Chapter-23 Electric Potential

Topics in the material

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5. Equipotential Surfaces
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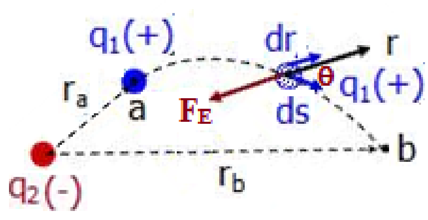
LEARNING GOALS

- To study and calculate electrical potential energy
- To define and study examples of electric potential
- To trace regions of equal potential as equipotential surfaces
- To find the electric field from electrical potential

Electric Potential Energy

A charged particle in an electric field has electric potential energy. It experiences a force as given by Coulomb's law. Electric potential energy is defined as the work done by Coulomb force when q_1 moves from a to b:

$$W = \int_{r_a}^{r_b} \vec{F}_E \cdot d\vec{s} = \int_{r_a}^{r_b} F_E \, ds \cos\theta = \int_{r_a}^{r_b} F_E \, dr \quad dr = \text{displacement along } \vec{r}$$



$$W = -\int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0} \frac{|q_1 q_2|}{r^2} dr$$

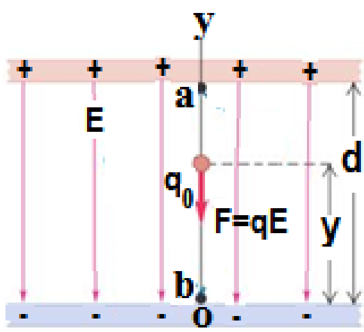
$$W = -\frac{1}{4\pi\varepsilon_0} |q_1 q_2| \int_{r_a}^{r_b} \frac{1}{r^2} dr = -\frac{1}{4\pi\varepsilon_0} |q_1 q_2| \left(-\frac{1}{r}\right)\Bigg|_{r_a}^{r_b}$$

$$W = -\frac{1}{4\pi\varepsilon_0} |q_1 q_2| \left(-\frac{1}{r_b} + \frac{1}{r_a}\right) = \frac{1}{4\pi\varepsilon_0} |q_1 q_2| \left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$W = \frac{1}{4\pi\varepsilon_0} |q_1 q_2| \left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

It is clear that the work depends only on the initial and final positions of q_1. In other words, the work done by the electric force is independent of path taken. The electric force is a conservative force.

Electric Potential Energy in a Uniform Field



A pair of charged parallel metal plates sets up a uniform electric field in downward with magnitude **E**.

The magnitude of force exerted by the electric field on a positive test charge q_0 in downward direction is

$F = q_0 E$

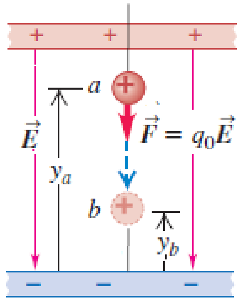
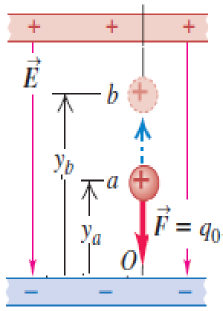
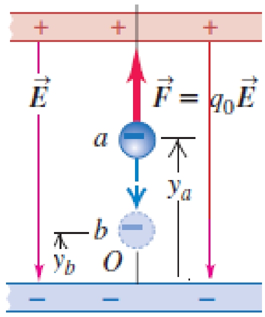
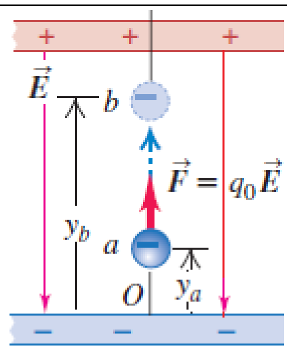
As the charge moves downward a distance 'd' from point 'a' to point 'b', the force on the test charge is constant and independent of its location.

So the work done by the electric force is the same for any path from 'a' to 'b' and is given by

$$W_{a\to b} = F\,d = q_0\,E\,d$$

This work is positive, since the force is in the same direction as the net displacement of the test charge.

An electric charge moving in an electric field

• Positive charge movement in the E field direction: – Field does positive work on charge. – K.E increases and U decreases	
• Positive charge movement opposite to the E field direction: – Field does negative work on charge. – K.E decreases and U increases	
• Negative charge movement in the E field direction: – Field does negative work on charge. – K.E decreases and U increases	
• Negative charge movement opposite to the E field direction: – Field does positive work on charge. – K.E increases and U decreases	

Electric Potential Energy of Two Point Charges

Let us consider an electric field caused by a single, stationary point charge 'q'. A test charge q_0 is moving from 'a' to 'b'.

The Coulomb's force F_r on q_0 is given by,

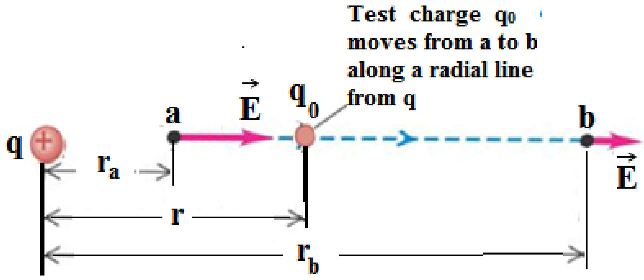
$$F_r = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r^2}$$

The work done on a test charge q_0 in moving from 'a' to 'b' by F_r is

$$W_{a\to b} = \int_{r_a}^{r_b} F_r dr = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r^2} dr \quad = \frac{q q_0}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2}$$

$$W_{a\to b} = \frac{q q_0}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_a}^{r_b} = \frac{q q_0}{4\pi\varepsilon_0}\left[-\frac{1}{r_b}+\frac{1}{r_a}\right]$$

$$W_{a\to b} = \frac{q q_0}{4\pi\varepsilon_0}\left[\frac{1}{r_a}-\frac{1}{r_b}\right]$$



In this case the work done by the electric force depends only on the endpoints.

$$W_{a\to b} = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r_a} - \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r_b}$$

$$W_{a\to b} = U_a - U_b$$

Where, $U_a = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r_a}$, potential energy when q_0 is at r_a from 'q'

$U_b = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r_b}$, potential energy when q_0 is at r_b from 'q'

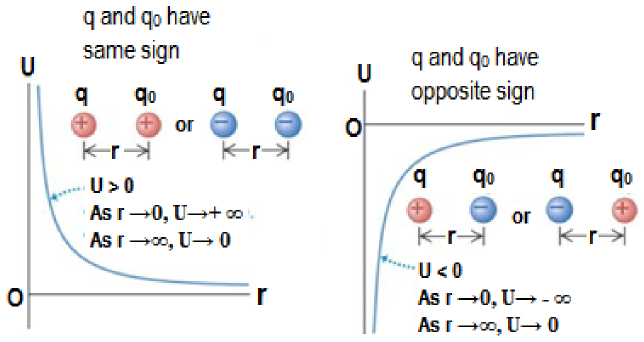
In general the electric potential energy for a system of two point charges q and q_0, separated by a distance r is written as $U = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r}$

For a conservative force we have

$$W_{a\to b} = U_a - U_b = -\left(U_b - U_a\right) = -\Delta U$$

Thus if, $W_{a\to b}$ is positive, then ΔU is negative and vice versa.

Potential energy curves: PE versus r



Graphs of the potential energy of two point charges q and q_0 versus their separation r is shown in the figure. It is clear from the graph that:

- q and q_0 have the same sign
  - $U > 0$
  - As $r \to 0$, $U \to +\infty$
  - As $r \to \infty$, $U \to 0$

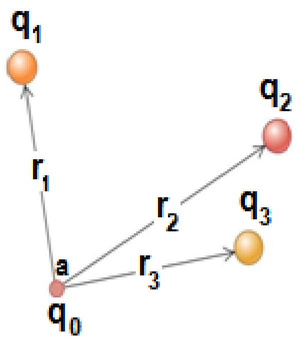
  Thus the potential energy between like charges increases sharply to positive (repulsive) values as the charges become close.

- q and q_0 have the opposite sign
  - $U < 0$
  - As $r \to 0$, $U \to -\infty$
  - As $r \to \infty$, $U \to 0$

  Unlike charges have potential energy becoming sharply negative as they become close (attractive).

Electric Potential Energy with Several Point Charges

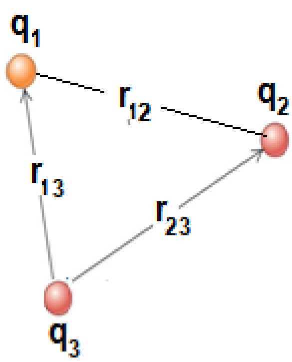
The potential energy associated with the test charge q_0 at point 'a' is the algebraic sum of the individual energies



$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_0}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_0}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q_3 q_0}{r} + \ldots\ldots$$

$$U = \frac{q_0}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots\ldots\right) \Rightarrow U = \frac{q_0}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

Above equation shows that we can always find a potential-energy function for any static electric field. So, for every electric field due to a static charge distribution, the force exerted by that field is conservative.



If the system contains many point charges then the total potential energy U is the sum of the potential energies of interaction for each pair of charges. We can write this as

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}} + \ldots \Rightarrow U = \frac{1}{4\pi\varepsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

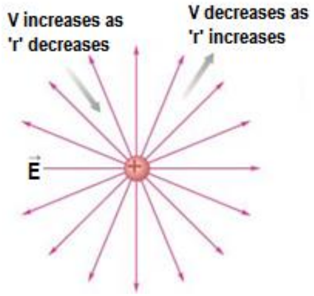
The electrical potential

Potential V at any point in an electric field is defined as the potential energy U per unit charge associated with a test charge q_0 placed at that point:

$$V = \frac{U}{q_0}$$

- Potential is a scalar quantity.
- The SI unit of potential, called one volt (1 V = 1 joule per coulomb)

The potential of 'a' with respect to 'b' is denoted by $V_{ab} = V_a - V_b$. This is also the potential difference between two points 'a' and 'b'. The potential difference between two points is often referred as voltage. It is defined as the work done by the electric force on a *unit* charge that moves from 'a' to 'b' in the field.



Thus,

$$V_{ab} = V_a - V_b = \frac{W_{a\to b}}{q_0} = \frac{U_a - U_b}{q_0}$$

An instrument that measures the difference of potential between two points is called a voltmeter.

Calculating Electric Potential

The potential V due to a single point charge is

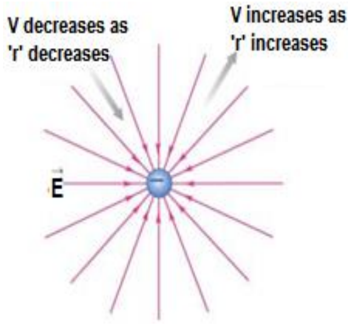
$$V = \frac{U}{q_0} \quad \Rightarrow \quad V = \left(\frac{1}{4\pi\varepsilon_0}\frac{q\,q_0}{r}\right)\left(\frac{1}{q_0}\right)$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Where r = distance from the point charge 'q' to the point at which the potential is evaluated.

- If q is positive, the potential that it produces is positive at all points;
- If q is negative, it produces a potential that is negative everywhere.
- V = 0 at an infinite distance from the point charge.
- The potential due to a collection of point charges is:

$$V = \frac{U}{q_0} \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$



Electron Volts

Electron volt is a unit of electrical energy. It is defined as the amount of electrical energy associated with an electron when it is placed inside an electrical field of potential difference of 1volt. So,

$W = q\,V$ => $1eV = (1.6 \times 10^{-19} C)(1 Volt)$ => $1eV = 1.6 \times 10^{-19} J$

Calculation of Electric Potential

1. Calculation of Electric Potential due to a charged conducting sphere

Let us consider a solid conducting sphere of radius R. q be the total charge contained in it. The charge conducting sphere can be considered as a point charge.

Outside the sphere:

Let us consider a point 'r' outside the sphere and keep a test charge q_0 there. The potential energy associated with q_0 is:

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q\,q_0}{r}$$

Now the potential V due to point charge is the potential energy U per unit charge associated with a test charge q_0 placed at that point.

$$V_{outside} = \frac{U}{q_0} \quad \Rightarrow \quad V_{outside} = \left(\frac{1}{4\pi\varepsilon_0}\frac{q\,q_0}{r}\right)\left(\frac{1}{q_0}\right) \quad \Rightarrow \quad V_{outside} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

On the surface of the sphere:

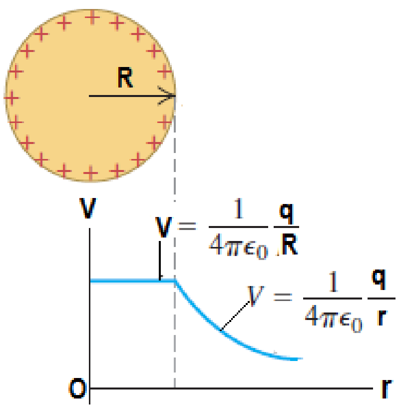
On the surface of the sphere, r = R.

$$V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

Inside the sphere:

Inside the sphere, $\vec{E}$ is zero everywhere. Hence no work is done on a test charge that moves from any point to any other point inside the sphere. So the potential is the same at every point inside the sphere and is equal to its value at the surface, i.e

$$V_{inside} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$



The variation of electric potential with the distance from the center of the charged conducting sphere is shown in the figure.

From the figure it is clear that

$$V_{outside} \ \alpha \ \frac{1}{r}$$

$$V_{inside} = V_{surface} = \text{constant}$$

2. Calculation of Electric Potential due to a uniformly charged ring.

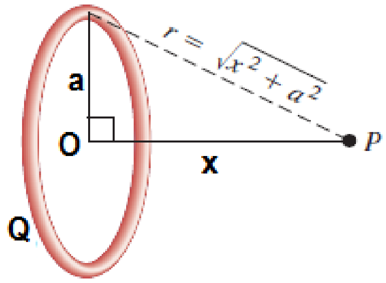
Let

Q = amount of charge distributed uniformly around a thin ring.

a = radius of the ring

Let us consider a point P on the axis of the ring that passes through the center and perpendicular to the plane of the ring.

x = distance of the point from the center of the ring on the axis.



Let dV be the potential at P due to the infinitesimal charge dq lying on the surface of the ring. So,

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

$$V = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \quad \Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{x^2+a^2}}\int dq$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$$

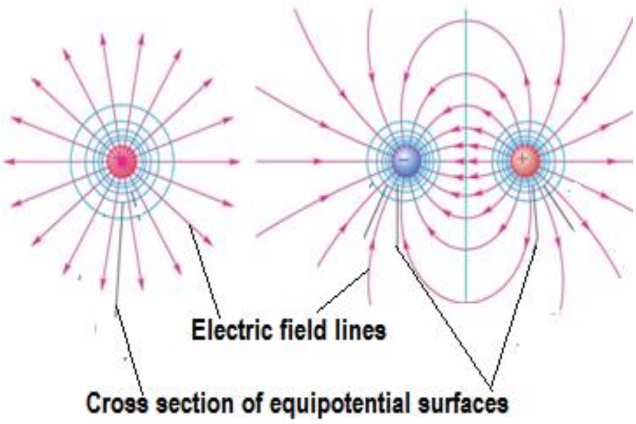
From the above expression it is clear that:

- At the center of the ring (x=0), $V_{center} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}$
- If x>>a, then $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x}$

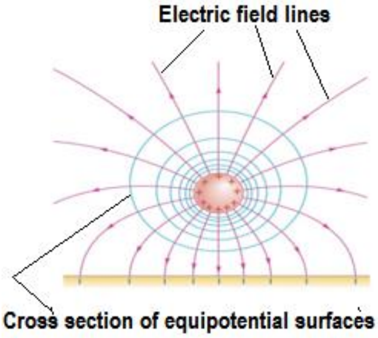
  This is the potential at a distance *x* from a point charge *Q*.

Equipotential surfaces and field lines

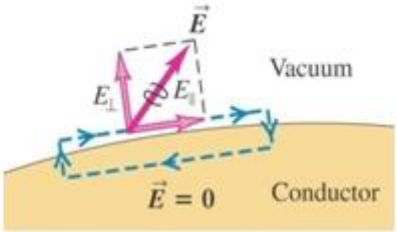
- An **equipotential surface** is a three-dimensional surface on which the electric potential V is the same at every point.
- If a test charge q_0 is moved from one point to other on such a surface, the electric potential energy ($U = q_oV$) remains constant.



- The work done in moving a test charge on the equipotential surface is zero.
- In a region where an electric field is present, we can construct an equipotential surface through any point. Equipotential surfaces for different potentials never intersect.
- Field lines & equipotential surfaces are <u>always</u> mutually perpendicular.
- Field lines & equipotential surfaces are shown in the figure.
- When all charges are at rest:
  - ✓ Surface of conductor is always an equipotential surface (shown in the figure below)
  - ✓ E field just outside conductor is always perpendicular to surface
  - ✓ Entire solid volume of conductor is at same potential.



- At all points on the surface of a conductor, the electric field just outside must be perpendicular to the surface. If had a tangential component, a net amount of work would be done on a test charge by moving it around a loop as shown here—which is impossible because the electric force is conservative.



Potential gradient

The electric field can be written as

$$\vec{E} = -\frac{dV}{d\vec{r}} \quad \Rightarrow \quad \vec{E} = -\left(\frac{dV}{dx}\hat{i} + \frac{dV}{dy}\hat{j} + \frac{dV}{dx}\hat{k}\right) \quad \Rightarrow \vec{E} = -\vec{\nabla}V$$

$\vec{\nabla}V$ is called the potential gradient

The components of the electric field are

$$E_x = -\frac{dV}{dx}, \quad E_y = -\frac{dV}{dy}, \quad E_z = -\frac{dV}{dz}$$

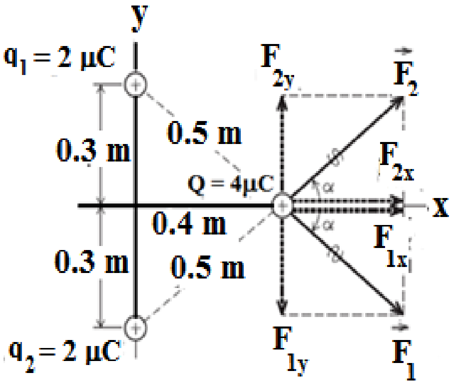
At each point, the potential gradient points in the direction in which V increases most rapidly with a change in position.

Hence at each point the direction of E is the direction in which V decreases most rapidly and is always perpendicular to the equipotential surface through the point.

Conceptual Problems:

Test Your Understanding of Section 23.1

Consider the system of three point charges as shown in figure.



a) What is the sign of the total potential energy of this system?

(i) positive; (ii) negative; (iii) zero.

b) What is the sign of the total amount of work you would have to do to move these charges infinitely far from each other?

(i) positive; (ii) negative; (iii) zero.

Answers: (a) (i), (b) (ii)

The three charges q_1, q_2 and q_3 are all positive. So all three pairs of individual potential energy are positive. Hence the total electric potential energy (U) is positive.

This means that it would take positive work to bring the three charges from infinity to the positions shown in the figure. Hence negative work to move the three charges from these positions back to infinity.

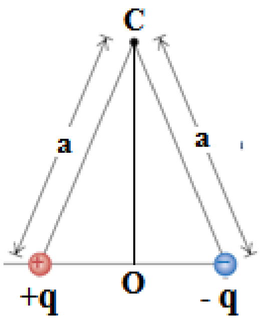
Test Your Understanding of Section 23.2

If the electric potential at a certain point is zero, does the electric field at that point have to be zero?

Answer: no

If V = 0 at a certain point, E does not have to be zero at that point.

For example, at any point on the perpendicular bisector line of an electric dipole. In figure the electric field at 'C' exists but V= 0.



In-class problems:

Example- 23.8: A charged conducting sphere

A solid conducting sphere of radius 'R' has a total charge 'q'. Find the electric potential everywhere, both outside and inside the sphere.

Solution: Given in the study material

Example-23.11 A ring of charge

Electric charge Q is distributed uniformly around a thin ring of radius ‘a’. Find the potential at a point ‘P’ on the ring axis at a distance ‘x’ from the center of the ring.

Solution: Given in the study material

Assignment problem:

23.14. A small particle has charge – 5 μC and mass 2 x 10^{-4}kg. It moves from point ‘A’ where the electric potential is V_A = + 200V, to point ‘B’ where the electric potential is V_B = + 800V. The electric force is the only force acting on the particle. The particle has speed 5m/s at point ‘A’. What is its speed at point ‘B’. Is it moving faster or slower at ‘B’ than at ‘A’? Explain.

Solution:

Applying conservation of energy to points A and B we get,

$K_A + U_A = K_B + U_B$

$$\frac{1}{2}m\,v_A^2 + qV_A = \frac{1}{2}m\,v_B^2 + qV_B$$

$$\frac{1}{2}m\,v_A^2 + qV_A = K_B + qV_B \quad \Rightarrow \quad \frac{1}{2}\left(2\text{ x }10^{-4}\text{kg}\right)\left(5\text{m/s}\right)^2 + qV_A - qV_B = K_B$$

$$\Rightarrow \quad K_B = \left(25\text{ x }10^{-4}\text{ J}\right) + q\left(V_A - V_B\right) \quad \Rightarrow \quad K_B = \left(25\text{ x }10^{-4}\text{ J}\right) + \left(-5\text{ x }10^{-6}\text{C}\right)\left(200\text{V} - 800\text{V}\right)$$

$$\Rightarrow \quad K_B = \left(25\text{ x }10^{-4}\text{ J}\right) + \left(30\text{ x }10^{-4}\text{J}\right)$$

$$\Rightarrow \quad \frac{1}{2}m\,v_B^2 = 55\text{ x }10^{-4}\text{ J} \quad \Rightarrow \quad v_B = \sqrt{\frac{2\text{ x }55\text{ x }10^{-4}\text{ J}}{\left(2\text{ x }10^{-4}\text{kg}\right)}} \quad \Rightarrow \quad v_B = 7.42\text{ m/s}$$

23.29 . A total electric charge of 3.5 nC is distributed uniformly over the surface of a metal sphere with a radius of 24 cm. If the potential is zero at a point at infinity, find the value of the potential at the following distances from the center of the sphere: (a) 48.0 cm; (b) 24.0 cm; (c) 12.0 cm.

Solution: charge = q = 3.5 nC, radius = R = 24cm

(a) Here, r = 48 cm

This is outside the sphere, so $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \left(9\text{ x }10^9\text{ N.m}^2\text{C}^{-2}\right)\frac{\left(3.5\text{ x }10^{-9}\text{C}\right)}{\left(48\text{ x }10^{-2}\text{m}\right)} = 65.6\text{V}$

(b) Here, r = 24 cm

This is on the sphere, so $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \left(9\text{ x }10^9\text{ N.m}^2\text{C}^{-2}\right)\frac{\left(3.5\text{ x }10^{-9}\text{C}\right)}{\left(24\text{ x }10^{-2}\text{m}\right)} = 131\text{V}$

(c) Here, r = 12.0 cm

This is inside the sphere. The potential has the same value as at the surface, 131 V.