Study Material
(Electromagnetism)

Chapter-32
Electromagnetic Waves

Topics covered:	• Maxwell's Equations and Electromagnetic Waves • Plane Electromagnetic Waves and Speed of Light
Conceptual Problems:	TYU 32.1
In Class Problems:	32.1

Maxwell's Equations and Electromagnetic Waves

Electromagnetic wave are the electromagnetic disturbance, consisting of time-varying electric and magnetic fields, that can propagate through space from one region to another, even through vacuum (free space). Such disturbances have the properties of a wave.

Maxwell suggested that an electromagnetic disturbance should propagate in free space with a speed equal to that of light and hence that light waves were likely to be electromagnetic in nature. At the same time, he discovered that the basic principles of electromagnetism can be expressed in terms of the four equations that we call Maxwell's equations. These four equations are

1) Gauss's law for electric fields;
2) Gauss's law for magnetic fields, showing the absence of magnetic monopoles;
3) Ampere's law, including displacement current; and
4) Faraday's law.

Maxwell's Equations of Electromagnetism is given in the tabular form

$\oint \vec{E} \cdot \overrightarrow{dA} = \frac{q_{enc}}{\varepsilon_0}$	Gauss's law for electric field
$\oint \vec{B} \cdot \overrightarrow{dA} = 0$	Non existence of magnetic monopole
$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \left(i_C + \varepsilon_0 \frac{d\varphi_E}{dt}\right)$	Modified Ampere's law
$\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d\varphi_B}{dt}$	Faraday's law

Ampere's law, including the displacement current discovered by Maxwell shows that a time-varying electric field acts as a source of magnetic field. This mutual interaction between the two fields is summarized in Maxwell's equations.
Faraday's law says that time varying magnetic field gives electric field.

Thus, when either an electric or a magnetic field is changing with time, a field of the other kind is induced in adjacent regions of space.

Generating Electromagnetic Radiation

According to Maxwell's equations, a point charge at rest produces a static **E** field but no **B** field; a point charge moving with a constant velocity produces both **E** and **B** fields. From Maxwell's equations we can get that every accelerated charge radiates electromagnetic energy.

When a point charge oscillate in simple harmonic motion it emit electromagnetic waves. Oscillating the charge up and down makes waves that propagate outward from the charge.

But the charge does not emit waves equally in all directions; the waves are strongest at 90^0 to the axis of motion of the charge, while there are no waves along this axis.

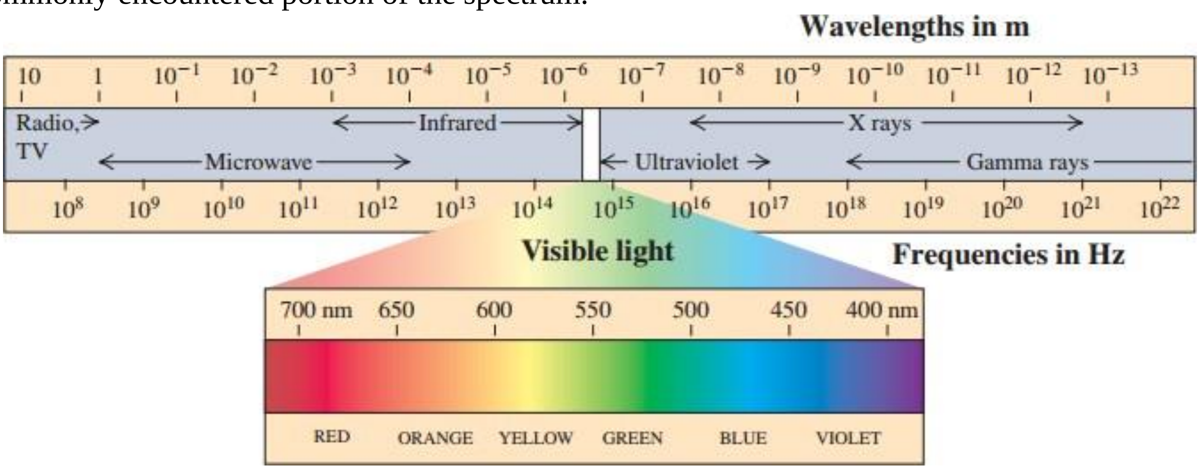
There is also a magnetic disturbance that spreads outward from the charge. Because the electric and magnetic disturbances spread or radiate away from the source.

Electromagnetic waves were first produced in the laboratory physicist Hertz. He used charges oscillating in L-C circuits. He detected the resulting electromagnetic waves with other circuits tuned to the same frequency.

In a radio transmitter, electric charges are made to oscillate along the length of the conducting antenna, producing oscillating field disturbances. Since many charges oscillate together in the antenna, the disturbances are much stronger than those of a single oscillating charge and can be detected at a much greater distance. In a radio receiver the antenna is also a conductor; the fields of the wave emanating from a distant transmitter exert forces on free charges within the receiver antenna, producing an oscillating current that is detected and amplified by the receiver circuitry.

The Electromagnetic Spectrum

The electromagnetic spectrum consists of electromagnetic waves of all frequencies and wavelengths. Figure below shows approximate wavelength and frequency ranges for the most commonly encountered portion of the spectrum.



- All electromagnetic waves travels with the same propagation speed (in vacuum) and is $c = 3 \times 10^8$ m/s.
- Electromagnetic waves may differ in frequency and wavelength but the relationship in vacuum holds for each.
- We can detect only a very small segment of this spectrum directly through our sense of sight. We call this range visible light.
- The approximate wavelengths for colors in the visible spectrum is shown in the table.

Wavelengths of Visible Light	
380-450 nm	Violet
450-495 nm	Blue
495-570 nm	Green
570-590 nm	Yellow
590-620 nm	Orange
620-750 nm	Red

Wavelengths of Invisible Light	
AM radio	5.4×10^5 Hz – 1.6×10^6 Hz
FM radio	8.8×10^7 Hz – 1.08×10^8 Hz
TV broadcast	Same as TV broadcast
cellular phones and wireless networks	Microwave
weather radar	Nearly 3×10^9 Hz
X-rays	10^{15} Hz to 10^{18} Hz
Gamma rays	10^{18} Hz to 10^{22} Hz

Uses of E.M Spectrum:

i) **Radio waves:** Its frequency range is 5×10^5 Hz to 10^9 Hz. These waves are used in a radio and T.V broadcasting system and for communication.

ii) **Microwaves:** Its frequency range is 10^9 Hz to 10^{11}Hz. These waves are used for a) Radar communication b) to study atomic and nuclear research. c) in a aircraft navigation. d) in micro wave ovens for cooking and warming of food

iii) **Infrared rays:** Its frequency ranger is $3X10^{11}$ Hz to $4X10^{14}$ Hz. These rays are used for i) Taking photographs in clouds and foggy conditions. ii) in treatment of muscular strains. iii) revealing the secret writings on ancient walls.

iv) **Visible radiations:** Its frequency range is $4X10^{14}$ Hz to $7.5X10^{14}$ Hz. These rays are used in i) In photography ii) In optical microscopes iii) In astronomy.

v) **Ultra-violet rays:** Its frequency range is $7.5X10^{14}$ Hz to $5X10^{15}$ Hz. These rays are used for i) To preserve the food stuffs ii) In making drinking water free from bacteria and germs iii) In sterilizing surgical instruments iv) In detecting invisible writings, forged documents and finger prints.

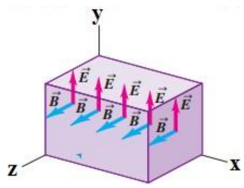
vi) **X-rays**: Its frequency range 10^{15} Hz to 10^{18} Hz. These rays are used in i) In medical diagnosis ii) For locating faults and cracks in big metallic bodies iii) In radio therapy to cure skin diseases , cancers and tumors iv) In location body fractures v) To study the crystal structure.

vii) **γ-rays**: Its frequency range is 10^{18} Hz to 10^{22} Hz . These rays are used in i) Treatment of cancer ii) To study the structure of the nucleus.

Plane Electromagnetic Waves and Speed of Light

Let us assume an electric field **E** that has only y-component and a magnetic field **B** with only Z-component.
Let us assume that both fields move together in the x-direction with a speed c.

Validity of Maxwell's equations for the propagation of em waves



- Maxwell's 1st and 2nd equations: As the electric and magnetic fields are perpendicular to each other at every instant and also perpendicular to the direction of propagation, Gauss law in electric field and the Gauss law in magnetic fields are satisfied.

Let us take as our Gaussian surface a rectangular box with sides parallel to xy, yz and zx plane. The box encloses no electric charge. The total electric flux and magnetic flux through the box are both zero,

If E-field and the B field has components along the direction of propagation the has x-component the electric flux and magnetic flux through the box would have been zero.

So, the electric and magnetic fields are perpendicular to each other at every instant. Thus Maxwell's 1st and 2nd equations are satisfied.

- Maxwell's 3rd equations (Faraday's law):
  Maxwell's 3rd equations (Faraday's law) will be satisfied if
  $E = c B$ -------------------(1)
  This equation is nothing but the em-wave in vacuum.
- Maxwell's 4th equations (Ampere's modified law):
  Maxwell's 4th equations (Ampere's modified law) will be satisfied if
  $B = \mu_0 \varepsilon_0 c E$ -------------------(2)
  This equation is also the em-wave in vacuum.
  Electromagnetic wave satisfied both the eq (1) and (2)

Thus,

$E = c\,B \quad \Rightarrow \quad E = c\,\mu_0\,\varepsilon_0\,c\,E$

$$\Rightarrow \quad c^2 = \frac{1}{\mu_0\varepsilon_0} \quad \Rightarrow c = \sqrt{\frac{1}{\mu_0\varepsilon_0}} \quad \Rightarrow c = \sqrt{\frac{1}{(4\pi \times 10^{-7}\ \text{N/A})(8.85 \times 10^{-12}\ \text{C}^2/\text{N.m}^2)}}$$

$c = 3 \times 10^8$ m/s

This is the speed of light in free space.

Thus all of Maxwell's equations are satisfied with the electromagnetic wave propagation.

The speed of light as obtained agree with the as defined by modern values.

Properties of Electromagnetic Waves

1. The electromagnetic wave is transverse in nature, i.e both E and B are perpendicular to the direction of propagation of the wave. The electric and magnetic fields are also perpendicular to each other. The direction of propagation is the direction of the vector product $\vec{E} \times \vec{B}$
2. There is a definite ratio between the magnitudes of E and B
   i.e $E = c\,B$ and $B = \mu_0\,\varepsilon_0\,c\,E$
3. The wave travels in vacuum with a definite and unchanging speed.
4. Unlike mechanical waves, which need the oscillating particles of a medium, electromagnetic waves require no medium.
5. $\vec{E}$ and $\vec{B}$ fields obey the superposition principle in waves just as in static situations: When two waves are superposed, the total $\vec{E}$ field at each point is the vector sum of the $\vec{E}$ fields of the individual waves, and similarly for the total $\vec{B}$ field.
6. Electromagnetic waves have the property of polarization.

Derivation of the Electromagnetic Wave Equation

We know that the differential form of the mechanical wave equation is

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\,\frac{d^2y}{dt^2} \qquad \text{----------(1)}$$

Where v = speed of propagation of the wave.

But the electromagnetic wave equation in terms of electric field is

$$\frac{d^2E}{dx^2} = \mu_0\,\varepsilon_0\,\frac{d^2E}{dt^2} \qquad \text{---------- (2)}$$

And the electromagnetic wave equation in terms of magnetic field is

$$\frac{d^2B}{dx^2} = \mu_0\,\varepsilon_0\,\frac{d^2B}{dt^2} \qquad \text{---------- (3)}$$

Compare these equations we get

$$\frac{1}{v^2} = \mu_0\varepsilon_0 \qquad \Rightarrow \quad v = \sqrt{\frac{1}{\mu_0\varepsilon_0}}$$

Here, v is the speed of electromagnetic waves.

Conceptual Problems:

Test Your Understanding of Section 32.1

a) Is it possible to have a purely electric wave propagate through empty space—-that is, a wave made up of an electric field but no magnetic field?

b) What about a purely magnetic wave, with a magnetic field but no electric field?

Answers: (a) D; (b) A; (c) B; (d) C

a) No.

A purely electric wave would have a varying electric field. Such a field necessarily generates a magnetic field through Ampere's law, so a purely electric wave is impossible.

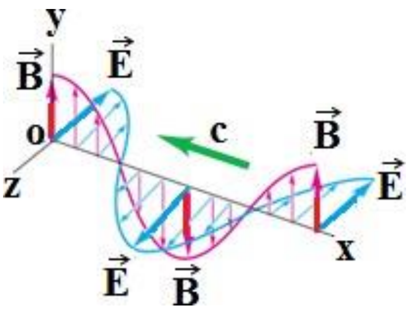
b) No

In the same way, a purely magnetic wave is impossible: The varying magnetic field in such a wave would automatically give rise to an electric field through Faraday's law..

In Class Problems: 32.1

Example 32.1 Electric and magnetic fields of a laser beam

A carbon dioxide laser emits a sinusoidal electromagnetic wave that travels in vacuum in the negative x-direction. The wavelength is 10.6 μm (in the infrared; see Fig.) and the E-field is parallel to the z-axis, with $E_{max} = 1.5\ \text{MV/m}$. Write vector equations for $\vec{E}$ and $\vec{B}$ as functions of time and position.



Solution:

Given that:

- The wave propagates in vacuum medium.
- Direction of propagation of electromagnetic wave = negative x-direction
- E-field is parallel to the z-axis

So, B must be along y- axis.
Let the E-vector and B-vectors are given by

$$\vec{E}(x,t) = \hat{k}\, E_{max} \cos(kx + \omega t) \quad \text{and} \quad \vec{B}(x,t) = \hat{j}\, B_{max} \cos(kx + \omega t)$$

From Faraday's law, we have

$$E_{max} = c B_{max} \Rightarrow B_{max} = \frac{E_{max}}{c} = \frac{1.5 \times 10^{6}\ \text{V/m}}{3.0 \times 10^{8}\ \text{m/s}} = 5.0 \times 10^{-3}\ \text{T}$$

The wave number k is given by

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\ \text{rad}}{10.6 \times 10^{-6}\ \text{m}} = 5.93 \times 10^{5}\ \text{rad/s}$$

$$\omega = c k = (3.0 \times 10^{8}\ \text{m/s})(5.93 \times 10^{5}\ \text{rad/m}) = 1.78 \times 10^{14}\ \text{rad/s}$$

Substituting these values into the above wave functions, we get

$$\vec{E}(x,t) = \hat{k}\, E_{max} \cos(kx + \omega t) = \hat{k}\,(1.5 \times 10^{6}\ \text{V/m}) \cos\left[(5.93 \times 10^{5}\ \text{rad/m})x + (1.78 \times 10^{14}\ \text{rad/s})t\right]$$

$$\vec{B}(x,t) = \hat{j}\, B_{max} \cos(kx + \omega t) = \hat{j}\left(5.0 \times 10^{-3}\ \mathrm{T}\right)\cos\left[\left(5.93 \times 10^{5}\ \mathrm{rad/m}\right)x + \left(1.78 \times 10^{14}\ \mathrm{rad/s}\right)t\right]$$