Chapter-24: Capacitance and Dielectrics

Topic

1. Capacitors and Capacitance
2. Capacitors in Series and Parallel
3. Energy Storage in Capacitors and Electric-Field Energy
4. Dielectrics
5. Conceptual Problems : TYU-24.1, TYU-24.2, TYU-24.3 & TYU-24.4
6. In class Problems : 24.2, 24.6, 24.10 & 24.11
7. Assignment Problems: 24.18 & 24.35

LEARNING GOALS

- To understand capacitors and calculate capacitance
- To analyze networks of capacitors
- To calculate the energy stored in a capacitor
- To examine dielectrics and how they affect capacitance

Capacitors and capacitance

- A capacitor is a device that stores electric potential energy and electric charge.
- Any two conductors separated by an "insulator" form a capacitor.
- To store energy in a capacitor, charge is transferred from one conductor to the other so that one has a negative charge and the other has an equal amount of positive charge. Work must be done to move the charges through the resulting potential difference between the conductors, and the work done is stored as electric potential energy.
- Capacitors have a large number of practical applications in devices such as electronic flash units for photography, pulsed lasers, air bag sensors for cars, and radio and television receivers.
- The capacitance depends on the sizes and shapes of the conductors and on the insulating material (if any) between them.
- The capacitance increases when an insulating material (a dielectric) is present. This happens because a redistribution of charge, called polarization, takes place within the insulating material. Polarization gives an added insight into the electrical properties of matter.
- Insulator will allow Electric field between the conductors,
- Insulator will not allow charge to flow from one conductor through itself to the other.
- The more charge you can hold, the larger the capacitor and "capacity"
- Capacitor can be charged by pushing the charge into it with a potential voltage "pressure"
- The electric field at any point in the region between the conductors is proportional to the magnitude Q of charge on each conductor.

  $Q \propto V_{ab}$ => $Q = C\,V_{ab}$ Where C is called the capacitance of the capacitor

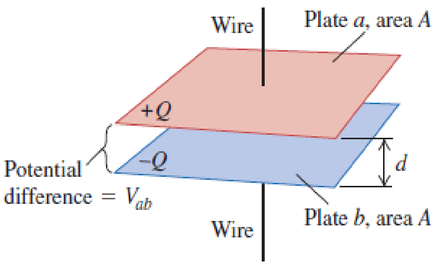
  $$\Rightarrow \quad C = \frac{Q}{V_{ab}}$$

  - Q = "charge stored"
  - V_{ab} = pressure that pushes and keeps charge there
  - C increases as Q increases. This indicates more capacity.
  - C decreases as V_{ab} increases. This means more pressure required to hold the charge there, so less effective in storing it temporarily.

- Units of Capacitance is Farad: Farad = Coulomb/Volt = Coulombs^2/Joule

Parallel-plate capacitor

Let σ = surface charge density of each plate i.e charge per unit area

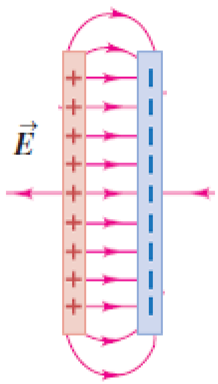


Then $\sigma = \dfrac{Q}{A}$

Where, Q = charge on each plate   A = area of each plate

The electric field magnitude is given by

$$E = \frac{\sigma}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{Q}{\varepsilon_0 A}$$



The electric field is uniform and the distance between the plates is ‘d’, so the potential difference (voltage) between the two plates is:

$$V_{ab} = E\,d \quad \Rightarrow \quad V_{ab} = \left(\frac{Q}{\varepsilon_0 A}\right) d$$

$$\text{So,}\ C = \frac{Q}{V_{ab}} \quad \Rightarrow \quad C = \frac{Q}{\left(\frac{Q}{\varepsilon_0 A}\right) d} \quad \Rightarrow \quad C = \frac{\varepsilon_0 A}{d}$$

This is the capacitance of a parallel-plate capacitor in vacuum.

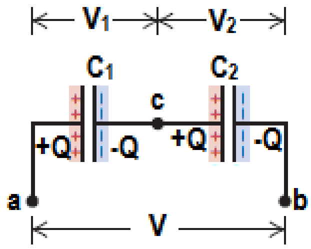
- C is engineered by controlling Area & distance.
- C **increases** with Area
- C **decreases** with separation
- units of ε_0 is $\dfrac{\text{Farads}}{\text{meter}}$ $\left[C = \dfrac{\varepsilon_0 A}{d} \Rightarrow \varepsilon_0 = \dfrac{C\,d}{A}\right]$
- Also units of ε_0 is $\dfrac{\text{Coulomb}^2}{\text{Newton. meter}^2}$ $\left[F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r^2} \Rightarrow \varepsilon_0 = \dfrac{1}{4\pi}\dfrac{q_1 q_2}{F\,r^2}\right]$

$$\frac{\text{Farads}}{\text{meter}} = \frac{\text{Coulomb}^2}{\text{Newton. meter}^2}$$

$$\Rightarrow \text{Farads} = \frac{\text{Coulomb}^2}{\text{Newton. meter}}$$

$$\Rightarrow \text{Farads} = \frac{\text{Coulomb}^2}{\text{Joule}}$$

Capacitors in series

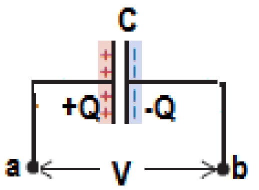


Capacitors are in series

- Capacitors are in **series** if connected one after the other
- **Charges are the same** on all plates in the series (even with different capacitances)
- Potential difference is divided across the branches.
- If V_1 and V_2 are the Potential differences across the branches, then $V = V_1 + V_2$

$$\Rightarrow \quad V = \frac{Q}{C_1} + \frac{Q}{C_2} \qquad ----(1)$$

If, C_s = Equivalent capacitance in a series connection, then



Equivalent Capacitor

$$V = \frac{Q}{C_s} \qquad ----(2)$$

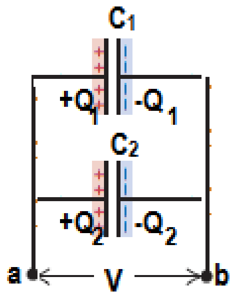
Comparing equation (1) and (2) we get

$$\frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} \quad \Rightarrow \quad \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} \qquad ----(3)$$

In general, $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$

- The reciprocal of the equivalent capacitance of a series combination equals the sum of the reciprocals of the individual capacitances.
- C_s is always LESS than the smallest capacitor in series

Capacitors in parallel



Capacitors are in parallel

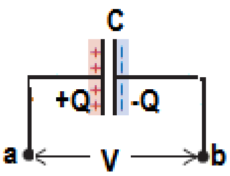
- Capacitors are connected in parallel between a and b. Potential difference **V is the same** for all the capacitors.
- Charges on the capacitors are not same.

$Q = Q_1 + Q_2$

=> $Q = C_1 V + C_2 V$

=> $Q = (C_1 + C_2)V$ ---------(1)

Let C_p is the equivalent capacitance of a parallel combination.



Equivalent Capacitor

Then, $Q = C_p V$ ---------(2)

Comparing the above equations we get

$C_p V = (C_1 + C_2)V \quad => \quad C_p = C_1 + C_2$

In general, $C_p = C_1 + C_2 + C_3 + \ldots\ldots$

- Thus, the equivalent capacitance of a parallel combination is the sum of the individual capacitances.
- C_p is always MORE than largest capacitor in parallel

Energy stored in a capacitor

The electric potential energy stored in a charged capacitor is equal to the amount of work required to charge it. When the capacitor is discharged, this stored energy is recovered as work done by electrical forces.

Let, after the charging of the capacitor is completed

Q = final charge in the capacitor and V = final potential difference

So, $V = \dfrac{Q}{C}$

Let q and v be the charge and potential difference, respectively, at any instant during the charging process;

Then $v = \dfrac{q}{C}$

At this stage the work dw is required to transfer an additional element of charge dq.

So, dw = v dq

$$\Rightarrow \; dw = \frac{q}{C}dq \quad \Rightarrow \; W = \int_0^Q \frac{q}{C}dq \quad \Rightarrow \; W = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$\Rightarrow \; W = \frac{1}{C}\left[\frac{Q^2}{2} - \frac{0^2}{2}\right] \quad \Rightarrow \; W = \frac{Q^2}{2C}$$

This work done is stored as potential energy in the capacitor. So the potential energy stored in a capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C}$$

- U increases with more stored charge;
- U is less for fixed charge on a larger capacitor
- Different forms of U

$$\rightarrow \quad U = \frac{Q^2}{2C} \;\Rightarrow\; U = \frac{Q^2}{2Q/V} \;\Rightarrow\; U = \frac{1}{2}QV$$

$$\rightarrow \quad U = \frac{(CV)^2}{2C} \;\Rightarrow\; U = \frac{1}{2}CV^2$$

- So the potential energy stored in a capacitor is

$$U = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

A capacitor can be charged by moving electrons directly from one plate to another. This requires work done against the electric field between the plates. Thus the energy is stored in the field in the region between the plates.

Energy density (u):

The energy per unit volume in the space between the plates of a parallel-plate capacitor with plate area 'A' and separation 'd' is called as **energy density,** denoted by 'u'
So,

$$u = \frac{\frac{1}{2}CV^2}{\text{volume}} \Rightarrow u = \frac{\frac{1}{2}CV^2}{Ad} \Rightarrow u = \frac{\frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2}{Ad} \Rightarrow u = \frac{\varepsilon_0 V^2}{2d^2}$$

$$\Rightarrow u = \frac{\varepsilon_0 E^2 d^2}{2d^2} \Rightarrow u = \frac{1}{2}\varepsilon_0 E^2$$

The energy density is $u = \frac{1}{2}\varepsilon_0 E^2$
This relationship of energy density is not only valid for a parallel-plate capacitor, but also valid for any capacitor in vacuum and indeed for any electric field configuration in vacuum.

This result has an interesting implication. We think of vacuum as space with no matter in it, but vacuum can nevertheless have electric fields and therefore energy. Thus "empty" space need not be truly empty after all.

Dielectrics

- Dielectric is non-conducting material. Most capacitors have dielectric between plates.
- Dielectric increases the capacitance and the energy density by a factor K.

  $C = K\, C_0$

  Where K = Dielectric constant of material

  C_0 = Capacitance of a capacitor when air/vacuum is between the plates.

  C = Capacitance of a capacitor when dielectric is between the plates.

- Dielectric reduces E field between the plates.
- Dielectric reduces VOLTAGE between plates by keeping Q constant.
- A dielectric increases the maximum possible potential difference between the plates of a capacitor.

Dielectric breakdown

- If the electric field is strong enough, partial ionization of dielectric material takes place, and the dielectric becomes a conductor. This is called dielectric breakdown.
- The dielectric strength is the maximum electric field the material can withstand before breakdown occurs.
- Many dielectric materials can tolerate stronger electric fields without breakdown compared to air. Thus using a dielectric allows a capacitor to sustain a higher potential difference and so store greater amounts of charge and energy.

Conceptual Problems:

Test Your Understanding - 24.1:

A capacitor has vacuum in the space between the conductors. If you double the amount of charge on each conductor, what happens to the capacitance?

(i) It increases; (ii) it decreases; (iii) it remains the same;
(iv) the answer depends on the size or shape of the conductors.

Answer: (iii)
The capacitance does not depend on the value of the charge *Q*. Doubling the value of Q causes the potential difference V_{ab} to double, so the capacitance $C = \frac{Q}{V_{ab}}$ remains the same. These statements are true no matter what the geometry of the capacitor.

Test Your Understanding - 24.2:

You want to connect a 4μF capacitor and an 8μF capacitor.

(a) With which type of connection will the 4μF capacitor have a greater potential difference across it than the 8μF capacitor?
(i) series; (ii) parallel; (iii) either series or parallel; (iv) neither series nor parallel.

(b) With which type of connection will the 4μF capacitor have a greater charge than the 8μF capacitor?
(i) series; (ii) parallel; (iii) either series or parallel; (iv) neither series nor parallel.

Answers: (a) (i), (b) (iv)

(a) In a series connection the two capacitors carry the same charge Q but have different potential differences $V_{ab} = \frac{Q}{C}$
Thus, the capacitor with the smaller capacitance C has the greater potential difference. Hence a 4μF capacitor will have a greater potential difference than an 8μF capacitor if the two are connected in series.

(b) In a parallel connection the two capacitors have the same potential difference V_{ab} but carry different charges $Q = C\,V_{ab}$

Thus the capacitor with the larger capacitance C has the greater charge. Hence the 8μF capacitor carries more charge than the 4μF capacitor no matter how they are connected. In a series connection they will carry the same charge, and in a parallel connection the 8μF capacitor will carry more charge.

Test Your Understanding - 24.3:

You want to connect a 4μF capacitor and an 8μF capacitor. With which type of connection will the 4μF capacitor have a greater amount of stored energy than the 8μF capacitor?

(i) series; (ii) parallel; (iii) either series or parallel; (iv) neither series nor parallel.

Answer: (i)

Capacitors connected in series carry the same charge Q.

We know that the amount of energy stored in a capacitor is $U = \frac{1}{2}\frac{Q^2}{C}$.

It shows that the capacitor with the smaller capacitance (C = 4 μF) has more stored energy in a series combination.

When capacitors are connected in parallel, they have the same potential difference V;

From the expression $U = \frac{1}{2}CV^2$ it is clear that: the capacitor with the larger capacitance (C = 8 μF) has more stored energy.

(If we had instead used $U = \frac{1}{2}CV^2$ to analyze the series combination, we would have to account for the different potential differences across the two capacitors.

Likewise, using $U = \frac{1}{2}\frac{Q^2}{C}$ to study the parallel combination would require us to account for the different charges on the capacitors.)

Test Your Understanding - 24.4:

The space between the plates of an isolated parallel-plate capacitor is filled by a slab of dielectric with dielectric constant K. The two plates of the capacitor have charges Q and – Q. You pull out the dielectric slab. If the charges do not change, how does the energy in the capacitor change when you remove the slab?

(i) It increases; (ii) it decreases; (iii) it remains the same.

Answer: (i)

Here Q remains the same, so we use $U = \frac{1}{2}\frac{Q^2}{C}$ for the stored energy.

We know that the dielectric lowers the capacitance by a factor of K.

Since U is inversely proportional to C, the stored energy increases by a factor of K. It takes work to pull the dielectric slab out of the capacitor because the fringing field tries to pull the slab back in (Figure). The work that you do goes into the energy stored in the capacitor.

In-class Problems:

Example 24.2: Properties of a parallel-plate capacitor

The plates of a parallel-plate capacitor in vacuum are 5.0mm. apart and 2.0m^2 in area. A10 kV potential difference is applied across the capacitor. Compute

a) the capacitance;

b) the charge on each plate; and

c) the magnitude of the electric field between the plates.

Solution: $A = 2.00\ \mathrm{m}^2$, $d = 5.0$ mm, $V = 10$ kV

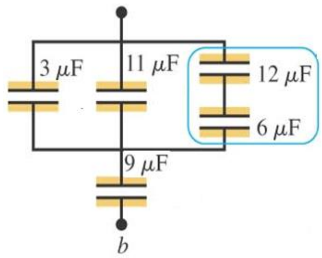
a) $$C = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85 \times 10^{-12}\,\mathrm{F/m}\right)\left(2.00\ \mathrm{m}^2\right)}{5 \times 10^{-3}\ \mathrm{m}} = 3.54 \times 10^{-9}\,\mathrm{F}$$

b) $$Q = CV = (3.54 \times 10^{-9}\ \mathrm{F})(10 \times 10^{3}\mathrm{V}) = 3.54 \times 10^{-5}\mathrm{C}$$

c) $$V = E\,d \;\Rightarrow\; E = \frac{V}{d} = \frac{10 \times 10^{3}\,\mathrm{V}}{5.0 \times 10^{-3}\mathrm{m}} = 2.0 \times 10^{-6}\,\mathrm{V/m}$$

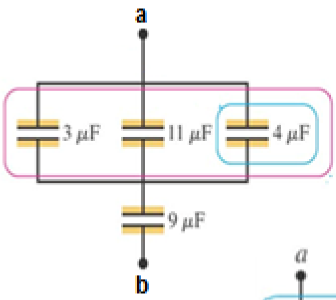
Example 24.6: A capacitor network

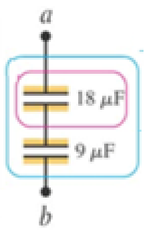
Find the equivalent capacitance of the five-capacitor network shown in figure.



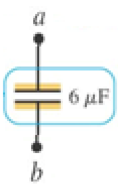
Solution:

Step-I $$C_s = \frac{C_1\,C_2}{C_1 + C_2} = \frac{12 \times 6}{12 + 6} = 4\,\mu\mathrm{F}$$



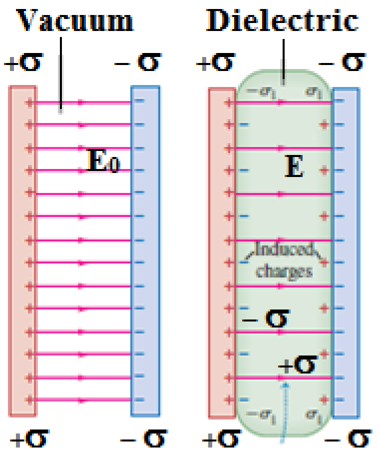


Step-II $$C_p = C_1 + C_2 + C_3 = (3 + 11 + 4)\mu\mathrm{F} = 18\ \mu\mathrm{F}$$



Step-III $$C_s = \frac{C_1\,C_2}{C_1 + C_2} = \frac{18 \times 9}{18 + 9} = 6\,\mu\mathrm{F}$$

Example 24.10: A capacitor with and without a dielectric



Suppose the parallel plates in figure, each have an area of 0.2 m^2 and are 0.01m apart. We connect the capacitor to a power supply, charge it to a potential difference V_0 = 3 kV and disconnect the power supply. We then insert a sheet of insulating plastic material between the plates, completely filling the space between them. We find that the potential difference decreases to 1.00 kV while the charge on each capacitor plate remains constant.

Find a) the original capacitance C_0

b) the magnitude of charge Q on each plate;

c) the capacitance C after the dielectric is inserted;

d) the dielectric constant K of the dielectric;

e) the permittivity of the dielectric;

f) the magnitude of the induced charge Q_i on each face of the dielectric;

g) the original electric field E_0 between the plates; and

h) the electric field E after the dielectric is inserted.

Solution: Given, $A = 0.2\ m^2$; $d = 0.01m$; $V_0 = 3\ kV$;

a) $$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ F/m)(0.2\ m^2)}{(0.1\ m)} = 1.77 \times 10^{-10} F$$

b) $$Q = C_0 V_0 = (1.77\ F)(3.0 \times 10^{12}\ V) = 0.531 \times 10^{-7}\ C$$

c) When the dielectric is inserted, Q is unchanged but the potential difference decreases to V = 1kV

Hence, the new capacitance is

$$C = \frac{Q}{V} = \frac{5.31 \times 10^{-7}\ C}{1.0 \times 10^{3}\ V} = 5.31 \times 10^{-10}\ F$$

d) The dielectric constant is

$$K = \frac{C}{C_0} = \frac{5.31 \times 10^{-10} F}{1.77 \times 10^{-10} F} = 3\ C$$

e) The permittivity is $\varepsilon = K\,\varepsilon_0 = 3 \times (8.85 \times 10^{-12}\ F/m) = 2.66 \times 10^{-11}\ F/m$

f) The induced charge is given

$$Q_i = Q\left(1 - \frac{1}{K}\right) = (5.31 \times 10^{-7}\ C)\left(1 - \frac{1}{3.0}\right) = 3.54 \times 10^{-7}\ C$$

(g) The magnitude of the uniform electric field between the plates is given by

$$E_0 = \frac{V_0}{d} = \frac{3000\ \text{V}}{1.0 \times 10^{-2}\text{m}} = 3.0 \times 10^5 \text{V/m}$$

(h) The magnitude of the electric field after the dielectric is inserted is

$$E = \frac{V}{d} = \frac{1000\ \text{V}}{1.0 \times 10^{-2}\text{m}} = 1.0 \times 10^5 \text{V/m}$$

Alternatively

$$E = \frac{E_0}{K} = \frac{3.0 \times 10^5 \text{V/m}}{3.0} = 1.0 \times 10^5 \text{V/m}$$

Inserting the dielectric

→ the capacitance increased by a factor of $K = 3$

→ electric field between the plates is reduced the by a factor of $\frac{1}{K} = \frac{1}{3}$

Above things happens by developing induced charges on the faces of the dielectric of magnitude $Q_i = Q\left(1 - \frac{1}{3}\right) = 0.667Q$

Example 24.11 Energy storage with and without a dielectric

Find the energy stored in the electric field of the capacitor in Example 24.10 and the energy density, both before and after the dielectric sheet is inserted.

Solution: Given, $A = 0.2\ \text{m}^2$; $d = 0.01\text{m}$; $V_0 = 3\ \text{kV}$;

U_0 = energies stored in the capacitor without dielectric

U = energies stored in the capacitor with dielectric in place

We know that

$$U_0 = \frac{1}{2}C_0V_0^2 = \frac{1}{2}\left(1.77 \times 10^{-10}\text{F}\right)\left(3000\text{V}\right)^2 = 7.97 \times 10^{-4}\ \text{J}$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(5.31 \times 10^{-10}\text{F}\right)\left(1000\text{V}\right)^2 = 2.66 \times 10^{-4}\ \text{J}$$

The energy density without dielectric is

$$u_0 = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\left(8.85 \times 10^{-12}\ \text{F/m}\right)\left(3.0 \times 10^5\ \text{N/C}\right)^2 = 0.398\ \text{J/m}^3$$

The energy density with dielectric is

$$u = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}\left(2.66 \times 10^{-11}\ \text{F/m}\right)\left(1.0 \times 10^5\ \text{N/C}\right)^2 = 0.133\ \text{J/m}^3$$

Comparing the energy densities we get

$$\frac{u_0}{u} = \frac{0.398\ \text{J/m}^3}{0.133\ \text{J/m}^3} = 3$$

Thus the energy density with the dielectric is one-third of the original energy density.

Alternative method:
Volume between the plates =V = $(0.2\ \text{m}^2)(0.01\ \text{m}) = 0.002\ \text{m}^3$

$$u_0 = \frac{U_0}{\text{volume}} = \frac{7.97 \times 10^{-4}\ \text{J}}{0.002\ \text{m}^3} = 0.398\ \text{J/m}^3$$

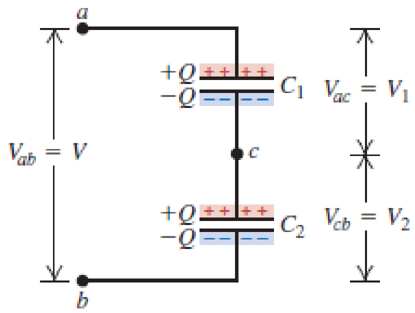
$$u = \frac{U}{\text{volume}} = \frac{2.66 \times 10^{-4}\ \text{J}}{0.002\ \text{m}^3} = 0.133\ \text{J/m}^3$$

Assignment Problem

24.18: In the figure, let C_1 = 3 μF, C_2 = 5 μF, and V_{ab} = +52 V. Calculate

a) the charge on each capacitor and

b) the potential difference across each capacitor.



Ans: C_1 = 3 μF, C_2 = 5 μF, and V_{ab} = +52 V.

Capacitors in series:

- The capacitors have the same charge Q.
- Their potential differences add:

a) $V_{ab} = V_{ac} + V_{cb}$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3\mu F} + \frac{1}{5\mu F} = \left(\frac{8}{15}\right)\frac{1}{\mu F} \Rightarrow C_{eq} = \left(\frac{15}{8}\right)\mu F = 1.876\ \mu F = 1.876 \times 10^{-3} \mu F$$

$Q = V_{ab} C_{eq} = (52\ \text{V})(1.876 \times 10^{-3}\ \text{F}) = 9.75 \times 10^{-5}$ C.

Thus, each capacitor has charge 9.75×10^{-5} C.

b) $$V_1 = \frac{Q}{C_1} = \frac{9.75 \times 10^{-5}\ \text{C}}{3 \times 10^{-6}\ \text{F}} = 32.5\ \text{V}, \qquad V_2 = \frac{Q}{C_2} = \frac{9.75 \times 10^{-5}\ \text{C}}{5 \times 10^{-6}\ \text{F}} = 19.5\ \text{V}$$

The capacitor with the smaller C has the larger V.

24.35: A 12.5 μF capacitor is connected to a power supply that keeps a constant potential difference of 24 V across the plates. A piece of material having a dielectric constant of 3.75 is placed between the plates, completely filling the space between them.

a) How much energy is stored in the capacitor before and after the dielectric is inserted?

b) By how much did the energy change during the insertion? Did it increase or decrease?

Ans.: $C_0 = 12.5\ \mu F$, $V = 24\ V$, $K = 3.75$

a) $C = K C_0 = 3.75\ (12.5\ \mu F) = 46.9\ \mu F$

$$U_{before} = \frac{1}{2} C_0 V^2 = \frac{1}{2}\left(12.5 \text{ x } 10^{-6} C\right)\left(24\ V\right)^2 = 3.6 \text{ x } 10^{-3}\ J$$

$$U_{after} = \frac{1}{2} C\, V^2 = \frac{1}{2} KC_0\ V^2 = K\left(\frac{1}{2} C_0\ V^2\right) = K\ U_{before} = 3.75 \text{ x } 3.6 \text{ x } 10^{-3}\ J = 13.5 \text{ x } 10^{-3}\ J$$

b) $\Delta U = 13.5 \text{ x } 10^{-3}\ J - 3.6 \text{ x } 10^{-3}\ J = 9.9 \text{ x } 10^{-3}\ J.$
Here the energy is increased.