Study Material
(Electromagnetism)

Chapter-31
Alternating Current

Topics covered:	• Phasors and Alternating Currents • Resistance and Reactance • The L-R-C Series Circuits • Power in Alternating-Current Circuits • Resonance in Alternating-Current Circuits
Conceptual Problems:	TYU 31.1, 31.2, 31.3, 31.4
In Class Problems:	31.1, 31.3, 31.4
Assignment Problems:	31.25, 31.27

Dr. Rajanikanta Parida
Department of Physics
ITER,SOA University, Bhubaneswar

Phasors and Alternating Currents

An ac source is a device that supplies a sinusoidally varying voltage (potential difference) or current. The usual circuit-diagram symbol for an ac source is 

A sinusoidal voltage might be represent as:

$v = V \cos \omega t$

Where, v = instantaneous potential difference

V = maximum potential difference = voltage amplitude

ω = angular frequency = $2\,\pi\,f$

f = frequency

Similarly, a sinusoidal current might be described as

$i = I \cos \omega t$

i = instantaneous current, and I = maximum current = current amplitude

Phasor diagrams:

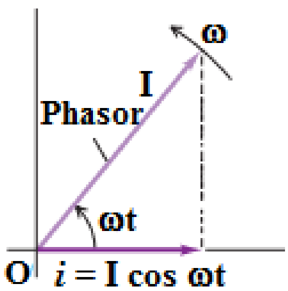
A phasor (rotating vector diagrams) is not a real physical quantity, but it helps to describe and analyze physical quantities that vary sinusoidally with time.

- As alternating current and voltage vary with time sinusoidally, these can be represented by phasor diagrams.
- The figure shows the phasor diagram for current.

  Length of phasor = maximum current I (Current amplitude).

  $$\left(\begin{array}{l}\text{Projection of the phasor}\\ \text{onto the horizontal axis}\end{array}\right) = (\text{instantaneous current})$$

  The phasor rotates counter-clockwise with constant angular speed ω.



Root-Mean-Square (rms) Values

Root mean square (rms) value of the the instantaneous current (I_{rms}) is the square root of the average (mean) value of i^2. Thus

$$I_{rms} = \sqrt{\left(i^2\right)_{av}}$$

Alternating current vary with time sinusoidally. The instantaneous current is given by

$i = I \cos \omega t$

Then, $i^2 = I^2 \cos^2 \omega t$

$$\Rightarrow \left(i^2\right)_{av} = I^2\left(\cos^2 \omega t\right)_{av}$$

$$\Rightarrow \left(i^2\right)_{av} = \frac{1}{2}I^2 \qquad \left[\because \left(\cos^2 \omega t\right)_{av} = \frac{1}{2}, \text{ over a complete cycle}\right]$$

$$\text{Thus,} \quad I_{rms} = \sqrt{\left(i^2\right)_{av}} = \sqrt{\frac{I^2}{2}} = \frac{I}{\sqrt{2}}$$

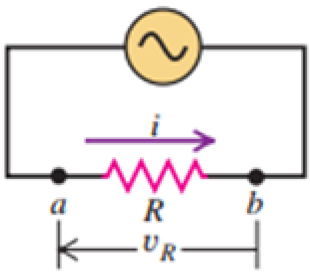
In the same way, the root-mean-square value of a sinusoidal voltage with amplitude (maximum value) V is

$$V_{rms} = \frac{V}{\sqrt{2}}$$

- Meters used for ac voltage and current measurements are nearly always calibrated to read rms values.
- Voltages and currents in power distribution systems are always described in terms of their rms values.
- The usual household power supply, “220-volt ac,” has an rms voltage of 220 V. The voltage amplitude is

  $$V = \sqrt{2}\, V_{rms} = \sqrt{2}\,(220\text{V}) = 311.08 \text{ V}$$

AC Circuit containing only Resistor



Let us consider an ac circuit containing only resistor.

R = resistance of the resistor

The sinusoidal current in the circuit is flowing in counter-clockwise direction. It is given by

$i = I \cos \omega t$ ------------------(1)

Here, I = current amplitude (maximum current)

From Ohm’s law, the instantaneous potential difference (voltage) v_R across the resistor is given by

$v_R = i\,R = I\,R \cos \omega t$ ------------------(2)

$\Rightarrow \quad v_R = V_R \cos \omega t$

Where, V_R = I R = Voltage amplitude (maximum voltage)

The above Eq. can be written in rms form as

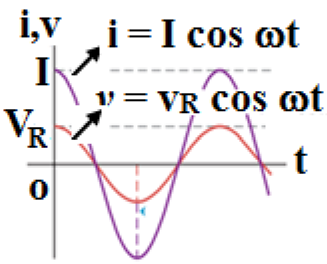
$(V_R)_{rms} = I_{rms}\, R$

From eq (1) and eq (2) it is clear that:

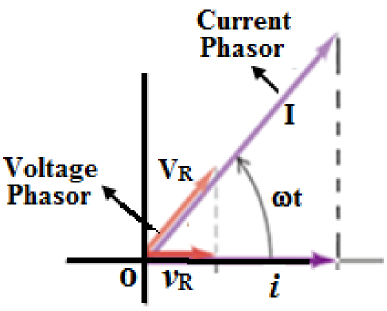
The current and voltage are both proportional to cos ωt

So the current i is in phase with the voltage v_R.

Figure in the right shows graphs of i and v_R as functions of time.

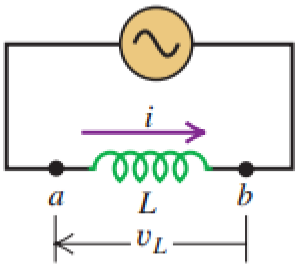


The phasor diagrams for current and voltage is shown in the figure.



- Because i and v_R are in phase and have the same frequency, the current and voltage phasors rotate together; they are parallel at each instant.
- The projections of current and voltage on the horizontal axis represent the respective instantaneous current and voltage.

AC Circuit containing only Inductor



Let us consider an ac circuit containing only Inductor.

L = self inductance of the inductor and have zero resistance

The sinusoidal current in the circuit is flowing in counter-clockwise direction. It is given by

$i = I \cos \omega t$ ------------------(1)

Here, I = current amplitude (maximum current)

The potential difference (v_L) appears across the inductor varies with time, the current also varies accordingly. This gives rise to a self-induced emf.

The induced emf in the direction of i is given by

$$\varepsilon = -L\frac{di}{dt}$$

If the current in the inductor is in the positive (counter-clockwise) direction from a to b and is increasing, then is positive and the induced emf is directed to the left to oppose the increase in current; hence point 'a' is at higher potential than is point 'b'.

Thus the potential difference (v_L) across the inductor is positive and is given by

$$v_L = +L\frac{di}{dt}$$

$$\Rightarrow \quad v_L = L\frac{d}{dt}(I\cos\omega t) = -I\omega L\sin\omega t \quad \Rightarrow \quad v_L = I\omega L\cos\left(\omega t + \frac{\pi}{2}\right)$$

$$\Rightarrow \quad v_L = V_L \cos\left(\omega t + \frac{\pi}{2}\right) \quad \text{----------------(2)}$$

Where, $V_L = I\,\omega L = I\,X_L$ -----------------(3)

The above Eq. can be written in rms form as

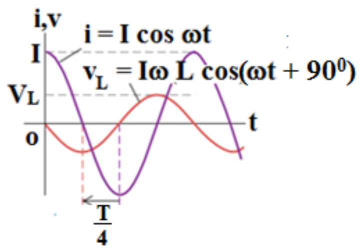
$(V_L)_{rms} = I_{rms}\,X_L$

Where, $X_L = \omega L$ = inductive reactance -----------------(4)

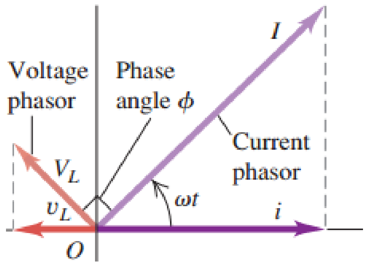
From eq (1) and eq (2) it is clear that:

The phase difference between the current i and voltage v_L is 90^0. The voltage leads the current by 90^0 in inductive circuit.

Figure in the right shows graphs of i and v_L as functions of time.



- Here, $v_L \propto \frac{di}{dt}$
- The points of maximum voltage on the graph correspond to maximum steepness of the current curve
- The points of zero voltage are the points corresponds to the maximum and minimum values of the current.
- The voltage peaks occur a quarter-cycle earlier than the current peaks.
- v_L leads i by $\pi/2$
- The phasor diagrams for current and voltage shows that v_L leads the current by $\pi/2$.



Inductive Reactance (X_L)

For an inductive circuit, the maximum voltage appears across the inductor is

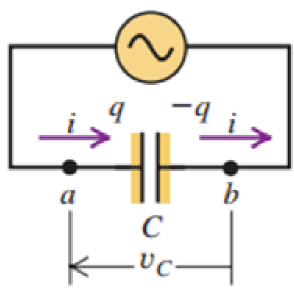
$V_L = I\omega L = I X_L$

Where, $X_L = \omega L$ = inductive reactance

The inductive reactance (X_L) opposes any change in the current through the inductor.

- For a given current amplitude, the voltage across the inductor is directly proportional to X_L.
- the inductive reactance (X_L) increase with increasing angular frequency (ω) and increasing inductance L.
- As X_L is proportional to frequency (ω), so a high-frequency voltage applied to the inductor gives only a small current, while a lower-frequency voltage of the same amplitude gives rise to a larger current.
- Inductors are used in some circuit applications, such as power supplies and radio-interference filters, to block high frequencies while permitting lower frequencies or dc to pass through. A circuit device that uses an inductor for this purpose is called a low-pass filter.

AC Circuit containing only Capacitor



Let us consider an ac circuit containing only capacitor.

C = capacitance of the capacitor

The sinusoidal current in the circuit is flowing in counter-clockwise direction. It is given by

$i = I \cos \omega t$ ------------------(1)

Here, I = current amplitude (maximum current)

The potential difference (v_C) appears across the capacitor is given by

$$v_C = \frac{q}{C} = \frac{\int i\,dt}{C} \qquad \left[\because\ i = \frac{dq}{dt} \Rightarrow dq = i\,dt \Rightarrow q = \int i\,dt\right]$$

$$\Rightarrow v_C = \frac{1}{C}\int I\cos\omega t\,dt = \frac{I}{\omega C}\sin\omega t$$

$$\Rightarrow v_C = \frac{I}{\omega C}\cos\left(\omega t - \frac{\pi}{2}\right)$$

$$\Rightarrow v_C = V_C\cos\left(\omega t - \frac{\pi}{2}\right) \qquad \text{----------------(2)}$$

Where, $V_C = \dfrac{I}{\omega C} = I X_C$ -----------------(3)

The above Eq. can be written in rms form as

$(V_C)_{rms} = I_{rms} X_C$

Where, $X_C = \dfrac{1}{\omega C}$ = capacitive reactance -----------(4)

From eq (1) and eq (2) it is clear that:

The phase difference between the current $\boldsymbol{i}$ and voltage $\boldsymbol{v_C}$ is 90^0. The current leads the voltage by 90^0 in capacitive circuit.

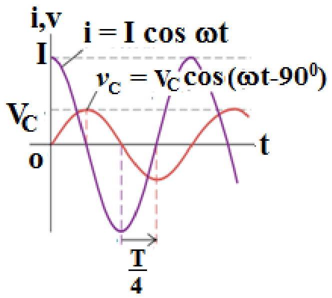
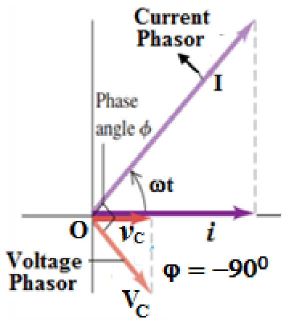


Figure in the right shows graphs of $\boldsymbol{i}$ and $\boldsymbol{v_C}$ as functions of time.

- We know that, $i = \dfrac{dq}{dt} = \dfrac{d(Cv_C)}{dt} = C\dfrac{dv_C}{dt}$. So, the instantaneous current is proportional to the rate of change of voltage.
- The current has its zero magnitude when the v_C curve is levels off at its maximum and minimum values.
- The points of maximum voltage on the graph correspond to maximum steepness of the current curve
- The points of zero voltage are the points corresponds to the maximum and minimum values of the current.
- The peaks of voltage occur a quarter-cycle after the corresponding current peaks, and we say that the voltage lags the current by 90^0. *i.e* v_C lags behinds the $\boldsymbol{i}$ by $\pi/2$
- The phasor diagrams for current and voltage shows that v_C lags behind the current by $\pi/2$.



Capacitive Reactance (X_C)

For a capacitive circuit, the maximum voltage appears across the capacitor is

$$V_C = \frac{I}{\omega C} = I X_C$$

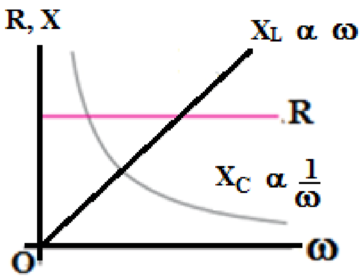
Where, $X_C = \dfrac{1}{\omega C}$ = capacitive reactance

- The capacitive reactance (X_C) of a capacitor is inversely proportional both to the capacitance (C) and to the angular frequency(ω)
  Thus, the greater the capacitance and the higher the frequency, the smaller the capacitive reactance
- Capacitors tend to pass high-frequency current and to block low-frequency currents and dc.
- A device that preferentially passes signals of high frequency is called a high-pass filter.

- For a given current amplitude, the voltage across the capacitor is inversely proportional to X_C.

Variation of R, X_L & X_C with ω

Figure in the right shows how the resistance of a resistor and the reactances of an inductor and a capacitor vary with angular frequency(**ω**).

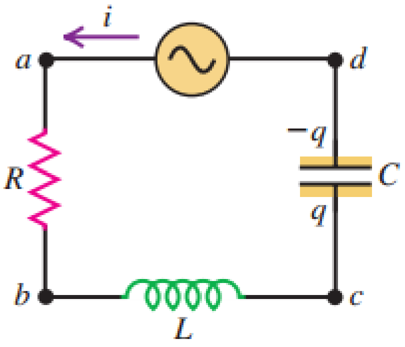


- Resistance (R) is independent of frequency,
- $X_L \;\alpha\; \omega$
- $X_C \;\alpha\; \dfrac{1}{\omega}$.
- For a dc circuit, frequency is zero. Thus,
  f = 0, => ω = 0 => $X_C = \infty$. So, there is no current through a capacitor.
  Again, f = 0, => ω = 0 => $X_L = 0$. So, there is no inductive effect
- If $\omega \to \infty$, then $X_L \to \infty$. the current through an inductor becomes vanishingly small.
  If $\omega \to \infty$, then $X_C \to 0$. => $v_C \to 0$, So, the current changes direction so rapidly that no charge can build up on either plate.

Circuit Elements with Alternating Current

Circuit Element	**Amplitude Relationship**	**Circuit Quantity**	**Phase of *V***
Resistor	$V_R = I\,R$	R	In phase with i
Inductor	$V_L = I\,X_L$	$X_L = \omega L$	Leads I by 90^0
Capacitor	$V_C = I\,X_C$	$X_C = \dfrac{1}{\omega C}$	Lags I by 90^0 **I C E (I leads the emf in capacitor circuit)**

AC Circuit containing L-R-C in Series



Let us consider an ac circuit containing resistor, inductor and capacitor. These are connected in series.

R = resistance of the resistor

L= self inductance of the inductor

C = capacitance of the capacitor

The sinusoidal current in the circuit is flowing in counter-clockwise direction. It is given by

$i = I \cos \omega t$ ------------------(1)

Here, I = current amplitude (maximum current)

The total potential difference (v) appears in the circuit can be written as

$v = V \cos(\omega t + \phi)$ ------------------(2)

Where, V = voltage amplitude, ϕ = phase difference between i and v

The magnitudes of V and ϕ can be obtained from the Phasor diagram.

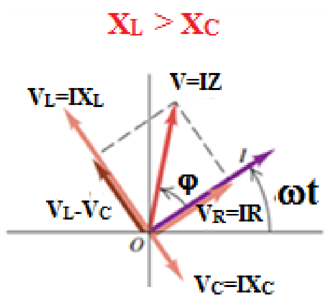
Phasor diagram

As R, L and C are connected in series, the **current** will be **same** in all the branches.

For resistor, the phasor V_R is in phase with the current phasor I,

For inductor, the phasor V_L leads the current phasor I by $\frac{\pi}{2}$

For capacitor, the phasor V_C lags the current phasor I by $\frac{\pi}{2}$



From the phasor diagram, the voltage amplitude across the ac circuit is

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\Rightarrow V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$\Rightarrow V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow V = IZ \quad \text{----------------------------------(3)}$$

The above Eq. can be written in rms form as

$V_{rms} = I_{rms} Z$

Where, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, Impedance of the LRC series circuit.

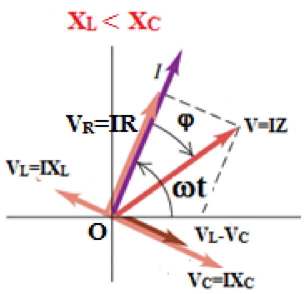
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad \text{----------------------------------(4)}$$

From the phasor diagram we have,

$$\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IX_R} = \frac{X_L - X_C}{X_R}$$

$$\tan\varphi = \frac{X_L - X_C}{X_R} = \frac{\omega L - \frac{1}{\omega C}}{R} \qquad \text{-----------------------------(5)}$$

Equation (5) represents the phase angle between voltage and current in L-R-C series circuit. From Eq. (5) it is clear that, the phase angle ϕ depends on frequency.



If $X_L < X_C$, then,

$X_L - X_C$ = -ve

⇨ $\tan\phi$ = -ve

⇨ ϕ = -ve (between 0 to -90^0)

Here, the voltage phasor (V) lags the current phasor (I) . It is shown in the figure in the right.

All of the expressions that we've developed for an L-R-C series circuit are still valid if one of the circuit elements is missing.

i) If the resistor is missing, then it become L-C circuit, We set R = 0 in all the expression. So, the impedance and phase angles will be

- $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \Rightarrow Z = \omega L - \frac{1}{\omega C}$
- $\tan\varphi = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{\omega L - \frac{1}{\omega C}}{0} = \infty \Rightarrow \varphi = 90^0$

ii) If the inductor is missing, then it become R-C circuit. We set L= 0. So, the impedance and phase angles will be

- $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \Rightarrow Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$
- $\tan\varphi = \frac{-\frac{1}{\omega C}}{R} = -ve \Rightarrow \varphi$ is between 0 to -90^0

iii) But if the capacitor is missing, we then it become R-L circuit. We set C = ∞. So, the impedance and phase angles will be

- $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \Rightarrow Z = \sqrt{R^2 + (\omega L)^2}$
- $\tan\varphi = \frac{\omega L}{R} = +ve \Rightarrow \varphi$ is between 0 to 90^0

Power in Alternating-Current Circuits

In an ac circuit, with L-R-C in series the current and voltages are given by

$\boldsymbol{i} = \mathbf{I \cos \omega t}$ **and**

$\boldsymbol{v} = \mathbf{V \cos (\omega t + \phi)}$

Then the instantaneous power is given by

$$p = v\, i$$

$$\Rightarrow \quad p = V \cos(\omega t + \phi)\ I \cos \omega t$$

$$\Rightarrow \quad p = [V \cos \omega t \cos \phi - \sin \omega t \sin \phi]\ I \cos \omega t$$

$$\Rightarrow \quad p = V I \cos^2\omega t \cos \phi - V I \sin \omega t \cos \omega t \sin \phi$$

$$\Rightarrow \quad p = V I \cos^2\omega t \cos \phi - \frac{V I}{2} \sin 2\omega t \sin \phi$$

$$\Rightarrow \quad p_{av} = V I \cos \phi \langle\cos^2\omega t\rangle - \frac{V I}{2} \sin \phi \langle\sin 2\omega t\rangle$$

$$\Rightarrow \quad p_{av} = V I \cos \phi \left(\frac{1}{2}\right) - \frac{V I}{2} \sin \phi\, (0)$$

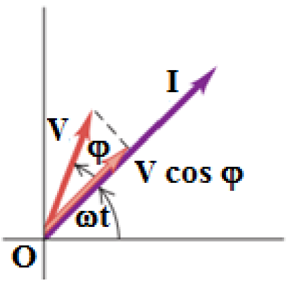
$$\Rightarrow \quad p_{av} = \frac{V I}{2} \cos \phi \quad \text{-------------------------------(1)}$$

$$\Rightarrow \quad p_{av} = \frac{V}{\sqrt{2}} \frac{I}{\sqrt{2}} \cos \phi = V_{rms} I_{rms} \cos \phi \quad \text{-------(2)}$$

Eq (2) represents the average power in an ac circuit and $\cos \phi$ is called the power factor of the circuit.

The power factor for L-R-C circuit is

$$\cos \varphi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$



For resistive circuit: $\boldsymbol{v}$ and $\boldsymbol{i}$ are in phase. $\phi = 0$, $\cos \phi = 1$

Thus, average power is $p_{av} = \frac{VI}{2} = V_{rms} I_{rms}$

And the power factor = $\cos \phi = 1$

For reactive circuit: Reactive circuit means, the circuit containing inductor and capacitor.

In these cases phase difference between $\boldsymbol{v}$ and $\boldsymbol{i}$ is $\phi = \pm 90^0$, $\cos \phi = 0$

Thus, average power is $p_{av} = 0$

And the power factor = $\cos \phi = 0$

In general: In general the average power is

$$p_{av} = \frac{V I}{2} \cos \phi = \frac{1}{2} I (V \cos \varphi) = \frac{1}{2} I (\text{component of V along I}) = \frac{1}{2} I (V_R)$$

Hence the above equation represents the average power dissipated in the resistor. On average there is no energy flow into or out of the inductor or capacitor, so none of p_{av} goes into either of these circuit elements.

Q. A low power factor is usually undesirable in power circuits. Justify.

Ans. For low power factor cos ϕ is less, i.e ϕ is large.

Under this situation a large current is needed to supply a given amount of power for a given potential difference. This results in large losses in the transmission lines.

Many types of ac machinery draw a lagging current; that is, the current drawn by the machinery lags the applied voltage. Hence the voltage leads the current, so $\phi > 0$ and $\cos\phi < 1$.

If a capacitor is connected parallel to the output then, the current drawn by the capacitor leads the voltage which compensates for the lagging current in the other branch of the circuit. The capacitor itself absorbs no net power from the line. So the power factor can be corrected toward the ideal value of 1.

Resonance in Alternating-Current Circuit

At resonance the current amplitude of is greatest at a particular frequency.

The expression for current in a L-R-C circuit is

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Where $Z = \sqrt{R^2 + (X_L - X_C)^2}$, Impedance of the LRC circuit

Thus the current I will be maximum if

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \text{minimum}$$

⇨ $$R^2 + (X_L - X_C)^2 = \text{minimum}$$

⇨ $$X_L - X_C = 0$$

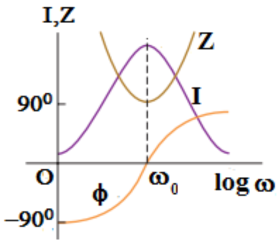
⇨ $$\omega_0 L - \frac{1}{\omega_0 C} = 0$$

⇨ $$\omega_0 L = \frac{1}{\omega_0 C}$$

⇨ $$\omega_0 = \sqrt{\frac{1}{LC}}$$

At this angular frequency (ω_0) the resonance peak occurs and is called the **resonance angular frequency**. At this angular frequency maximum current will be appeared in the circuit for a given source voltage amplitude; in other words, the frequency $\left(f_0 = \frac{\omega_0}{2\pi}\right)$ to which the circuit is "tuned."

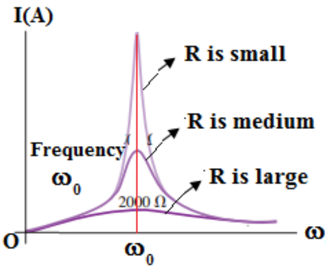
The variation of I and Z with angular frequency in the log scale is shown in the adjoining figure. Figure shows that



- Z is minimum at **resonance angular frequency** ω_0.
- I is maximum at **resonance angular frequency** ω_0.
- $\phi = 0$, at **resonance angular frequency** ω_0.

The variation of I with angular frequency for different values of resistance (R) is shown in the adjoining figure. Figure shows that

For small values of R, the resonance peak is sharp and also the current amplitude is greatest at resonance angular frequency ω_0.

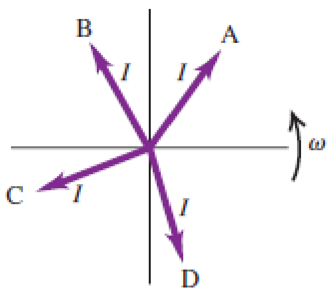


Conceptual Problems:

Test Your Understanding of Section 31.1

The figure shows four different current phasors with the same angular frequency. At the time shown, which phasor corresponds to

a) a positive current that is becoming more positive;
b) a positive current that is decreasing toward zero;
c) a negative current that is becoming more negative;
d) a negative current that is decreasing in magnitude toward zero?



Answers: (a) D; (b) A; (c) B; (d) C

For each phasor, the actual current is represented by the projection of that phasor onto the horizontal axis. The phasors all rotate counter-clockwise around the origin with angular speed ω so at the instant shown:
the projection of phasor A is positive but trending toward zero;
the projection of phasor B is negative and becoming more negative;
the projection of phasor C is negative but trending toward zero; and
the projection of phasor D is positive and becoming more positive.

Test Your Understanding of Section 31.2

An oscillating voltage of fixed amplitude is applied across a circuit element. If the frequency of this voltage is increased, will the amplitude of the current through the element

i) increase, ii) decrease or iii) remain same

if it is

a) a resistor b) an inductor or c) a capacitor.

Answers: (a) (iii); (b) (ii); (c) (i)

For a resistor, $V_R = IR \quad \Rightarrow \quad I = \dfrac{V_R}{R}$

The maximum voltage V_R and resistance R do not change with frequency, so the maximum current I remains constant.

For an inductor, $V_L = I X_L \quad \Rightarrow \quad I = \dfrac{V_L}{X_L} \quad \Rightarrow \quad I = \dfrac{V_L}{\omega L}$

The maximum voltage V_L and inductance L are constant, so the maximum current I decreases as the frequency increases.

For a capacitor, $V_C = I X_C \quad \Rightarrow \quad I = \dfrac{V_C}{X_C} \quad \Rightarrow \quad I = \dfrac{V_C}{1/\omega C} \quad \Rightarrow \quad I = \omega C V_C$

The maximum voltage V_C and capacitance C are constant, so the maximum current increases as the frequency increases.

Test Your Understanding of Section 31.3

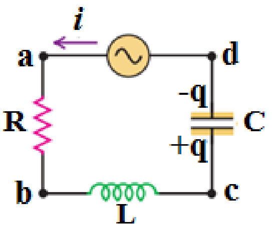
Rank the following ac circuits in order of their current amplitude, from highest to lowest value.

(i) the circuit in the adjoining figure

(ii) the circuit in the adjoining figure with the capacitor and inductor both removed;

(iii) the circuit in the adjoining figure with the resistor and capacitor both removed;

(iv) the circuit in the adjoining figure with the resistor and inductor both removed.



Answer: (iv), (ii), (i), (iii)

For the circuit

$$I = \frac{V}{Z} = \frac{50V}{500\ \Omega} = 0.01\ A$$

If the capacitor and inductor are removed so that only the ac source and resistor remain, the circuit is like that shown in Fig. 31.7a; then

$$I = \frac{V}{R} = \frac{50V}{300\ \Omega} = 0.17\ A$$

If the resistor and capacitor are removed so that only the ac source and inductor remain, the circuit is like that shown in Fig. 31.8a; then

$$I = \frac{V}{X_L} = \frac{50V}{600\ \Omega} = 0.083\ A$$

Finally, if the resistor and inductor are removed so that only the ac source and capacitor remain, the circuit is like that shown in Fig. 31.9a; then

$$I = \frac{V}{X_C} = \frac{50V}{200\ \Omega} = 0.25\ A$$

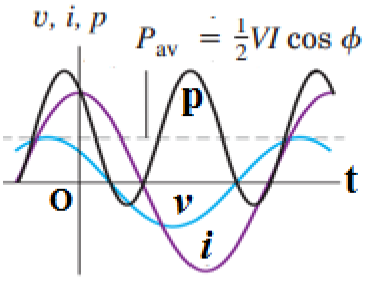
Test Your Understanding of Section 31.4

Figure in the right shows that during part of a cycle of oscillation, the instantaneous power delivered to the circuit is negative. This means that energy is being extracted from the circuit.

a) Where is the energy extracted from?

(i) the resistor; (ii) the inductor; (iii) the capacitor; (iv) the ac source;

(v) more than one of these.



(b) Where does the energy go?
(i) the resistor; (ii) the inductor; (iii) the capacitor; (iv) the ac source;
(v) more than one of these.

Answers: (a) (v); (b) (iv)

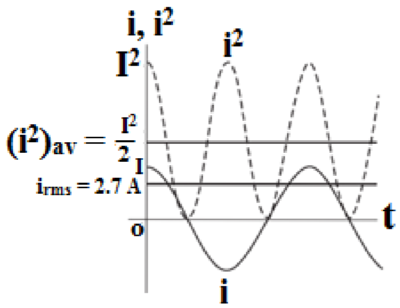
The energy cannot be extracted from the resistor, since energy is dissipated in a resistor and cannot be recovered. Instead, the energy must be extracted from either the inductor (which stores magnetic field energy) or the capacitor (which stores electric field energy). Positive power means that energy is being transferred from the ac source to the circuit, so negative power implies that energy is being transferred back into the source.

In Class Problems: 31.1, 31.3, 31.4

Example 31.1 Current in a personal computer

The plate on the back of a personal computer says that it draws 2.7 A from a 120-V, 60-Hz line. For this computer, what are

a) the average current,
b) the average of the square of the current, and
c) the current amplitude?



Solution:

a) The average of any sinusoidally varying quantity, over any whole number of cycles, is zero.

b) Given that, $I_{rms} = 2.7$ A
$I_{rms} =$ the square root of the mean(average) of the square of the current, . $\left(i^2\right)_{av}$

$I_{rms} = \sqrt{\left(i^2\right)_{av}}$

$\left(i^2\right)_{av} = \left(I_{rms}\right)^2 = (2.7\ A)^2 = 7.3\ A^2$

c) The maximum value of the instantaneous current (current amplitude) is
$I = \sqrt{2}\, I_{rms} = \sqrt{2}\,(2.7\ A) = 3.8\ A$
Graphs of i and i^2 versus time t is shown.

Q: Why would we be interested in the average of the square of the current?

Ans: We know that the rate at which energy is dissipated in a resistor R is i^2R
This rate varies if the current is alternating.
So it is best described by its average value, $\left(i^2\right)_{av} R = \left(I_{rms}\right)^2 R$

Example 31.3 A resistor and a capacitor in an ac circuit

A 200 Ω resistor is connected in series with a 5.0 μF capacitor. The voltage across the resistor is $v_R = (1.2\ V)\cos\left(2500\ rad/s\right)t$ (shown in the figure)

a) Derive an expression for the circuit current.
b) Determine the capacitive reactance of the capacitor.
c) Derive an expression for the voltage across the capacitor.

Solution: $\omega = 2500$ rad/s, $R = 200\ \Omega$

a) $$i = \frac{v_R}{R} = \frac{(1.2\ V)\cos(2500\ rad/s)\,t}{200\ \Omega} = (6.0 \times 10^{-3}\ A)\cos(2500\ rad/s)\,t$$

b) $$X_C = \frac{1}{\omega C} = \frac{1}{(2500\ rad/s)(5.0 \times 10^{-6}\ F)} = 80\ \Omega$$

c) $$V_C = I X_C = (3.0 \times 10^{-3}\ A)(80\ \Omega) = 0.48\ V$$

The instantaneous capacitor volt-age is given by

$$v_C = V_C \cos(\omega t - 90^0) = (0.48\ V)\cos[(2500\ rad/s)\,t - \pi/2\ rad]$$

Example 31.4 An L-R-C series circuit

In a series L-R-C circuit, suppose, $R = 300\ \Omega$, $L = 60$ mH, $C = 0.5\ \mu F$, $V = 50$ V, and $\omega = 10{,}000$ rad/s. Find the reactances X_L and X_C, the impedance Z, the current amplitude I, the phase angle ϕ, and the voltage amplitude across each circuit element.

Solution:

$$X_L = \omega L = (10{,}000\ rad/s)(60 \times 10^{-3}\ H) = 600\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(10{,}000\ rad/s)(0.5 \times 10^{-6}\ F)} = 200\ \Omega$$

The impedance Z of the circuit is then

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(300\ \Omega)^2 + (600\ \Omega - 200\ \Omega)^2} = 500\ \Omega$$

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left[\frac{(600\ \Omega - 200\ \Omega)}{300\ \Omega}\right] = 53^0$$

The voltage amplitudes V_R, V_L, and V_C across the resistor, inductor, and capacitor, respectively, are

$V_R = I R = (0.1\ A)(300\ \Omega) = 30\ V$

$V_L = I X_L = (0.1\ A)(600\ \Omega) = 60\ V$

$V_C = I X_C = (0.1\ A)(200\ \Omega) = 20\ V$

From the above it is clear that

- As $X_L > X_L$, hence the voltage amplitude across the inductor is greater than that across the capacitor.
- $\phi = 53^0$ and is positive.
- The voltage leads the current by 53^0
- The source voltage amplitude $V = 50$ V
  The sum of the voltage amplitudes across the separate circuit elements =30V+60V+ 20V
  Thus, $50\ V \neq 30V + 60V + 20V$
  Instead, V = vector sum of the V_R, V_L, and V_C phasors.

Solution: $\omega = 2500$ rad/s, $R = 200\ \Omega$

a) $i = \dfrac{v_R}{R} = \dfrac{(1.2\text{ V})\cos(2500\text{ rad/s})\,t}{200\ \Omega} = (6.0 \times 10^{-3}\text{ A})\cos(2500\text{ rad/s})\,t$

b) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(2500\text{ rad/s})(5.0 \times 10^{-6}\text{ F})} = 80\ \Omega$

c) $V_C = I X_C = (3.0 \times 10^{-3}\text{ A})(80\ \Omega) = 0.48\text{ V}$

The instantaneous capacitor volt-age is given by

$v_C = V_C \cos(\omega t - 90^0) = (0.48\text{ V})\cos[(2500\text{ rad/s})\,t - \pi/2\text{ rad}]$

Assignment Problems: 31.25, 31.27

31.25. An L-R-C series circuit is connected to a 120-Hz ac source that has $V_{rms} = 80$ V. The circuit has a resistance of 75 Ω and an impedance at this frequency of 105 Ω. What average power is delivered to the circuit by the source?

Solution: $V_{rms} = 80$ V, $R = 75\ \Omega$, $Z = 105\ \Omega$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{(80\text{ V})}{105\ \Omega} = 0.762\text{ A}$$

$$\tan\varphi = \frac{R}{Z} = \frac{75\ \Omega}{105\ \Omega} = 0.714$$

$p_{av} = V_{rms}\ I_{rms} \cos\phi = (80\text{ V})(0.762\text{ A})(0.714) = 435\text{ W}$

31.27. In an L-R-C series circuit the source is operated at its resonant angular frequency. At this frequency, the reactance X_C of the capacitor is 200 Ω and the voltage amplitude across the capacitor is 600 V. The circuit has R = 300 Ω . What is the voltage amplitude of the source?

Solution: $V_C = 600$ V, $X_C = 200\ \Omega$, $R = 300\ \Omega$

As the circuit is operated at resonance, $Z = R = 300\ \Omega$

Current across the capacitor = $I = \dfrac{V_C}{X_C} = \dfrac{(600\text{ V})}{200\ \Omega} = 3\text{ A}$

Voltage amplitude of the source = $V = I Z = I R = (3\text{ A})(300\ \Omega) = 900\text{ V}$