Q. 23. (2017, Set-2)

$\Rightarrow$ As the degree of each vertex is at least 3, then

$$\sum_{v \in V} d(v) \geq 3n$$

$$\Rightarrow 2 \times e(G) \geq 3n \quad \text{(using degree sum)}$$

$$\Rightarrow 50 \geq 3n.$$

$$\Rightarrow n_{max} = 16 \quad \text{(as } n \text{ is integer)}$$

— x —

Q. 30 (2022)

G is one simple disconnected graph with $n(G) = 10$.

$\therefore$ G will have maximum number of edges if

$$G = K_9 \cup K_1$$

$$\therefore e(K_9) = \frac{9 \times 8}{2} = 36$$

$$e(K_1) = \frac{1 \times 0}{2} = 0$$

$$\therefore e(G) = 36 + 0 = 36$$

— x —

Q. 28 (2015, set 2).

$\Rightarrow$ Let, $G'$ be the complement of the graph $G$.

For self-complementary $\quad G \cong G'$

$$n(G) = n(G') = n.$$

$$e(G) = e(G') = e.$$

$$\therefore \quad K_n = G \cup G'$$

$$\Rightarrow e(K_n) = e + e.$$

$$\Rightarrow e = \frac{n(n-1)}{4}$$

$$\Rightarrow 4 \mid n \quad \text{or} \quad 4 \mid n-1$$

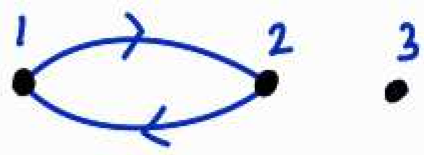
$$\Rightarrow n = 4k \quad \text{or} \quad 4k+1$$

—— x ——

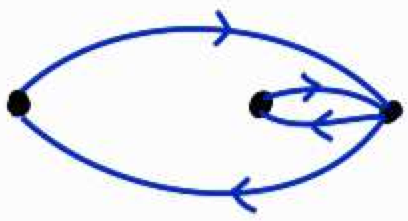
Q.3 (2014, Set 1)

A) Always not true.

$G$:



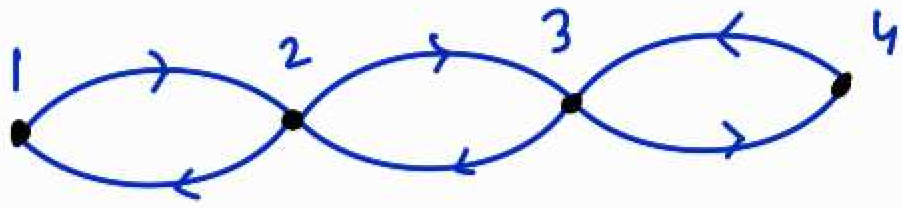

$G_1$:

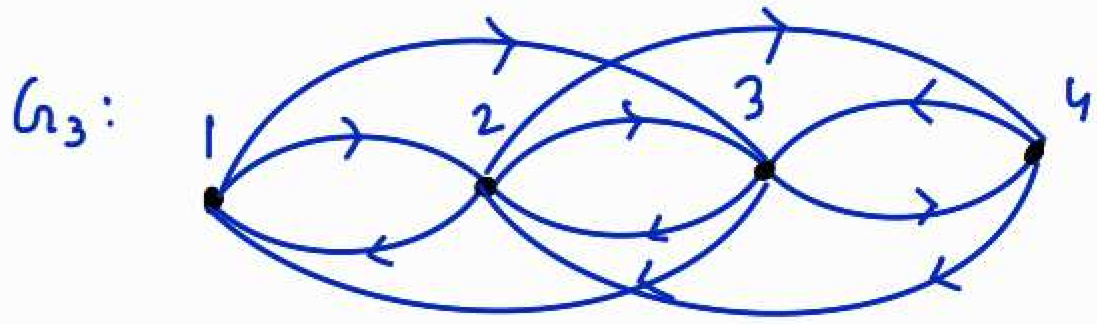


B) True: $u$-$v$ path in $G$ is the $v$-$u$ path in $G_2$ and $v$-$u$ path in $G$ is the $u$-$v$ path in $G_2$.

C) Always not true.

$G$:






D) Not true.

— $\alpha$ —

Q. 51. (2014, set 1).

$\Rightarrow$ Let us find the no. of non-zero entries in the Adjacency matrix of $G$. $(A_{144 \times 144})$

Case 1: $a \neq 1, 12$ $\quad b \neq 1, 12$

$\therefore$ # of possible values for $a = 10$

$\therefore$ # of " " " $b = 10$.

$\therefore$ # of total rows considered here is: $10 \times 10 = \boxed{100}$.

if $a \neq 1$ and $12$ then $c$ can be selected in 3 ways such that $|a-c| \leq 1$

Ily $d$ can be selected in 3 ways s.t. $|b-d| \leq 1$

$\therefore$ # of non-zero entries in row $(a, b)$ is

$= 3 \times 3 - 1$ (as self-loop is not allowed)

$= \boxed{8}$

Case 2: $a = 1$ or $12$, $b \neq 1, 12$.

$\therefore$ # of rows considered here is $= 2 \times 10 = \boxed{20}$

if $a = 1$ or $12$ then $c$ can be selected in 2 ways such that $|a-c| \leq 1$

$\therefore$ # of non-zero entries in row $(a, b)$ is

$= 2 \times 3 - 1$ (as self-loop is not allowed)

$= \boxed{5}$

<u>Case 3:</u> $a \neq 1, 12$ , $b = 1$ or $12$

$\therefore$ # of rows considered here is $= 10 \times 2 = \boxed{20}$

if $b = 1$ or $12$ then $d$ can be selected in 2 ways such that $|b-d| \leq 1$

$\therefore$ # of non-zero entries in row $(a, b)$ is

$= 3 \times 2 - 1$ (as self-loop is not allowed)

$= \boxed{5}$

<u>Case 4:</u> $a = 1$ or $12$ , $b = 1$ or $12$

$\therefore$ # of rows considered here is $= 2 \times 2 = \boxed{4}$

$\therefore$ # of non-zero entries in row $(a, b)$ is

$= 2 \times 2 - 1$ (as self-loop is not allowed)

$= \boxed{3}$

$\therefore$ Total no. of non-zero entries in $A_{144 \times 144}$ is

$= 8 \times 100 + 5 \times 20 + 5 \times 20 + 3 \times 4$

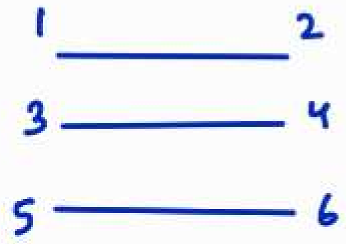
$= 1012$

$\therefore \quad e(G_2) = \frac{1012}{2} = 506$

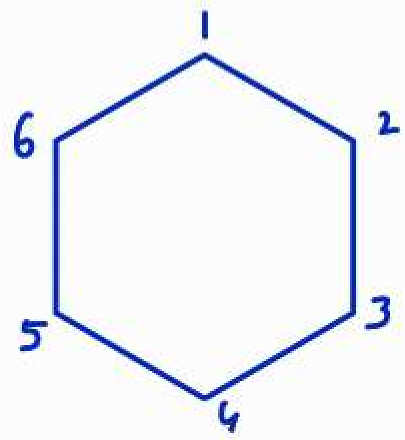
— x —

Q. 52. (2014, set 1)

A.




B.




C.

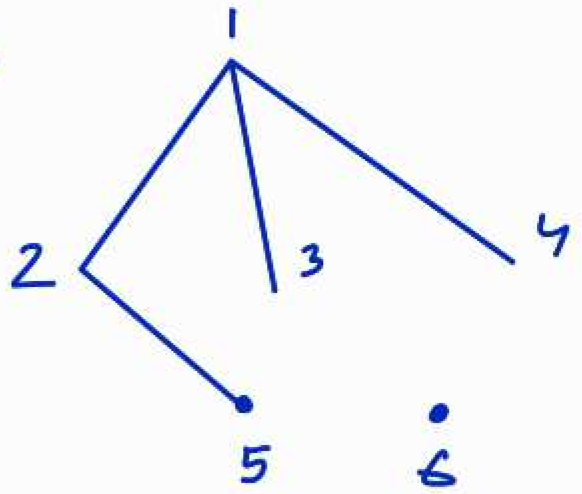
$d: 3, 3, 3, 1, 0, 0$

$d': \quad 2, 2, 0, 0, 0$

$d'': \quad 1, -1, 0, 0$

$\therefore$ Not graphic.

D.




— α —

Q. 3 (2014, set 2)

Maximum no of edges in a bipartite graph of order $n$ is : $\frac{1}{4} n^2$

$\therefore$ for $n = 12$, $\frac{1}{4} n^2$

$= \frac{1}{4} \times 144$

$= 36$.

$n_1 \times n_2$ | $n_1 + n_2 = n$

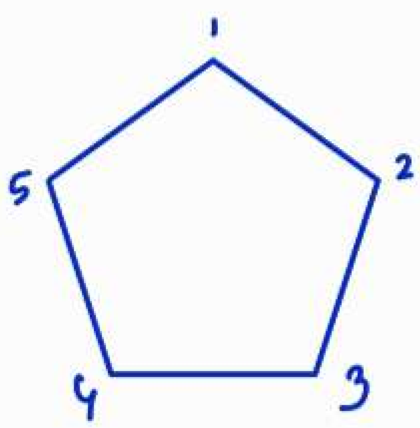
$= \frac{n}{2} \times \frac{n}{2}$ | $n_1 = n_2 = \frac{n}{2}$

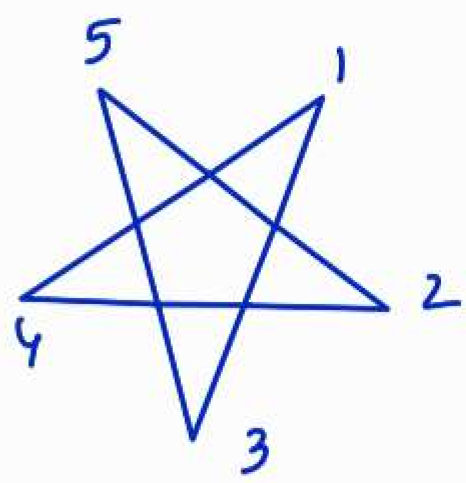
$= \frac{n^2}{4}$.

— α —

Q. 36 (2014, set 2)

$n = 5$.




— α —

Q. 25. (2013)

\# of odd degree vertices is even. True.

Sum of degrees of all vertices is even. True.

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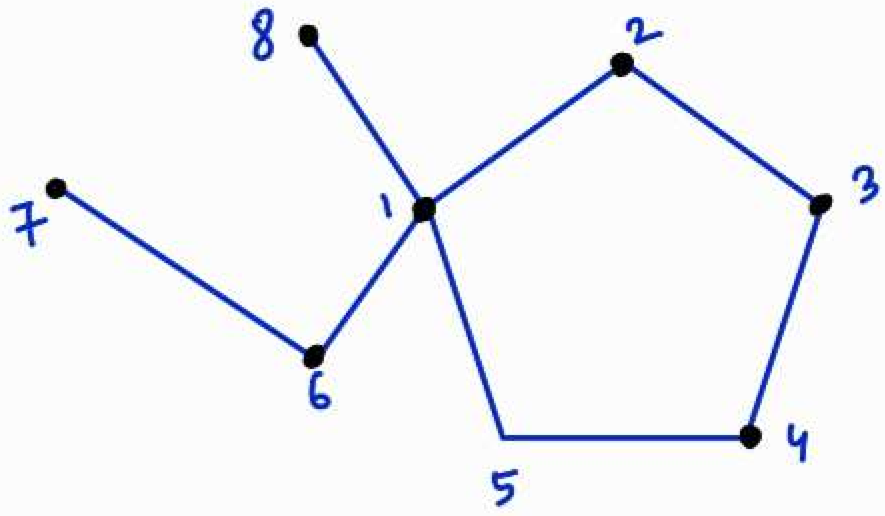
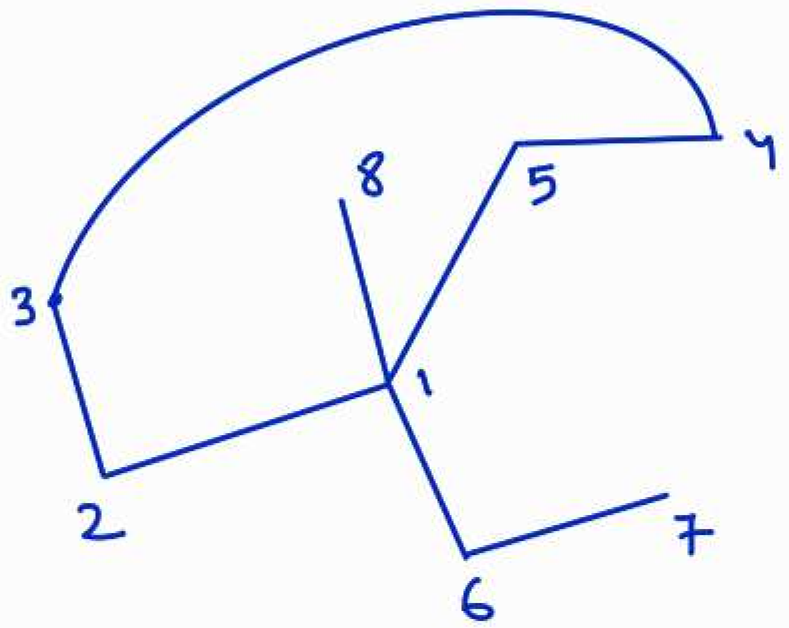
Q. 26 (2012)

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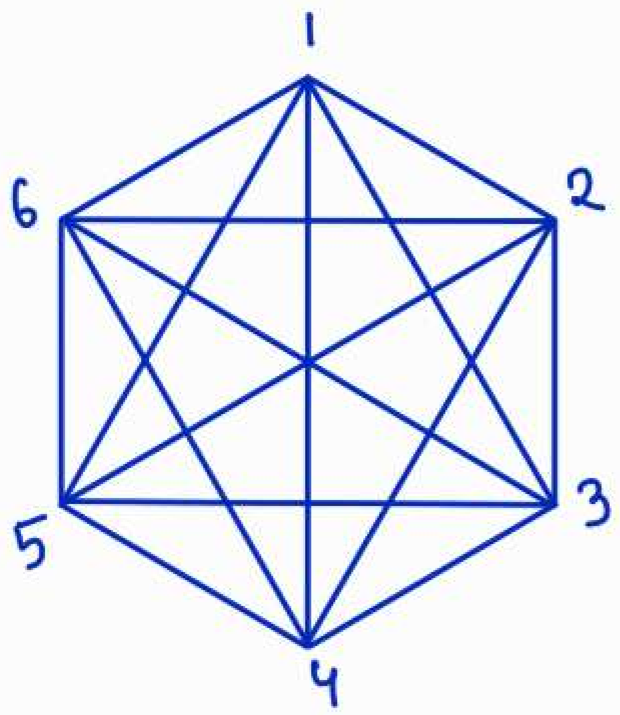
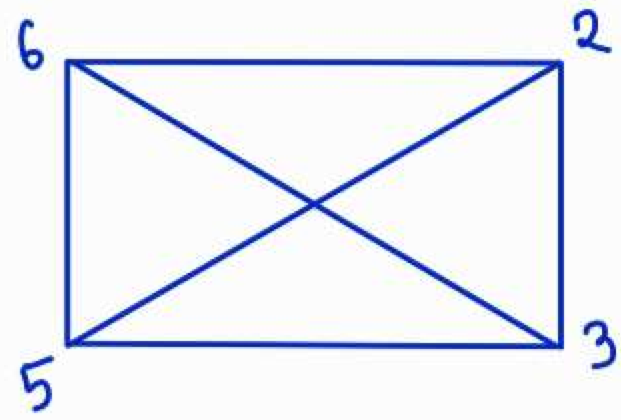
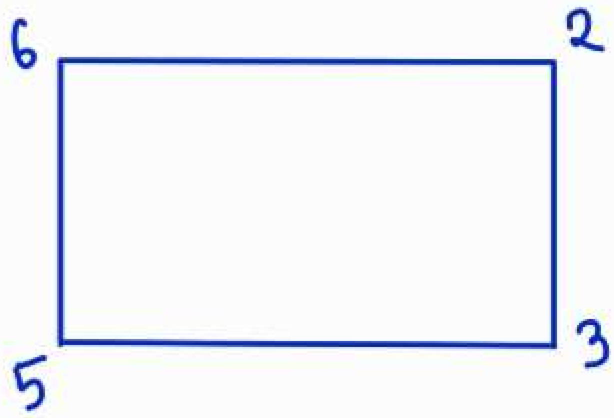
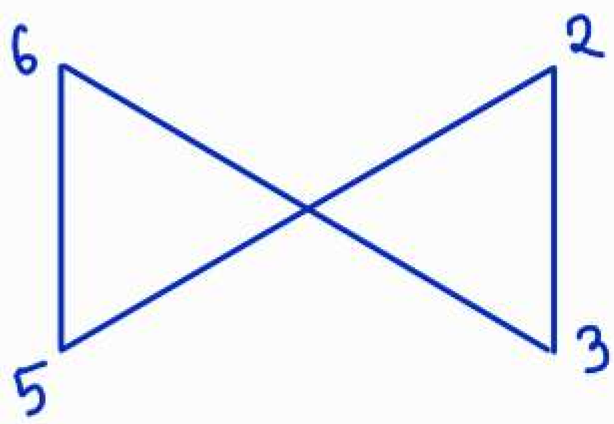
Q. 38: (2012)



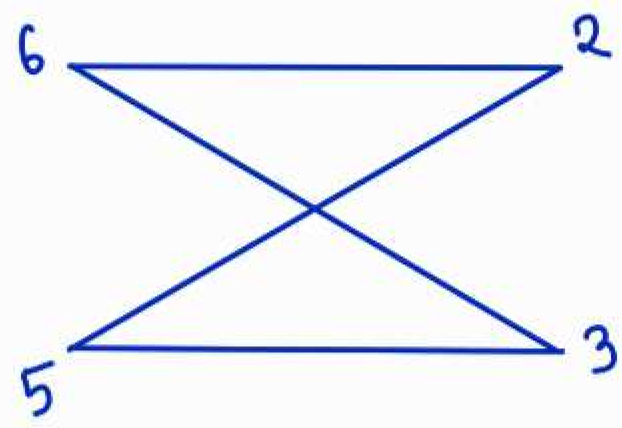



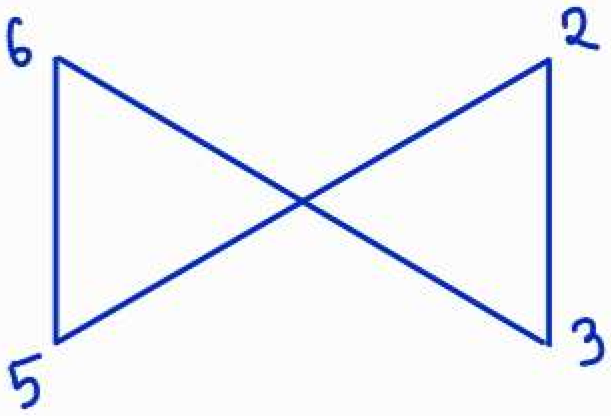

2 3 5 6 2



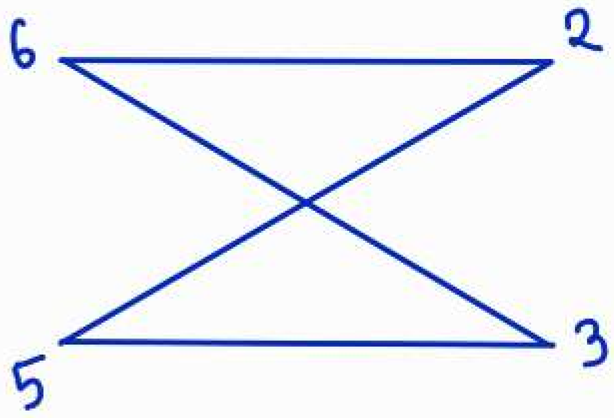

2 3 6 5 2



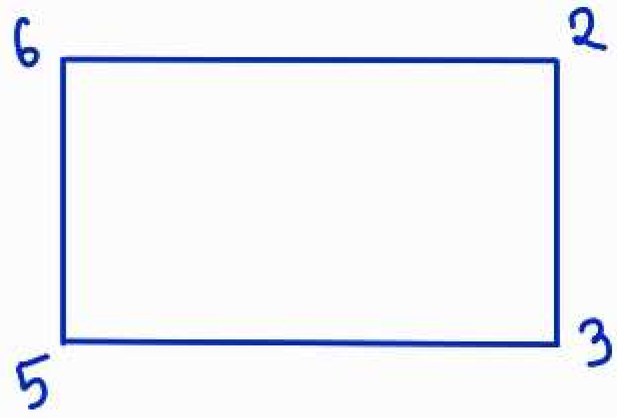

2 5 3 6 2




25632




26352




26532

$\therefore$ # of distinct cycles of length 4 in G is

$$= {}^{6}C_{4} \times 3$$

$$= 45.$$

— $\alpha$ —