# Chapter-26: Direct Current Circuits

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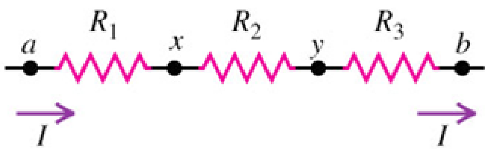
## LEARNING GOAL

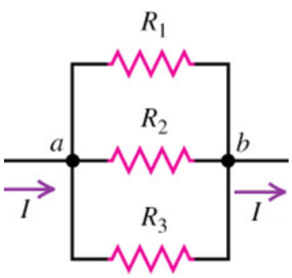
- To study resistors in series and parallel
- To consider Kirchhoff's Rules
- To see the design and learn about the use of electronic measuring instruments
- To mentally assemble R-C circuits
- To study the applications of circuits in household wiring

## Resistors in series




- Resistors are in series if they are connected one after the other so the current is the same in all of them.
- The equivalent resistance of a series combination is the sum of the individual resistances: $R_S = R_1 + R_2 + R_3 + \ldots$
- Series Resistors have resistance LARGER than the largest value present.

## Resistors in parallel




- Resistors are in parallel if they are connected so that the potential difference must be the same across all of them.
- The equivalent resistance of a parallel combination is given by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots..$$

- Parallel Resistors have resistance SMALLER than the smallest value present.

## Kirchhoff's Rules

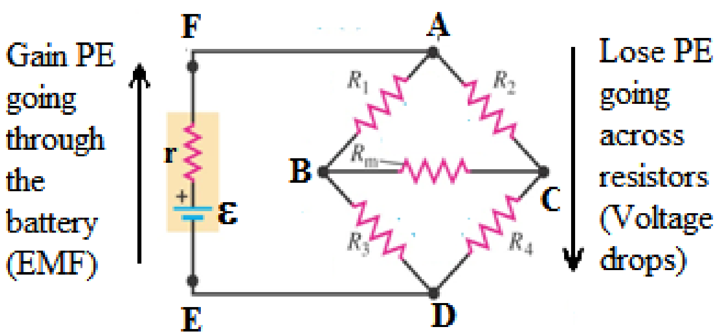
- Kirchhoff's junction Law (Kirchhoff's current Law):

  The algebraic sum of the currents into any junction is zero: $\Sigma I = 0$.

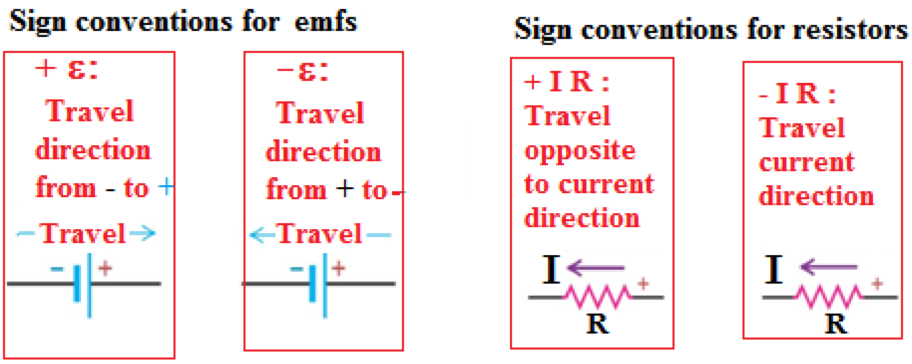
- Conservation of Charge in time (steady state currents)

  Kirchhoff's loop Law (Kirchhoff's voltage Law): The algebraic sum of the **potential differences** in any loop must equal zero: $\Sigma V = 0$.

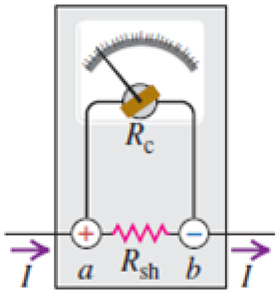
- Conservation of Energy.



Sign convention for the loop rule




## Ammeters




An ammeter always measures the current passing through it.

An ideal ammeter would have zero resistance, so including it in a branch of a circuit would not affect the current in that branch.

Real ammeters always have some finite resistance, but it is always desirable for an ammeter to have as little resistance as possible.

An ammeter is designed by connecting a resistor in parallel with a galvanometer. So that some of the current bypasses the meter coil. The parallel resistor is called a shunt resistor ( $R_{sh}$).

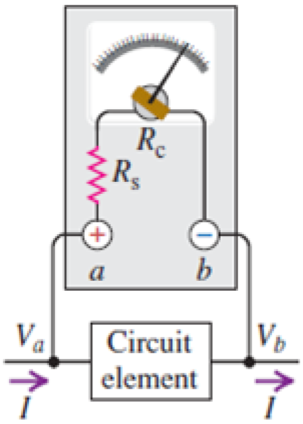
$R_c$ = Resistance of the galvanometer
$R_{sh}$ = Shunt Resistor
$I_a$ = Ammeter Current
$I_{fs}$ = current through the galvanometer for full scale deflection for ammeter current $I_a$.
$I_a - I_{fs}$ = Current through the shunt resistor.

Since the shunt resistor is connected parallel to the galvanometer, we have

Potential difference across the galvanometer = Potential difference across the shunt resistor

$I_{fs} R_c = (I_a - I_{fs}) R_{sh}$

## Voltmeters




Voltmeter is a device used to measure potential difference or the voltage between two points. An ideal voltmeter would have infinite resistance, so connecting it between two points in a circuit would not alter any of the currents.

Real voltmeters always have finite resistance, but a voltmeter should have large enough resistance that connecting it in a circuit does not change the other currents appreciably.

A voltmeter is designed by connecting a resistor $R_s$ in series with the galvanometer Then only a fraction of the total potential difference appears across the galvanometer coil and the remainder appears across $R_s$.

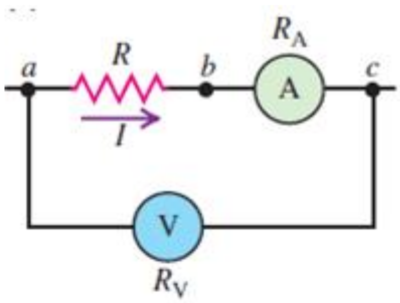
For a voltmeter with full-scale reading $V_{fs}$ we should have

$V_{fs} = I_{fs} (Rc + Rs)$

## Ammeters and Voltmeters in Combination

A voltmeter and an ammeter can be used together to measure resistance and power.
The resistance R of a resistor is given by

$$R = \frac{V_{ab}}{I}$$

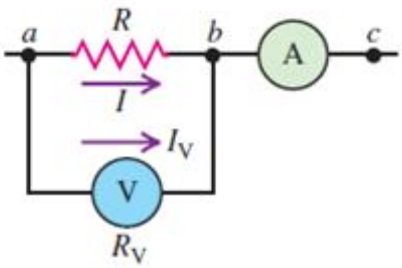



$V_{ab}$ = potential difference between its terminals
I = current between the terminals

The power input P to any circuit element is given by

$P = V_{ab}\, I$

In principle, the most straightforward way to measure R or P is to measure $V_{ab}$ and I simultaneously.

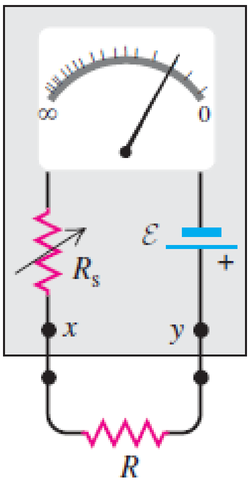



When ammeter A reads the current I in the resistor R, the Voltmeter V, however, reads the sum of the potential difference $V_{ab}$ across the resistor and the potential difference $V_{bc}$ across the ammeter.

If we transfer the voltmeter terminal from c to b, as in Fig. 26.16b, then the voltmeter reads the potential difference $V_{ab}$ correctly, but the ammeter now reads the sum of the current I in the resistor and the current IV in the voltmeter.

Either way, we have to correct the reading of one instrument or the other unless the corrections are small enough to be negligible.

## Ohmmeters



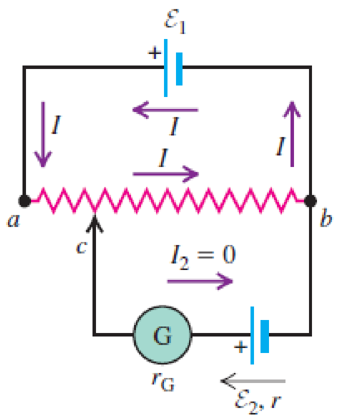

- Ohmmeter measures the resistance of a resistor.
- It uses a meter, a resistor, and a source (often a flashlight battery) connected in series
- The resistance R to be measured is connected between terminals x and y.
- The series resistance $R_s$ is variable; it is adjusted so that when terminals x and y are short-circuited (that is, when R = 0 ), the meter deflects full scale.

- When nothing is connected to terminals x and y, so that the circuit between x and y is open (that is, when $R \to \infty$ ), there is no current and hence no deflection.
- For any intermediate value of R the meter deflection depends on the value of R, and the meter scale can be calibrated to read the resistance R directly. Larger currents correspond to smaller resistances, so this scale reads backward compared to the scale showing the current.

## The Potentiometer

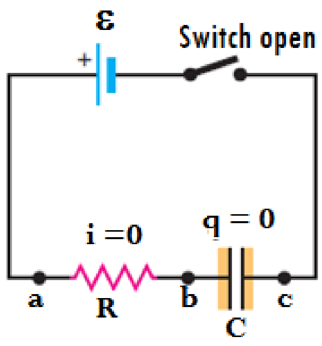
The potentiometer is an instrument that can be used to measure the emf of a source without drawing any current from the source. The term potentiometer is also used for any variable resistor. In addition to this it also has a number of other useful applications. Essentially, it balances an unknown potential difference against an adjustable, measurable potential difference.

The principle of the potentiometer is shown schematically in the figure.




- A resistance wire ab of total resistance $R_{ab}$ is permanently connected to the terminals of a source of known emf $\varepsilon_1$.
- A sliding contact 'c' is connected through the galvanometer 'G' to a second source whose emf $\varepsilon_2$ is to be measured.
- As contact 'c' is moved along the resistance wire, the resistance $R_{bc}$ between points c and b varies.
- If the resistance wire is uniform, $R_{bc}$ is proportional to the length of wire between c and b.
- To determine the value of $\varepsilon_2$, contact 'c' is moved until a position is found at which the galvanometer shows no deflection; this corresponds to zero current passing through $\varepsilon_2$.
- With $I_2 = 0$ Kirchhoff's loop rule gives $\varepsilon_2 = I R_{bc}$
- With $I_2 = 0$ the current I produced by the emf $\varepsilon_1$ has the same value no matter what the value of the emf $\varepsilon_2$. We calibrate the device by replacing $\varepsilon_2$ by a source of known emf; then any unknown emf $\varepsilon_2$ can be found by measuring the length of wire cb for which $I_2 = 0$.
- But this to work, $V_{ab}$ must be greater than $\varepsilon_2$.

## R-C Circuits: Charging a Capacitor




In an **R-C circuit** a resistor and a capacitor are connected in series. The circuit consists of a battery (or power supply) with constant emf ε and zero internal resistance ( r=0). We also neglect the resistance of all the connecting conductors.

Let $v_{ab}$= potential difference across the resistor R

$v_{bc}$ = potential difference across the capacitor

Applying kirchhoff's law, we get

$v_{ab} + v_{bc} = \varepsilon$ -------------- (1)

Initially i.e at t = 0

Capacitor was uncharged. So , $v_{bc} = 0$
From eqn (1) we have $v_{ab} = \varepsilon$

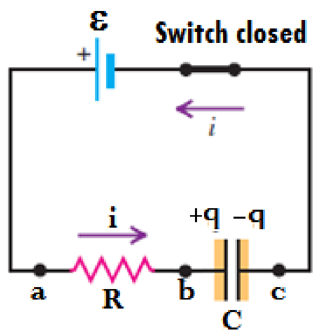
Initial current ($I_0$) through the resistor R, (by Ohm's law): $I_0 = \frac{V_{ab}}{R} = \frac{\varepsilon}{R}$

As the capacitor charges, then:
$v_{bc}$ increases and $v_{ab}$ decreases, corresponding current decreases.

Capacitor is fully charged
After a long time the capacitor becomes fully charged, then

- Current decreases to zero,
- $v_{ab} = 0$
- $v_{bc} = \varepsilon$ i.e the entire battery emf ($\varepsilon$ ) appears across the capacitor.

At any instant of time t:




After the switch has been closed let
q = charge on the capacitor at some time t and
i = current in the circuit at some time t

So, $v_{ab} = iR$ and $v_{bc} = \frac{q}{C}$

Putting this value in eqn (1)

$v_{ab} + v_{bc} = \varepsilon$

$\Rightarrow \; iR + \frac{q}{C} = \varepsilon$

$\Rightarrow \; iR = \varepsilon - \frac{q}{C} \quad \Rightarrow \; i = \frac{\varepsilon}{R} - \frac{q}{RC} \quad ----(2)$

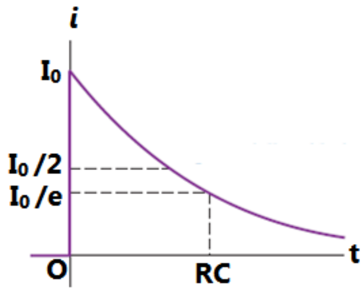
## Calculation of instantaneous current (i)

The instantaneous current i is given by

$$i = \frac{dq}{dt} = \frac{d}{dt}Q_f\left(1-e^{-\frac{t}{RC}}\right) \quad \Rightarrow i = \frac{d}{dt}Q_f - \frac{d}{dt}Q_f e^{-\frac{t}{RC}}$$

$$\Rightarrow i = 0 - Q_f e^{-\frac{t}{RC}}\left(-\frac{1}{RC}\right) \qquad \Rightarrow i = \frac{Q_f}{RC}e^{-\frac{t}{RC}}$$

$$\Rightarrow i = \frac{\varepsilon}{R}e^{-\frac{t}{RC}} \quad (\text{using eqn 3}) \quad \Rightarrow i = I_0\, e^{-\frac{t}{RC}} \quad -----(5)$$




The current decreases exponentially with time as the capacitor charges.

## Time Constant

The time at which the current decreases to 1/e (0.368) time the original value is called the time constant.

So, at time constant ($\tau$), $i = I_0/e$

Putting this value in eqn- 5 we get

$$i = I_0\, e^{-\frac{t}{RC}}$$

$$\Rightarrow \frac{I_0}{e} = I_0\, e^{-\frac{\tau}{RC}} \quad \Rightarrow \quad e^{-1} = e^{-\frac{\tau}{RC}}$$

$$\Rightarrow 1 = \frac{\tau}{RC} \qquad \Rightarrow \tau = RC$$

Thus time constant is $\tau = RC$. This is also called relaxation time.

The unit is RC is second

At this time the charged stored in the capacitor is given by

$$q = \varepsilon C\left(1-e^{-\frac{t}{RC}}\right)$$

$$\Rightarrow q_\tau = \varepsilon C\left(1-e^{-\frac{\tau}{RC}}\right) \qquad \Rightarrow q_\tau = Q_f\left(1-e^{-\frac{RC}{RC}}\right)$$

$$\Rightarrow q_\tau = Q_f\left(1-e^{-1}\right) \qquad \Rightarrow q_\tau = Q_f\left(1-0.368\right)$$

$$\Rightarrow q_\tau = 0.632Q_f$$

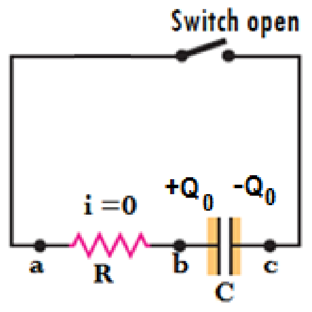
Thus in time $t = \tau = RC$ the capacitor charge has reached 0.632 times its final value $Q_f$.

When $\tau$ is small, the capacitor charges quickly.

When $\tau$ is larger, the charging takes more time.

If the resistance is small, it's easier for current to flow, and the capacitor charges more quickly. If R is in ohms and C in farads, is in seconds.

## Discharging a Capacitor




Let the capacitor is fully charged to $Q_0$.

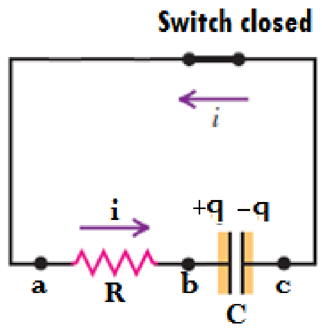
Now the battery is removed from the R-C circuit and the circuit is open.

When the circuit is closed let the time t = 0.

Thus at t = 0, $q = Q_0$

Then the capacitor starts discharging through the resistor.

### Calculation of instantaneous charge (q) during discharging




Let q = charge in the capacitor at any instant of time t.

Applying Kirchhoff's loop rule we get,

$$V_{ab} + V_{bc} = 0$$

$$\Rightarrow iR + \frac{q}{C} = 0 \qquad \Rightarrow iR = -\frac{q}{C}$$

$$\Rightarrow i = -\frac{q}{RC} \qquad --------(1)$$

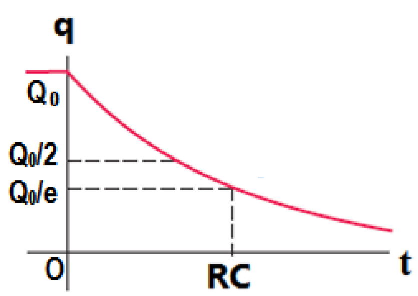
$$\Rightarrow \frac{dq}{dt} = -\frac{q}{RC} \qquad \Rightarrow \frac{dq}{q} = -\frac{dt}{RC}$$

$$\Rightarrow \int_{Q_0}^{q} \frac{dq}{q} = -\int_0^t \frac{dt}{RC}$$

$$\Rightarrow [\ln q]_{Q_0}^{q} = -\frac{t}{RC} \Rightarrow \ln q - \ln Q_0 = -\frac{t}{RC}$$

$$\Rightarrow \ln\frac{q}{Q_0} = -\frac{t}{RC}$$

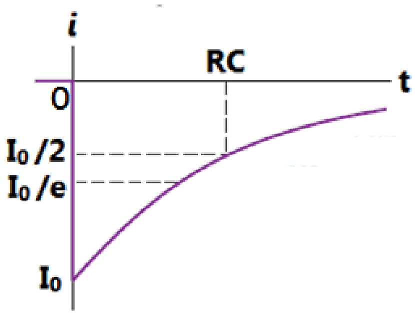
$$\Rightarrow \frac{q}{Q_0} = e^{-\frac{t}{RC}} \qquad \Rightarrow q = Q_0\, e^{-\frac{t}{RC}}$$




Variation of charge with time during discharging of capacitor is shown in the graph.

The charge decreases exponentially with time as the capacitor discharges.

## Calculation of instantaneous current (*i*) during discharging




The instantaneous current (i) is given by

$$i = \frac{dq}{dt} = \frac{d}{dt}\left(Q_0\, e^{-\frac{t}{RC}}\right)$$

$$\Rightarrow\ i = Q_0\, e^{-\frac{t}{RC}}\left(-\frac{1}{RC}\right)$$

$$\Rightarrow\ i = -\frac{Q_0}{RC} e^{-\frac{t}{RC}} \quad \Rightarrow\ i = I_0\, e^{-\frac{t}{RC}} \qquad (\text{using eqn} -1)$$

Variation of current with time during discharging of capacitor is shown in the graph.

The current decreases exponentially as the capacitor discharges. (The current is negative because its direction is opposite to that in figure.

## Power (P) in R-C circuit:

When the capacitor is charging, the instantaneous power supplied by the battery in the R-C circuit is

$P = \varepsilon i$

From the R-C circuit we have

$$V_{ab} + V_{bc} = \varepsilon$$

$$\Rightarrow\ \varepsilon = iR + \frac{q}{C} \quad \Rightarrow\ i\varepsilon = i^2R + i\frac{q}{C} \quad \Rightarrow\ P = i^2R + i\frac{q}{C}$$

$i^2R$ = power dissipated in the resistor

$i\dfrac{q}{C}$ = power stored in the capacitor.

$$\text{Thus, } \begin{pmatrix}\text{Power supplied}\\ \text{by the battery}\end{pmatrix} = \begin{pmatrix}\text{power dissipated}\\ \text{in the resistor}\end{pmatrix} + \begin{pmatrix}\text{power stored}\\ \text{in the capacitor}\end{pmatrix}$$

## Energy concept

The total energy supplied by the battery during charging of the capacitor is $\varepsilon\, Q_f$

But the energy stored in the capacitor is $\dfrac{\varepsilon\, Q_f}{2}$

Thus the energy dissipated in the resistor is $\dfrac{\varepsilon\, Q_f}{2}$

Thus, of the energy supplied by the battery, exactly half is stored in the capacitor, and the other half is dissipated in the resistor.

## Conceptual Problem:

### Test Your Understanding of Section 26.4

The energy stored in a capacitor is equal to $\frac{q^2}{2C}$. When a capacitor is discharged, what fraction of the initial energy remains after an elapsed time of one time constant?

(i) 1/e (ii) $1/e^2$ (iii) 1 - 1/e (iv) $(1 - 1/e)^2$

(v) Answer depends on how much energy was stored initially.

**Answer: (ii)**

After one time constant, t = RC and the initial charge $Q_0$ has decreased to

$$q = Q_0\, e^{-\frac{t}{RC}} \;\Rightarrow\; q = Q_0\, e^{-\frac{RC}{RC}} \;\Rightarrow\; q = Q_0\, e^{-1} \;\Rightarrow\; q = \frac{Q_0}{e}$$

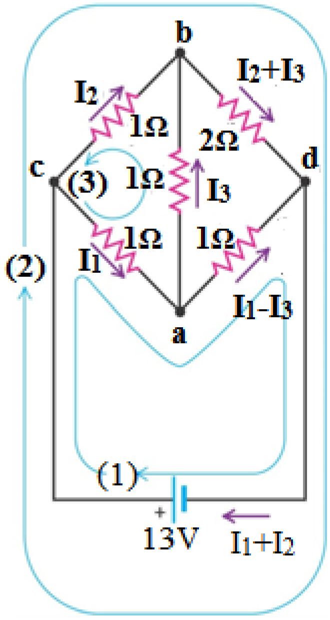
Initial energy stored = $\frac{Q_0^2}{2C}$ Energy stored after t = RC is $\frac{(Q_0/e)^2}{2C} = \left(\frac{1}{e^2}\right)\frac{Q_0^2}{2C} = 0.135\left(\frac{Q_0^2}{2C}\right)$

Thus the energy is decreased by a fraction of 0.135 of its initial energy. This result doesn't depend on the initial value of the energy.

## In class Problem

### Example 26.6 A complex network

Adjoin figure shows a "bridge" circuit. Find the current in each resistor and the equivalent resistance of the network of five resistors.




Solution:

Applying Kirchhoff's loop rule to the three loops we get:

$13\text{ V} - I_1(1\,\Omega) - (I_1 - I_3)(1\,\Omega) = 0 \quad \Rightarrow \; 2I_1 - I_3 = 13$

$-I_2(1\,\Omega) - (I_2 + I_3)(2\,\Omega) + 13\text{ V} = 0 \quad \Rightarrow \; 2I_2 - 2I_3 = -13$

$-I_1(1\,\Omega) - I_3(1\,\Omega) + I_2(1\,\Omega) = 0 \quad \Rightarrow \; I_1 + I_3 = I_2$

Solving above equations we get, $I_1 = 6\text{A}$ $I_2 = 5\text{A}$ $I_3 = -1\text{A}$

The equivalent resistance of the network is

$$R_{eq} = \frac{13\text{V}}{11\text{A}} = 1.2\Omega$$

## Example 26.8 Designing an ammeter

What shunt resistance is required to make the 1.0mA, 20 Ω meter into an ammeter with a range of 0 to 50 mA?

**Solution:**

The ammeter must handle a maximum current = $I_a = 50 \times 10^{-3}$A

The coil resistance $R_c = 20\ \Omega$

The meter shows full-scale deflection when the current through the coil is $I_{fs} = 1.0 \times 10^{-3}$A

$$I_{fs}R_c = R_s\left(I_a - I_{fs}\right)$$

$$\Rightarrow R_s = \frac{I_{fs}R_c}{I_a - I_{fs}} = \frac{\left(1 \times 10^{-3}\,A\right)\left(20\,\Omega\right)}{\left(50 \times 10^{-3}\,A - 1 \times 10^{-3}\,A\right)} = 0.408\,\Omega$$

## Example 26.9 designing a voltmeter

What series resistance is required to make the 1.00-mA, 20 Ω meter into a voltmeter with a range of 0 to 10.0 V?

**Solution:**

The maximum allowable voltage across the voltmeter $V_v$ = 10V

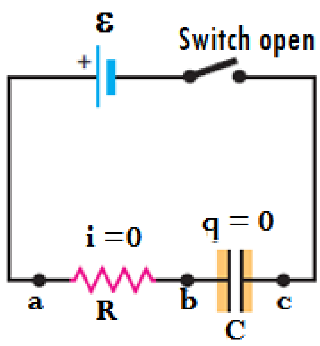
The coil resistance $R_c = 20\ \Omega$

Current through the coil is $I_{fs} = 1.0 \times 10^{-3}$A

$$R_s = \frac{V_v}{I_{fs}} - R_c = \frac{10\,V}{1 \times 10^{-3}\,A} - 20\,\Omega = 9980\,\Omega$$

## Example 26.12 Charging a capacitor

A 10 MΩ resistor is connected in series with a 1.0 μF capacitor and a battery with emf 12.0 V. Before the switch is closed at time t = 0, the capacitor is uncharged.




a) What is the time constant?

b) What fraction of the final charge $Q_f$ is on the capacitor at t = 46 s?

c) What fraction of the initial current ($I_0$) is still flowing at t = 46 s?

**Solution:**

R = 10 MΩ = $10 \times 10^{6}$ Ω, C = 1.0 μF = $1.0 \times 10^{-6}$ F, ε = 12.0 V.

a) $t = RC = (10 \times 10^{6}\ \Omega)(1.0 \times 10^{-6}\ F) = 10$ s

b) $q = Q_f\left(1 - e^{-\frac{t}{RC}}\right) \Rightarrow \frac{q}{Q_f} = 1 - e^{-\left(\frac{t}{RC}\right)} \Rightarrow \frac{q}{Q_f} = 1 - e^{-\left(\frac{46\,s}{10\,s}\right)} = 0.99$

After time t = 46 s the capacitor is 99% charged

=> After time = 4.6 x time constant, the capacitor is 99% charged

The circuit would charge more rapidly if we reduced the time constant by using a smaller resistance.

c) $i = i_0 e^{-\frac{t}{RC}} \Rightarrow \frac{i}{i_0} = e^{-\left(\frac{46\,s}{10\,s}\right)} = 0.010$

After time t = 46 s the current has decreased to 1.0% of its initial value

=> After time = 4.6 x time constant, current has decreased to 1.0% of its initial value

## Example 26.13 Discharging a capacitor

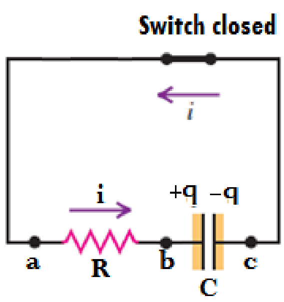
The resistor and capacitor of Example 26.12 are reconnected as shown in figure. The capacitor has an initial charge of 5.0 μC and is discharged by closing the switch at t = 0.

a) At what time will the charge be equal to 0.5 μC?

b) What is the current at this time?

Solution: $R = 10\ M\Omega = 10 \times 10^6\ \Omega$, $C = 1.0\ \mu F = 1.0 \times 10^{-6}\ F$, $\varepsilon = 12.0\ V$.

Initial charge of the capacitor $Q_0 = 5.0 \times 10^{-6} C$




a) $q = Q_0\, e^{-\left(\frac{t}{RC}\right)} \Rightarrow \frac{q}{Q_f} = e^{-\left(\frac{t}{RC}\right)}$

$$\Rightarrow \ln\left(\frac{q}{Q_0}\right) = -\frac{t}{RC} \Rightarrow t = -RC\ln\left(\frac{q}{Q_0}\right)$$

$$\Rightarrow t = -(10\,s)\ln\left(\frac{0.5\,\mu C}{5\,\mu C}\right) = 23\,s$$

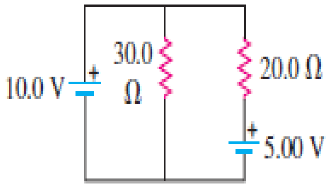
b) $i = I_0\, e^{-\frac{t}{RC}} \Rightarrow i = \frac{\varepsilon}{R} e^{-\frac{t}{RC}} \Rightarrow i = \frac{\varepsilon C}{RC} e^{-\frac{t}{RC}} \Rightarrow i = \frac{-Q_0}{RC} e^{-\frac{t}{RC}}$

$$\Rightarrow i = -\left(\frac{5 \times 10^{-6} C}{10\,s}\right) e^{-\left(\frac{23\,s}{10\,s}\right)} = -5 \times 10^{-8} A$$

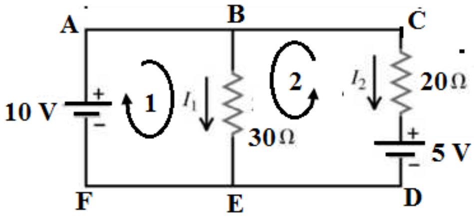
The current is negative because i has the opposite sign when the capacitor is discharging than when it is charging.

## Assignment Problem

26.26: The batteries shown in the circuit in the figure have negligibly small internal resistances. Find the current through (a) the 30 Ω resistor; (b) the 20 Ω resistor; (c) the 10 V battery.




Ans: a) Applying KVL to the loop (1), ABEFA




$\Sigma IR = \Sigma \varepsilon$

$(30\ \Omega)I_1 = 10\ V$

$I_1 = 0.333\ A$

b) Applying KVL to the loop (3): ABCDEFA

$\Sigma IR = \Sigma \varepsilon$

$(20\ \Omega)I_2 = +10\ V - 5\ V$

$(20\ \Omega)I_2 = 5\ V$

$I_2 = 0.25\ A.$

c) $I_1 + I_2 = 0.333\ A + 0.250\ A = 0.583\ A$

26.48 A 1.5 µF capacitor is charging through a 12.0 Ω resistor using a 10.0-V battery. What will be the current when the capacitor has acquired one-fourth of its maximum charge? Will it be of the maximum current?

**Solution:** The maximum current in the circuit is at the beginning and is given by

$$i_0 = \frac{\varepsilon}{R} = \frac{10V}{12\Omega} = 0.833\ A$$

The charge is increasing and at any instant it is given by

$$q = Q_f\left(1 - e^{-\frac{t}{RC}}\right) \quad \Rightarrow \quad \frac{Q_f}{4} = Q_f\left[1 - e^{-\left(\frac{t}{RC}\right)}\right]$$

$$\Rightarrow \quad \frac{1}{4} = 1 - e^{-\left(\frac{t}{RC}\right)} \quad \Rightarrow \quad e^{-\left(\frac{t}{RC}\right)} = \frac{3}{4} \quad \Rightarrow \quad e^{-\left(\frac{t}{RC}\right)} = \frac{3}{4}$$

At this situation the current will be

$$i = i_0 e^{-\frac{\tau}{RC}} \quad \Rightarrow \quad i = i_0\frac{3}{4} = (0.833\ A)\frac{3}{4} = 0.625\ A$$