the following inequality

$$\begin{aligned}
& -(2 - \beta)^2 \cdot \frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - N \cdot \frac{\beta^2 \cdot \theta_E^2}{4\hat{\theta}_j^2} - \frac{2 \cdot (2 - \beta) \cdot \beta \cdot \theta_E \cdot K}{4\hat{\theta}_j^2} - N + \frac{(2 - \beta) \cdot K}{\hat{\theta}_j} + \frac{N \cdot \beta \cdot \theta_E}{\hat{\theta}_j} > \\
& -\frac{\sum_i x_i^2}{4\theta_j^2} - N \cdot \frac{\theta_E^2}{4\hat{\theta}_j^2} - \frac{2 \cdot \theta_E \cdot K}{4\theta_j^2} - N + \frac{K}{\theta_j} + \frac{N \cdot \theta_E}{\theta_j}
\end{aligned} \tag{48}$$

where $\sum_i x_i = K$ denotes the polarization level.

12.4.1 When the event is neutral ($\theta_E = 0$)

Given $\theta_E = 0$, equation 35 is reduced to

$$-(2 - \beta)^2 \cdot \frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - N + \frac{(2 - \beta) \cdot K}{\hat{\theta}_j} > -\frac{\sum_i x_i^2}{4\theta_j^2} - N + \frac{K}{\theta_j}$$

or,

$$-(2 - \beta)^2 \cdot \frac{\sum_i x_i^2}{4\hat{\theta}_j^2} + \frac{(2 - \beta) \cdot K}{\hat{\theta}_j} > -\frac{\sum_i x_i^2}{4\theta_j^2} + \frac{K}{\theta_j} \tag{49}$$

The first terms on either side resembles the loss in utility due to increased spread of readers around the point 0, denoted by $\sum_i x_i^2$. The introduction of the new role of θ_E assuages the loss by a factor $0 < (2 - \beta)^2 < 1$, given $\beta > 1$. Given $\theta_E = 0$, both θ_L and $\hat{\theta}_L$ are negative while θ_R and $\hat{\theta}_R$ are positive.

- When $K = 0$, then the new policy is effective iff the following holds

$$-(2 - \beta)^2 \cdot \frac{\sum_i x_i^2}{4\hat{\theta}_j^2} > -\frac{\sum_i x_i^2}{4\theta_j^2} \tag{50}$$

- When $K = -1$, then the new policy is effective iff the following holds

$$-(2 - \beta)^2 \cdot \frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - \frac{(2 - \beta)}{\hat{\theta}_j} > -\frac{\sum_i x_i^2}{4\theta_j^2} - \frac{1}{\theta_j} \tag{51}$$

- When $K = 1$

$$-(2 - \beta)^2 \cdot \frac{\sum_i x_i^2}{4\hat{\theta}_j^2} + \frac{(2 - \beta)}{\hat{\theta}_j} > -\frac{\sum_i x_i^2}{4\theta_j^2} + \frac{1}{\theta_j} \tag{52}$$

12.4.2 When event favours the left ($\theta_E = -1$)

When the event favours the left, overall reader utility can increase if the following inequality holds (follows from equation 35).

$$\begin{aligned}
 -(2 - \beta)^2 \cdot \frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - N \cdot \frac{\beta^2}{4\hat{\theta}_j^2} + \frac{2 \cdot (2 - \beta) \cdot \beta \cdot K}{4\hat{\theta}_j^2} - N - \frac{(2 - \beta) \cdot K}{\hat{\theta}_j} - \frac{N \cdot \beta}{\hat{\theta}_j} > \\
 - \frac{\sum_i x_i^2}{4\theta_j^2} - N \cdot \frac{1}{4\theta_j^2} + \frac{2 \cdot K}{4\theta_j^2} - N + \frac{K}{\theta_j} - \frac{N}{\theta_j}
 \end{aligned} \tag{53}$$

- When $K = 0$, then the new policy is effective iff the following holds

$$-(2 - \beta)^2 \cdot \frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - N \cdot \frac{\beta^2}{4\hat{\theta}_j^2} - \frac{N \cdot \beta}{\hat{\theta}_j} > - \frac{\sum_i x_i^2}{4\theta_j^2} - N \cdot \frac{1}{4\theta_j^2} - \frac{N}{\theta_j} \tag{54}$$

Upon simplifying terms,

$$\frac{\sum_i x_i^2}{4} \left[\frac{1}{\theta_j^2} - \frac{(2 - \beta)^2}{\hat{\theta}_j^2} \right] > -N \left[\frac{\beta}{\hat{\theta}_j} - \frac{1}{\theta_j} \right] + 0.25N \left[\frac{\beta^2}{\hat{\theta}_j^2} - \frac{1}{\theta_j^2} \right] \tag{55}$$

- When $K = -1$, then the new policy is effective iff the following holds

$$\begin{aligned}
 -(2 - \beta)^2 \cdot \frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - N \cdot \frac{\beta^2}{4\hat{\theta}_j^2} - \frac{2 \cdot (2 - \beta) \cdot \beta}{4\hat{\theta}_j^2} + \frac{(2 - \beta)}{\hat{\theta}_j} - \frac{N \cdot \beta}{\hat{\theta}_j} > - \frac{\sum_i x_i^2}{4\theta_j^2} - N \cdot \frac{1}{4\theta_j^2} \\
 - \frac{2}{4\theta_j^2} - \frac{1}{\theta_j} - \frac{N}{\theta_j}
 \end{aligned} \tag{56}$$

Simplifying terms

$$\begin{aligned}
 \frac{\sum_i x_i^2}{4} \left[\frac{1}{\theta_j^2} - \frac{(2 - \beta)^2}{\hat{\theta}_j^2} \right] + 0.5 \left[\frac{1}{\theta_j^2} - \frac{(2 - \beta)\beta}{\hat{\theta}_j^2} \right] > N \left[\frac{\beta}{\hat{\theta}_j} - \frac{1}{\theta_j} \right] + 0.25N \left[\frac{\beta^2}{\hat{\theta}_j^2} - \right. \\
 \left. \frac{1}{\theta_j^2} \right] - \left[\frac{1}{\theta_j} + \frac{2 - \beta}{\hat{\theta}_j} \right]
 \end{aligned} \tag{57}$$

- When $K = 1$, then the new policy is effective iff the following holds

$$\begin{aligned}
& -(2 - \beta)^2 \cdot \frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - N \cdot \frac{\beta^2}{4\hat{\theta}_j^2} + \frac{2 \cdot (2 - \beta) \cdot \beta}{4\hat{\theta}_j^2} - \frac{(2 - \beta)}{\hat{\theta}_j} - \frac{N \cdot \beta}{\hat{\theta}_j} > \\
& - \frac{\sum_i x_i^2}{4\theta_j^2} - N \cdot \frac{1}{4\theta_j^2} + \frac{2}{4\theta_j^2} + \frac{1}{\theta_j} - \frac{N}{\theta_j}
\end{aligned} \tag{58}$$

$$\begin{aligned}
& \frac{\sum_i x_i^2}{4} \left[\frac{1}{\theta_j^2} - \frac{(2 - \beta)^2}{\hat{\theta}_j^2} \right] - 0.5 \left[\frac{1}{\hat{\theta}_j^2} - \frac{(2 - \beta)\beta}{\hat{\theta}_j^2} \right] > -N \left[\frac{\beta}{\hat{\theta}_j} - \frac{1}{\theta_j} \right] + \\
& 0.25N \left[\frac{\beta^2}{\hat{\theta}_j^2} - \frac{1}{\theta_j^2} \right] + \left[\frac{1}{\theta_j} + \frac{2 - \beta}{\hat{\theta}_j} \right]
\end{aligned} \tag{59}$$