Chapter - 2

Q. 38 (2019)

(I) G has a minimum spanning tree, if no two edges of G have the same weight.

$\Rightarrow$ Let, G has two MST, T_1 and T_2.

Let, $e_1 \in E(T_1)$ such that $e_1 \notin E(T_2)$

$e_2 \in E(T_2)$ such that $e_2 \notin E(T_2)$

Without loss of any generality, suppose $w(e_1) \leq w(e_2)$.

Now, $T_2 - e_2 + e_1$ is also a spanning tree.

$$w(T_2 - e_2 + e_1) = w(T_2) - w(e_2) + w(e_1) \leq w(T_2)$$

This is contradicting the fact that, T_2 is MST.

$\therefore$ We have unique MST.

(II)

Q.49 (2020)

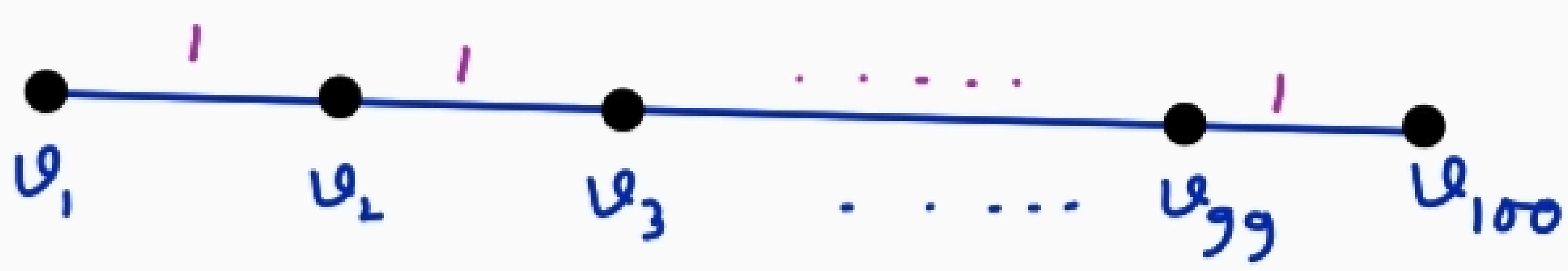
$G = (V, E)$

$V = \{v_1, v_2, \dots, v_{100}\}$

$E = \{(v_i, v_j) \mid 1 \le i < j \le 100\}$.

$w_{ij} = |i - j|$ weight of the edge joining v_i and v_j

$\therefore$ MST is :

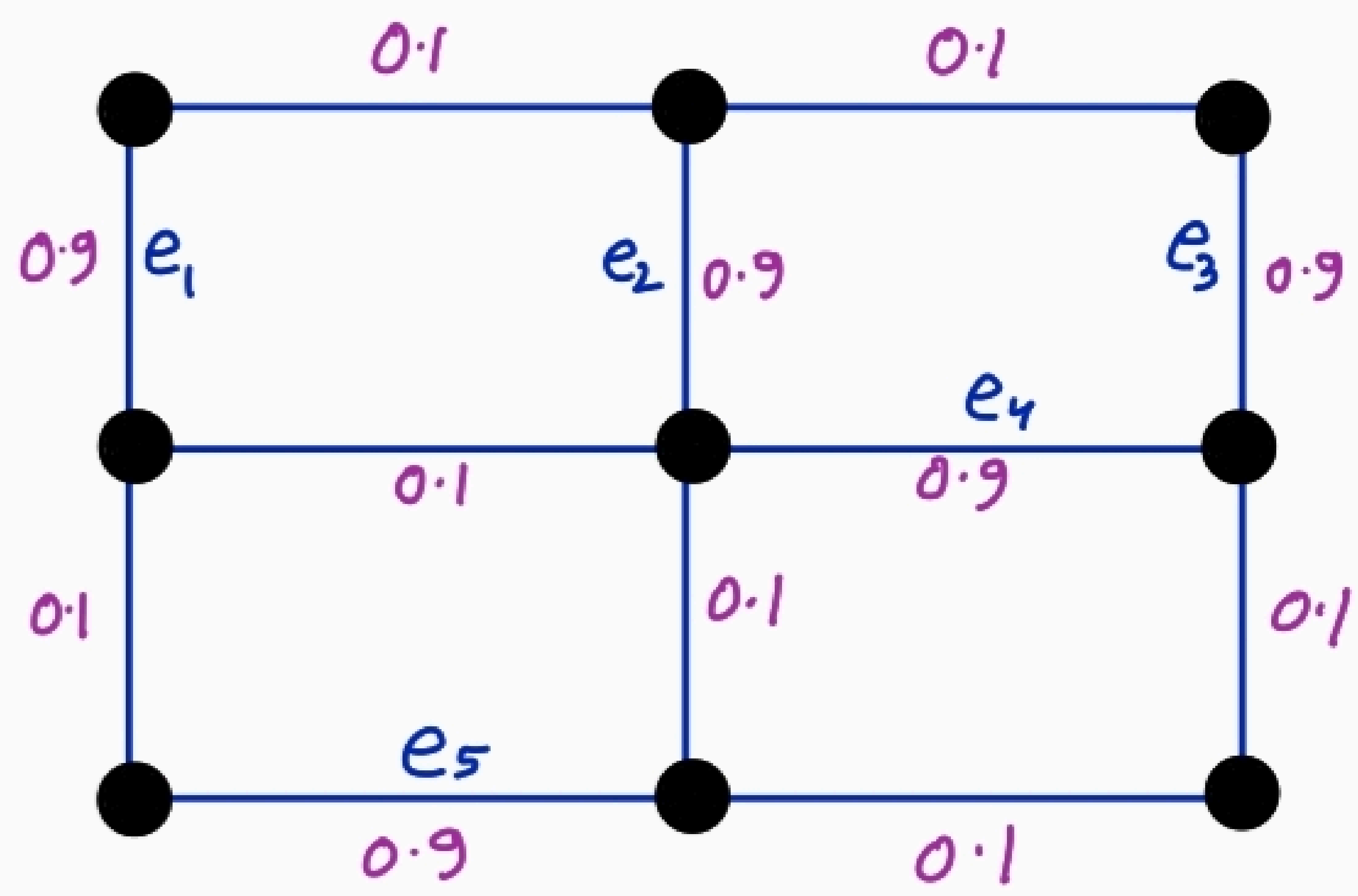


$\therefore$ Total weight $= |1-2| + |2-3| + |3-4| + \dots + |99-100|$

$= 99$

Q.17. (2021)

G:



$n(G) = 9$ $\quad\quad$ $n(MST) = 9$

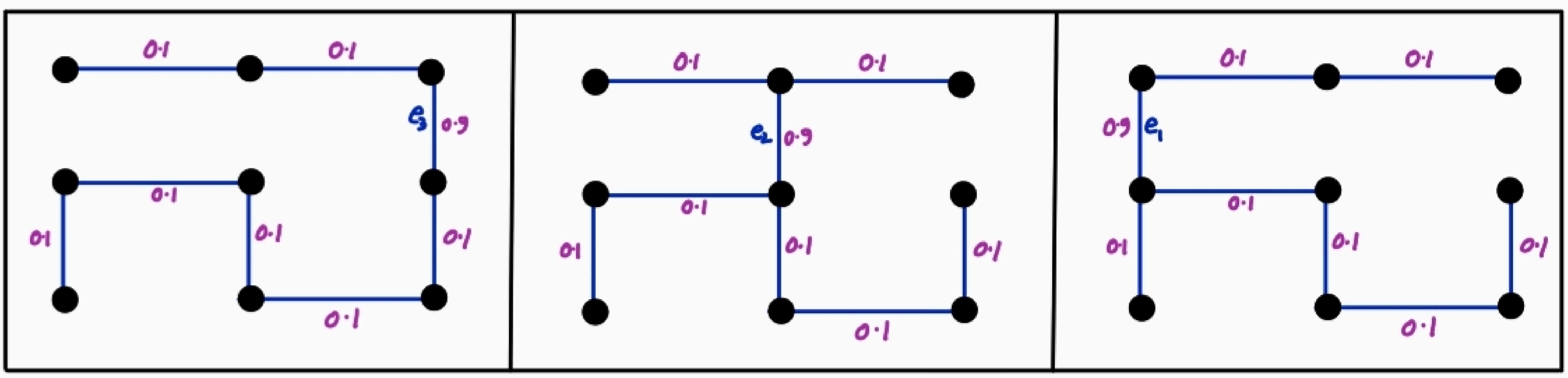
$e(G) = 12$ $\quad\quad$ $e(MST) = 9-1=8$

$\therefore$ We have to remove 4 edges from G to obtain MST

In G, there are 5 edges with weight 0·9

We have to remove both e_4 and e_5 and 2 more edges from e_1, e_2 and e_3 to get the MST.

As we can select 2 edges from e_1, e_2 and e_3 in ${}^3C_2 = 3$ ways, we will have 3 distinct MST.



MST_1 $\quad\quad$ MST_2 $\quad\quad$ MST_3

Q. 36 (2021)

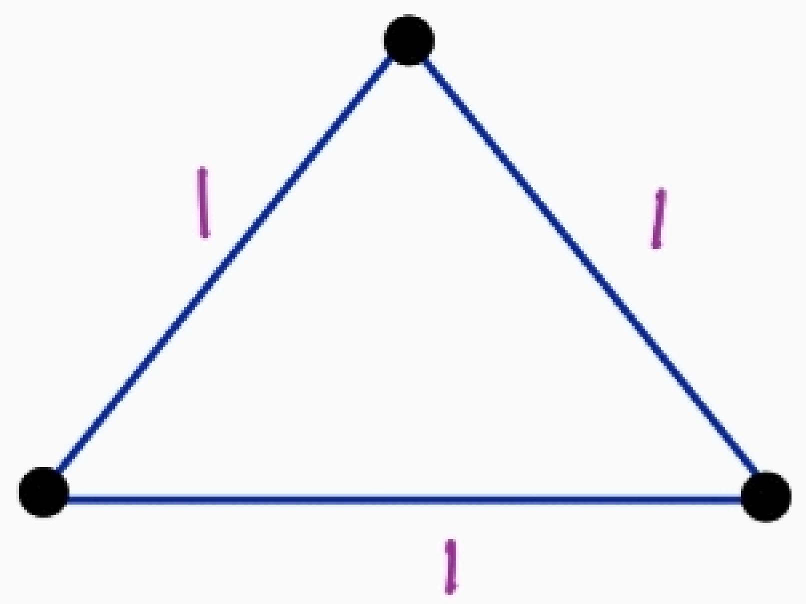
Out of syllabus.

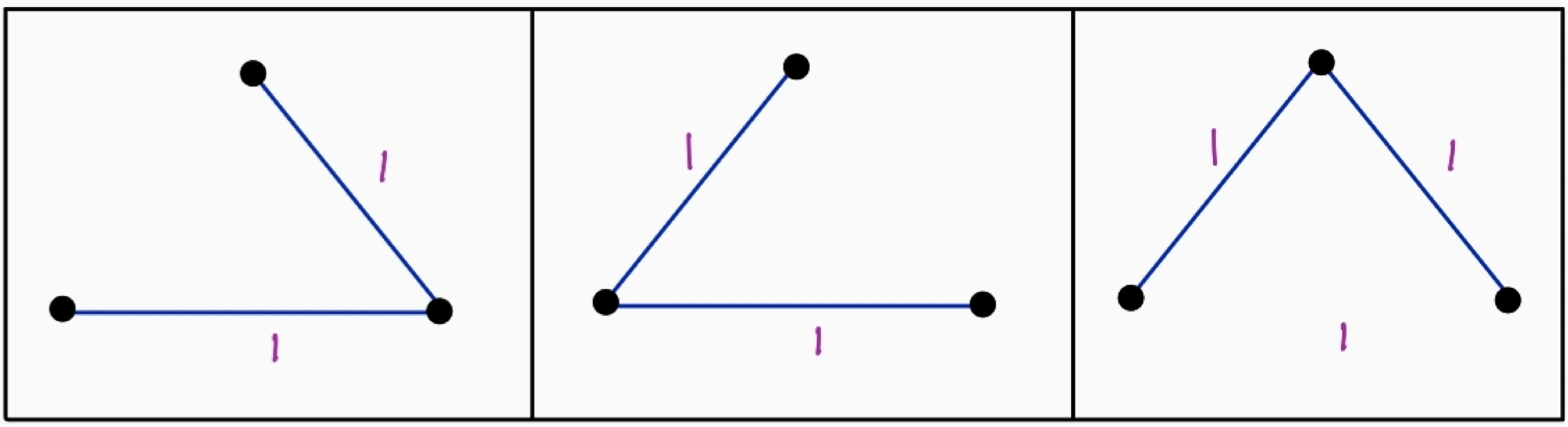
Q. 41 (2021)

Out of syllabus.

Q. 1. (2021)

S_1: False.



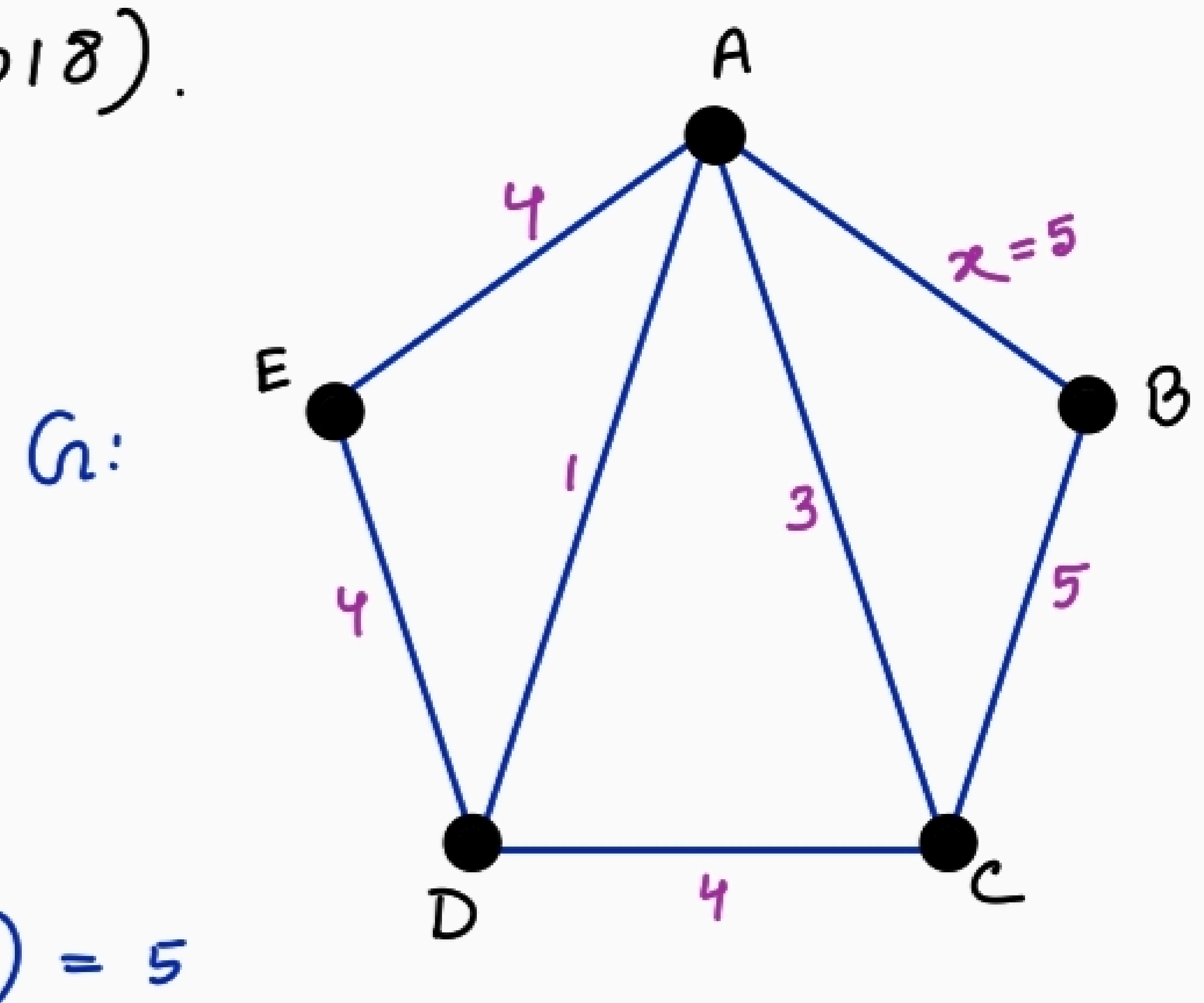


MST_1 MST_2 MST_3

There is no common edge.

S_2: True. done in Q. 38. (2019).

Q.47. (2018).

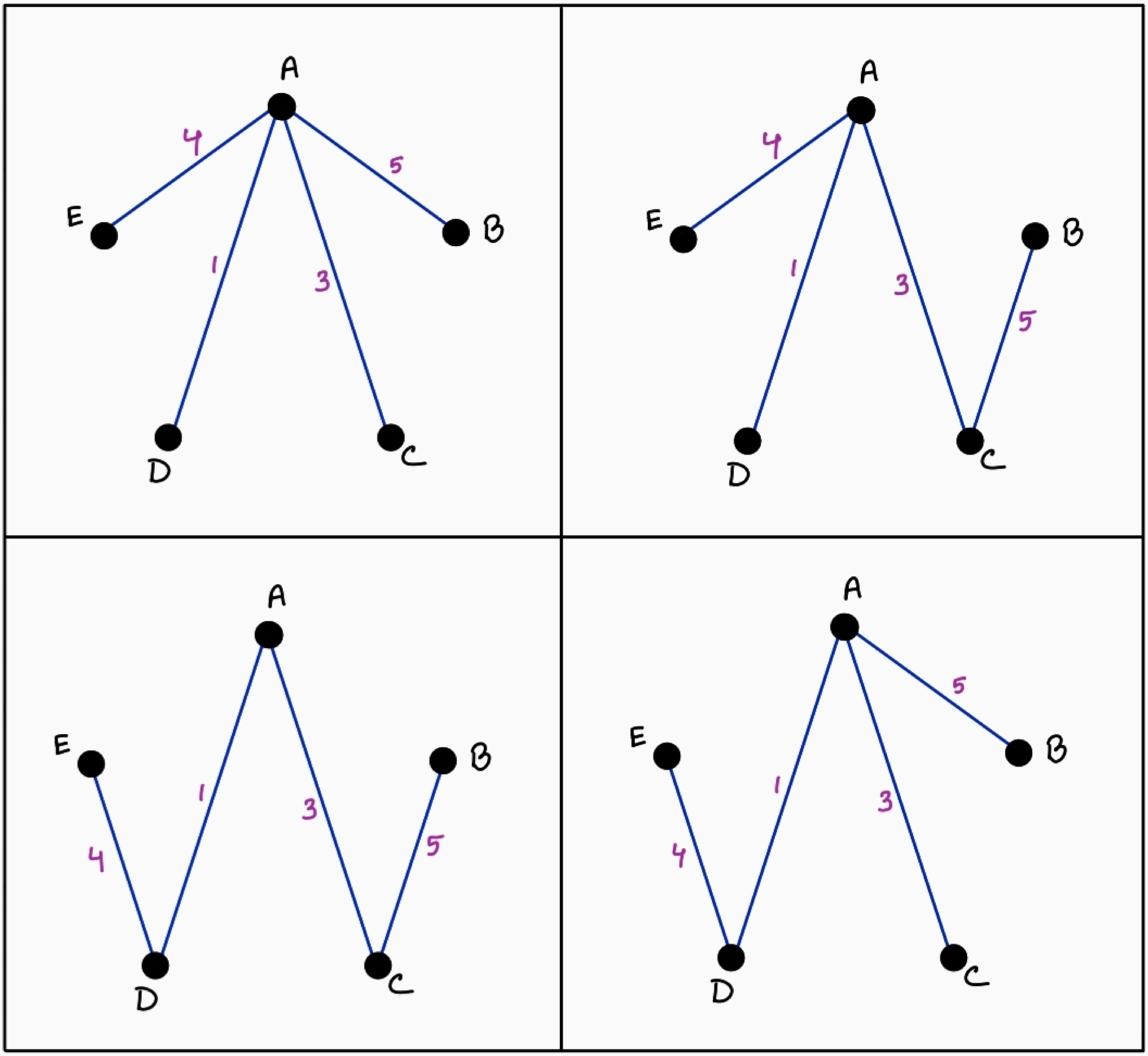


(if $x<5$, then we have to remove BC from ABC. if $x>5$ then we have to remove AB. But if $x=5$, we will have 2 options)

$n(G) = 5$

$e(G) = 7$

We have to remove 3 edges to get MST.



Q.20. (2017)

$n(T) = 10.$

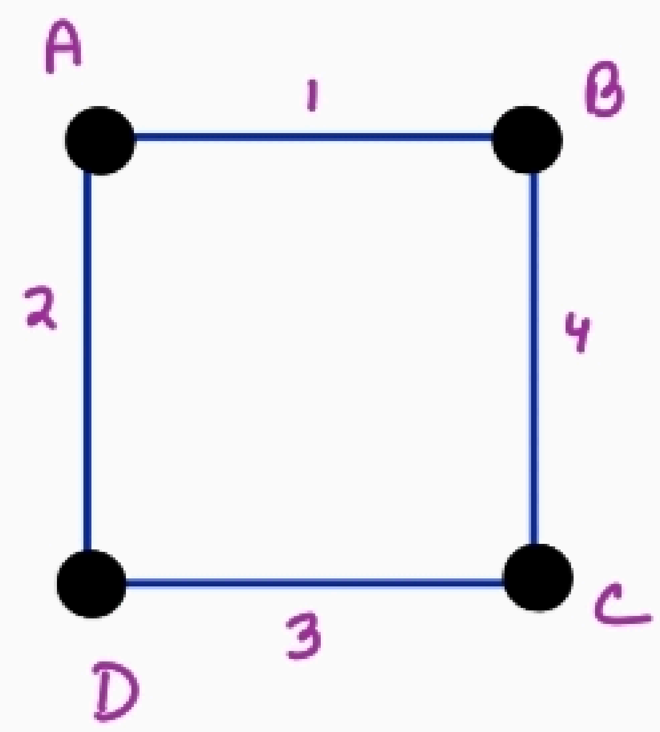
$\therefore e(T) = 9.$

$$\therefore \sum_{v \in V(T)} d(v) = 2 \times 9 = 18$$

Q. 26 (2017)

(I): True. done in Q.38. (2019).

(II): False.

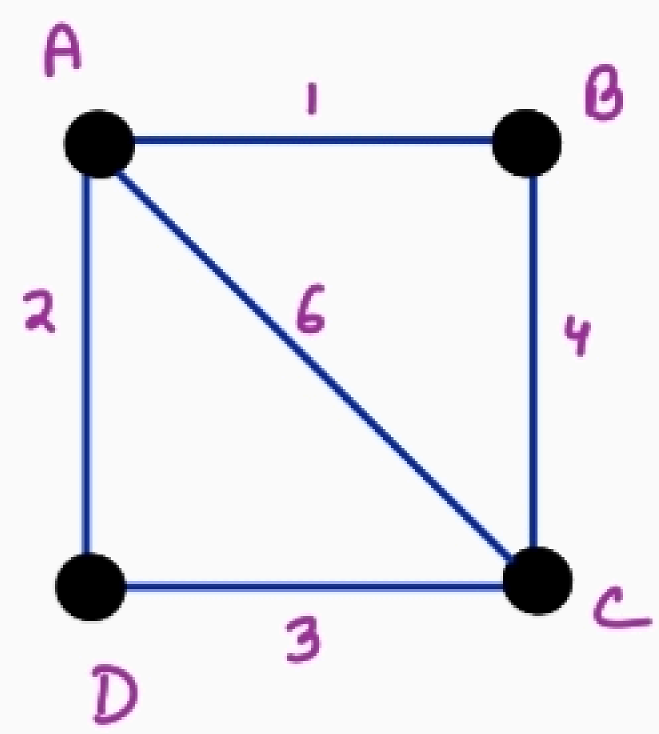


Shortest A-C path : A − B − C $w = 5$
A − D − C $w = 5$.

Q.14 (2016)

(P): True.

(Q): False

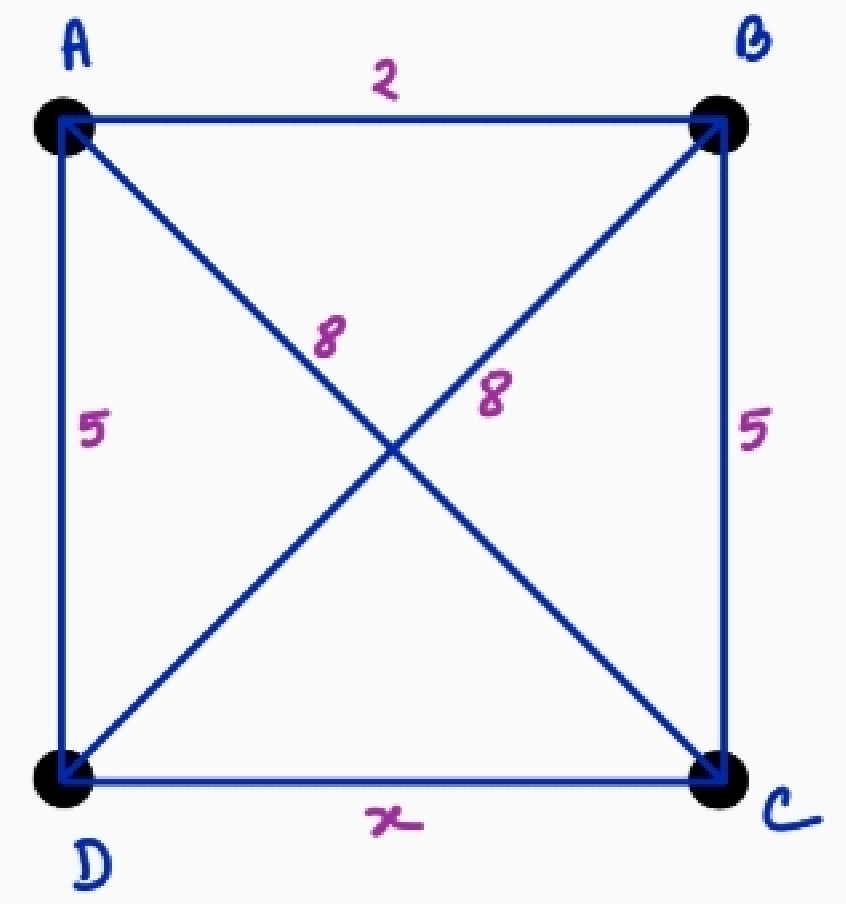


Shortest A-C path : A — B — C $w = 5$
A — D — C $w = 5$.

If we increase the weights of each edge by 6, then.

A — B — C $w = 17$
A — D — C $w = 17$
A — C $w = 12$

Q. 38. (2016)



$$w = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 2 & 8 & 5 \\ B & 2 & 0 & 5 & 8 \\ C & 8 & 5 & 0 & x \\ D & 5 & 8 & x & 0 \end{array}$$

Shortest paths:

A-B : A — B $w = 2$

A-C : A — B — C $w = 7$
A — D — C $w \le 7$, if $x \le 2$

A-D : A - D $w = 5$

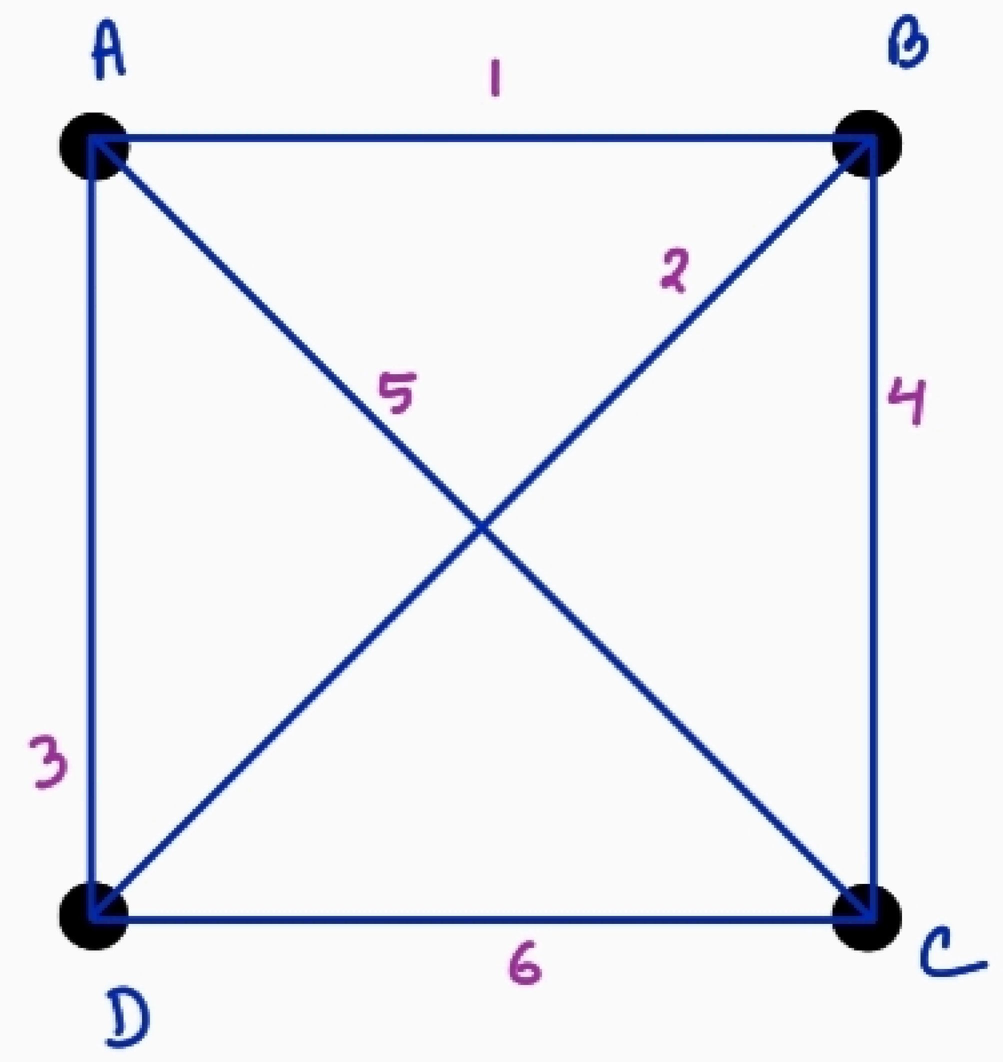
B-C : B — C $w = 5$

B-D : B - A — D $w = 7$
B - C — D $w \le 7$ if $x \le 2$

C-D : C — B — A — D $w = 12$
C — D $w \le 12$ if $x \le 12$

$\therefore$ Maximum possible value for x is 12.

Q. 39 (2016)

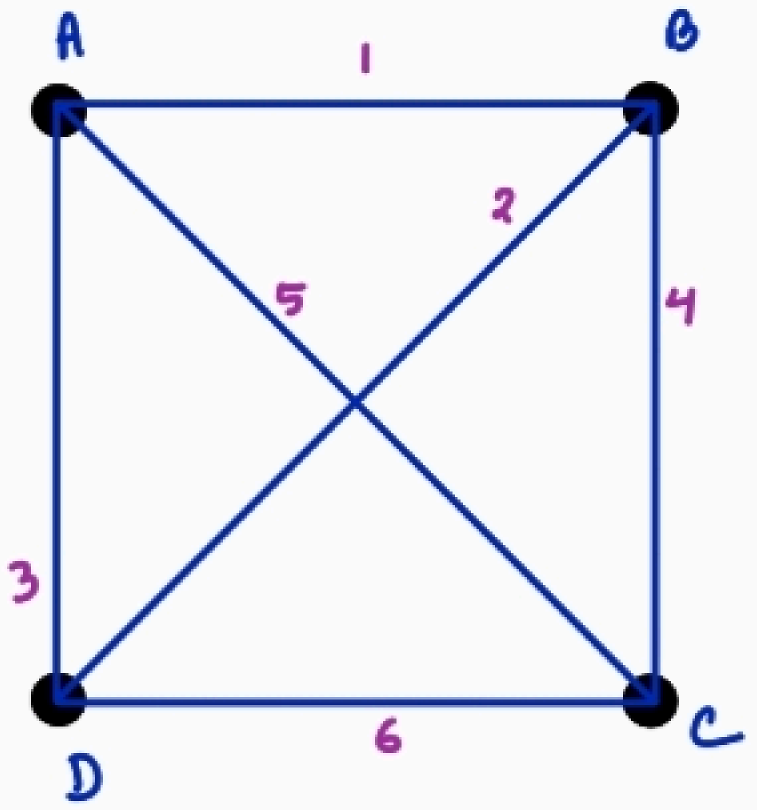


This is the case when we will get the maximum possible weight for a MST.

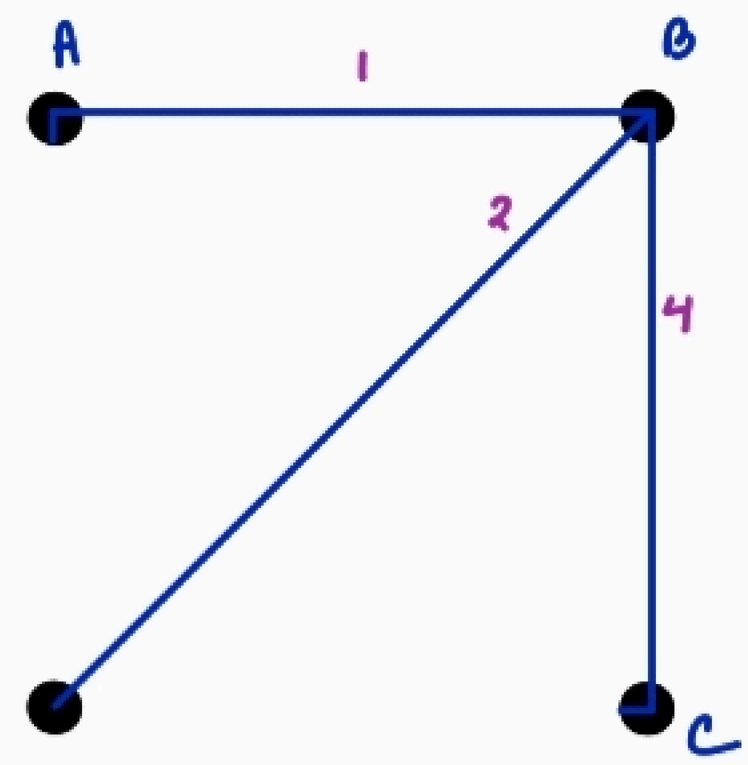
$w = 7$

Q. 40 (2016)

(I): False.



G



MST of G.

Here AD is the lightest edge of the cycle A-C-D-A, but not included in the MST.

(II) True.

Q. 10 (2015).

A binary tree is a rooted tree in which every node has at most two children.

def, n_1 : # of leaf nodes (no children)

n_2 : # of nodes with on children.

n_3 : # of nodes with two children.

$n = n_1 + n_2 + n_3$ (total # of vertices in T).

∴ From degree sum formula,

$$1 \times n_1 + 2 \times n_2 + 3 \times n_3 - 1 = 2 \times (n-1)$$

$$\Rightarrow 20 + 2n_2 + 3n_3 - 1 = 2 \times 20 + 2n_2 + 2n_3 - 2$$

($n_1 = 20$ given)

$$\Rightarrow n_3 = 19$$

Q. 50 (2015).

bridge ≈ cut-edge.

∴ B is true.

Q.25 (2015)

def, n_1 : # of leaf nodes (no children) = 200

n_2 : # of nodes with on children.

n_3: # of nodes with two children.

$n = n_1 + n_2 + n_3$ (total # of vertices in T).

∴ From degree sum formula,

$$1 \times n_1 + 2 \times n_2 + 3 \times n_3 - 1 = 2 \times (n-1)$$

$$\Rightarrow 200 + 2n_2 + 3n_3 - 1 = 2 \times 200 + 2n_2 + 2n_3 - 2$$

($n_1 = 200$ given)

$$\Rightarrow n_3 = 199$$

Q.40 (2015).

$n(G) = 100$

$e(G) = 300$.

$w(MST) = 500$.

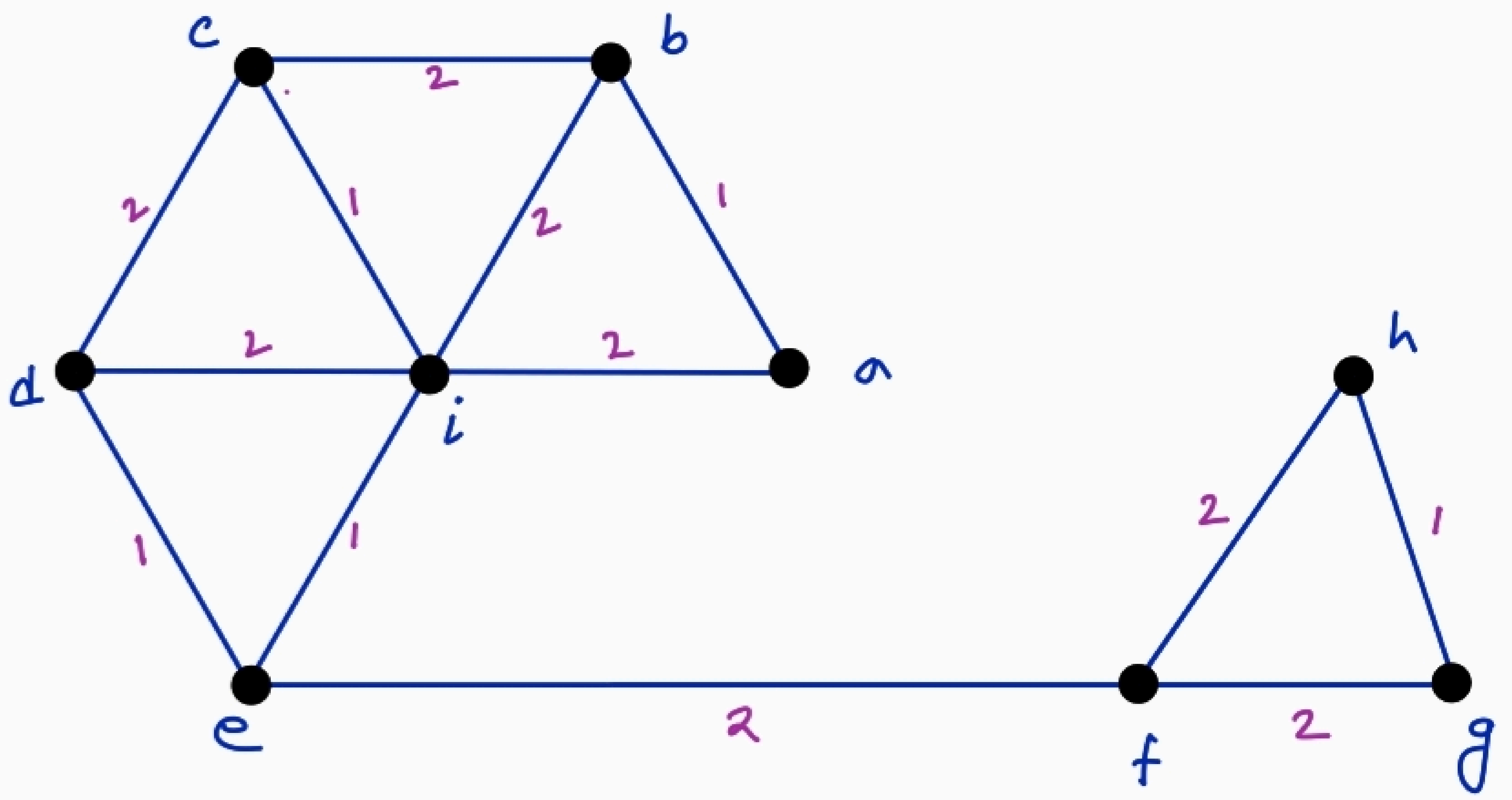
$n(MST) = 99$.

∴ New weight of a MST $= 500 + 99 \times 5$

$= 995.$

— x —

Q. (2014, set-2)

G: 

$n(G) = 9$

$e(G) = 13$

$n(MST) = 9$

$e(MST) = 9-1 = 8$

We can remove one edge from $\{hf, fg\}$. (2 ways)

We have to remove both cd and di

We can remove two edges from $\{cb, bi, ai\}$ (3 ways)

$\therefore$ Total # of MST $= 2 \times 3 = 6$

Q. 51 (2014)

$G: \; G_1, G_2, \ldots, G_k$

Each of G_i is a tree.

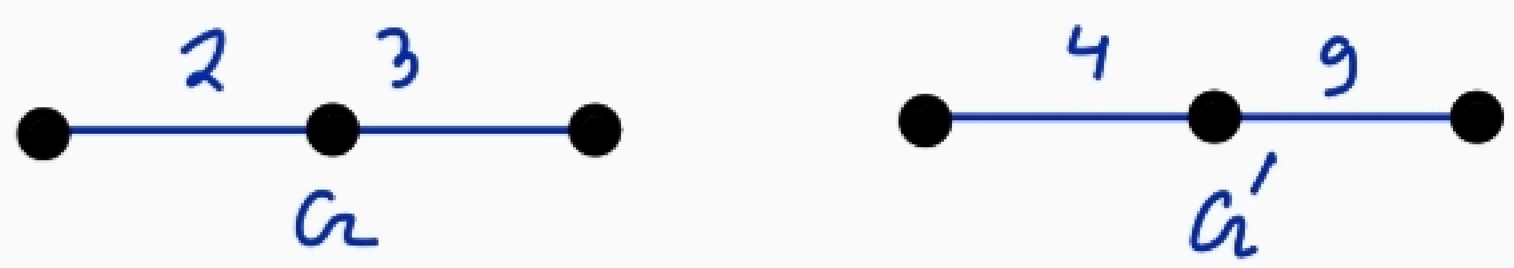
$$\therefore \; e(G_i) = n(G_i) - 1$$

$$\therefore \; \sum_{i=1}^{k} e(G_i) = \sum_{i=1}^{k} \left[n(G_i) - 1 \right]$$

$$\Rightarrow e(G) = n - k$$

Q. 29 (2012)

A.



(False)

$t = 5, \quad t' = 13 \qquad \therefore t' \neq t^2$

B.

(False)

$\therefore t = 2, \quad t' = 4. \qquad \therefore t' \not< t^2$

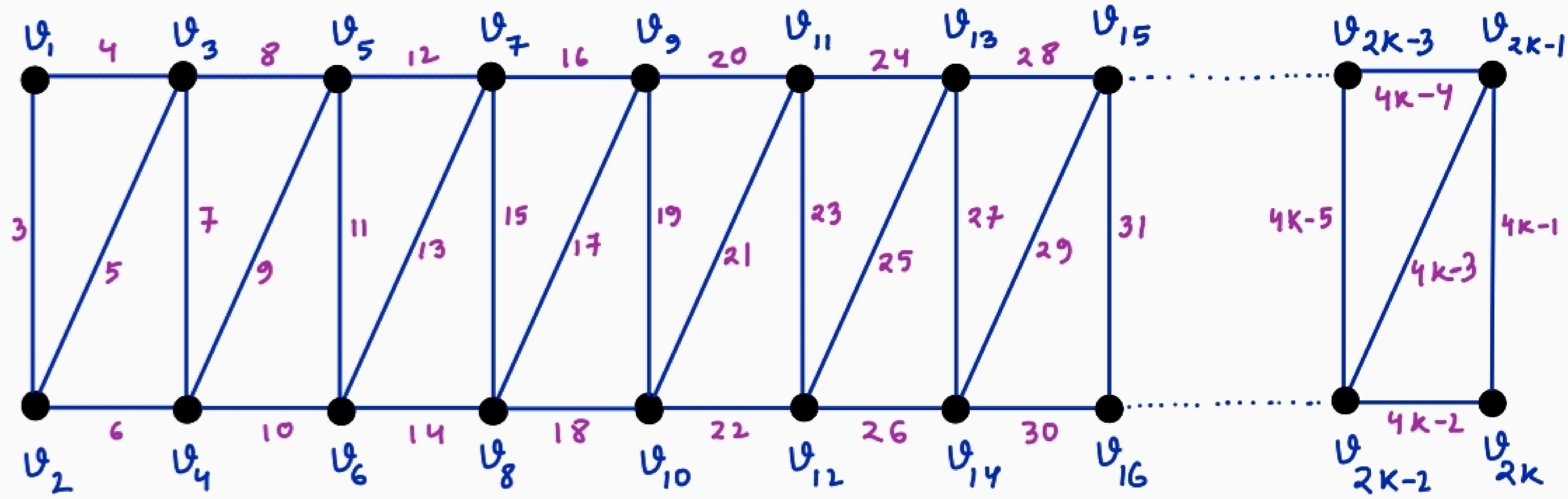
C.

(False)

Here, $t' = t^2 = 4$ and $T' = T$

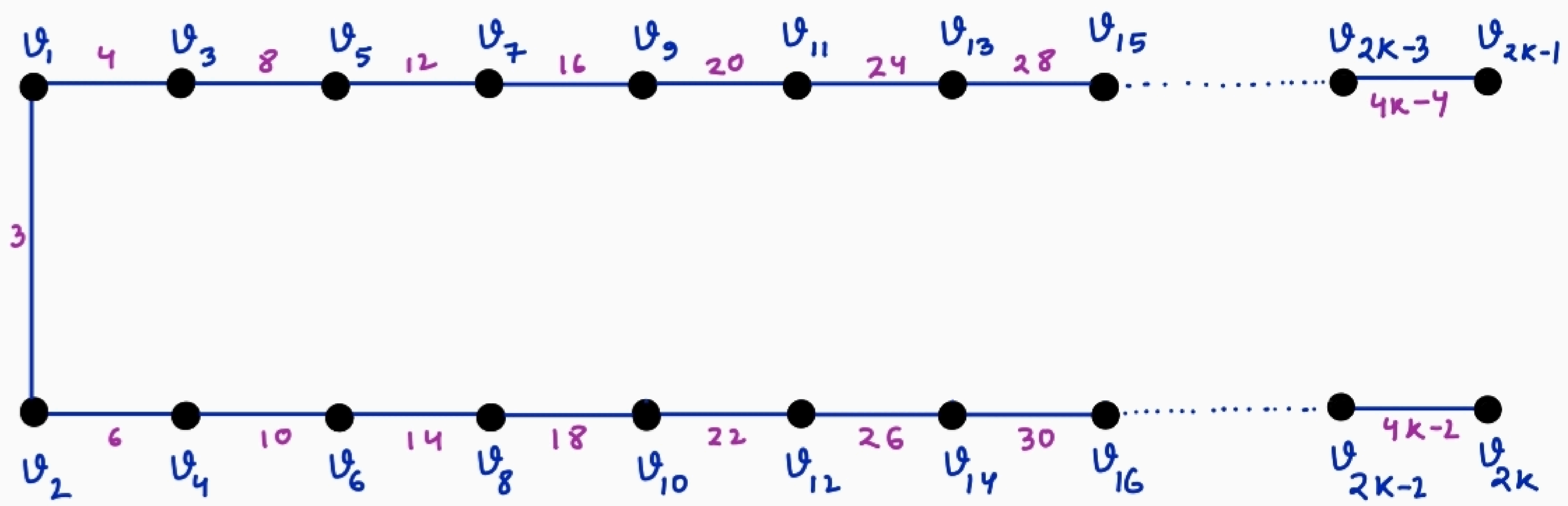
Q. 40. (2012) D. SACET

Q. 54 (2011)



G

$\begin{pmatrix} n = 2k \text{ if even} \\ = 2k-1 \text{ if odd} \end{pmatrix}$.



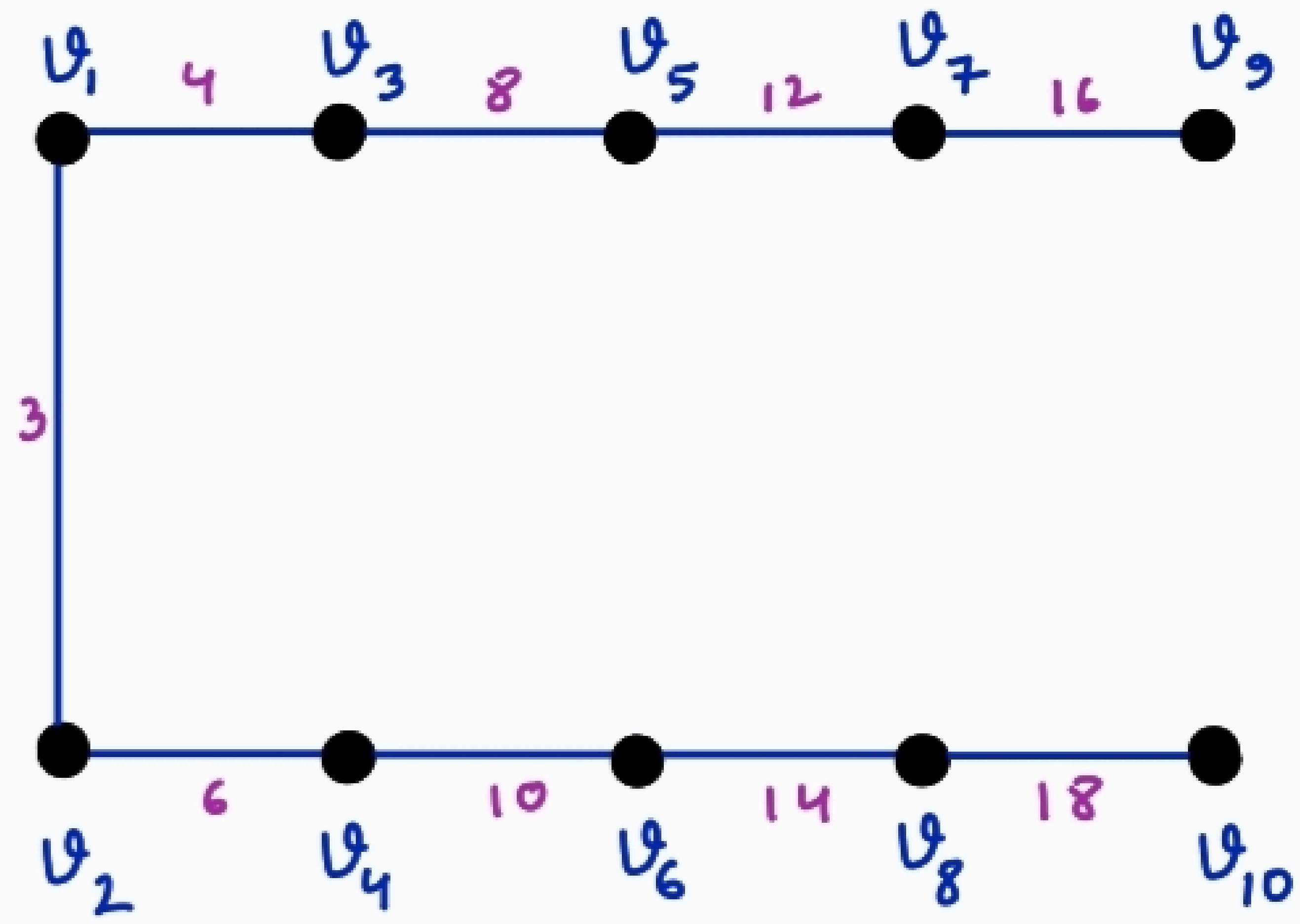
$$\therefore \text{Total weight of MST} = 3 + 4 + 6 + 8 + \cdots + 2(n-1)$$

$$= 1 + 2\big(1 + 2 + 3 + \cdots + (n-1)\big)$$

$$= 1 + n(n-1)$$

$$= n^2 - n + 1$$

Q.55 (2011)



Length of the path from v_5 to v_6 is

$$= 8 + 4 + 3 + 6 + 10$$

$$= 31$$