Chapter 26 DIRECT CURRENT CIRCUITS

In this chapter, learning goals are

1. To study the resistors in series and parallel (Analyze)
2. To learn Kirchoff's Rules (KCL and KVL)
3. To learn about the design & use of electronic measuring instruments, such as Ammeter, Voltmeter, Ohmmeter, Potentiometer
4. R-C circuits (charging and discharging a capacitor (Analysis)

Principal concern in this chapter is with direct current (dc) circuits, in which the direction of the current does not change with time. Flashlights and automobile wiring systems are examples of dc circuits, whereas household electrical power is supplied in the form of Alternating current (ac), in which the current oscillates back and forth.

Resistors in series

Here $V_{ab} = V_{ax} + V_{xy} + V_{yb}$
$= I(R_1 + R_2 + R_3)$
$\Rightarrow R_{ab} = \frac{V_{ab}}{I}$
$= R_1 + R_2$

Between a & b,

- Resistors are connected one after another in series connection.
- current remains same in all of them.

⊙ The equivalent resistance,

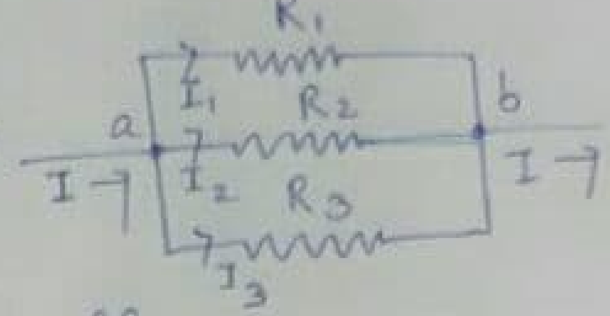
$$R_s = R_1 + R_2 + R_3 + \cdots,$$

sum of the individual resistances.

<u>Note</u>: Series Resistors have Equivalent resistance Larger than the largest value of resistance present in the series combination.

Resistors in parallel

⊙ The resistors are in parallel between a and b so that the potential difference must be same across all of them.



Here $I = I_1 + I_2 + I_3 = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

or $\frac{I}{V_{ab}} = \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

<u>Note</u>: The equivalent resistance (R_p) is always less than any individual resistance.

Kirchoff's Rules

To compute the currents in the network (which can't be reduced to simple series-parallel combination of resistors) Kirchoff's Rules are used.

① <u>First statement</u>: Kirchoff's Junction Rule (Kirchoff's Current Law)

The algebric sum of the currents into any junction is zero.

That is $\Sigma I = 0$

<u>Second statement</u> Kirchhoff's Loop Rule
or (Kirchhoff's voltage law).

The algebraic sum of the potential differences in any loop (associated with emfs and resistive elements) must equal to zero.

That is $\Sigma V = 0$

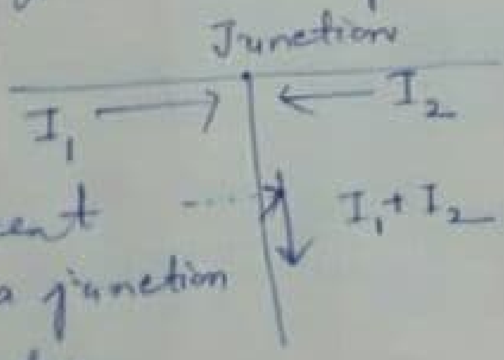
The junction Rule is based on conservation of electric charge.

The Loop Rule states that electrostatic force is conservative.

<u>Note</u>: ① <u>Junction</u> is a point in a circuit where 3 or more conductors meet.

② <u>Loop</u> is any closed conducting path.

③ Kirchhoff's <u>junction rule</u> states that as much current flows into a junction as flows out of it.



The current leaving a junction equals the current entering it.

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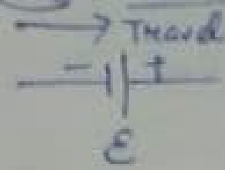
<u>Sign Conventions for the Loop Rule</u>

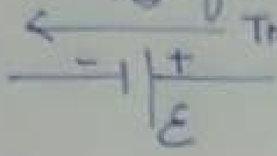
We need some sign conventions in applying the loop rule.

(a) Sign conventions for emfs

(+ε): If Travel direction is Travel from − to +

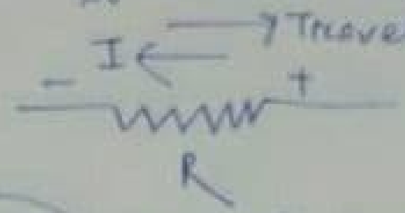


(−ε): If Travel direction is from + to −

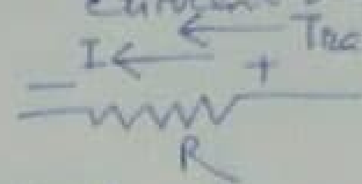


(b) Sign conventions for resistors

(+IR): Travel opposite to current direction



(−IR) Travel in current direction



In each part of the figure, "Travel" is the direction that we imagine going ~~go~~ around the loop, which is not necessarily the direction of the current.

Steps to use these sign conventions to apply Kirchoff's Loop Rule to any network:

(1) First assume a direction for the current in each branch of the circuit and mark it on a diagram of the circuit.

(2) Starting at any point in the circuit, imagine travelling around a loop, adding emfs and IR terms as we come to them.

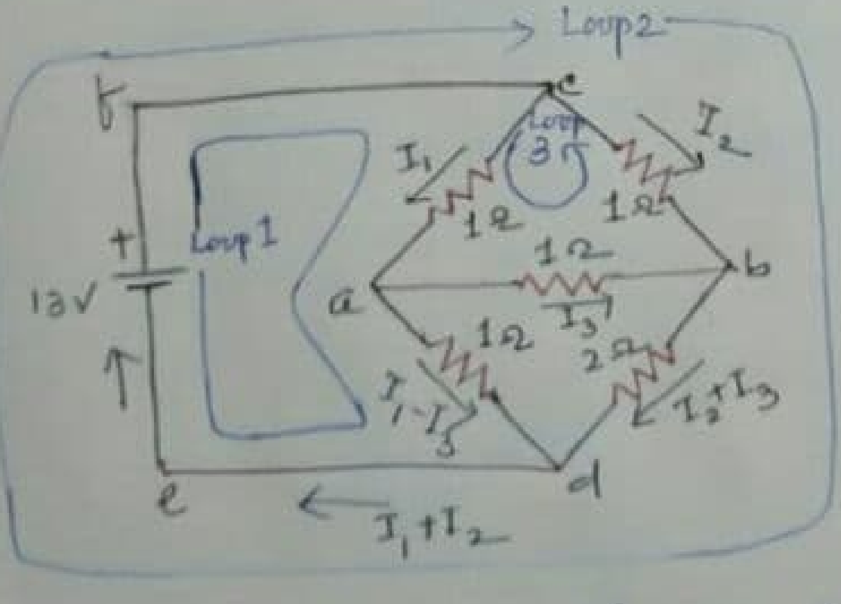
(3) When we travel through a source in the direction from − to +, the emf is considered to be positive, and when we travel from + to −, the emf is considered to be negative.

(4) When we travel through a resistor in the same direction as assumed current, the IR terms is negative, because the current goes in the direction of decreasing potential.

When we travel through a resistor in the direction opposite to the assumed current, the IR term is positive because this represents a rise of potential.

Kirchoff's two rules are used to solve a wide variety of network problems. From these rules a number of independent equations are obtained which is equal to the number of unknowns so that the equations are solved simultaneously to get the required solution.

Example



The figure shows a bridge circuit. Find the current in each resistor and the equivalent resistance of the network of five resistors.

Solution : Here the points a, b, c, d are junctions but points e, f are not.

The blue lines show possible Loops.

There are 5 unknown currents through the 5 resistors but applying junction rule to junctions a and b, we can represent them in terms of 3 unknown currents I_1, I_2 & I_3 as shown in the figure.

Apply Loop rule to the 3 loops (Using sign convention)

<u>Loop 1</u> $13 - I_1(1) - (I_1 - I_3)1 = 0$ — ① $\Rightarrow 13 = 2I_1 - I_3$

<u>Loop 2</u> $-I_2(1) - (I_2 + I_3)2 + 13 = 0$ — ②

<u>Loop 3</u> $-I_1(1) - I_3(1) + I_2(1) = 0$ — ③

from equ^n ③, $I_2 = I_1 + I_3$, substitute this into equ^n ② to eliminate I_2,

We have, $13 = 3I_1 + 5I_3$ — ④

Equ^n ① × 3, $6I_1 - 3I_3 = 39$

Equ^n ④ × 2, $6I_1 + 10I_3 = 26$

$-13I_3 = 13 \Rightarrow \boxed{I_3 = -1A}$

Using $I_3 = -1A$ in ①, $13 = 2I_1 + 1 \Rightarrow \boxed{I_1 = 6A}$

Using I_1 & I_3 in ③, $I_2 = \cancel{18 - 5} \Rightarrow 6 - 1$

$\Rightarrow \boxed{I_2 = 5A}$

-ve value of the 'I_3' $\Rightarrow$ Its direction is opposite to the direction we assumed.

Total current through the network is $(I_1 + I_2) = 11A$. The equivalent resistance of the network is $R_{eq} = 13V/11A = 1.2\Omega$

⑥

Electrical Measuring Instruments

① Ammeters:

A current measuring instrument, which always measures the current passing through it.

An Ideal ammeter would have zero resistance, so that including it in a branch of a circuit would not affect the current in that branch.

Real ammeters always have some finite resistance, but it is desirable for an ammeter to have as little resistance as possible.

Designing an ammeter

Any meter (Galvanometer) can be used to measure currents that are larger than its full-scale reading by connecting a resistor parallel with it (Fig. a), so that some of the current bypasses the meter coil.

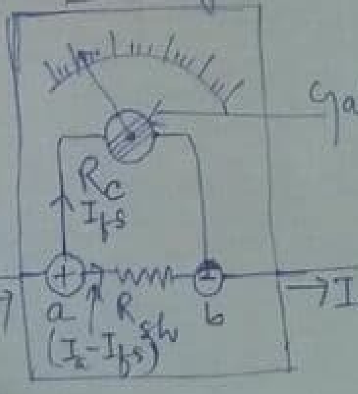
The parallel resistor is called a shunt-resistor or a shunt, denoted as 'R_{sh}'.

Let us make a meter with full-scale current 'I_{fs}' and coil resistance 'R_c' into an ammeter with full scale reading I_a.

To determine the shunt resistance R_{sh} needed, note that 'I_{fs}' is the current through the galvanometer, for full scale deflection for ammeter current I_a.

The current through the shunt is $(I_a - I_{fs})$.

Fig (a) moving coil Ammeter



Shunt is connected parallel to the galvanometer
So, potential difference across the galvanometer
= Potential diff. across the shunt
$= V_{ab}$

$\Rightarrow \boxed{I_{fs} R_c = (I_a - I_{fs}) R_{sh}}$ → for an ammeter

Example on Designing an Ammeter: What shunt Resistance is required to make the 1.00 mA, 20.0 Ω meter into an ammeter with a range of 0 to 50.0 mA ?

Solⁿ Since the meter is used as an ammeter, the internal connections are as shown in fig. ⓐ.

Given

$I_{fs} = 1.00\,mA = 10^{-3}\,A$

$R_c = 20\,\Omega$, $I_a = 50\,mA = 50 \times 10^{-3}\,A$

$$\Rightarrow R_{sh} = \frac{I_{fs} R_c}{I_a - I_{fs}} = \frac{10^{-3} A\,(20\,\Omega)}{(50 \times 10^{-3} - 1 \times 10^{-3})\,A}$$

$$= \frac{20}{49}\,\Omega = 0.408\,\Omega$$

Note: The resistance of the ammeter can be obtained by $R_{eq} = R_c \parallel R_{sh}$

$$\Rightarrow R_{eq} = \left[\frac{1}{R_c} + \frac{1}{R_{sh}}\right]^{-1} = 0.400\,\Omega$$

At full-scale deflection,

$I_a = 50\,mA$ (current through the ammeter)

$I_{fs} = 1\,mA$ (" " " Galvanometer)

$(I_a - I_{fs}) = 49\,mA$ (" " " shunt)

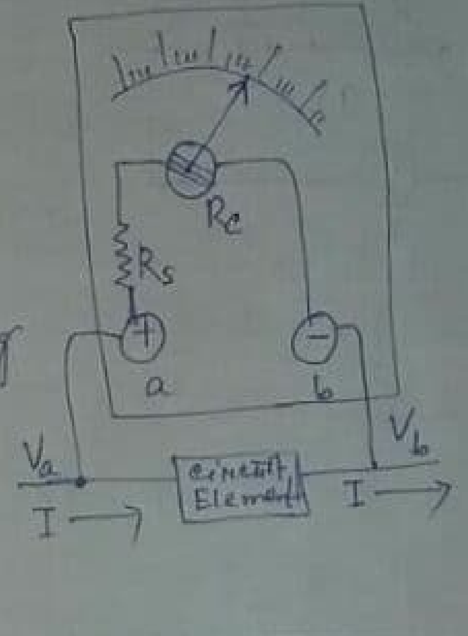
(2) Voltmeters The same meter may also be used to measure potential difference or voltage.

A voltage measuring device is called a voltmeter, which always measures the potential difference between two points.

Ideal voltmeters have infinite resistance, so connecting it between two points in a circuit would not alter any of the currents.

Real voltmeters always have finite resistance.

fig. (6) Moving-coil Voltmeter



For a voltmeter with full-scale reading V_V, we need a series resistor R_s such that,

$$V_V = I_{fs}(R_c + R_s) \Rightarrow \boxed{R_s = \frac{V_V}{I_{fs}} - R_c}$$

→ for a voltmeter

Example Designing a voltmeter

What series resistance is required to make the 1.00 mA (I_{fs}), 20.0 Ω (R_c) meter into a voltmeter with a range of 0 to 10.0 V?

Sol^n: $V_V = 10V$

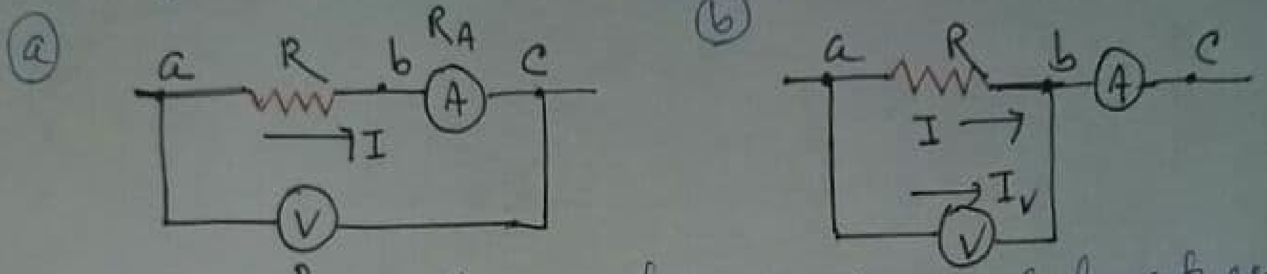
$$R_s = \frac{V_V}{I_{fs}} - R_c = \frac{10V}{1\times10^{-3}A} - 20\Omega = 10^4\Omega - 20\Omega$$

$$= 10{,}000\,\Omega - 20\Omega = 9980\,\Omega$$

Note: The equivalent resistance of the Voltmeter is, $(R_s + R_c) = 9980\Omega + 20\Omega$
$= 10{,}000\,\Omega$.

Ammeters and voltmeters in Combination

Fig. Ammeter-Voltmeter method for measuring resistance



A voltmeter and ammeter can be used together to measure resistance and power.

The resistance 'R' of a resistor $= \frac{V_{ab}\text{(Between its terminals)}}{I}$

Power input 'P' to any circuit is given by

$$P = V_{ab} I$$

So, the most straightforward way to measure R or P is to measure V_{ab} and I simultaneously.

In fig (a), Ammeter reads 'I' in the resistor, but voltmeter reads sum of V_{ab} and V_{bc}.

If we transfer the voltmeter terminals from 'c' to 'b' (shown in fig (b)), then the voltmeter reads V_{ab} correctly, pd. across the resistor, but now ammeter reads sum of I & I_V.

Either way, we have to correct the reading of one instrument or the other unless the corrections are small enough to be negligible.

Ohmmeters

This is an alternative method for measuring resistance. It consists of a meter, a resistor, and a source (often a flashlight battery) connected in series as shown in the figure.

The resistance R to be measured is connected between terminals x and y.

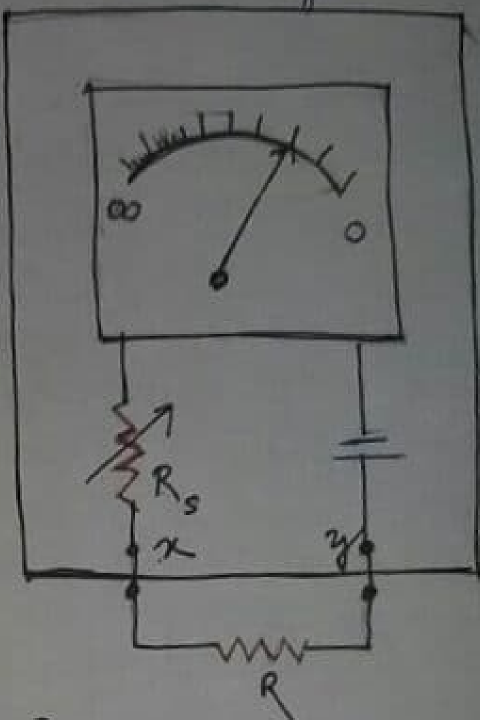


Fig. Ohmmeter circuit

R_s is a variable resistance.

To use the ohmmeter, first connect x directly to y and adjust R_s until the meter reads zero. Then connect 'x' and 'y' across the resistor R and read the scale.

When $R=0$, terminals x & y are short-circuited. R_s is adjusted so that the meter deflects full scale.

When $R \to \infty$, nothing is connected to terminals x and y i.e. circuit between x and y is open, so there is no current and hence no deflection.

For any intermediate value between 0 to ∞, the meter deflection depends on the value of R. Larger currents correspond to smaller resistance, so this scale reads backward compared to the scale showing the current (while current measurement is done).

The Potentiometer:

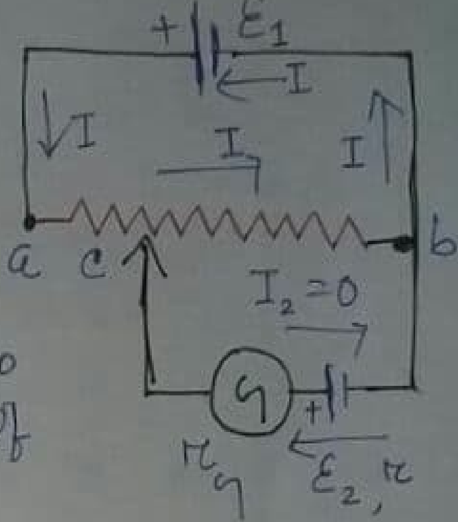
It is an instrument that can be used to measure the emf of a source without any current drawing from the source.

It also balances an unknown potential difference against an adjustable, measurable pot. difference.

Fig. a.

circuit symbol for potentiometer

fig b. A potentiometer circuit



Principle: A resistance wire ab of resistance R_{ab} is permanently connected to the terminals of a source of known emf $\mathcal{E}_1$.

A sliding contact 'c' is connected through the galvenometer G to a second source whose emf $\mathcal{E}_2$ is to be measured ($\mathcal{E}_2$ is unknown).

As the contact 'c' is moved along the resistance wire, the resistance R_{cb} between the points c and b varies and R_{cb} is proportional to the length of the wire between c and b.

To determine the value of $\mathcal{E}_2$ contact c is moved until a position is found at which the galvenometer shows no deflection, which implies zero current passing through $\mathcal{E}_2$.

So $I_2 = 0$, & Kirchoff's loop rule gives

$\mathcal{E}_2 = IR_{cb}$, Unknown emf $\mathcal{E}_2$ can be found out.

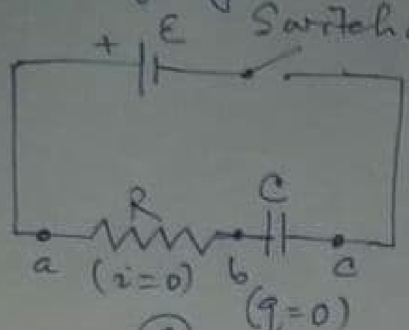
Note: V_{ab} must be greater than $\mathcal{E}_2$ for this to work.

* Only theory questions are expected from this part.

R-C circuits

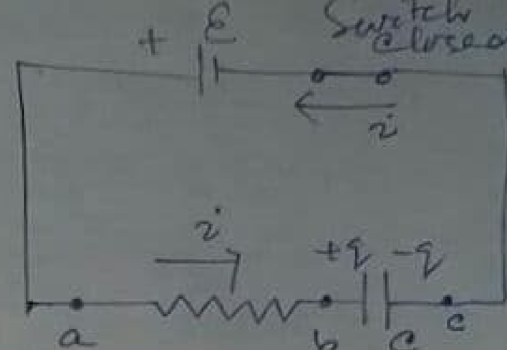
Many devices incorporate circuits in which a capacitor is alternately charged and discharged. These include flashing traffic lights, automobile turn signals and electronic flash units.

Charging a Capacitor



(a)

(capacitor initially uncharged)
(Just before the switch is closed, the charge q is zero.)



(b) When the switch is closed at $t=0$, the charge on the capacitor increases over time while the current decreases. The current jumps from zero to $\frac{\varepsilon}{R}$. As time passes, q approaches Q_f and the current i approaches zero.

A circuit in which a resistor and a capacitor in series is called an R-C circuit.
The circuit consists of a battery with constant emf ε and zero internal resistance. Also the resistance of all the connecting conductors are neglected.
Initially at $t=0$, capacitor is uncharged
$\Rightarrow V_{bc}=0$ (Potential difference across the capacitor)
& $V_{ab}=\varepsilon$ (Applying Kirchhoff's law)

The initial current (I_0) through the resistor R (by Ohm's law), $I_0 = \frac{V_{ab}}{R} = \frac{\mathcal{E}}{R}$

As the capacitor charges, then

V_{bc} increases and V_{ab} decreases, (pot. diff. across the resistor)

Corresponding current decreases.

The sum of these two voltages is constant and equal to $\mathcal{E}$.

$\Rightarrow$ $V_{ab} + V_{bc} = \mathcal{E}$ —— (1)

Capacitor is fully charged after a long time then, current decreases to zero.

$V_{ab} = 0$ and $V_{bc} = \mathcal{E}$, the entire battery emf ($\mathcal{E}$) appears across the capacitor.

At any instant of time 't',

when the switch is closed, let q = charge on the capacitor & i = current in the circuit

Instantaneous potential differences v_{ab} and v_{bc} are $v_{ab} = iR$, $v_{bc} = q/C$

Using Kirchoff's loop rule, we find

$$\mathcal{E} - iR - q/C = 0 \quad \text{—— (2)}$$

potential drops by an amount 'iR' as we travel from a to b and by q/C as we travel from b to c.

$$\Rightarrow \quad i = \frac{\mathcal{E}}{R} - \frac{q}{RC} \quad \text{—— (3)}$$

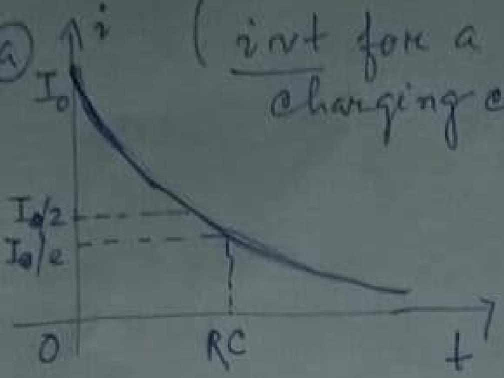
As the charge 'q' increases, the term q/RC becomes larger & the capacitor charge approaches its ~~the~~ final value Q_f. The current decreases & becomes zero.

Note Q_f does not depend on R (Equⁿ 4)

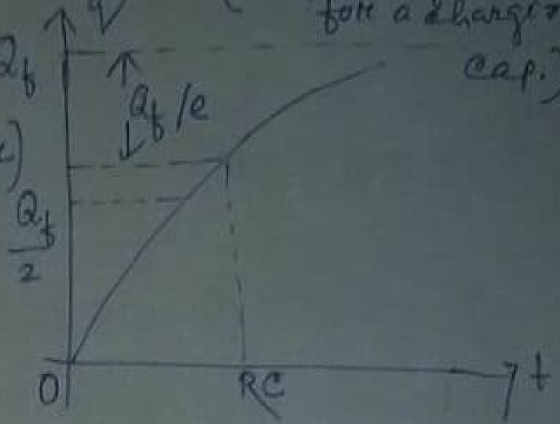
As the current decreases and becomes zero, $i = 0$, Use this in ③ $\Rightarrow \frac{\mathcal{E}}{R} = \frac{Q_f}{RC}$

$$\Rightarrow \boxed{Q_f = C\mathcal{E}} \quad - ④$$

fig. ⓐ (i vs t for a charging capacitor)



ⓑ (cap. charge vs time for a charging cap.)



The above figure shows current 'i' and capacitor charge 'q' as functions of time for the circuit (Page 13).

In fig. ⓐ, current decreases exponentially with time as the capacitor charges.

In fig. ⓑ, the charge on the capacitor increases exponentially with time toward the final value Q_f.

Calculation of Instantaneous current (i) and charge (q) as functions of time

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} \quad (\text{From } 3)$$

$$= -\frac{1}{RC}(q - C\mathcal{E})$$

Rearranging, $\frac{dq}{q - C\mathcal{E}} = \frac{-dt}{RC}$

Integrating, $\int_0^q \frac{dq}{q - C\mathcal{E}} = \int_0^t \frac{-dt}{RC}$

$\Rightarrow \ln\left(\frac{q-C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$

Taking inverse logarithm, (or exponentiating)

$\frac{q-C\mathcal{E}}{-C\mathcal{E}} = e^{-t/RC}$ $\Rightarrow q = C\mathcal{E}\left[1-e^{-t/RC}\right]$

$= Q_f\left[1-e^{-t/RC}\right]$ — (3)

So, $i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC}$ — (4)

In eqn (3) and (4), charge and current both are exponential function of time.

<u>Time constant</u> (τ): The time at which the current decreases to $\frac{1}{e}$ (0.368) times the original value is called the time constant.

At this time, the capacitor charge has reached $(1-\frac{1}{e}) = 0.632$ of its final value $Q_f (= C\mathcal{E})$.

So at time constant (τ), $i = I_0/e$,

putting this in eqn (4), $\frac{I_0}{e} = I_0 e^{-t/RC}$

$\Rightarrow \frac{1}{e} = e^{-t/RC}$ $\Rightarrow e^{-1} = e^{-t/RC}$ $\Rightarrow 1 = \frac{\tau}{RC}$

$\Rightarrow \boxed{\tau = RC}$, So time constant is $\tau = RC$.

This is called <u>relaxation time</u>.

Unit of τ is second.

At this time, the charge stored in the capacitor is given by

$q_\tau = C\mathcal{E}\left(1-e^{-\tau/RC}\right) = Q_f\left(1-e^{-RC/RC}\right)$

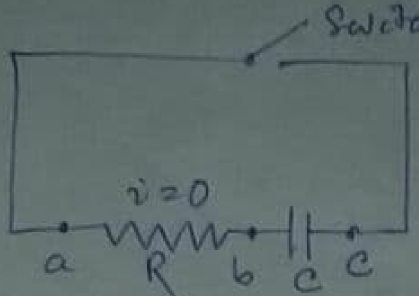
$= Q_f(1-e^{-1}) = Q_f(1-0.368) = 0.632\, Q_f$

Thus in time $t = \tau$, the capacitor charge has reached 0.632 times its final value Q_f.

When τ is small, the capacitor charges quickly.
When τ is larger, the charging takes more time.

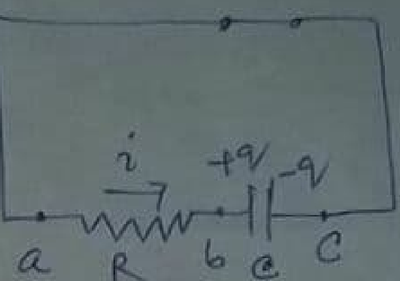
Discharging a Capacitor :

Fig. ⓐ Capacitor initially charged



(Before the switch is closed at time $t=0$, the capacitor charge is Q_0 and the current is zero)

ⓑ Discharging the capacitor



(At time t after the switch is closed, the capacitor charge is q and the current is i)

Let the capacitor is fully charged to Q_0.
Now the battery is removed from R-C circuit and the circuit is open.
When the circuit is closed let the time $t=0$.

So, at $t=0$, $q=Q_0$
The capacitor then discharges through the resistor and its charge eventually decreases to zero.

Calculation of i and q (time-varying current and charge) at some instant :

Now $\mathcal{E}=0$, $\Rightarrow V_{ab}+V_{bc}=0$

$\Rightarrow iR+\frac{q}{C}=0 \Rightarrow iR=-q/C \Rightarrow i=\frac{-q}{RC}$ —①

$$\Rightarrow \frac{dq}{dt} = \frac{-q}{RC} \Rightarrow \frac{dq}{q} = -\frac{dt}{RC}$$

$$\Rightarrow \int_{Q_0}^{q} \frac{dq}{q} = -\int_0^t \frac{dt}{RC}$$

$$\Rightarrow [\ln q]_{Q_0}^{q} = -\frac{t}{RC} \Rightarrow (\ln q - \ln Q_0) = -t/RC$$

$$\Rightarrow \ln \frac{q}{Q_0} = -\frac{t}{RC} \Rightarrow \frac{q}{Q_0} = e^{-t/RC}$$

$$\Rightarrow \boxed{q = Q_0 e^{-t/RC}}$$ (R-C circuit, discharging capr)

The instantaneous current 'i' is given by

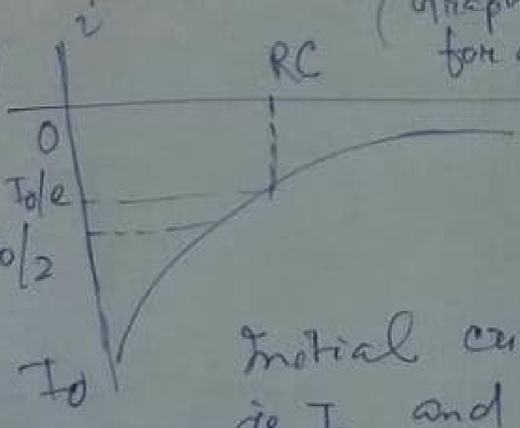
$$i = \frac{dq}{dt} = \frac{d}{dt}\left[Q_0 e^{-t/RC}\right]$$

$$= Q_0 e^{-t/RC}\left(-\frac{1}{RC}\right)$$

$$= -\frac{Q_0}{RC} e^{-t/RC}$$

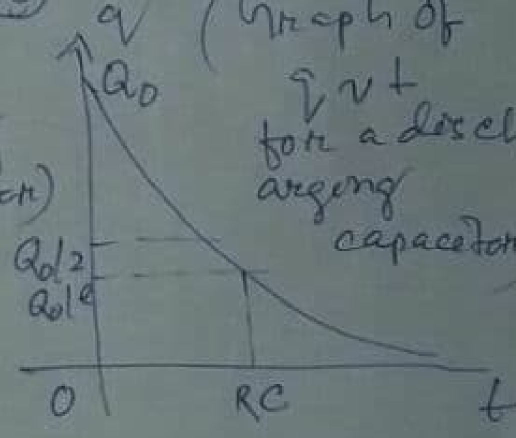
$$\Rightarrow \boxed{i = I_0 e^{-t/RC}}$$

fig. (a)



(Graph of i v t for a discharging capacitor)

(b)



(Graph of q v t for a discharging capacitor)

Initial current is I_0 and the initial capacitor charge is Q_0.
Both i and q asymptotically approach zero.

Power in R-C circuit

When the capacitor is charging, the instantaneous rate at which the battery delivers energy to the circuit is,

$$P = \mathcal{E} i$$

We have for RC circuit, $V_{ab} + V_{bc} = \mathcal{E}$

$\Rightarrow \mathcal{E} = iR + q/C \Rightarrow \mathcal{E} i = i^2 R + \frac{iq}{C}$

$\Rightarrow P = i^2 R + iq/C$

i^2R: Power dissipated in the resistor

iq/C: power stored in the capacitor

Energy: The total energy supplied by the battery during charging of the capacitor equals the battery emf $\mathcal{E} Q_f$.

Energy stored in the capacitor is $\frac{\mathcal{E} Q_f}{2}$

& Energy dissipated in the resistor is $\frac{\mathcal{E} Q_f}{2}$

Example 1

A $10\,M\Omega$ resistor is connected in series with a $1.0\,\mu F$ capacitor and a battery with emf $12.0\,V$. Before the switch is closed at time $t=0$, the capacitor is uncharged.

(a) What is the time constant?

(b) What fraction of the final charge Q_f is on the capacitor at $t = 46\,s$?

(c) What fraction of the initial current (I_0) is still flowing at $t = 46\,s$?

Sol^n: (a) $\tau = RC = (10 \times 10^{6}\,\Omega)(1.0 \times 10^{-6} F) = 10\,s$

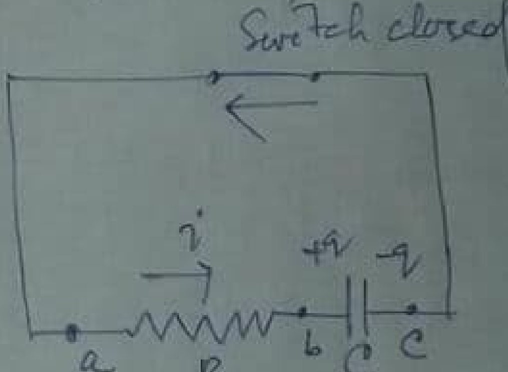
(b) $q = Q_f(1 - e^{-t/RC}) \Rightarrow \frac{q}{Q_f} = 1 - e^{-\left(\frac{46s}{10s}\right)}$
$= 0.99$

After $t = 46s$ the cap^r is 99% charged.

(c) $i = I_0 e^{-t/RC} \Rightarrow \frac{i}{I_0} = e^{-\frac{46s}{10s}} = 0.01$

$\Rightarrow$ After $t = 46s$, the current has decreased to 1% of its initial value.

Example (2) The resistor and capacitor are connected as shown in figure. The capacitor has an initial charge of 5.0 μC and is discharged by closing the switch at $t=0$.
(a) At what time will the charge be equal to 0.5 μC?
(b) What is the current at this time?



Sol^n (a) $q = Q_0 e^{-t/RC}$
$\Rightarrow \ln(q/Q_0) = -t/RC$
$\Rightarrow t = -RC \ln(q/Q_0)$
$= -(10\,Sec) \ln \frac{0.5\,\mu C}{5\,\mu C} = 23\,s$

(b) $i = I_0 e^{-t/RC} = \frac{\mathcal{E}}{R} e^{-t/RC} = \frac{\mathcal{E}C}{RC} e^{-t/RC}$
$= -\frac{Q_0}{RC} e^{-t/RC} = -\left(\frac{5 \times 10^{-6} C}{10s}\right) e^{-\frac{23s}{10s}}$
$= -5 \times 10^{-8} A$ ('i' is -ve because capacitor is discharging
'i' has opposite sign when the capacitor is discharging than when it is charging.