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Sources of Magnetic Field

Magnetic field of a moving charge

$$B = \frac{\mu_0}{4\pi} \frac{qv \sin\phi}{r^2}$$

$$\mu_0 = 4\pi \times 10^{-7} \, T \cdot \frac{m}{A}$$

$$E = \frac{Kq}{r^2}$$

$$\frac{1}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2} \quad - 1$$

$$1T = \frac{1N \cdot m}{A}$$

$$1G = \text{CGS unit}$$

$$1T = 10^4 G$$

$$c = \frac{1}{\sqrt{\mu_0 \cdot \varepsilon_0}}$$

Magnetic Field of a current element

The total magnetic field provided by several moving charges is the vector sum of magnetic field produced by individual charges

$$\vec{B} = \vec{B_1} + \vec{B_2} + \ldots$$

$$= \sum_{i=1}^{n} dB_i$$

Let us consider a current carrying conductor having A = cross-sectional area.

l = length

dl – short length segment of current carrying conductor

$dQ = Nq$

$N = nV$

q = charge

n = no. of charges

$nV = nAdl$

v_d = velocity of charges moving in the conductor

$$dB = \frac{\mu_0}{4\pi} \, dQ \frac{v_d}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{nAdl \, q v_d}{r^2}$$

$$nAqv_d = I$$

$$dB = \frac{\mu_0}{4\pi} \frac{Idl}{r^2} \sin\phi$$

$$dB = \frac{\mu_0}{4\pi} I \frac{(\vec{dl} \times \hat{r})}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{I \vec{dl} \times \vec{r}}{r^3}$$

Biot and Savart Law

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I(\vec{dl} \times \hat{r})}{r^2}$$

Magnetic Field of a straight current carrying conductor

Let us consider

I = current

x = distance on $\perp$ bisector

$l = 2a$ = length of the conductor

$\phi \quad dl = dy$

$$B = \frac{\mu_0}{4\pi}\int \frac{I\,dl \sin\phi}{r^2}$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{dl \sin\phi}{r^2}$$

$$\sin\phi = \sin(\pi - \phi)$$

$$r^2 = x^2 + y^2 \qquad = \frac{x}{\sqrt{x^2+y^2}}$$

$$B = \frac{\mu_0 I}{4\pi}\int \frac{dy}{x^2+y^2} \cdot \frac{x}{\sqrt{x^2+y^2}}$$

$$\frac{\mu_0}{4\pi} I \int_{-a}^{a} \frac{x\,dy}{(x^2+y^2)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi} \frac{2a}{x\sqrt{x^2+a^2}}$$

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$$B = \frac{\mu_0 I}{2\pi x}$$

$$x = r$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B = \frac{\mu_0 I}{4\pi} \cdot \frac{2a}{x\sqrt{x^2+a^2}}$$

$$a \to \infty \quad \sqrt{x^2+a^2} = \sqrt{a^2} = a$$

$$B = \frac{\mu_0 I}{2\pi x} \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

$$x = 0 \quad B =$$

Force between two parallel conductor

The force per unit length exist between two parallel current carrying conductor is given by

$$F = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

FORCES BETWEEN TWO PARALLEL CONDUCTORS

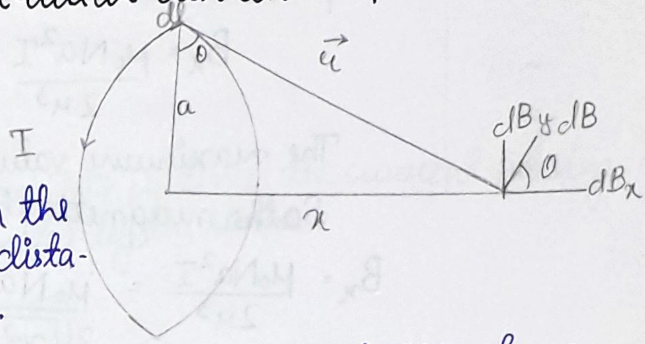
$$I : B = \frac{\mu_0 I_1}{2\pi r}$$

$$F = I_2 L B$$
$$= \frac{\mu_0 I_1 I_2}{2\pi r}$$

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Magnetic field of a circular current loop

$$dB = \frac{\mu_0 I dl}{4\pi r^2} \quad - 1$$



Let us take a point P on the axis of the loop. at the distance x from the centre.

The magnetic field at P due to the current element dl is shown above

The components of the vector are

$$dB_x = B\cos\theta = \frac{\mu_0 I dl \cos\theta}{4\pi r^2} \Rightarrow \frac{\mu_0 I dl}{4\pi r^2}\frac{a}{r} = \frac{\mu_0 a I dl}{4\pi r^3} \quad - 2$$

$$dB_y = B\sin\theta = \frac{\mu_0 I \, dl \sin\theta}{4\pi r^2} \Rightarrow \frac{\mu_0 I \, dl}{4\pi r^2 r}$$

$$= \frac{\mu_0 I x dl}{4\pi r^3}$$

The x- component of the total field B, is obtained by integrating eq^n 1

$$B_x = \int dB_x$$

$$= \int \frac{\mu_0 I a \, dl}{4\pi r^3}$$

$$= \frac{\mu_0 I a}{4\pi r^3} \int dl$$

$$= \frac{\mu_0 a I (2\pi a)}{4\pi r^3}$$

$$= \frac{\mu_0 a^2 I}{2 r^3} \quad -4$$

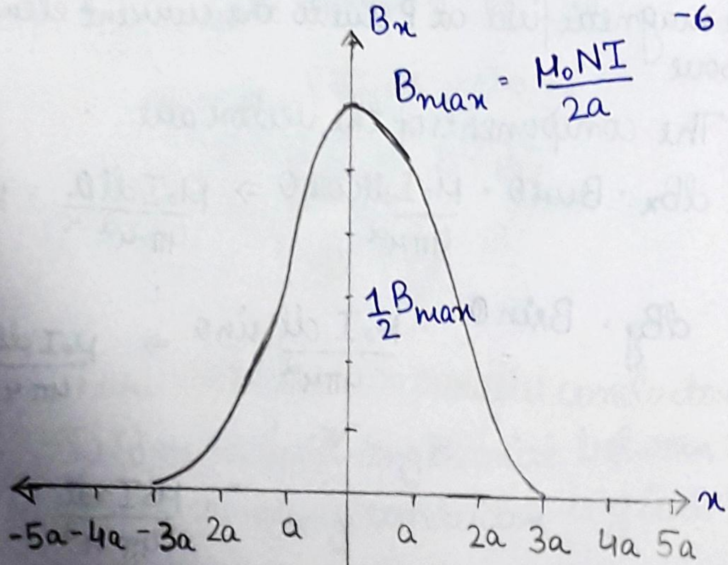
It represents the total field B on the axis of a circular loop.

If a coil consists of N loops, then the total field is

$$B_x = \frac{\mu_0 N a^2 I}{2 r^3} \quad - 5$$

The maximum value of the maximum field (x = 0)
So the magnetic field at the centre of the coil is

$$B_x = \frac{\mu_0 N a^2 I}{2 r^3} = \frac{\mu_0 N a^2 I}{2(\sqrt{x^2 + a^2})^3} = \frac{\mu_0 N a^2 I}{2 a^3} = \frac{\mu_0 N I}{2a} \quad -6$$



$$B_x = \frac{\mu_0}{2(x^2+a^2)^{3/2}}$$

$$\frac{NIa^2\pi}{\pi}$$

$$= \frac{NIA}{\pi} = \frac{\mu}{\pi}$$

$$\mu = NIA$$

$$B_x = \frac{\mu_0 \mu}{2\pi (x^2+a^2)^{3/2}}$$

$x = 0$ $a \rightarrow$

$$B_x = \frac{\mu_0 NI}{2a}$$

$x = \infty$

$B_x = 0$

Ampere's Law

The line integral of B equal to μ_0 times the current passing through the area bounded by the surface.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- The line integral does not depend on the shape of the path or on the position of the wire inside it.
- If the current in a wire is opposite, the integral has the opposite sign
- If the path does not enclose the wire, then the net change in θ during the trip around the integration path is zero.
- If $\oint \vec{B} \cdot d\vec{l} = 0$ then, it does not mean than $\vec{B} = 0$ everywhere along ~~the~~ the path, only that the total current through an area bounded by the path is zero.

$$I_{encl} = I_1 - I_2 - I_3$$

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$$\oint \vec{B}.d\vec{l} = \mu_0 I_{net}$$

$$I_{net} = I_{enclo}$$
$$= I_1 - I_2 + I_3 + I_4 - I_5$$
$$= \sum_{i=i}^{N} I_i$$

Application of Ampere's Law

1. Field of long cyclinder conductor

Find the magnetic field as a function of the distance r from the conductor axis for points both inside (r > R) and outside the conductor (R > R).

Solution

$$\oint B.d\vec{l} = \mu_0 I$$

Let R = radius of cyclinder

I = current

L = length

r = radius of amperican surface both

Apply the ampere's law

To determine the magnetic field of the conductor

$$\oint \vec{B}.d\vec{l} = \mu_0 I$$

i outside the conductor (r>R)

$$\int B.dl = \mu_0 I$$

$$B\int_0^{2\pi r} dl = \mu_0 I$$

$$\Rightarrow B.2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

ii At the surface (r = R)

$$\int_0^{2\pi R} B.dl = \mu_0 I$$

$B.2\pi R = \mu_0 I$

$B = \dfrac{\mu_0 I}{2\pi R}$

iii at the Internal point ($r<R$)

$J = J'$

$\dfrac{I}{A} = \dfrac{I'}{A'}$

$\dfrac{I}{\pi R^2} = \dfrac{I'}{\pi R'^2}$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

$B \cdot \int_0^{2\pi r} dl = \mu_0 \dfrac{I r^2}{R^2}$

$B . 2\pi \cancel{r} = \dfrac{\mu_0 I r^{\cancel{2}}}{R^2}$

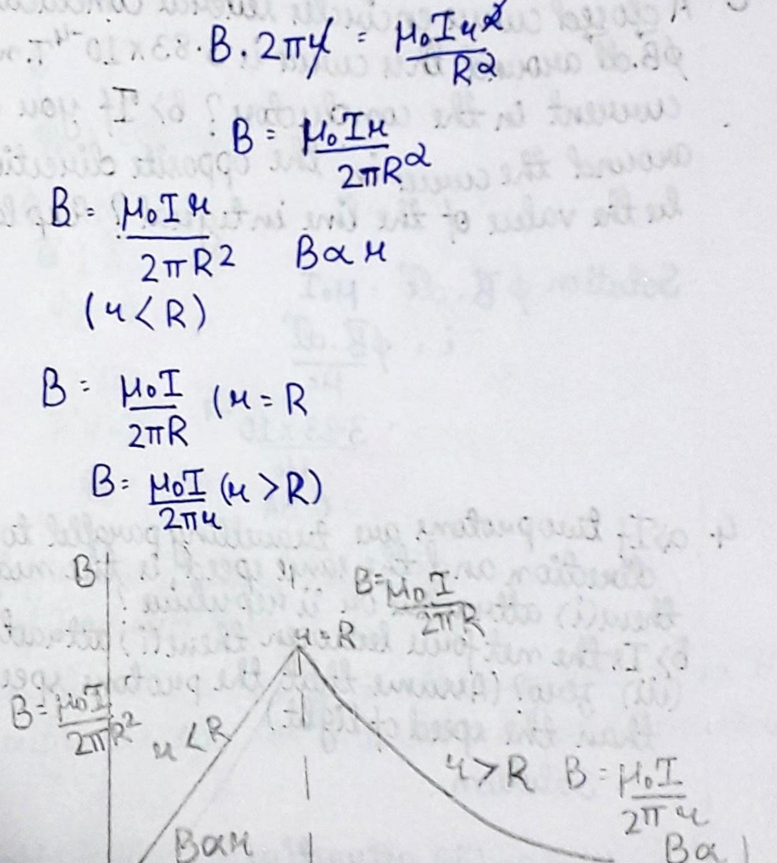
$B = \dfrac{\mu_0 I r}{2\pi R^2}$

$B = \dfrac{\mu_0 I r}{2\pi R^2}$ $\quad B \propto r$

($r<R$)

$B = \dfrac{\mu_0 I}{2\pi R}$ ($r = R$

$B = \dfrac{\mu_0 I}{2\pi r}$ ($r > R$)



2. Two long, parallel wires are separated by a distance of 2.50 cm. The force per unit length each other wire exerts on the other is 4.00×10^{-5} N/m and the wires repel each other. The current is one wire repel each other. The current in one wire is 0.600 A. a> What is the current in the second wire. b> Are the two currents in the same direction or in the opposite direction

Solution

$r = 2.50$ cm

$= 2.50 \times 10^{-2}$ m

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r} = 4.0 \times 10^{-5} \text{ N/m}$$

$$I_2 = \frac{F}{L} \frac{2\pi r}{\mu_0 I_1}$$

$$= 4.0 \times 10^{-5} \times \frac{2\pi \times 2.50 \times 10^{-2}}{4\pi \times 10^{-7} \times 0.60}$$

$= 8.33$ A

b. opposite

3. A closed curve encircles several conductors. The line integral $\oint \vec{B} \cdot d\vec{l}$ around this curve is 3.83×10^{-4} T.m a> What is the net current in the conductor? b> If you were to integrate around the curve in the opposite direction, what would be the value of the line integral? Explain

Solution $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

$$i = \frac{\oint \vec{B} \cdot d\vec{l}}{\mu_0}$$

$$= \frac{3.83 \times 10^{-7}}{\mu_0}$$

$= 0.3$ A

4. a> If two protons are travelling parallel to each other in the same direction and the same speed, is the magnetic force between them (i) attractive or ii repulsive?
b> Is the net force between them (i) attractive (ii) repulsive or (iii) zero? (Assume that the protons' speed is much slower than the speed of light)

Solution.

a (i) attractive

b (ii) repulsive