Assignment - 1

Q.1 The area of a triangle formed by the tips of vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is – (a) $\frac{1}{2}(\vec{a}-\vec{b}).(\vec{a}-\vec{a})$ (b) $\frac{1}{2}\left|(\vec{a}-\vec{b})\times(\vec{a}-\vec{c})\right|$ (c) $\frac{1}{2}\left|\vec{a}\times\vec{b}\times\vec{c}\right|$ (d) $\frac{1}{2}(\vec{a}\times\vec{b}).\vec{c}$

Ans (b) $\frac{1}{2}\left|(\vec{a}-\vec{b})\times(\vec{a}-\vec{c})\right|$

Q.2. If $\vec{a}$ and $\vec{b}$ are two arbitary vectors with magnitudes a and b, respectively, $|\vec{a}\times\vec{b}|^2$ will be equal to – (a) $a^2b^2-(\vec{a}.\vec{b})^2$ (b) $ab-\vec{a}.\vec{b}$ (c) $a^2b^2+(\vec{a}.\vec{b})^2$ (d) $ab+\vec{a}.\vec{b}$

Ans (a) $a^2b^2-(\vec{a}.\vec{b})^2$

$|\vec{a}\times\vec{b}|^2 = (\vec{a}\times\vec{b}).(\vec{a}\times\vec{b})$

Let, $\vec{a}\times\vec{b} = (|\vec{a}|.|\vec{b}|\sin\theta)\hat{n}$

($\vec{a}$, $\vec{b}$, $\hat{n}$ form a triad),

$$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2\sin^2\theta\,(\hat{n}.\hat{n})$$

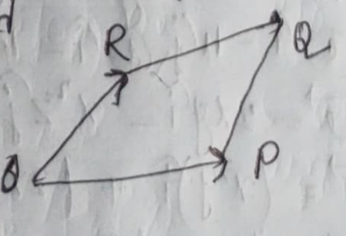
$$= |\vec{a}|^2|\vec{b}|^2(1-\cos^2\theta)$$

$$= |\vec{a}|^2|\vec{b}|^2\left[1-\left(\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right)^2\right]$$

$$= a^2b^2-(\vec{a}.\vec{b})^2 \quad \text{— (a)}$$

Q.3 For the parallelogram OPQR shown in the sketch, $\vec{OP} = a\hat{i}+b\hat{j}$ and $\vec{OR} = c\hat{i}+d\hat{j}$, The area of of the parallelogram is – (a) ad – bc (b) ac + bd (c) ad + bc (d) ab – bd

Ans (a) ad – bc



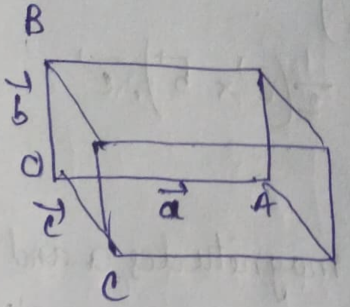
$$\vec{Area} = \vec{OP}\times\vec{OR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & 0 \\ c & d & 0 \end{vmatrix} = (ad-bc)\hat{k}$$

$|\vec{Area}| = ad - bc$

Q.4 Half of the volume of the parallelepiped formed by the tips of vectors, $\vec{a}$, $\vec{b}$ and $\vec{c}$ representing three sides

meeting at a corner is – (a) $\frac{1}{2}(\vec{a}-\vec{b}).(\vec{a}-\vec{c})$
(b) $\frac{1}{2}(\vec{a}-\vec{b}) \times (\vec{a}-\vec{c})$ (c) $\frac{1}{2}|\vec{a}\times\vec{b}\times\vec{c}|$ (d) $\frac{1}{2}(\vec{a}\times\vec{b}).\vec{c}$

Ans (d) $\frac{1}{2}(\vec{a}\times\vec{b}).\vec{c}$



$\vec{b}\times\vec{c} = bc\sin\theta\,\hat{n} = bc\,\hat{n}$
$\vec{a}.(\vec{b}\times\vec{c}) = a.\,bc\,\hat{n} = abc$

Q.5 The angle (in degree) between two planes vectors $\vec{a} = \frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}$ and $\vec{b} = -\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}$ is – (a) 30 (b) 60 (c) 90 (d) 120

Ans $\vec{a}.\vec{b} = \left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right).\left(-\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right) = -\frac{3}{4}+\frac{1}{4} = -\frac{1}{2}$
$|\vec{a}| = 1$ and $|\vec{b}| = 1$
$\theta = \cos^{-1}\left(\frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}\right) = \cos^{-1}\left(-\frac{1}{2}\right) = 120°$

$\boxed{\text{(d) } 120°}$

Q.6 The inner (dot) product of two non zero vectors $\vec{P}$ and $\vec{Q}$ is zero. The angle (degrees) between the two vector is – (a) 0 (b) 30 (c) 90 (d) 120°

Ans (c) 90°
$\vec{P}.\vec{Q} = 0 \Rightarrow |\vec{P}||\vec{Q}|\cos\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90°$

Q.7 Velocity vector of a flow field is given as $\vec{V} = 2xy\hat{i} - x^2z\hat{j}$. The velocity at (1,1,1) is parallel to (a) $4\hat{i}-2\hat{j}$ (b) $4\hat{i}-\hat{k}$ (c) $\hat{i}-4\hat{j}$ (d) $\hat{i}-4\hat{k}$

Ans $\vec{v} = 2xy\hat{i} - x^2z\hat{j}$
$\vec{F} = (x^2y)\hat{i} + (2xz)\hat{j} + (3yz^2)\hat{k}$
velocity vector = curl $\vec{F}$ = $\nabla\times\vec{F}$

$$\text{curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & -x^2z & 0 \end{vmatrix} = [0-(-x^2)]\hat{i} + [0-0]\hat{j} + [-2xz-2x]\hat{k}$$
$$= x^2\hat{i} - (2xz+2x)\hat{k}$$

At point (1, 1, 1)
curl $\vec{F} = \hat{i} - 4\hat{k}$
$\boxed{(a)\ \hat{i} - 4\hat{k}}$

Q.8 If A (0,4,3), B (0,0,0) and C (3,0,4) are three points defined in x, y, z co-ordinate system, then which of the following vector is perpendicular to both vectors $\vec{AB}$ and $\vec{BC}$.
Ans (a) $16\hat{i} + 9\hat{j} - 12\hat{k}$ (b) $16\hat{i} - 9\hat{j} + 12\hat{k}$ (c) $16\hat{i} - 9\hat{j} - 12\hat{k}$ (d) $16\hat{i} + 9\hat{j} + 12\hat{k}$
Ans $\vec{AB} = 0\hat{i} - 4\hat{j} - 3\hat{k}$, $\vec{BC} = 3\hat{i} + 0\hat{j} + 4\hat{k}$

$$\vec{AB} \times \vec{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -4 & -3 \\ 3 & 0 & 4 \end{vmatrix} = -16\hat{i} - 9\hat{j} + 12\hat{k} \quad (d)$$

Q.9 The vector that is NOT perpendicular to the vector $(\hat{i} + \hat{j} + \hat{k})$ and $(\hat{i} + 2\hat{j} + 3\hat{k})$ is - (a) $\hat{i} - 2\hat{j} + \hat{k}$ (b) $-\hat{i} + 2\hat{j} - \hat{k}$ (c) $0\hat{i} + 0\hat{j} + 0\hat{k}$
(d) $4\hat{i} + 3\hat{j} + 5\hat{k}$
Ans $(4\hat{i} + 3\hat{j} + 5\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 4 + 3 + 5 = 12$
$(4\hat{i} + 3\hat{j} + 5\hat{k}) \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 4 + 6 + 12 = 25$

(d) $4\hat{i} + 3\hat{j} + 5\hat{k}$

Q.10 If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three orthogonal vectors, Given that $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$, the vector $\vec{c}$ is parallel to –
(a) $\hat{i} + 2\hat{j} + 3\hat{k}$ (b) $2\hat{i} + \hat{j}$ (c) $2\hat{i} - \hat{j}$ (d) $4\hat{k}$
Ans (c) $2\hat{i} - \hat{j}$

Answer the following Questions:

1. Determine wheather the points A(1,3,2), B(3,-1,6), C(5,2,0) and D(3,6,-4) lie in the same plane.

Ans For the four points A, B, C, D to be co-planar, they should lie on same plane which implies,
$AD \cdot (AC \times AB) = 0$

$AB = 2\hat{i} - 4\hat{j} + 4\hat{k}$, $AC = 4\hat{i} - \hat{j} - 2\hat{k}$ and $AD = 2\hat{i} + 3\hat{j} - 6\hat{k}$

$$AB \times AC = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 4 \\ 4 & -1 & -2 \end{vmatrix} = \hat{i}(8+4) + \hat{j}(16+4) + \hat{k}(-2+16)$$
$$= 12\hat{i} + 20\hat{j} - 14\hat{k}$$

$AD \cdot (AB \times AC) = 24 + 60 - 84 = 0$

Therefore points A, B, C, D are coplanar.

Q.12 Find a vector equation and parametric equation for the line segment that joins P to Q. $P(0,-1,1)$, $Q\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$

Ans Given, $P(0,-1,1)$ and $Q\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$

$$\overrightarrow{PQ} = \left\langle \frac{1}{2} - 0, \frac{1}{3} + 1, \frac{1}{4} - 1 \right\rangle = \left\langle \frac{1}{2}, \frac{4}{3}, -\frac{3}{4} \right\rangle = \langle a, b, c \rangle$$

Let, $(x_0, y_0, z_0) = (0, -1, 1)$

Vector equation, $\vec{r} = \vec{r_0} + t\vec{v}$

$$\vec{r} = (0\hat{i} - \hat{j} + \hat{k}) + t\left(\frac{1}{2}\hat{i} + \frac{4}{3}\hat{j} - \frac{3}{4}\hat{k}\right)$$

parametric equation, $x = 0 + \frac{1}{2}t$

$y = -1 + \frac{4}{3}t$

$z = 1 + \left(-\frac{3}{4}\right)t$

Q.13 Determine wheather the lines L_1 and L_2 are parallel, skew or intersecting. If they interset from the point of intersection

$L_1: x = 3+2t, y = 4-t, z = 1+3t$

$L_2: x = 1+4s, y = 3-2s, z = 4+5s$

Ans check for parallel Lines:

$\frac{2}{4} = \frac{1}{2}$

$\frac{-1}{-2} = \frac{1}{2}$ $\quad\therefore$ Here, $\frac{1}{2} = \frac{1}{2} \neq \frac{3}{5}$, so the lines are not parallel.

$\frac{3}{5}$

Check for intersecting lines:

$3+2t = 1+4s$ — (i)

$4-t = 3-2s$ — (ii)

$1+3t = 4+5s$ — (iii)

From (i) we get, $3+2t = 1+4s$

$\Rightarrow t = 2s-1$ — (iv)

Putting (iv) in equation (ii)

$4-(2s-1) = 3-2s$

$\Rightarrow 3 = 3$

$\therefore$ The lines L_1 and L_2 don't intersect each other

$\therefore$ L_1 and L_2 lines are skew.

Q14 Find the velocity, acceleration and speed of a particle with the given position function, $r(t) = t^2\hat{i} + 2t\hat{j} + \ln t\,\hat{k}$

Ans $\vec{r}(t) = t^2\hat{i} + 2t\hat{j} + \ln t\,\hat{k}$

velocity $= v(t) = \frac{d}{dt}(\vec{r}(t)) = \frac{d}{dt}(t^2\hat{i} + 2t\hat{j} + \ln t\,\hat{k})$

$= 2t\hat{i} + 2\hat{j} + \frac{1}{t}\hat{k}$

acceleration $= a(t) = \frac{d}{dt}(\vec{v}(t)) = \frac{d}{dt}(2t\hat{i} + 2\hat{j} + \frac{1}{t}\hat{k})$

$= 2\hat{i} - \frac{1}{t^2}\hat{k}$

speed $= |\vec{v}(t)| = \sqrt{(2t)^2 + 2^2 + \left(\frac{1}{t}\right)^2} = \sqrt{4t^2 + 4 + \frac{1}{t^2}}$

15. Find the position vector of the particle that has given acceleration and the specified and the specified initial velocity and position, $\vec{a}(t) = t\hat{i} + e^t\hat{j} + e^{-t}k$ $v(0) = \hat{k}$, $r(0) = \hat{j} + \hat{k}$

Ans $v(t) = \int a(t)\,dt = \int (t\hat{i} + e^t\hat{j} + e^{-t}k)\,dt$

$= \int t\,dt\,\hat{i} + \int e^t\,dt\,\hat{j} + \int e^{-t}\,dt\,\hat{k}$

$$= \frac{t^2}{2}\hat{i} + e^{t}\hat{j} - e^{-t}\hat{k} + c$$

$$\vec{v}(0) = \frac{0^2}{2}\hat{i} + e^{0}\hat{j} - e^{-0} + c = \hat{k}$$

$$= \hat{j} - \hat{k} + c = \hat{k}$$

$$\Rightarrow c = 2\hat{k} + \hat{j}$$

$$\vec{v}(t) = \frac{t^2}{2}\hat{i} + e^{t}\hat{j} - e^{-t}\hat{k} + 2\hat{k} - \hat{j}$$

$$= \frac{t^2}{2}\hat{i} + (e^{t} - 1)\hat{j} + (2 - e^{-t})\hat{k}$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \int \frac{t^2}{2}dt\,\hat{i} + \int (e^{t}-1)\hat{j} + \int (2-e^{-t})\hat{k}$$

$$= \frac{t^3}{6}\hat{i} + (e^{t} - t)\hat{j} + (2t + e^{-t})\hat{k} + c_2$$

$$\vec{r}(0) = \frac{0}{6}\hat{i} + (e^{0} - 0)\hat{j} + (2\times 0 + e^{-0})\hat{k} + c_2 = \hat{j} + \hat{k}$$

$$\Rightarrow \hat{j} + \hat{k} + c_2 = \hat{j} + \hat{k}$$

$$\Rightarrow c_2 = 0$$

$$\vec{r}(t) = \frac{t^3}{6}\hat{i} + (e^{t} - t)\hat{j} + (2t + e^{-t})\hat{k}$$

Q.16 Find the curvature of $r(t) = \langle t, t^2, t^3 \rangle$ at the point $(1,1,1)$

Ans $r(t) = \langle t^1, t^2, t^3 \rangle$

$\therefore r'(t) = \langle 1, 2t, 3t^2 \rangle$

$\therefore r''(t) = \langle 0, 2, 6t \rangle$

$|r'(t)| = \sqrt{1 + 4t^2 + 9t^4}$

$$r'(t) \times r''(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} = 6t^2\hat{i} - 6t\hat{j} + 2\hat{k}$$

$|r'(t) \times r''(t)| = \sqrt{36t^4 + 36t^2 + 4} = 2\sqrt{9t^4 + 9t^2 + 1}$

At point $P = \langle 1,1,1 \rangle$

Now, let us find the value for t which $r(t) = P$, clearly we get, $t = 1$

we know that the formula of curvature of a curve is,

$$K = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$$= \frac{2\sqrt{9t^4+9t^2+1}}{(1+4t^2+9t^4)^{3/2}}$$

Apply, $t = 1$ as we calculated earlier, we get,

$$K = \frac{2\sqrt{9 \cdot 1^4 + 9 \cdot 1^2 + 1}}{(1+4 \cdot 1^2 + 9 \cdot 1^4)^{3/2}}$$

$$= \frac{\sqrt{36+36+4}}{14^{3/2}}$$

$$= \frac{2\sqrt{19}}{14\sqrt{14}}$$

$$= \frac{1}{7}\sqrt{\frac{19}{14}} = 0.17$$

17. Find the length of the curve $r(t) = 12t\hat{i} + 8t^{3/2}\hat{j} + 3t^2\hat{k}, 0 \le t \le 1$

Ans $r(t) = 12t\hat{i} + 8t^{3/2}\hat{j} + 3t^2 K$

$\therefore r'(t) = 12\hat{i} + 12\sqrt{t}\,\hat{j} + 6t\hat{k}$

$\therefore |r'(t)| = \sqrt{144 + 144t + 36t^2} = \sqrt{36(t+2)^2} = 6|t+2|$

$= 6(t+2)$. as $0 \le t \le 1$

Then, $L = \int_0^1 |r'(t)|dt = \int_0^1 6(t+2)dt = \left[3t^2 + 12t\right]_0^1 = 15 - 0 = 15$

18. Find equations of the normal plane and osculating plane of the curve at the given point.

$x = 2\sin 3t,\ y = t,\ z = 2\cos 3t;\ (0, \pi, -2)$

Ans: Let us find the equations of the normal and osculating plane of the curve,

$$r(t) = 2\sin 3t\,\hat{i} + t\,\hat{j} + 2\cos 3t\,\hat{k}$$

at $(0, \pi, -2)$, we will first compute the tangent vector and unit normal vector, we have,

$$r'(t) = (2\sin 3t)'\hat{i} + t'\hat{j} + (2\cos 3t)'\hat{k}$$

$$= 6\cos 3t\,\hat{i} + \hat{j} - 6\sin 3t\,\hat{k}$$

$$|r'(t)| = \sqrt{(6\cos 3t)^2 + 1^2 + (-6\sin 3t)^2}$$

$$= \sqrt{36\cos^2 3t + 1 + 36\sin^2 3t} = \sqrt{36+1} = \sqrt{37}$$

It follows that,

$$T(t) = \frac{6\cos 3t}{\sqrt{37}}\hat{i} + \frac{1}{\sqrt{37}}\hat{j} - \frac{6\sin 3t}{\sqrt{37}}\hat{k}$$

$$T'(t) = \frac{1}{\sqrt{37}}(-18\sin 3t)\hat{i} - \frac{1}{\sqrt{37}} \cdot 18\cos 3t\,\hat{k}$$

$$|T'(t)| = \sqrt{\left(-\frac{18\sin 3t}{\sqrt{37}}\right)^2 + \left(-\frac{18\cos 3t}{\sqrt{37}}\right)^2} = \sqrt{\frac{18^2}{37}} = \frac{18}{\sqrt{37}}$$

The normal unit vector is,

$$N(t) = -\frac{18\sin 3t}{\sqrt{37}} \cdot \frac{\sqrt{37}}{18}\hat{i} - \frac{18\cos 3t}{\sqrt{37}} \cdot \frac{\sqrt{37}}{18}\hat{k} = -\sin 3t\,\hat{i} - \cos 3t\,\hat{k}$$

The point $(0, \pi, -2)$ is obtained for $t = \pi$, so to the normal plane at this point will have the normal vector,

$$r'(\pi) = 6\cos 3\pi\,\hat{i} + \hat{j} - 6\sin 3\pi\,\hat{k} = \langle -6, 1, 0 \rangle$$

So the equation of the normal plane is,

$$-6(x-0) + 1(y-\pi) + 0(z+2) = 0$$

$$\Rightarrow -6x + y - \pi = 0$$

$$\Rightarrow -6x + y = \pi$$

To find the osculating plane $(0, \pi, -2)$ we first note that the normal vector of this plane is $B(\pi)$:

$$B(\pi) = T(\pi) \times N(\pi)$$

$$= \left\langle \frac{6\cos 3\pi}{\sqrt{37}}, \frac{1}{\sqrt{37}}, \frac{-6\sin 3\pi}{\sqrt{37}} \right\rangle \times \left\langle -\sin 3\pi, 0, -\cos 3\pi \right\rangle$$

$$= \left\langle -\frac{6}{\sqrt{37}}, \frac{1}{\sqrt{37}}, 0 \right\rangle \times \langle 0, 0, 1 \rangle$$

$$= \left(-\frac{6}{\sqrt{37}}\hat{i} + \frac{1}{\sqrt{37}}\hat{j} \right) \times \hat{k}$$

$$= -\frac{6}{\sqrt{37}}(\hat{i} \times \hat{k}) + \frac{1}{\sqrt{37}}(\hat{j} \times \hat{k})$$

$$= \frac{6}{\sqrt{37}}\hat{j} + \frac{1}{\sqrt{37}}\hat{i}$$

$$= \left\langle \frac{1}{\sqrt{37}}, \frac{6}{\sqrt{37}}, 0 \right\rangle$$

So the equation of the osculating plane at $(0, \pi, -2)$ is,

$$\frac{1}{\sqrt{37}}(x - 0) + \frac{6}{\sqrt{37}} \cdot (y - \pi) + 0(z + 2) = 0$$

$$\Rightarrow \frac{1}{\sqrt{37}} \cdot x + \frac{6}{\sqrt{37}} \cdot y - \frac{6}{\sqrt{37}}\pi = 0$$

$$\Rightarrow x + 6y = 6\pi$$

19. A projectile is fired with an initial speed of 200 m/s and angle of elevation $60°$. Find (a) the range of the projectile, (b) the maximum height reached and (c) the speed at impact.

Ans $|v(0)| = 200$ m/s and since the angle of elevation is $60°$, a unit vector in the direction of the velocity is,

$(\cos 60°)\hat{i} + (\sin 60°)\hat{j} = \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$. Thus $v(0) = 200\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$

$$= 100\hat{i} + 100\sqrt{3}\hat{j}$$

and ~~we~~ if we set up the axes so that the projectile starts at the origin, then $r(0) = 0$. Ignoring air resistance, the only force is that due to gravity, so

$F(t) = ma(t) = -mg\hat{j}$ where $g = 9.8$ m/s². Thus $a(t) = -9.8\hat{j}$

and by integrating, we have $v(t) = -9.8t\,\hat{j} + c$;

$100\hat{i} + 100\sqrt{3}\hat{j} = v(0) = c$, $v(t) = 100\hat{i} + (100\sqrt{3} - 9.8t)\hat{j}$

Again integrating,

$r(t) = 100t\,\hat{i} + (100\sqrt{3}t - 4.9t^2)\hat{j} + D$ where $0 = r(0) = D$.

Thus the position function of the projectile is,

$r(t) = 100t\hat{i} + (100\sqrt{3}t - 4.9t^2)\hat{j}$

(a) Parametric equations for the projectile are,

$x(t) = 100t$, $y(t) = 100\sqrt{3}t - 4.9t^2$.

The projectile reaches the ground when $y(t) = 0$ and $(t > 0)$.

$\Rightarrow 100\sqrt{3}t - 4.9t^2 = t(100\sqrt{3} - 4.9)$

$\Rightarrow t = \dfrac{100\sqrt{3}}{4.9} = 35.3$ sec

So the range is, $x\left(\dfrac{100\sqrt{3}}{4.9}\right) = 100 \times \dfrac{100\sqrt{3}}{4.9} = 3535$ metre

(b) The maximum height is reached when $y(t)$ has a critical number (or equivalently, when the vertical component of velocity is 0): $y'(t) = 0$

$\Rightarrow 100\sqrt{3} - 9.8t = 0$

$\Rightarrow t = 17.7$ sec

Maximum height, $y\left(\dfrac{100\sqrt{3}}{9.8}\right) = 100\sqrt{3} \cdot \dfrac{100\sqrt{3}}{9.8} - 4.9 \cdot \left(\dfrac{100\sqrt{3}}{9.8}\right)^2$

$= 1531$ metre

(c) From part (a) impact occurs at $t = \dfrac{100\sqrt{3}}{4.9}$ s. Thus the velocity at impact is,

$v\left(\dfrac{100\sqrt{3}}{4.9}\right) = 100\hat{i} + \left[100\sqrt{3} - \left(\dfrac{100\sqrt{3}}{4.9}\right) \times 9.8\right]\hat{j}$

$= 100\hat{i} - 100\sqrt{3}\hat{j}$

and speed $= \left|v\left(\dfrac{100\sqrt{3}}{4.9}\right)\right| = \sqrt{10{,}000 + 30{,}000} = 200$ m/s

20. Find the tangential and normal components of the acceleration vector.

$r(t) = e^t\hat{i} + \sqrt{2}t\hat{j} + e^{-1}\hat{k}$

Ans $r(t) = e^t\hat{i} + \sqrt{2}t\hat{j} + e^{-1}\hat{k}$

$\therefore r'(t) = e^t\hat{i} + \sqrt{2}\hat{j}$

$|r'(t)| = \sqrt{e^{2t} + 2}$