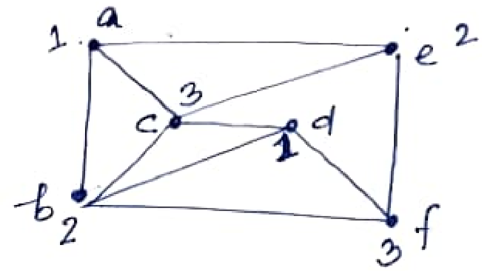
Ch. 5

Q. 36. Q. 18 (GATE 2018)

$\chi(G) = 3$



Ch. 6

Q. 37. Q. 16 (GATE 2021)

$n = 8$

$f = 5$

Euler's formula : $n - e + f = 2$

$\Rightarrow 8 - e + 5 = 2$

$\Rightarrow e = 11$

$\therefore$ No. of edges is 11.

Q. 38. Q. 3. (GATE 2016, SET 2)

According to Four Color Theorem The minimum number of colours that is sufficient to vertex colour any planar graph is 4.

Q. 39. Q. 54 (GATE 2015, SET 1)

$n = 10$

edges covering each face $= 3$

As every edge is shared by 2 faces

$2e = 3f$

$\Rightarrow f = \frac{2}{3} e$

By Euler's formula

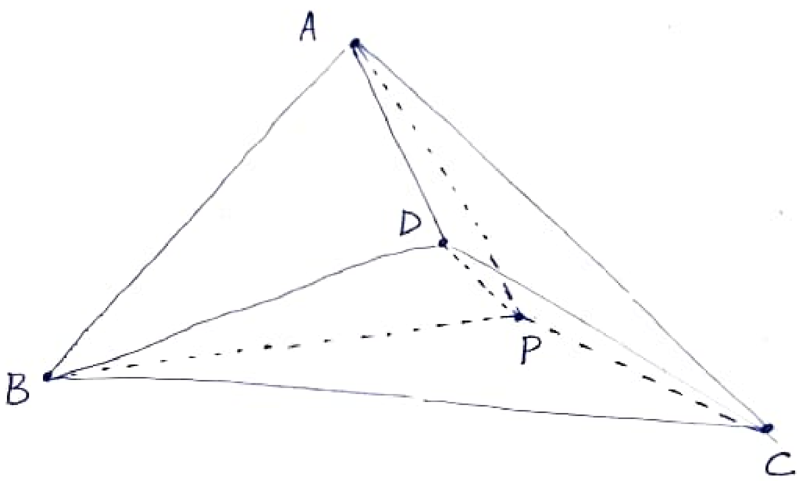
$$n - e + f = 2$$

$$\Rightarrow 10 - e + \frac{2}{3}e = 2$$

$$\Rightarrow e = 24$$

So the number of edges in G is 24.

Q. 40. Q. 10 (GATE 2014, SET 1)



ABCD is a tetrahedron.

P is a point inside connecting to each corner A, B, C & D of the tetrahedron. So we have the internal planes ABP, APC, BPC, DPC, DPB and DPA which are 6 in number.

Q. 41 Q. 52 (GATE 2014, SET 3)

Not in course

Q. 42 Q. 17 (GATE 2012)

$n = 10$

$e = 15$

By Euler's formula

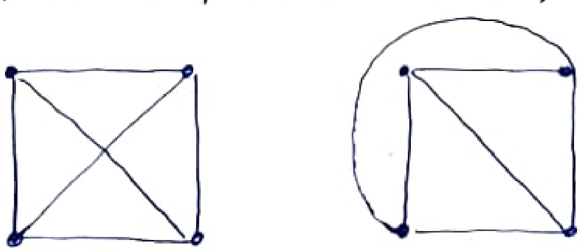
$n - e + f = 2$

$\Rightarrow 10 - 15 + f = 2$

$\Rightarrow f = 7$

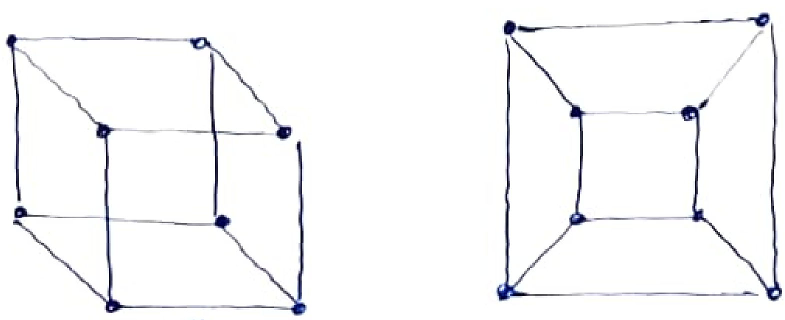
Out of the 7 faces one must be unbounded. So the no. of bounded faces is 6.

Q. 43 Q. 17 (GATE 2011)



K_4

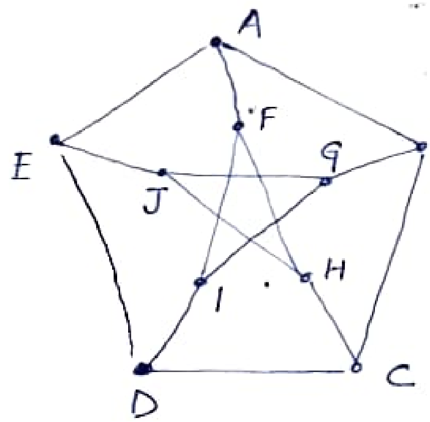
planar



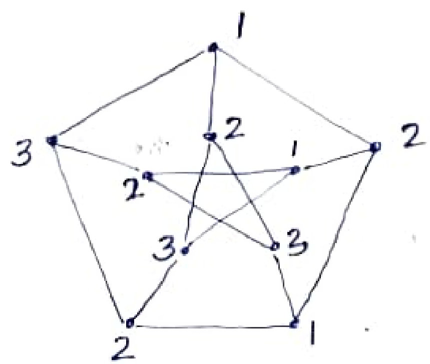
Q_3

planar

Q.44 (GATE 2022)



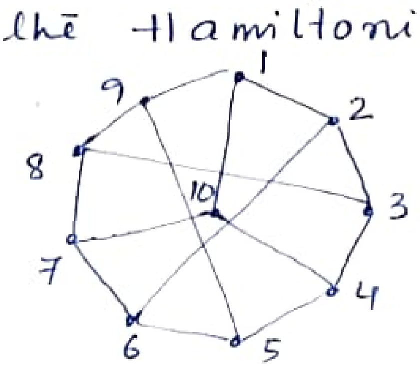
A : Chromatic number of the graph is 3.



B : The Petersoon graph has a Hamiltonian path but not a Hamiltonian cycle.

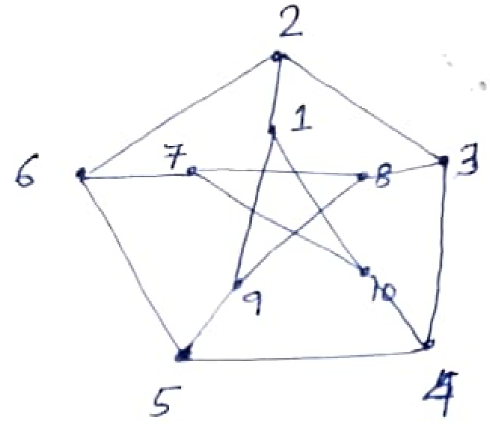
Hamiltonian path : E - A - F - I - D - C - B - G - J - H

C : The following graph is isomorphic to the Hamiltonian graph



1 ↔ F	6 ↔ E
2 ↔ A	7 ↔ J
3 ↔ B	8 ↔ G
4 ↔ C	9 ↔ I
5 ↔ D	10 ↔ H

Redraw



D: The size of the largest independent set of the Peterson graph is not 3 but 4.

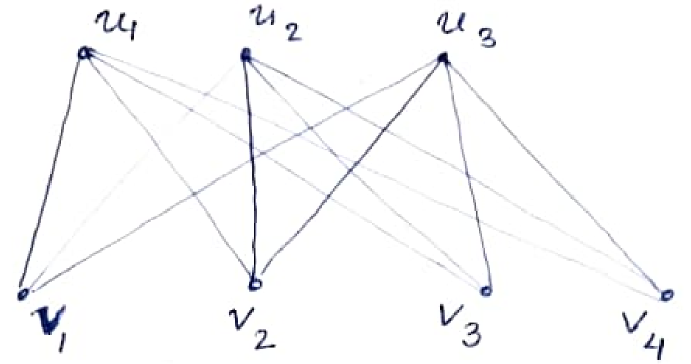
$\{A, J, I, C\}$ is an independent set of the Peterson graph of size 4.

Q.45. Q.12 (GATE 2019)

In an undirected complete graph of order n there are $n!$ possible ways to visit every node. But from these $n!$ ways there are n different places or nodes to start and 2 different directions to travel. (clockwise & anticlockwise)

So there are $\frac{n!}{2n} = \frac{(n-1)!}{2}$ distinct Hamiltonian cycles.

Q.46 Q.52 (GATE 2020)



From u_1 4 edges of different colors (I, II, III, IV) are incident with v_1, v_2, v_3 and v_4. Similarly from u_2 4 edges of different colors (II, III, IV, I) are incident with v_1, v_2, v_3 & v_4 and so on using the 4 colors (III, IV, I, II) ~~and (IV, I, II, III)~~ on the edges from u_3. Now adding a new vertex s which is adjacent to all the 7 vertices we will require 7 colors including the 4 colors I, II, III & IV to edge color the graph.

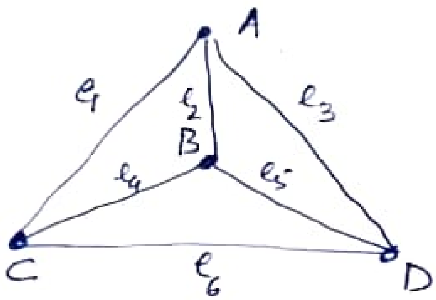
So minimum number of colors required to edge color G is the maximum degree of the graph i.e. max(4, 5, 7) = 7.

Q.47 Q.26 (GATE 2013)

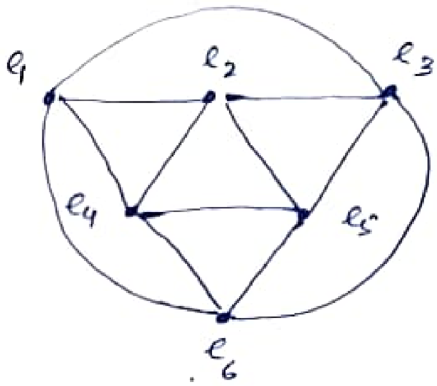
P: TRUE

Every edge in the given cycle graph will become a vertex in L(G) and every vertex of the cycle graph will become an edge in L(G).

Q: FALSE



$K_4 \rightarrow$ clique



Not a clique as e_1 not adjacent to e_5

R: FALSE

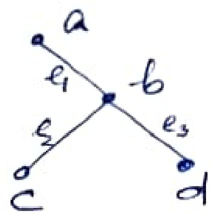
Let G be a graph with 5 vertices and 9 edges. Let the degree of one vertex be 2 and the rest be 4. So L(G) has 9 vertices and 25 edges. For a planar graph $e \le 3v - 6$. But in L(G)

$$e = 25 \nleq 3 \times 9 - 6 = 21$$

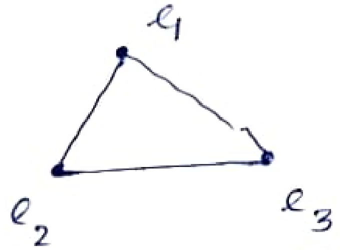
So L(G) is non planar.

S : FALSE

Let T be a tree as follows:



L (T)



L (T) is a cycle graph which is not a tree.