Study Material

(Electromagnetism)

Chapter-29

Electromagnetic Induction

Topics covered :	• Faraday's Law • Lenz's Law • Motional Electromotive Force • Induced Electric Fields Magnetic Fields • Displacement Current & Maxwell's Equations
Conceptual Problems :	TYU 29.2, 29.3, 28.7
In Class Problems :	29.5, 29.11
Assignment Problems :	29.2, 29.41

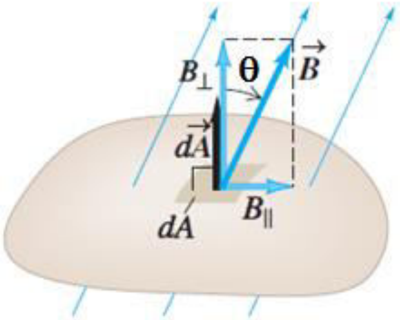
Dr. Rajanikanta Parida

Department of Physics

ITER,SOA University, Bhubaneswar

Magnetic Flux:

The magnetic flux ($d\phi_B$) linked with an infinitesimal area element dA placed in a magnetic field B (shown in the figure in right), is



$$d\varphi_B = \vec{B}\,.\,\vec{dA} = B\,dA\cos\theta$$

θ = angle between $\vec{B}$ and $\vec{dA}$

The total magnetic flux φ_B is through a finite area is

$$\varphi_B = \int_{\text{surface}} \vec{B}.\vec{dA}$$

For uniform *B*-field and a flat surface: $\varphi_B = \vec{B}.\vec{A} = B\,A\cos\theta$

Calculating the flux of a uniform magnetic field through a flat area

1. $\vec{B}$ and $\vec{A}$ are parallel, i.e $\theta = 0$, (Figure-1) then $\varphi_B = \vec{B}.\vec{A} = B\,A$
2. $\vec{B}$ and $\vec{A}$ are perpendicular, i.e $\theta = 90^0$, (Figure-2) then $\varphi_B = \vec{B}.\vec{A} = 0$
3. $\vec{B}$ and $\vec{A}$ are at an angle θ (Figure-3) then $\varphi_B = \vec{B}.\vec{A} = B\,A\cos\theta$

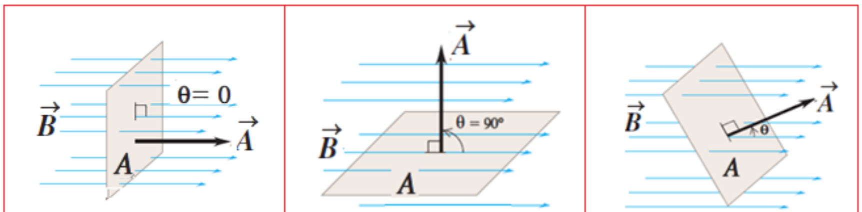


Figure - 1 Figure - 2 Figure - 3

Faraday's Law

The time rate of change of magnetic flux through the loop is directly propertional to the induced emf. It is given by

- emf induced in a single loop: $\varepsilon = -\dfrac{d\varphi_B}{dt}$
- for N loops: $\varepsilon = -N\dfrac{d\varphi_B}{dt}$ ---------(1)

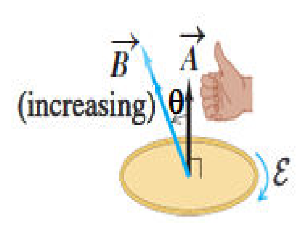
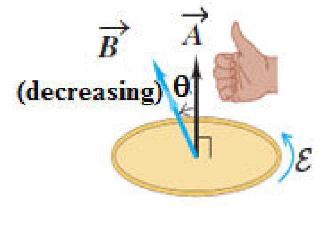
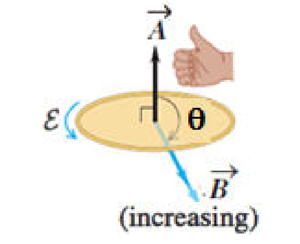
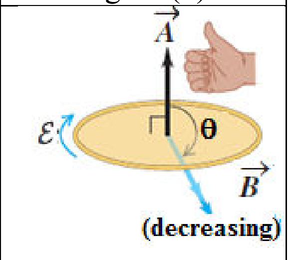
Direction of Induced emf

We can find the direction of an induced emf or current by using Eq. (1) together with some simple sign rules. The procedure is given below:

1. Define a positive direction for the vector area $\vec{A}$
2. From the directions of $\vec{A}$ and the magnetic field $\vec{B}$ determine the sign of the magnetic flux φ_B and its rate of change of the magnetic flux $\dfrac{d\varphi_B}{dt}$
3. Determine the sign of the induced emf or current.
   - If the flux is increasing, (i.e $\dfrac{d\varphi_B}{dt}$ is positive) , then the induced emf or current is negative;

- If the flux is decreasing, (i.e $\frac{d\varphi_B}{dt}$ is negative) then the induced emf or current is positive.

4. Now, determine the direction of the induced emf or current using the right hand.
   - Curl the fingers of the right hand around the $\vec{A}$ vector, with the right thumb in the direction of A.
   - If the induced emf or current in the circuit is positive, it is in the same direction as your curled fingers;
   - if the induced emf or current is negative, it is in the opposite direction.

Figure (a)	Figure (b)	Figure (c)	Figure (d)
			
φ_B is more positive	φ_B is less positive	φ_B is more negative	φ_B is less negative.
=> φ_B is increasing => $\varphi_B > 0$ => $\frac{d\varphi_B}{dt} > 0$	=> φ_B is decreasing => $\varphi_B > 0$ => $\frac{d\varphi_B}{dt} < 0$	=> φ_B is decreasing => $\varphi_B < 0$ => $\frac{d\varphi_B}{dt} < 0$	=> φ_B is increasing => $\varphi_B < 0$ => $\frac{d\varphi_B}{dt} > 0$
=> $\varepsilon < 0$ emf is negative	=> $\varepsilon > 0$ emf is positive	=> $\varepsilon > 0$ emf is positive	=> $\varepsilon < 0$ emf is negative
=> emf is opposite to the direction of the curled fingers of the right hand when the right thumb points along $\vec{A}$.	=> emf is in the same direction of the curled fingers of the right hand when the right thumb points along $\vec{A}$.	=> emf is in the same direction of the curled fingers of the right hand when the right thumb points along $\vec{A}$.	=> emf is opposite to the direction of the curled fingers of the right hand when the right thumb points along $\vec{A}$.

Lenz's Law

The direction of induced emf is such that it opposes the change from which it is produced.

An induced current always opposes any change in magnetic flux through a circuit.

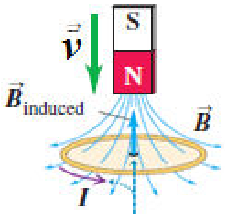
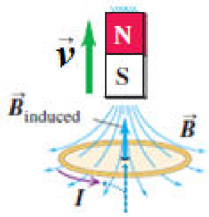
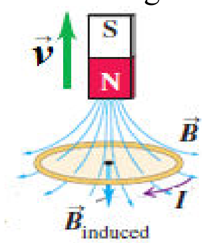
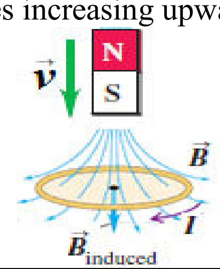
- The Lenz's law gives the direction of an induced current; but the magnitude of the current depends on the resistance of the circuit.
- If the circuit resistance is more, then the induced current will be less that appears to oppose any change in flux and the easier it is for a flux change to take effect.
- If the loop is made out of an insulator, there would be almost no induced current in response to changes in the flux through the loop.

- Conversely, the less the circuit resistance, the greater the induced current and the more difficult it is to change the flux through the circuit.

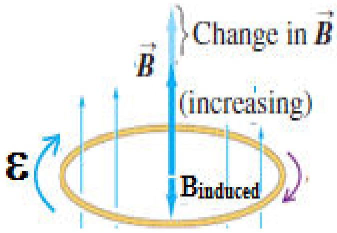
- As long as there is relative motion between magnet and loop, an induced current flows in the loop. Once there is no relative motion, the induced current very quickly decreases to zero because of the nonzero resistance in the loop.

- The extreme case occurs when the resistance of the circuit is zero. Then the induced current in the loop will continue to flow even after the induced emf has disappearedi.e even after the magnet has stopped moving relative to the loop. Exotic materials called superconductors do indeed have zero resistance

Directions of induced currents as a bar magnet moves along the axis of a conducting loop. If the bar magnet is stationary, there is no induced current.

(a) Motion of magnet causes increasing downward flux through loop.  (b) Motion of magnet causes decreasing upward flux through loop. 	The induced magnetic field is upward to oppose the flux change. To produce this induced field, the induced current must be counterclockwise as seen from above the loop.
(c) Motion of magnet causes decreasing downward flux through loop.  d) Motion of magnet causes increasing upward flux through loop. 	The induced magnetic field is downward to oppose the flux change. To produce this induced field, the induced current must be clockwise as seen from above the loop.

Lenz's law and the direction of induced current



The figure in the right shows that the magnetic field through a coil is uniform. The magnitude of the field is increasing, so there is an induced emf.
We can use Lenz's law to determine the direction of the resulting induced current.
The induced current due to the change in is clockwise, as seen from above the loop. The added field causes is downward, opposing the change in the upward field B

Lenz's law is in consequence to conservation of energy

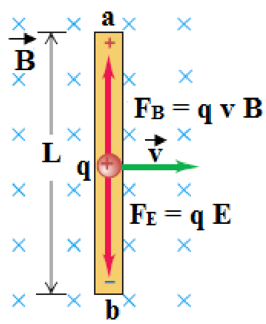
The change in magnetic flux through a stationary circuit may be due to

- a varying magnetic field,
- motion of the conductors that make up the circuit,

Due to the change in flux in the circuit, an induced current flows. The induced current opposes the change in flux through the circuit.

Lenz's law is also directly related to energy conservation. If the induced current in the direction opposite to that given by Lenz's law,
For example, the magnetic force on the rod would accelerate it to ever increasing speed with no external energy source, even though electric energy is being dissipated in the circuit. This would be a clear violation of energy conservation and doesn't happen in nature.

Motional Electromotive Force



Let us consider a conductor, moves in a magnetic field. In this situation an emf is induced. This is due to the fact that the magnetic forces acts on the mobile charges present in the conductor.

Figure shown in the right is a rod moving in a constant velocity $\vec{v}$ inside an uniform magnetic field $\vec{B}$ which is directed into the plane of the paper. A charged particle 'q' in the rod will now experiences a magnetic force and is given by
$\vec{F}_B = q\left(\vec{v} \times \vec{B}\right)$
The magnitude of this magnetic force is
$\mathbf{F_B = q\ v\ B}$
As 'q' is positive, the direction of $\vec{F}_B$ is upward.

Now, $\vec{F}_B$ causes the free charges in the rod to move:

- Creating an excess of positive charge at the upper end a and
- Creating negative charge at the lower end b.
- This creates an electric field $\vec{E}$ within the rod, in the direction from **a** toward **b** (opposite to $\vec{F}_B$). The magnitude of the electric force is: $\mathbf{F_E = q\ E}$
- Charge continues to accumulate at the ends of the rod until becomes large enough for the downward electric force F_E to cancel exactly the upward magnetic force F_B.
- When the charges are in equilibrium then,

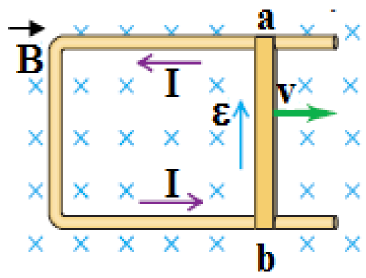
$$q\,E = q\,v\,B$$
$$\Rightarrow \mathbf{E = v\,B}$$

The magnitude of the potential difference is V_{ab} is,

$$V_{ab} = E\,L$$
$$\Rightarrow Vab = v\,B\,L$$

Here, point **a** is at higher potential than point **b**

Now let, the moving rod slides along a stationary U-shaped conductor, forming a complete circuit (shown in the figure).



No magnetic force acts on the charges in the stationary U-shaped conductor, but the charge that was near points **a** and **b** redistributes itself along the stationary conductor, creating an electric field within it. This field establishes a current in the direction shown.

The moving rod has become a source of electromotive force; within it, charge moves from lower to higher potential, and in the remainder of the circuit, charge moves from higher to lower potential.

ε = motional electromotive force

Absolute value of emf induced in bar translating in uniform $\vec{B}$ perpendicular to the plane in which the bar moves is

$$\varepsilon = B\,L\,v$$

Motional emf (ε), length (L) and velocity (v) are perpendicular to uniform magnetic field ($\vec{B}$)

The induced current is given by

$$I = \frac{\varepsilon}{R}$$

Where, R = total circuit resistance of U-shaped conductor and the sliding rod.

Then $\varepsilon = I\,R$

$$\Rightarrow B\,L\,v = I\,R$$
$$\Rightarrow I = \frac{B\,L\,v}{R}$$

Motional emf: General Form

We can generalize the concept of motional emf for a conductor with any shape, moving in any magnetic field, uniform or not.

Let dε = motional emf due to an element $d\vec{\ell}$ of the conductor,

Then, we have $d\varepsilon = \left(\vec{v} \times \vec{B}\right) \cdot d\vec{\ell}$

For any closed conducting loop, the total motional emf is

$$\varepsilon = \oint \left(\vec{v} \times \vec{B}\right) \cdot d\vec{\ell} \qquad \text{---------------(1)}$$

This equation is same as the faraday's law of electromagnetic induction, i.e

$$\varepsilon = -\frac{d\varphi_B}{dt} \qquad \text{---------------- (2)}$$

Eq (1) and (2) are equivalent.

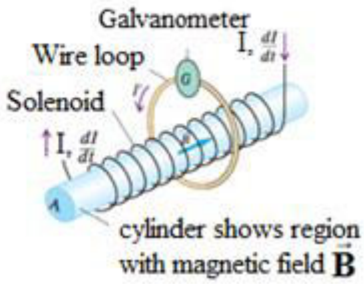
Eq. (1) is an alternative formulation of Faraday's law.

For moving conductors case Eq (1) is convenient.

But when we have stationary conductors in changing magnetic fields, Eq. (1) cannot be used; in this case, is the only correct way to express by Eq (2).

Induced Electric Fields

- When a conductor moves in a magnetic field, an induced emf developed due to the magnetic forces on charges in the conductor.
- But an induced emf also occurs when there is a changing flux through a stationary conductor.



Let's consider a thin solenoid with following specification:
A = cross-sectional area
n = no. of turns per unit length
G = galvanometer (measures the current in the loop).
I = current in the winding of the solenoid

The current I in the solenoid set up a magnetic field along the solenoid axis. The magnitude magnetic field B is given by

The windings of a long solenoid carry a current I that is increasing at a rate $\frac{dI}{dt}$.

The magnetic flux in the solenoid is increasing at a rate $\frac{d\varphi_B}{dt}$

This changing flux passes through a wire loop. So, emf is induced around the loop. It is given by

$$\varepsilon = -\frac{d\varphi_B}{dt} = -\frac{d(BA)}{dt} = -\frac{d(\mu_0 n I A)}{dt} = -\mu_0 n A \frac{dI}{dt} \qquad \text{-------------} (1)$$

If the total resistance of the loop is *R*, the induced current in the loop is given by

$$I = \frac{\varepsilon}{R} \qquad \text{-------------} (2)$$

The changing magnetic flux induces **electric field** in the conductor i.e the changing magnetic field acts as a source of electric field.

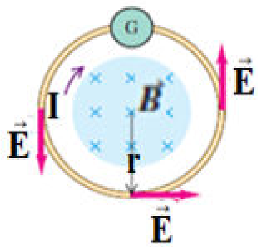
Again, when the charge 'q' goes once around the loop, the total work done on it by the electric field must be equal to qε. i.e the electric field in the loop is not conservative (because the line integral of around a closed path is not zero). It is given by

$$\oint \vec{E} \cdot \vec{dl} = \varepsilon \qquad \text{-------------} (3)$$

From Faraday's law we get,

$$\varepsilon = -\frac{d\varphi_B}{dt}$$

Thus, $\oint \vec{E} \cdot \vec{dl} = -\frac{d\varphi_B}{dt}$ ------------- (4)



This equation is valid only if the path around which we integrate is stationary.

Let us consider a stationary circular loop of radius ***r*** as shown in the figure.

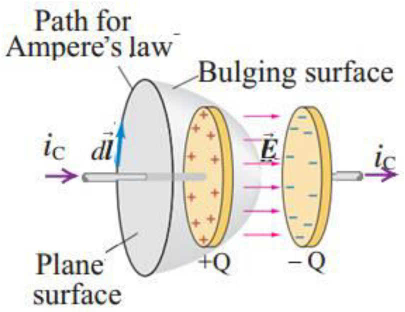
Applying equation (4) to the figure, we get

$$\oint \vec{E}.\vec{dl} = \left|\frac{d\varphi_B}{dt}\right| \quad => \quad 2\pi\, r\, E = \left|\frac{d\varphi_B}{dt}\right|$$

$$=> \quad E = \frac{1}{2\pi\, r}\left|\frac{d\varphi_B}{dt}\right| \qquad \text{------------- (5)}$$

The electric field has the same magnitude at every point on the circle and is tangent to it at each point.

Displacement Current & Maxwell's Equations Magnetic Flux



We know that a varying magnetic field gives rise to an induced electric field.
Again a varying electric field gives rise to a magnetic field. This explains the existence of radio waves, gamma rays, and visible light, as well as all other forms of electromagnetic waves.

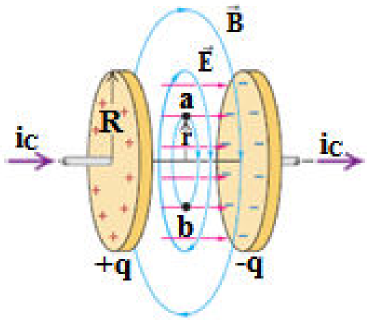
Let us consider a capacitor having capacitance C. The instantaneous charge is

$q = C\,v$ --------------------(1)

where v = instantaneous potential difference,

ε = permittivity of the material inside the capacitor

For a parallel-plate capacitor, the capacitance is given by



$$C = \frac{\varepsilon A}{d} \qquad \text{--------------------(2)}$$

Where, A = plate area and d = spacing between the plates

The potential difference (v) between plates is

$v = E\,d$ --------------------(3)

where, E = electric field magnitude between plates.

Substituting eq. (2) & (3) in (1) we get

$$q = C\,v \quad \Rightarrow \quad q = \left(\frac{\varepsilon A}{d}\right)(Ed) \quad \Rightarrow \quad q = \varepsilon A E \quad \Rightarrow \quad q = \varepsilon\,\varphi_E \qquad \text{-------------(4)}$$

Where, ϕ_E = Electrical flux through the surface = E A

As the capacitor charges, the current is given by

$$i = \frac{dq}{dt} \quad \Rightarrow \quad i = \frac{d(\varepsilon\,\varphi_E)}{dt} \quad \Rightarrow \quad i = \varepsilon\frac{d\varphi_E}{dt} \qquad \text{--------------------(5)}$$

This is a fictitious current in the region between the plates and is called as displacement current. Thus the displacement current is given by

$$i_D = \varepsilon\frac{d\varphi_E}{dt} \qquad \text{--------------------(5)}$$

Thus we can imagine that, due to time rate of change of electric flux gives some current, which we call as displacement current i_D. Therefore the Ampere's law, has to be modified and we have to include this fictitious current, along with the real conduction current in Ampere's law.

Thus, the generalized Ampere's law is given by

$$\oint \vec{B}.\ d\vec{\ell} = \mu_0 \left(i_C + i_D \right) \qquad \text{--------------------(6)}$$

The fictitious current was invented by Maxwell The corresponding displacement current density is given by

$$J_D = \frac{\varepsilon}{A}\frac{d\varphi_E}{dt} \quad \Rightarrow \quad J_D = \frac{\varepsilon}{A}\frac{d(EA)}{dt} \quad \Rightarrow \quad J_D = \varepsilon\frac{dE}{dt}$$

Maxwell's Equations of Electromagnetism

Maxwell's Equations of Electromagnetism is given in the tabular form

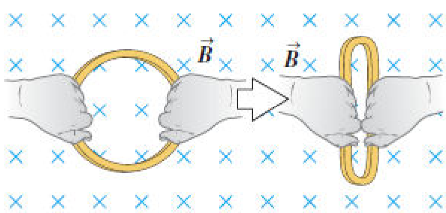
$\oint \vec{E}.\ d\vec{A} = \frac{q_{enclose}}{\varepsilon_0}$	Gauss's law for electric fields
$\oint \vec{B}.\ d\vec{A} = 0$	Gauss's law for electric fields
$\oint \vec{B}.\ d\vec{\ell} = \mu_0 \left(i_C + \varepsilon_0 \frac{d\varphi_E}{dt} \right)$	Modified Ampere's law
$\oint \vec{E}.\ d\vec{\ell} = -\frac{d\varphi_B}{dt}$	Faraday's Laws

Conceptual Problems

Test Your Understanding of Section 29.2

The figure at right shows a wire coil being squeezed in a uniform magnetic field.

a) While the coil is being squeezed, is the induced emf in the coil
   (i) clockwise, (ii) counterclockwise, or (iii) zero?
b) is the induced emf in the coil
   (i) clockwise, (ii) counter-clockwise, or (iii) zero?



Answers: (a) (i), (b) (iii)

(a) Initially there is magnetic flux into the plane of the page, which we call positive.

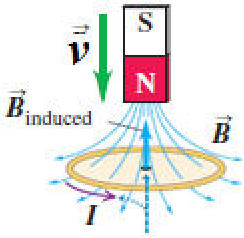
While the loop is being squeezed, the flux is becoming less positive $\frac{d\varphi_B}{dt} < 0$

So the induced emf is positive $\left[\varepsilon = -\frac{d\varphi_B}{dt} > 0 \right]$

If the thumb of the right hand is into the page, then curl fingers is clockwise, so this is the direction of positive induced emf.

(b) Since the coil's shape is no longer changing, the magnetic flux is not changing and there is no induced emf

Test Your Understanding of Section 29.3



a) Suppose the magnet in the figure were stationary and the loop of wire moved upward. Would the induced current in the loop be
   (i) in the same direction as shown in figure,
   (ii) in the direction opposite to that shown in figure, or
   (iii) zero?

b) Suppose the magnet and loop of wire in figure both moved downward at the same velocity. Would the induced current in the loop be
   (i) in the same direction as shown in figure
   (ii) in the direction opposite to that shown in figure or
   (iii) zero?

Answers: (a) (i), (b) (iii)

a) The flux through the loop will change and will increase as loop approaches to the stationary magnet. So the induced emf and induced current direction will be same as original.

b) Since the magnet and loop are moving together, withe same velocity, the relative motion between them is zero. So, the flux through the loop will not change and no emf is induced.

Test Your Understanding of Section 29.4

The earth's magnetic field points toward (magnetic) north. For simplicity, assume that the field has no vertical component (as is the case near the earth's equator).

a) If you hold a metal rod in your hand and walk toward the east, how should you orient the rod to get the maximum motional emf between its ends?
   (i) east-west; (ii) north-south; (iii) up-down;
   (iv) you get the same motional emf with all of these orientations.

b) How should you hold it to get ***zero*** emf as you walk toward the east?
   (i) east-west; (ii) north-south; (iii) up-down; (iv) none of these.

c) In which direction should you travel so that the motional emf across the rod is *zero* no matter how the rod is oriented?
   (i) west; (ii) north; (iii) south; (iv) straight up;
   (v) straight down.

Answers: (a) (iii); (b) (i) or(ii); (c) (ii) or(iii)

a) If the rod should be hold vertically and move towards east, to get the maximum motional emf. In this situation length of the rod is perpendicular to both the magnetic field and the direction of motion. With this orientation, $\vec{L}$ and $\left(\vec{v} \times \vec{B}\right)$ are parallel to each other.

b) If the rod should be hold in horizontal orientation and move towards east, then $\vec{L}$ will be perpendicular to $(\vec{v} \times \vec{B})$ and no emf will be induced.

c) If you walk due north or south, and no emf will be induced for any orientation of the rod.

Test Your Understanding of Section 29.5

If you wiggle a magnet back and forth in your hand, are you generating an electric field? If so, is this electric field conservative?

Answers: Yes electric field is generated. No, electric field is not conservative

The magnetic field at a fixed position changes as you move the magnet. Such induced electric fields are not conservative.

Test Your Understanding of Section 29.7

a) Which of Maxwell's equations explains how a credit card reader works?

b) Which one describes how a wire carrying a steady current generates a magnetic field?

Answers: (a) Faraday's law, (b) Ampere's law

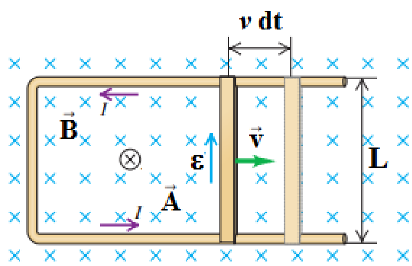
a) A credit card reader works by inducing currents in the reader's coils as the card's magnetized stripe is swiped.

b) Ampere's law describes how currents of all kinds (both conduction currents and displacement currents) give rise to magnetic fields.

In Class Problems

Example 29.5 Generator III: The slidewire generator

Figure shows a U-shaped conductor in a uniform magnetic field $\vec{B}$ perpendicular to the plane of the figure and directed into the page. We lay a metal rod (the "slide wire") with length L across the two arms of the conductor, forming a circuit, and move it to the right with constant velocity $\vec{v}$. This induces an emf and a current, which is why this device is called a slide wire generator. Find the magnitude and direction of the resulting induced emf.



Solution:

As the rod is moving the area of the loop (bounded by the moving rod) is increasing. Therefore, the magnetic flux changes.

So, an emf is induced in this expanding loop.

The magnetic field is uniform over the area of the loop,

So, the flux linked with the loop is $\phi_B = B\,A\cos\theta$.

We choose the area vector $\vec{A}$ into the page, in the same direction of $\vec{B}$.

So, $\theta = 0$

Thus flux $\phi_B = B\,A$

The direction of the induced emf will be in clock-wise around the loop (Using right-hand rule: thumb points into the page and curl fingers gives the direction of the emf).

From Faraday's law, we have,

$$\varepsilon = -\frac{d\varphi_B}{dt} = -\frac{d(B\,A)}{dt} = -B\frac{dA}{dt}$$

But the increase in area dA = (length) x (breadth) = L (v dt)

$$\frac{dA}{dt} = L\,v$$

Putting this value in the above equation we get,

$$\varepsilon = -B\,L\,v$$

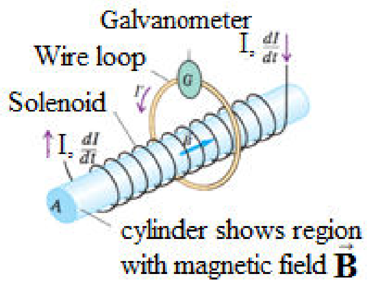
The minus sign tells us that the emf is directed counter-clockwise around the loop. The induced current is also counter-clockwise, as shown in the figure.

Example 29.11 Induced electric fields

Suppose the long solenoid in Fig. 29.17a has 500 turns per meter and cross-sectional area 4.0 cm^2. The current in its windings is increasing at 100 A/s.

a) Find the magnitude of the induced emf in the wire loop outside the solenoid.

b) Find the magnitude of the induced electric field within the loop if its radius is 2.0 cm.



Solution:

The windings of a long solenoid carry a current I that is increasing at a rate $\frac{dI}{dt}$.

The magnetic flux in the solenoid is increasing at a rate $\frac{d\varphi_B}{dt}$

This changing flux passes through a wire loop. So, emf is induced around the loop. It is given by

$$\varepsilon = -\frac{d\varphi_B}{dt} = -\frac{d(BA)}{dt} = -\frac{d(\mu_0\, n\, I\, A)}{dt} = -\mu_0\, n\, A\frac{dI}{dt}$$

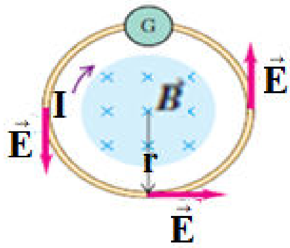
$$\varepsilon = -\left(4\pi\,x\,10^{-7}\ \text{Wb/A.m}\right)(500\ \text{turns})\left(4\,x\,10^{-3}\ m^2\right)(100\ \text{A.s}) = -25\,x\,10^{-6}\ V$$

(b) We know that

$$\varepsilon = \oint \vec{E}\,.\,d\vec{\ell} \quad \Rightarrow \quad \varepsilon = E\oint d\ell \quad \Rightarrow \quad \varepsilon = E\ 2\pi r$$

$$\Rightarrow \quad E = \frac{|\varepsilon|}{2\pi r} = \frac{25\,x\,10^{-6}\ V}{2\pi\left(2\,x\,10^{-2}\ m^2\right)}\varepsilon = -\left(2\,x\,10^{-4}\ V/m\right)$$

In the figure the magnetic flux is into the plane of the paper and is increasing. According to the right-hand rule for induced emf a positive emf would be clockwise around the loop; the negative sign of shows that the emf is in the counter-clockwise direction.



Assignment Problems

29.2: The magnetic flux associated with a metal loop of resistance 4.0 Ω, varies with respect to time as $\varphi = 2t^3 + 5t^2 - 3t + 2$. Find the heat generated in loop between t = 3s and t = 7s.

Ans: Induced emf is given by

$$\varphi = 2t^3 + 5t^2 - 3t + 2$$

$$\varphi_3 = 2(3)^3 + 5(3)^2 - 3(3) + 2 = 92\ \text{Wb}$$

$$\varphi_7 = 2(7)^3 + 5(7)^2 - 3(7) + 2 = 912\ \text{Wb}$$

$$\Delta\varphi = \varphi_7 - \varphi_3 = (912\ \text{Wb}) - (92\ \text{Wb}) = 820\ \text{Wb}$$

$\Delta t = 7 - 3 = 4$ s

$$\text{Induced emf} = \varepsilon = \frac{\Delta\varphi}{\Delta t} = \frac{820\ \text{Wb}}{4\text{s}} = 205\ \text{V}$$

$$\text{Heat generated} = H = \varepsilon^2 Rt = \left(\frac{\varepsilon^2}{R}\right)t = \frac{(205)^2}{4}4 = (205)^2\ \text{J}$$

29.41: The electric flux through a certain area of a dielectric is $(8.76 \times 10^3\ \text{V.m/s}^4)\ t^4$.The displacement current through that area is 12.9 pA at time t = 26.1 ms. Calculate the dielectric constant for the dielectric.

Ans: $\varphi = (8.76 \times 10^3\ \text{V.m/s}^4)\ t^4$

$i_D = 12.9\ \text{pA} = 12.9 \times 10^{-12}\ \text{A}$

$t = 26.1 \times 10^{-3}$ s

We know that

$$I_D = \varepsilon\frac{d\varphi_E}{dt} = \varepsilon\frac{d}{dt}\left(8.76 \times 10^3 \text{V.m/s}^4\right)t^4$$

$$\Rightarrow I_D = K\varepsilon_0 \left(8.76 \times 10^3 \text{V.m/s}^4\right)\left(4t^3\right) \qquad \varepsilon_0 = 8.854\ 10^{-12}\ \text{F/m}$$

$$\Rightarrow I_D = K\left(8.854\ 10^{-12}\ \text{F/m}\right)\left(8.76 \times 10^3 \text{V.m/s}^4\right)\left(4t^3\right)$$

$$\Rightarrow K = \frac{12.9 \times 10^{-12}\ \text{A}}{\left(8.854\ 10^{-12}\ \text{F/m}\right)\left(8.76 \times 10^3 \text{V.m/s}^4\right)4\left(26.1 \times 10^{-3}\text{s}\right)^3} = 2.34$$