MID-SEMESTER EXAMINATION, MARCH-2017
INTRODUCTORY GRAPH THEORY (CSE 1004)

Programme: B.Tech(CSE & CST) **Semester:2nd**
Full Marks: 30 **Time: 2 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Analyze graphs and graph models, connected graphs, multigraphs, digraphs and regular graphs as well as discuss the degree sequence of graphs.	L3, L3, L3, L3, L3, L3	1(a),1(b), 1(c),2(a), 2(b),2(c)	2,2, 2,2, 2,2
Discuss and analyze the isomorphism of graphs.	L2, L2, L3	3(a),3(b), 3(c)	2,2, 2
Define bridges, trees, spanning trees and study its various concepts and apply the Kruskal's and Prim's algorithms to find the minimum spanning tree of a connected weighted graph.	L3, L3, L3	4(a), 4(b), 4(c)	2,2, 2
Define cut-vertices, blocks and analyze the connectivity of graphs.	L2, L3, L3	5(a), 5(b), 5(c)	2,2, 2

*Bloom's taxonomy levels: Knowledge (L1), Comprehension (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1. (a) Define a complete graph and determine the size of a complete graph of order n. Draw a complete graph of order 6. 2

   (b) Let R be the relation defined on the vertex set of a graph G by uRv, where $u, v \in V(G)$, if u is connected to v, that is, if G contains a $u-v$ path. Then show that R is an equivalence relation. 2

   (c) A digraph D has a vertex set {-3, 3, 6, 12} and $(i, j) \in D$ if $i \neq j$ and $i \mid j$, that is, j is a multiple of i. Draw the digraph D. 2

2. (a) Show that if G is a disconnected graph containing exactly 2 odd vertices, then these odd vertices must be in the same component of G. 2

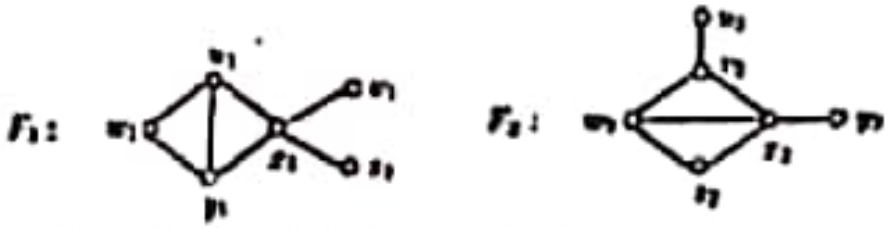
(b) Show that if G and $\bar{G}$ are both r-regular for some non-negative integer r, then G has odd order. 2

(c) Determine the integer $x(0 \le x \le 7)$, for which the sequence $7, 6, 5, 4, 3, 2, 1, x$ is graphical. 2

3. (a) Give an example of two different non-isomorphic graphs of order 4 and size 4. 2

(b) Let G_1 and G_2 be two graphs having the same degree sequence. If G_1 contains a vertex of degree 2 that is adjacent to a vertex of degree 3 and a vertex of degree 4, while G_2 contains a vertex of degree 2 that is adjacent to two vertices of degree 3, then can we conclude that G_1 is not isomorphic to G_2. Explain your answer. 2

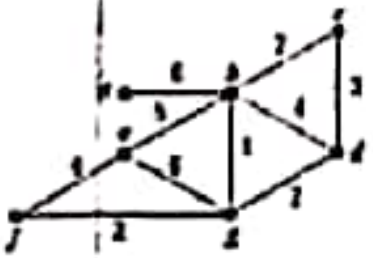
(c) Determine whether the given pair of graphs F_1 and F_2 are isomorphic or not. 2



4. (a) Prove that a tree of order n has size $n-1$. 2

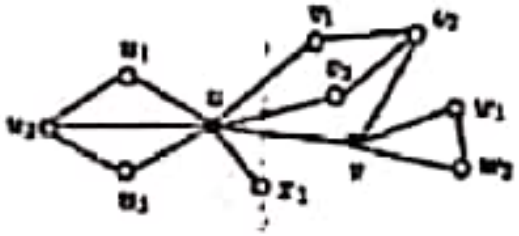
(b) A tree T of order 21 has only vertices of degree 1, 3, 5 and 6. If T has exactly 15 end-vertices and one vertex of degree 6, then determine the number of vertices of T having degree 5. 2

(c) Determine the minimum spanning tree of the given weighted graph using Kruskal's algorithm. 2



5. (a) Give a counterexample to the following statement. 2
"If G is a connected graph containing only even vertices, then G contains no cut-vertices."

(b) Determine the cut-vertices, bridges and blocks of the given graph. 2



(c) Prove that if a vertex v lies in every $u-w$ path in a connected graph G, where u and w are two vertices distinct from v in G, then v is a cut-vertex of G. 2

"End of Questions"