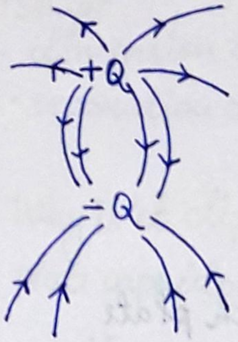
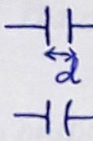
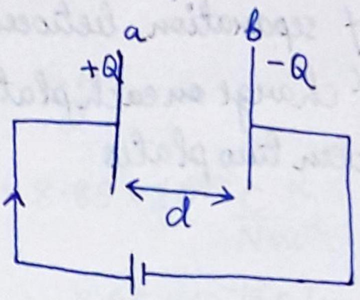
6.05.21

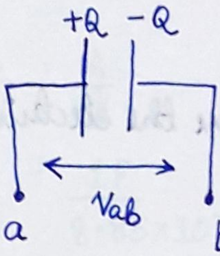
Capacitance and Dielectrics

Capacitors and Capacitance









$$C = \frac{Q}{V_{ab}}$$

$$Q \propto V_{ab}$$

$$V \propto Q$$

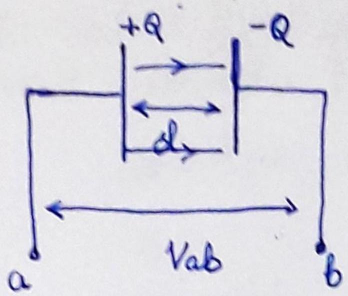
$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r}$$

$$1F = 1\,\text{farad} = \frac{1C}{1V}$$

$$1\mu F = 10^{-6} F$$

$$1pF = 10^{-12} F$$

Parallel plate capacitor



A - cross-sectional area of each plate

d - distance of separation between two plates

Q - amount of charge on each plate

V_{ab} - p.d between two plates

$$\sigma = \frac{Q}{A}$$

$$P = \frac{Q}{V}$$

Apply gauss law to define the electric field

$$E = \frac{\sigma}{\varepsilon_0}$$

$$= \frac{Q}{\varepsilon_0 A}$$

$$V_{ab} = Ed = \frac{Qd}{\varepsilon_0 A}$$

$$C = \frac{Q}{V_{ab}} = \frac{Q}{\frac{Qd}{\varepsilon_0 A}}$$

$$C = \frac{\varepsilon_0 A}{d}$$

$$\varepsilon_0 = 8 \cdot 85 \times 10^{-12} \, C^2/Nm^2$$

$C \propto A$, $C \propto \frac{1}{d}$, $C \propto$ medium.

The capacitance of a capacitor depends on
- shape
- dimension and
- separation of What the capacitor is made of

Why is it difficult to prepare 1F capacitance of a parallel plate capacitor placed in vacuum.

1F

$$C = \frac{\varepsilon_0 A}{d}$$

$$\varepsilon = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$$

$$= 8.85 \times 10^{-12} F/m$$

$$\frac{C}{\varepsilon_0} = \frac{A}{d}$$

$$= \frac{1F}{8.85 \times 10^{-12} F/m} = \frac{A}{d}$$

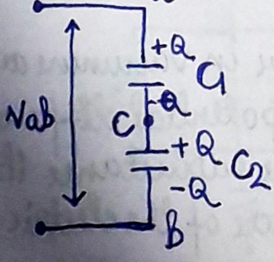
$$\frac{A}{d} \approx 10^{12} m$$

$d = 1m$ $\quad$ $A = 10^{12} m^2$

$A = 1m^2$ $\quad$ $d = 10^{-12} m$

Capacitors in series and parallel

Capacitors in series



$$V_{ab} = V_{ac} + V_{cb} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$C = \frac{Q}{V}$$

$$\frac{V_{ab}}{Q} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$V = \frac{Q}{C}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

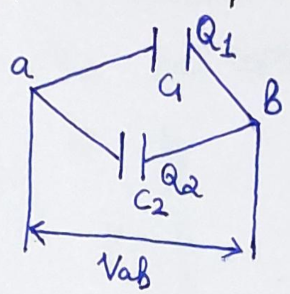
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

If $C_1 = C_2 = C_3 = \cdots C_n$

$$\frac{1}{C} = \frac{N}{C_1}$$

$$C = \frac{C_1}{N}$$

Capacitance in parallel



$$Q = Q_1 + Q_2$$

$$C = \frac{Q}{V_{ab}} = Q = CV_{ab}$$

$$Q = C_1 V_{ab} + C_2 V_{ab}$$

$$\frac{Q}{V_{ab}} = C_1 + C_2$$

$$C = C_1 + C_2$$

$$C = C_1 + C_2 + C_3 \cdots$$

If $C_1 = C_2 = C_3 = \cdots C_n$

$$C = NC_1$$

1. The plates of a parallel-plate capacitor in vacuum are 5.00 apart and 2.00 m² in area. A 10.0 kV potential difference is applied across the capacitor. Compute (a) the capacitance (B) the charge on each plate and (c) the magnitude of the electric field between the plates

Solution

$d = 5.00\,mm$

$= 5 \times 10^{-3}\,m$

$A = 2\,m^2$

$V = 10\,kV$

a) $C = \varepsilon_0 \frac{A}{d}$

$= \frac{8.85 \times 10^{-12} \times 2}{5 \times 10^{-3}}$

$= 3.54 \times 10^{-9}\,F$

b) $C = \frac{Q}{V}$

$Q = CV$

$= 3.54 \times 10^{-9} \times 10 \times 10^{3}$

$= 3.54 \times 10^{-5}\,C$

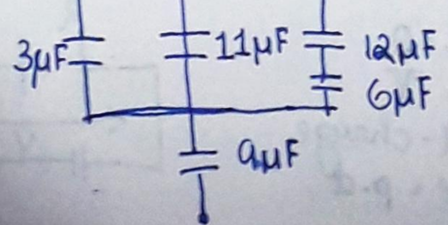
c) $E = \frac{V}{d}$

$= \frac{10 \times 10^{3}}{5 \times 10^{-3}}$

$= 2 \times 10^{6}\,V/m$

$E = \frac{Q}{\varepsilon_0 A} = 2 \times 10^{6}\,V/m$

2: Find the equivalent capacitance of the five-capacitor network



Solution

$\frac{1}{C_1} = \frac{1}{12} + \frac{1}{6}$

$= 4\mu F$

$C_2 = 3\mu F + 11\mu F + 4\mu F$

$= 18\mu F$

$\frac{1}{C_3} = \frac{1}{9} + \frac{1}{18}$

$= 6\mu F$

3. A capacitor has vacuum in the space between the conductors. If you double the amount of charge on each conductor, what happens to the capacitance. (i) increases (ii) decreases (iii) it remains the same (iv) the answer depends on the size and shape of the conductors

Solution

$C = \frac{Q}{V_{ab}} \Rightarrow C' = \frac{Q'}{V_{ab}'}$

$Q = 2Q$

$Q \propto V_{ab}$

$V_{ab}' = 2V_{ab}$

$C' = \frac{2Q}{2V_{ab}}$

$= \frac{Q}{V_{ab}} = C$

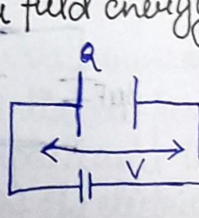
$\therefore$ iii remains the same

8.05.21 Energy storage in capacitors & electric field energy.

$C = \frac{Q}{V}$

Q - charge

V = p.d



$$V = \frac{Q}{C}$$

At an intermediate time

Let q = amount of charge stored in each plate

$$V = q/C$$

dq - charge

$$dW = V \cdot dq$$

$$V = q/C$$

$$dW = \frac{q}{C}\,dq$$

$$\int_0^W dW = W = \frac{1}{C}\int_0^Q q\,dq$$

$$W = \frac{Q^2}{2C}$$

$$C = \frac{Q}{V}$$

$$W = \frac{1}{2}QV$$

$$Q = CV \qquad W = \frac{1}{2}CV^2$$

$$W = U$$

Electric field energy = energy density $= u = \frac{U}{\text{Volume}}$

$$= \frac{\frac{1}{2}CV^2}{Ad}$$

$$C = \frac{\varepsilon_0 A}{d}$$

$$u = \frac{\frac{1}{2}\frac{\varepsilon_0 A}{d} V^2}{Ad}$$

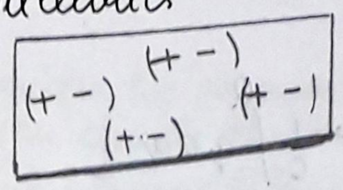
$$u = \frac{1}{2}\varepsilon_0 \frac{V^2}{d^2}$$

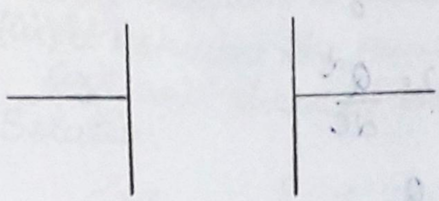
$$E = \frac{V}{d}$$

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Increase in u.

Dilectrics





$$C_0 = \frac{Q}{V_0}$$

Q = charge

V_0 = p.d

Separation b/w 2 plates has dielectrics K

$$C = \frac{Q}{V}$$

$$V < V_0$$

$$K = \frac{C}{C_0}$$

$$C_0 = \frac{\varepsilon_0 A}{d}$$

$$C = K\varepsilon_0 \frac{A}{d}$$

$$\frac{C}{C_0} = K$$

$$C = KC_0$$

$$C_0 = \frac{Q}{V_0}$$

$$C = \frac{Q}{V}$$

$$\frac{C}{C_0} = \frac{V_0}{V}$$

$$\frac{V_0}{V} = K$$

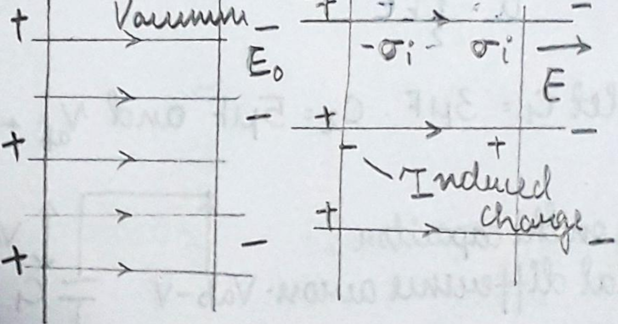
$$V = \frac{V_0}{K}$$

$$V_0 = E_0 d$$

$$V = E d$$

$$\frac{E_0}{E} = K$$

Induced charge and Polarisation



Vaccum

$E_0 = \frac{\sigma}{\varepsilon_0}$

Dielectrics

$E = \frac{\sigma - \sigma_i}{\varepsilon_0}$

$\frac{E}{E_0} = \frac{\sigma - \sigma_i}{\sigma}$

$\frac{\sigma_i}{\sigma} = 1 - \frac{1}{K}$

$\sigma_i = \left(1 - \frac{1}{K}\right)\sigma$

$\varepsilon = K\varepsilon_0$

$C = KC_0$

$= \frac{K\varepsilon_0 A}{d}$

$Q_i = 1 - \frac{1}{K}$

Energy density in presence of dillectrics

$u_0 = \frac{1}{2}\varepsilon_0 E^2$

$u = \frac{1}{2}K\varepsilon_0 E^2$

$u = \frac{1}{2}\varepsilon E^2$

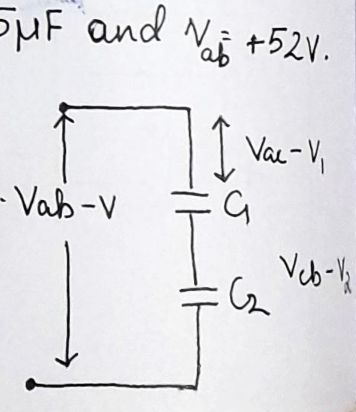
In the figure let $C_1 = 3\mu F$. $C_2 = 5\mu F$ and $V_{ab} = +52V$.

Calculate

a the charge on the capacitor

b the potential difference across.

each capacitor



$C_1 = 3\times10^{-6}F$

$C_2 = 5\times10^{-6}F$

$V_{ab} = 52V$

a)

$C = \frac{Q}{V}$

$Q = CV$

$C = \frac{C_1C_2}{C_1+C_2}$

$= 1.88\times10^{-6}$

$Q = 1.88\times10^{-6}\times52$

$= 97.76\times10^{-6}C$

$V_1 = \frac{Q}{C_1} = \frac{97.76\times10^{-6}}{3\times10^{-6}} = 32.58\times10^{-6+6}V$

$= 32.58V$

$V_2 = \frac{Q}{C_2} = \frac{97.76\times10^{-6}}{5\times10^{-6}} = 19.552V$