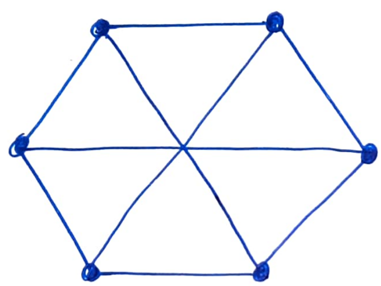
ASSIGNMENT - 3

Q1. How many perfect matching does the graph K_6 have? Draw all perfect matching.

Ans.

K_6 :



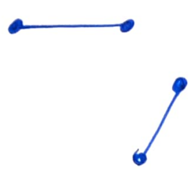
No. of perfect matching $= \dfrac{(2 \times 3)!}{2^3 \times 3!} = 15$

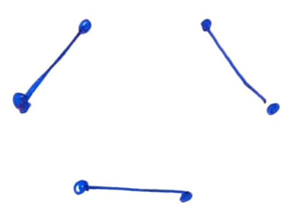
where, $n = 3$

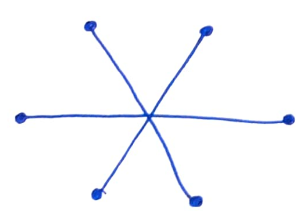
$\Uparrow$

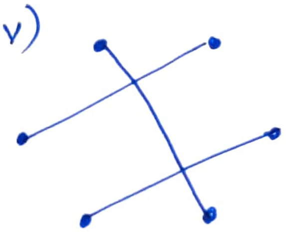
$2n = 6$

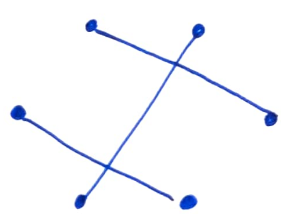
$\Rightarrow n = 3$

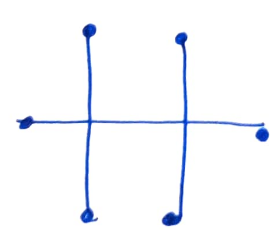
i) 

ii) 

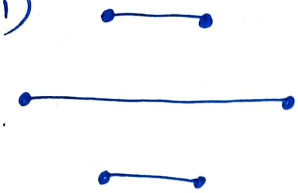
iii) 

iv) 

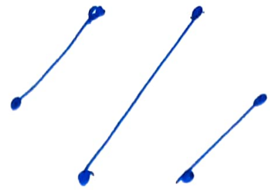
v) 

vi) 

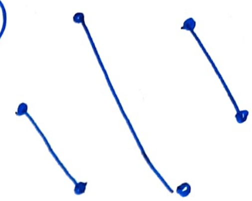
vii)



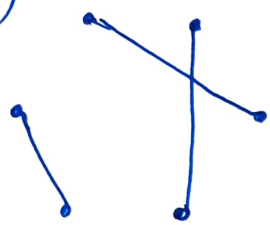
viii)



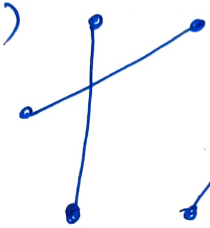
ix)



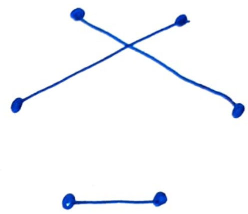
x)



xi)



xii)



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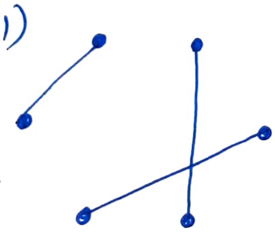
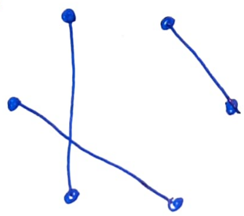
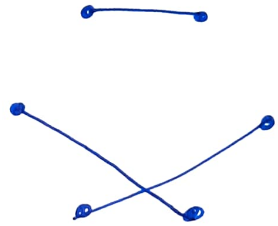


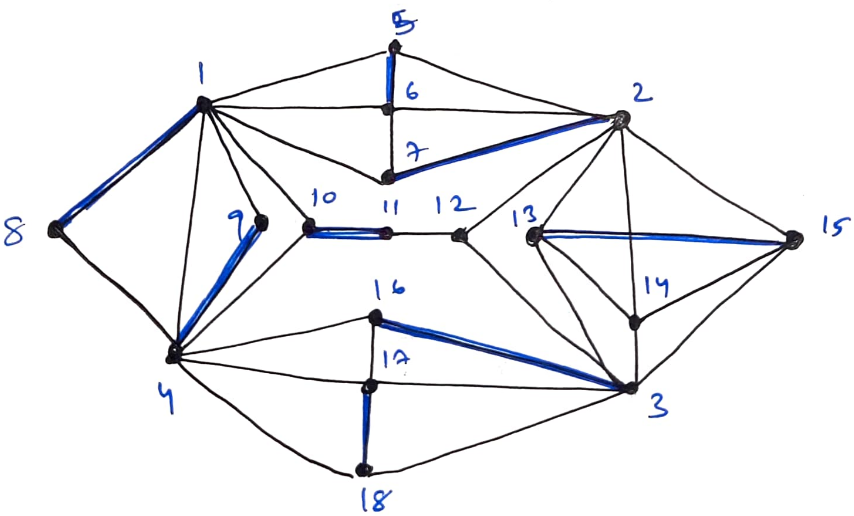
xiv)



xv)



Q2. Determine whether the given graph has 1-factor or not.



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Ans The max^m size of a matching is 8.

ie. $\{ V_1V_8, V_4V_9, V_5V_6, V_2V_7, V_{10}V_{11}, V_{13}V_{15}, V_{16}V_3, V_{17}V_{18} \}$.

Let $S = \{1, 2, 3, 4\}$.

$G - S = \{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$.

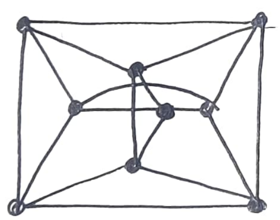
$o(G - S) = 6 > |S|$

By Tutte's 1-factor theorem, we conclude that G has no 1-factor.

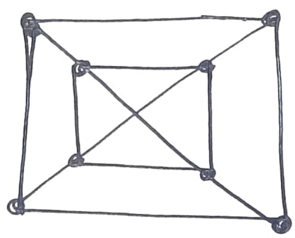
(Ans).

Q3. Compute the chromatic no., clique no. and the independence no. of the graphs below:

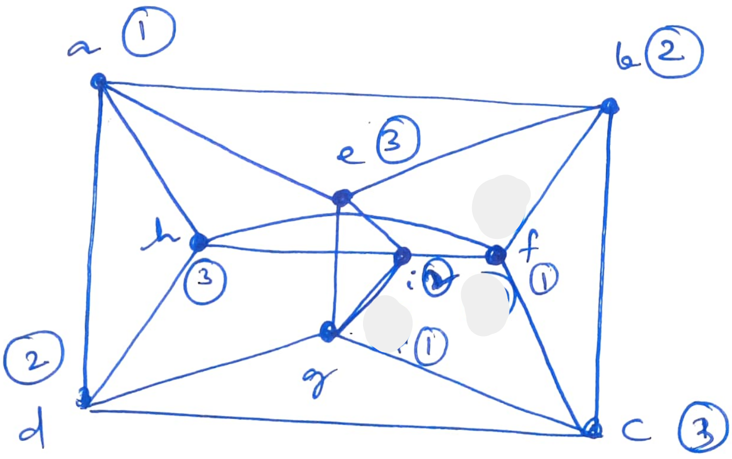
G_1:



G_2:



Ans. 1) G_1:



$\omega(G) = 3$

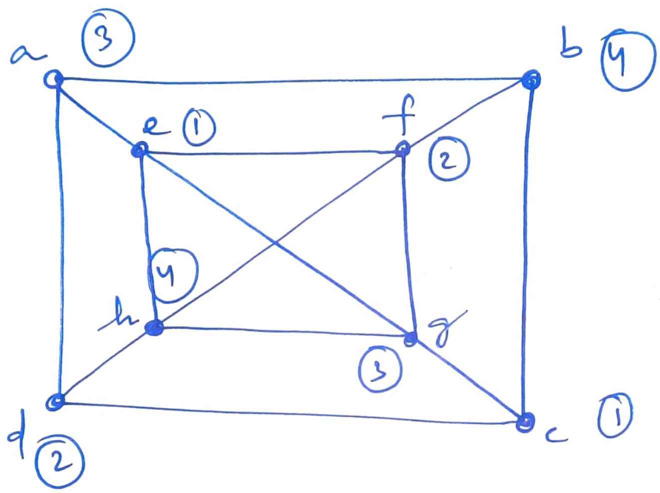
$\gamma(G) = 3$

$\chi(G) \geq (\omega(G) = 3)$

$\chi(G) \geq \dfrac{n(G)}{\alpha(G)} = 3$

$\therefore \chi(G) = 3$

ii) G_2:



$\omega(G) = 4$ $\{e, f, g, h\}$ $\rightarrow$ clique no.

$\alpha(G) = 3$ $\{a, f, c\}$ $\rightarrow$ independence no.

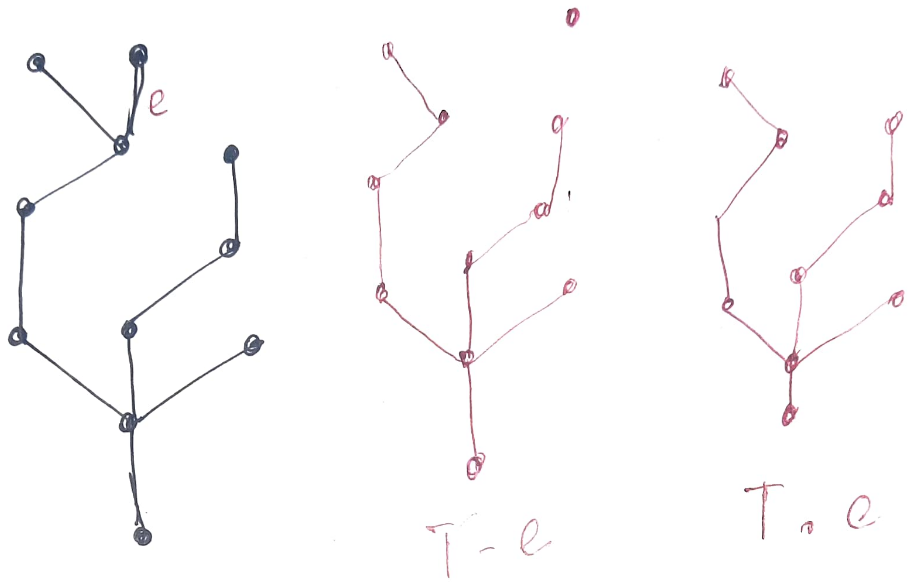
$\therefore \chi(G) \geq (\omega(G) = 4)$

$\chi(G) \geq \left(\dfrac{n(G)}{\alpha(G)} = \dfrac{8}{3}\right)$.

$\therefore \chi(G) = 4.$

Q4 a) Compute the chromatic polynomial of wheel $C_n \vee K_1$.

b) Compute the chromatic polynomial of the given tree using chromatic reccurrance.



Ans.

a) If a vertex x of $G \vee K_1$ which is adjacent to every other vertex is colored with particular color then other vertices cannot be colored with same color.

If vertex x can be colored in k ways, then $\chi(G, k-1)$ gives a proper coloring to the rest.

$$\chi(G \vee K_1, k) = k(G, k-1).$$

Now;

$$\chi(C_n \vee K_1, k) = k\,\chi(C_n, k-1)$$
$$= k\left[(k-2)^n + (-1)^n (k-2)\right] \text{ (Ans)}$$

b) $\chi(T; k) = k(k-1)^{n-1}$

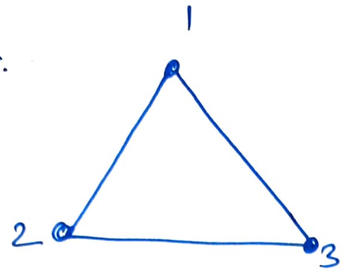
$$\chi(T; k) = \underline{k(k-1)^{10}}$$

By chromatic recurrence

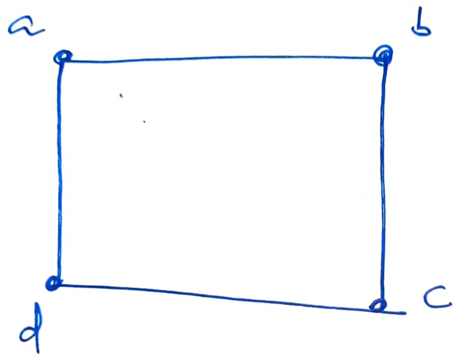
$$\chi(T_{11}; k) = k\,\underline{\chi(T_{10}; k)} - \chi(T_{11} \cdot e; k)$$
$$= k \cdot k(k-1)^9 - \chi(T_{10}; k)$$
$$= k^2(k-1)^9 - k(k-1)^9$$
$$= (k-1)^9(k^2-k)$$
$$= (k-1)^9 \cdot k(k-1)$$
$$= \underline{k(k-1)^{10}}$$

Q5. Find the cartesian product of C_4 and C_3 and compute $\chi(C_4 \square C_3)$.

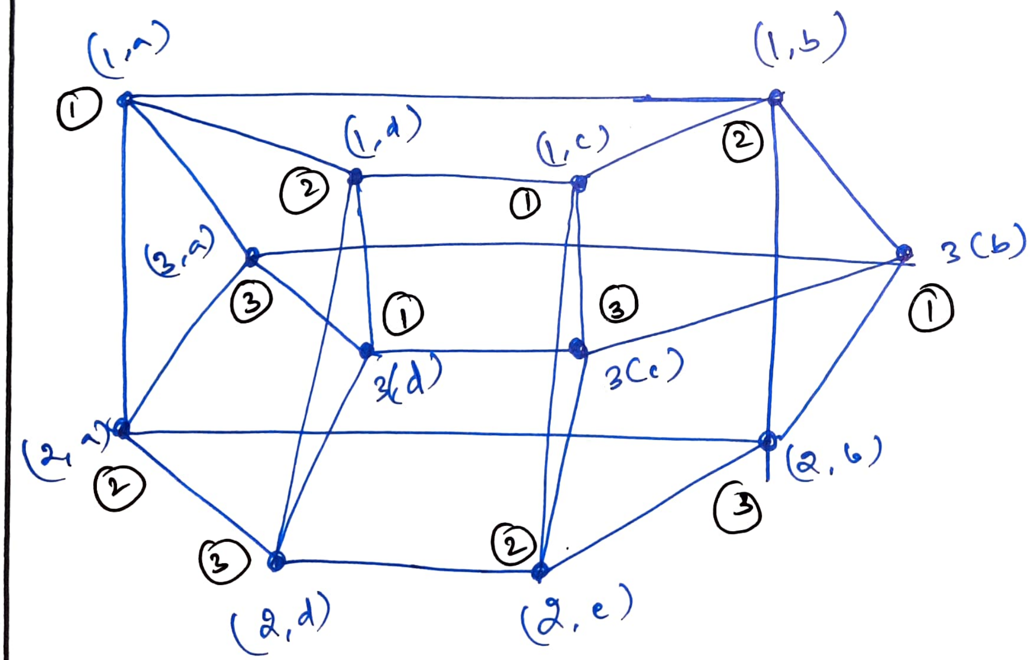
Ans. C_3 :



C_4 :



$C_3 \square C_4 = C_4 \square C_3$ =



$\therefore \chi(C_4 \square C_3) = 3$ (Ans)

Name:—————————— Regd. Number:——————————