Study Material

(Electromagnetism)

Chapter -27

Magnetic Field and Magnetic Forces

Topics covered:	• Magnetic Field • Magnetic Field Lines and Magnetic Flux • Motion of n of Charged Particles in a Magnetic Field • Magnetic Force on a Current-Carrying Conductor
Conceptual Problems:	TYU 27.2, 27.3, 27.4, 27.6
In Class Problems:	27.2, 27.7
Assignment Problems:	27.5, 27.15

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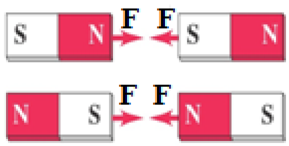
Magnetic poles

If a bar magnet, is free to rotate, one end points north. This end is called a North Pole or N pole; the other end is a South Pole or S pole.

Opposite poles attracts

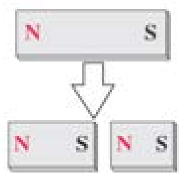


When a magnetized rod is floated on water or suspended by a string from its center, it tends to line itself up in a north-south direction. The needle of an ordinary compass is just such a piece of magnetized iron.

Like poles repels

Forces between magnetic poles mimic forces between charges. Opposite poles attract each other, and like poles repel each other. But, either pole of a permanent magnet will attract a metal like iron.

Breaking a bar magnet does not separate its poles. Each piece has a north and South Pole, even if the pieces are different sizes (The smaller the piece, the weaker its magnetism.) but not two isolated poles.

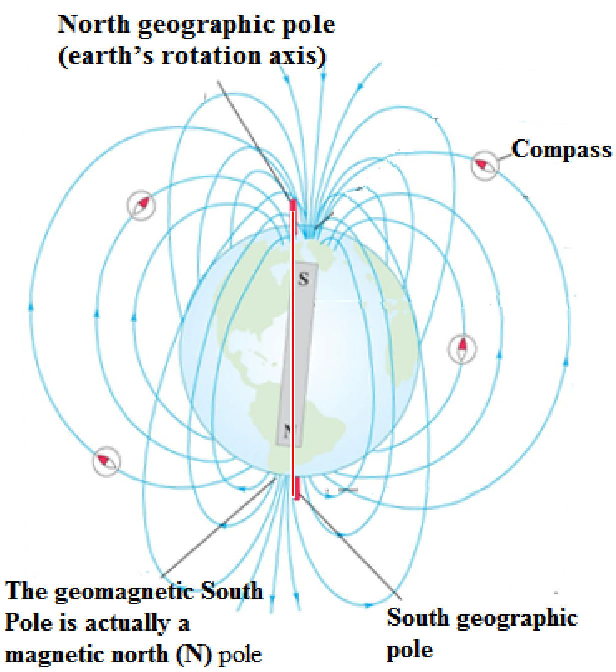


There is no experimental evidence for magnetic monopoles.

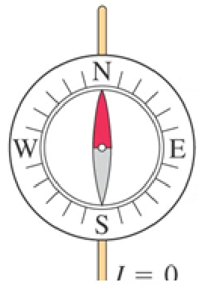
In contrast to electric charges, magnetic poles always come in pairs and can't be isolated.

Magnetic field of the earth

- The earth itself is a magnet.
- Its geographic north pole is close to a magnetic south pole. Due to this the north pole of a compass needle points north.
- The earth's magnetic axis is not quite parallel to its geographic axis (the axis of rotation), so a compass reading deviates somewhat from geographic north. This deviation, which varies with location, is called magnetic declination or magnetic variation.
- Also, the magnetic field is not horizontal at most points on the earth's surface; its angle up or down is called magnetic inclination. At the magnetic poles the magnetic field is vertical.

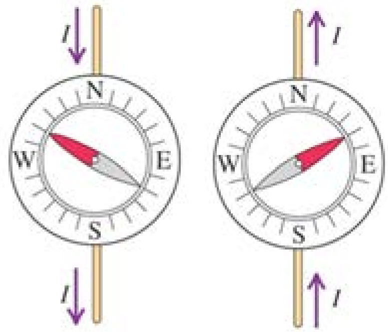


Electric current and magnets



In 1820, Hans Oersted discovered that a current-carrying wire causes a compass to deflect.
This discovery revealed a connection between moving charge and magnetism.

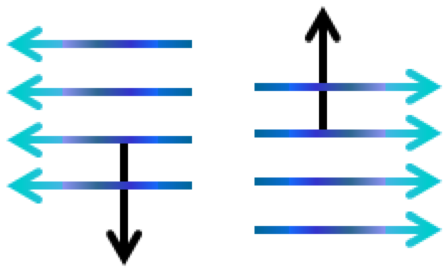
We'll find a RIGHT-HAND RULE applies to identify the direction of a magnetic field from a current-carrying wire.



Right Thumb in direction of current

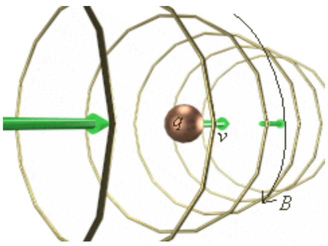
Right Hand Fingers curl in direction of Magnetic field.

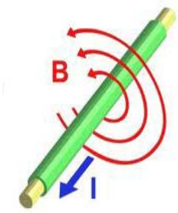
The magnetic field

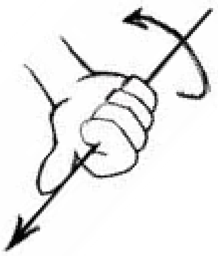


A moving charge (or current) creates a *magnetic field* in the surrounding space (in addition to its electric *field*). Magnetic fields denoted with letter "B" and measured in Tesla or Gauss (10^{-7} Tesla)

The magnetic field exerts a force on any other moving charge or current that is present in the field.







Tesla = Newton-second/ Coulomb-meter

Tesla = Newton/Amp-meter

A magnetic field exerts a force on any other moving charge - or current - that is present in the field.

The magnetic force on a moving charge:

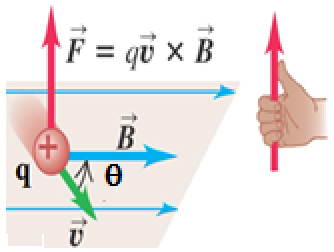
Let us consider a uniform magnetic field of strength B. A charge particle having charge ‘q’ is allowed to move with a velocity ‘v’.
Then the magnitude of the magnetic force on the moving charge ‘q’ is given by

$F = |q|\, v\, B \sin\theta$ Where, θ = angle between $\vec{v}$ and $\vec{B}$

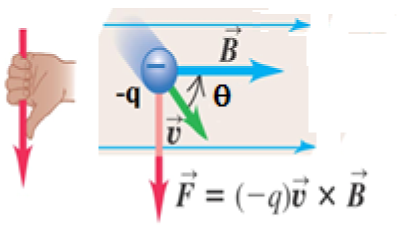
$$\vec{F} = |q|\left(\vec{v} \times \vec{B}\right)$$

Direction of the magnetic force on a moving charge ‘q’ is along the direction of $\left(\vec{v} \times \vec{B}\right)$ i.e perpendicular to both the velocity and magnetic field (B).

Direction of the magnetic force on a positive and a negative charge is shown below.



The right-hand rule gives the direction of the force on a positive charge.

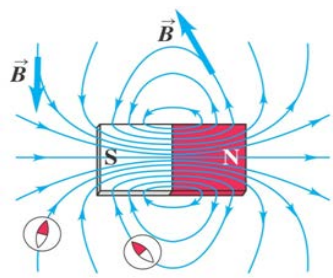


The left-hand rule gives the direction of the force on a negative charge.

From the figure it is clear that two charges of equal magnitude but opposite signs moving in the same direction in the same field will experience magnetic forces in opposite directions.

Magnetic field lines

Magnetic field is represented by magnetic field lines. Magnetic field lines starts from north and ends at the south pole (i.e magnetic field lines point away from N poles and toward S poles). Some properties of the magnetic field lines are:



- The tangent to the magnetic field line at a point gives the direction if magnetic field (B) at that point.
- If the field lines are close together, the field magnitude is large;
- If the field lines are far apart, then the field magnitude is small.
- Field lines never intersect each other because the direction of B at each point is unique,

Magnetic Flux and Gauss's Law for Magnetism

We define the magnetic flux $d\phi_B$ through a elements of area dA is

$\phi_B = B\cos\theta\, dA$

The total magnetic flux through the surface is the sum of the contributions from the individual area elements.

Thus

$$\varphi_B = \oint B\cos\theta\, dA$$

The total magnetic flux through a closed surface is always zero.

$$\varphi_B = \oint B\cos\theta\, dA = 0$$

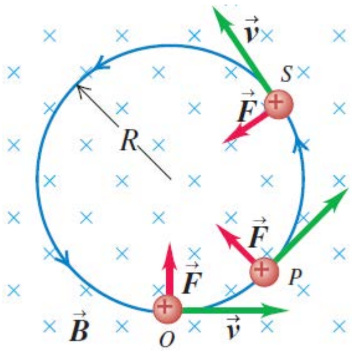
This equation is called Gauss's law for magnetism. It also follows from this equation that magnetic field lines always form closed loops.

Magnetic field lines have no ends. Unlike electric field lines that begin and end on electric charges, magnetic field lines never have end points; such a point indicates the presence of a monopole. Magnetic field lines begin at the north pole of a magnet and end at a south pole. But the field lines of a magnet actually continue through the interior of the magnet and form closed loops.

The SI unit of magnetic flux is Weber (1 Wb)

1 Wb = 1 T . m2 = 1 N . m/A

Motion of charged particles in a magnetic field



Let us consider a uniform magnetic field of strength B (into the plane of the paper).

A charge particle having charge 'q' is allowed to move with a velocity 'v'.

Then the magnitude of the magnetic force on the moving charge 'q' is given by

$$\vec{F} = |q|\left(\vec{v} \times \vec{B}\right)$$

$F = qvB\sin\theta$ Where, θ = angle between $\vec{v}$ and $\vec{B}$

$F = q\,vB$ If $\theta = 90^0$.

Direction of the magnetic force on a moving charge 'q' is along the direction of $\left(\vec{v} \times \vec{B}\right)$ i.e perpendicular to both the velocity and magnetic field (B).

As the force ($\vec{F}$) and velocity $\left(\vec{v}\right)$ are perpendicular each other, the particle will move in a circular path.

Necessary centripetal force is given by the magnetic force experienced by the particle.

Thus, centripetal force = magnetic force

$$\frac{mv^2}{R} = q\,v\,B \qquad \text{Where, } R = \text{radius of the circular path.}$$

$$R = \frac{mv}{q\,B}$$

This is the radius of a circular orbit in a magnetic field)

This equation can be written as

$$R = \frac{mv}{qB} = \frac{p}{qB}$$ Where, p = mv, momentum of the particle.

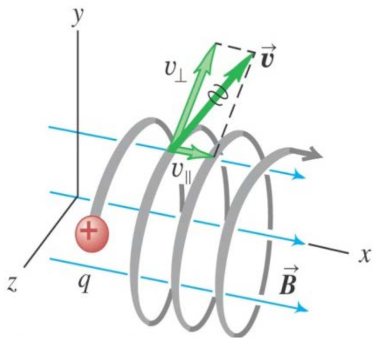
If the charge is negative, the particle moves clockwise around the orbit in the above figure.
The angular speed ω of the particle is:

$$\omega = \frac{v}{R} = \frac{v}{mv/qB} = \frac{qB}{m}$$

The number of revolutions per unit time is called the frequency of revolution (f). It is given by

$$f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

This frequency *f* is independent of the radius R of the path.



Special case:

Case I: If the initial velocity is **not** perpendicular to the magnetic field (here magnetic field is uniform).

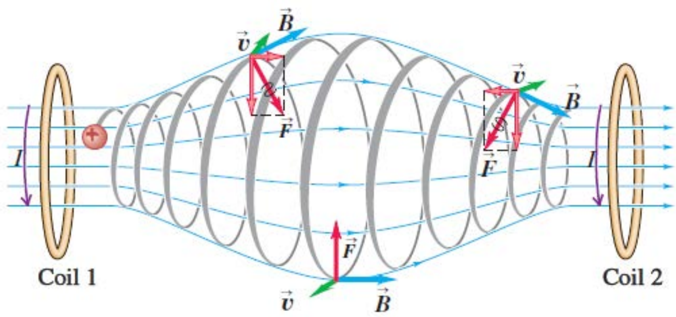
The velocity has two components. The velocity component parallel to the field is constant because there is no force parallel to the field. Then the particle moves in a helix as shown in the figure.
The radius of the helix is given by

$$R = \frac{mv}{qB}$$

Where, v is now the component of velocity perpendicular to the B field.

Case II: Motion of a charged particle in a **non-uniform magnetic** field

a) Magnetic bottle: Magnetic bottle consists of two circular coils carrying current separated by some distance.



Particles near either end of the region experience a magnetic force toward the center of the region.
Particles with appropriate speeds spiral repeatedly from one end of the region to the other and back.
Because charged particles can be trapped in such a magnetic field, it is called a magnetic bottle. This technique is used to confine very hot plasmas with temperatures of the order of 10^6 K.

b) Van Allen radiation belts around the earth

The earth's non-uniform magnetic field traps charged particles coming from the sun in doughnut-shaped regions around the earth. These regions are called the Van Allen radiation belts. This is one way of containing an ionized gas that has a temperature of the order of 10^6 K, which would vaporize any material container.

Application:

1. Cyclotron is a device which produce accelerated particle. The particle move in nearly circular path. While revolving in nearly circular path the particle increases its energy and orbital radii but not their angular speed or frequency. Cyclotron frequency is given by

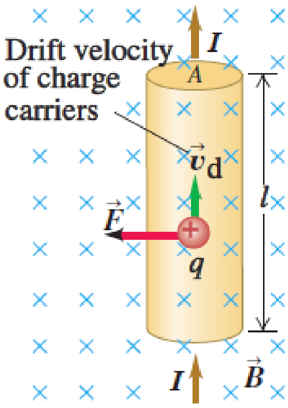
$$f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

2. Magnetron is a device which produce microwave radiation for microwave ovens and radar systems, emits radiation with a frequency equal to the frequency of circular motion of electrons in a vacuum chamber between the poles of a magnet and is given by

$$f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

The magnetic force on a current-carrying conductor

Let us consider a uniform magnetic field of strength B directed into the plane of the paper. A current carrying conductor is kept inside the magnetic field.



ℓ = length of the conducting wire
A = Area of the conducting wire
i = current, flows from bottom to top.

Let's assume that the moving charges are positive.
Then the magnitude of the magnetic force on a single moving charge of the conductor is given by

$$\vec{F} = q\left(\vec{v}_d \times \vec{B}\right)$$

Here the drift velocity $\vec{v}_d$ is upward and is, perpendicular to $\vec{B}$. Thus $\theta = 90^0$

Thus the magnitude of the force is

$$\mathbf{F = q\, v_d\, B}$$

Let n = number of charges per unit volume
Thus, total number of charges contained in conductor = n (volume of the conductor) = n A ℓ.
The total force F on all the moving charges in the conductor is

$$\mathbf{F = (nA\,\ell)\,(q\,v_d\,B) = (nA\,q\,v_d)\,(\ell\,B) = I\,\ell\,B}$$

In the vector form it can be written as:

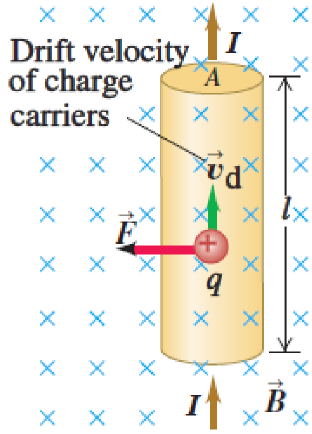
$$\vec{F} = i\left(\vec{\ell} \times \vec{B}\right)$$

If the conductor is not straight, we can divide it into infinitesimal segments d ℓ .
The force $d\vec{F}$ on each segment is

$$d\vec{F} = i\left(d\vec{\ell} \times \vec{B}\right)$$

Total force can be obtained by integrating the above equation over the total length of the conductor.

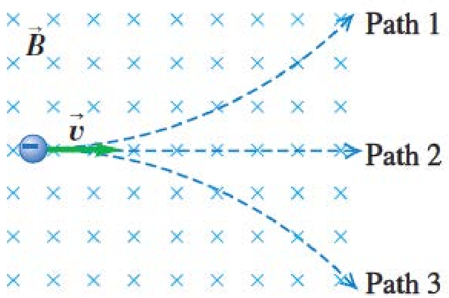
If the charge particles are negative:



If the moving charges are negative, such as electrons in a metal, then an upward current corresponds to a down-ward drift velocity.
But because q is now negative, the direction of the force F is the same as before. Thus, Eqs. $\vec{F} = i\left(\vec{\ell} \times \vec{B}\right)$ and $d\vec{F} = i\left(d\vec{\ell} \times \vec{B}\right)$ are valid for both positive and negative charges and even when both signs of charge are present at once. This happens in some semiconductor materials and in ionic solutions.

Conceptual problem: TYU-27.2, 27.3, 27.4, 27.6

Test Your Understanding of Section 27.2



The figure at right shows a uniform magnetic field $\vec{B}$ directed into the plane of the paper (shown by the X's). A particle with a negative charge moves in the plane. Which of the three paths—1, 2, or 3—does the particle follow?

Answer: path 3

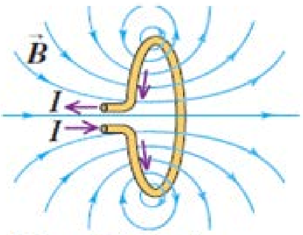
Applying the right-hand rule to the vectors $\vec{v}$ (which points to the right) and $\vec{B}$ (which points into the plane of the figure) says that the force $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$ on a positive charge would point upward. Since the charge is negative, the force points downward and the particle follows a trajectory that curves downward.

Test Your Understanding of Section 27.3

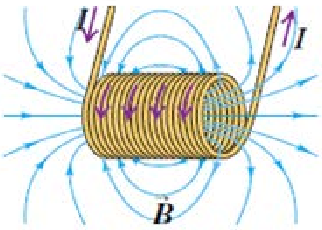
Imagine moving along the axis of the current-carrying loop shown in the figure, starting at a point well to the left of the loop and ending at a point well to the right of the loop.

(a) How would the magnetic field strength vary as you moved along this path?
  - (i) It would be the same at all points along the path;
  - (ii) it would increase and then decrease;
  - (iii) it would decrease and then increase.

(b) Would the magnetic field direction vary as you moved along the path?



Magnetic fields of a current-carrying loop



Magnetic fields of a current-carrying coil (solenoid)

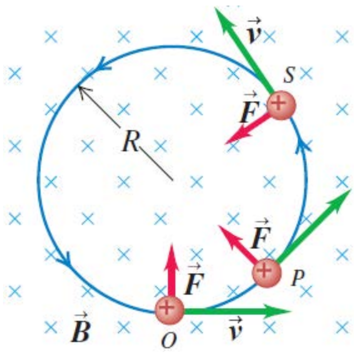
Answers: (a) (ii), (b) no

The magnitude of $\vec{B}$ would increase as you moved to the right, reaching a maximum as you pass through the plane of the loop.

As you moved beyond the plane of the loop, the field magnitude would decrease. You can tell this from the spacing of the field lines: The closer the field lines, the stronger the field.

The direction of the field would be to the right at all points along the path, since the path is along a field line and the direction of $\vec{B}$ at any point is tangent to the field line through that point.

Test Your Understanding of Section 27.4



(a) If you double the speed of the charged particle in Figure while keeping the magnetic field the same (as well as the charge and the mass), how does this affect the radius of the trajectory?
   (i) The radius is unchanged;
   (ii) the radius is twice as large;
   (iii) the radius is four times as large;
   (iv) the radius is as large;
   (v) the radius is as large.

(b) How does this affect the time required for one complete circular orbit?
   (i) The time is unchanged;
   (ii) the time is twice as long;
   (iii) the time is four times as long;
   (iv) the time is as long;
   (v) the time is as long.

Answers: (a) (ii), (b) (i)

The radius of the orbit is directly proportional to the speed,
So doubling the particle speed causes the radius to double as well.
The particle has twice as far to travel to complete one orbit but is traveling at double the speed, so the time for one orbit is unchanged. This result also follows from Eq. (27.12), which states that the angular speed ω is independent of the linear speed v. Hence the time per orbit $T=2\pi/\omega$, likewise does not depend on v.

Test Your Understanding of Section 27.6

The figure shows a top view of two conducting rails on which a conducting bar can slide. A uniform magnetic field is directed perpendicular to the plane of the figure as shown. A battery is to be connected to the two rails so that when the switch is closed, current will flow through the bar and cause a magnetic force to push the bar to the right. In which orientation, A or B, should the battery be placed in the circuit?

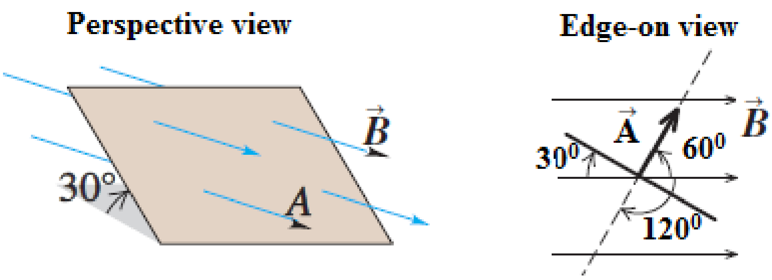
Answer: A

This orientation will cause current to flow clockwise around the circuit and hence through the conducting bar in the direction from the top to the bottom of the figure. From the right-hand rule, the magnetic force $\vec{F} = I\left(\vec{\ell} \times \vec{B}\right)$ on the bar will then point to the right.

In Class Problems: Example: 27.2, 27.7

Example 27.2 Magnetic flux calculations

Below figure is a perspective view of a flat surface with area 3cm^2 in a uniform magnetic field $\vec{B}$. The magnetic flux through this surface is +0.90 mWb. Find the magnitude of the magnetic field and the direction of the area vector $\vec{A}$.



Solution: $A = 3\ \text{cm}^2,\ \varphi_B = +\ 0.90\ \text{m.Wb},\ \theta = 60^0$

$$\varphi_B = A\,B\cos\theta$$

$$B = \frac{\varphi_B}{A\ \cos\theta} = \frac{0.9 \times 10^{-3}\ \text{Wb}}{\left(3 \times 10^{-4}\text{m}^2\right)\cos 60^0} = 6\ \text{T}$$

Example 27.7 Magnetic force on a straight conductor

A straight horizontal copper rod carries a current of 50A from west to east in a region between the poles of a large electromagnet. In this region there is a horizontal magnetic field toward the northeast (that is, 45° north of east) with magnitude 1.20 T.

(a) Find the magnitude and direction of the force on a 1.00-m section of rod.

(b) While keeping the rod horizontal, how should it be oriented to maximize the magnitude of the force? What is the force magnitude in this case?

Solution: $i = 50\ \text{A},\ \ B = 1.2\ \text{T}$

$$\vec{\ell} = (1.00\ \text{m})\hat{i}$$

$$\vec{B} = (1.20\ \text{T})\ (\cos 45°)\hat{i} + (\sin 450)\hat{j}$$

$$\vec{F} = I\left(\vec{\ell} \times \vec{B}\right)$$

$$\Rightarrow \quad \vec{F} = I \; (1.00 \text{ m})\hat{i} \times (1.20 \text{ T})\left[(\cos 45°)\hat{i} + (\sin 45^0)\hat{j}\right]$$

$$\Rightarrow \quad \vec{F} = I \; (1.00 \text{ m})(1.20 \text{ T})\left[(\cos 45°)(\hat{i} \times \hat{i}) + (\sin 45^0)(\hat{i} \times \hat{j})\right]$$

$$\Rightarrow \quad \vec{F} = (50\text{A})(1.00 \text{ m})(1.20 \text{ T})\left[0 + \frac{1}{\sqrt{2}}\hat{k}\right]$$

$$\Rightarrow \quad \vec{F} = (42.4 \text{ N})\hat{k}$$

Assignment Problems: 27.5, 27.15

27.5: An electron experiences a magnetic force of magnitude 4.60 x 10^{-15} N when moving at an angle of 60^0 with respect to a magnetic field of magnitude 3.50 x 10^{-3} T. Find the speed of the electron.

Ans: Magnitude of the charge of electron = 1.6 x 10^{-19} C.

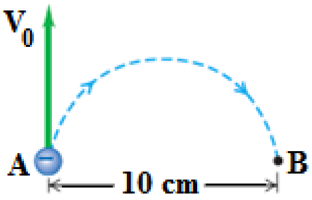
$F = q \, v \, B \sin\theta$

$$v = \frac{F}{q\,B\sin\theta} = \frac{4.6 \times 10^{-15} \text{ N}}{\left(1.6 \times 10^{-19} \text{ C}\right)\left(3.5 \times 10^{-3} \text{ T}\right)\left(\sin 60^0\right)} = 9.49 \times 10^6 \text{ m/s}$$

27.15: An electron at point in figure has a speed of v_0 = 1.41 x 10^6 m/s. Find

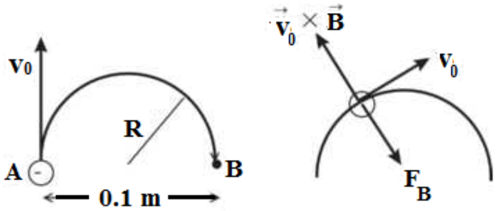
a) the magnitude and direction of the magnetic field that will cause the electron to follow the semicircular path from A to B and

b) the time required for the electron to move from A to B.



Ans: a) The necessary centripetal force is provided by the magnetic force experienced by the electron.

Thus,



$$F_B = \frac{mv^2}{R}$$

$$q\,v\,B\sin\theta = \frac{mv^2}{R}$$

$$q\,v\,B = \frac{mv^2}{R} \quad \Rightarrow \quad B = \frac{m\,v}{q\,R} = \frac{(9.1 \times 10^{-31}\text{kg})(1.41 \times 10^6 \text{ m/s})}{(1.6 \times 10^{-31} \text{ C})(0.05 \text{ m})} = 1.6 \times 10^{-4} \text{ T}$$

$$\Rightarrow \quad B = 1.6 \times 10^{-4} \text{ T}$$

b) The time is given by

$$t = \frac{\text{length of the semicircular path}}{\text{speed}}$$

$$t = \frac{\pi R}{v_0} = \frac{\pi(0.05 \text{ m})}{\left(1.41 \times 10^6 \text{ m/s}\right)} = 1.11 \times 10^{-7}\text{s}$$