# Study Material

## (Electromagnetism)

## Chapter-28

# Sources of Magnetic Field

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|---|---|---|
| **Topics covered** | : | • Magnetic Field of a Current Element<br>• Magnetic Field of a Straight Current-Carrying Conductor<br>• Magnetic Field of a Circular Current Loop<br>• Ampere's Law<br>• Applications of Ampere's Law |
| **Conceptual Problems** | : | TYU 28.2, 28.5, 28.6 |
| **In Class Problems** | : | 28.3, 28.8 |
| **Assignment Problems** | : | 28.33, 28.44 |

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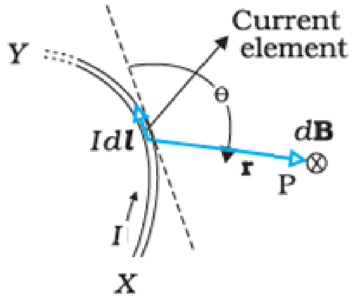
## Magnetic Field of a Current Element

The relation between current and the magnetic field it is given by the Biot-Savart's law. Figure in the right shows a finite conductor XY carrying current I.




Consider an infinitesimal element dl of the conductor. The magnetic field dB due to this element at a point P which is at a distance r from it is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,d\ell \sin\theta}{r^2}$$

Where, θ = angle between **dl** and the displacement vector **r**

$\mu_0$ the is known as the permeability of free space (or vacuum).

$$\mu_0 = 4\pi \text{ x } 10^{-7} \text{ T.m/A}$$

This is called Biot-Savart's law.

In vector form, Biot-Savart's law can be witten as

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I\,\overrightarrow{d\ell} \text{ x } \hat{r}}{r^2}$$

Direction of is along the direction of . We can use the right hand rule for this. In the above figure the direction of is into the plane of the paper.

We can use Biot-Savart's law to find the total magnetic field B at any. Thus

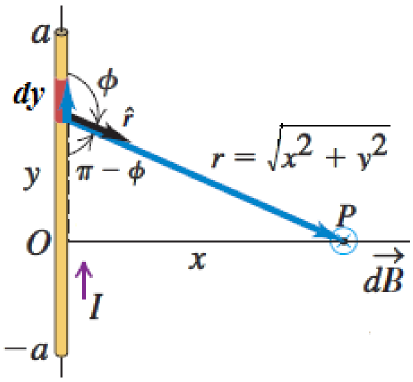
$$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{I\,\overrightarrow{d\ell} \text{ x } \hat{r}}{r^{\cdot}}$$

## Magnetic Field of a Straight Current-Carrying Conductor

Fro Biot-Savart's law, the magnetic field due to current edlement dy is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dy \sin\varphi}{r^2} \quad \text{-------------(1)}$$

From the figure we have,




- $r = \sqrt{x^2 + y^2}$
- $\sin\varphi = \sin(\pi - \varphi) = \frac{x}{r} \quad \Rightarrow \quad x = r\sin\varphi$
- $\tan(\pi - \varphi) = \frac{x}{y} \quad \Rightarrow \quad -\tan\varphi = \frac{x}{y}$

$$\Rightarrow \quad y = -x\cot\varphi \quad \Rightarrow \quad dy = x\,\text{cosec}^2\varphi\, d\varphi$$

Putting these values in eqn (1) we get,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dy \sin\varphi}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dy\sin\varphi}{r^2} = \frac{\mu_0}{4\pi}\frac{I\left(x\,\mathrm{cosec}^2\varphi\,d\varphi\right)\sin\varphi}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,x\sin\varphi\,d\varphi}{r^2\sin^2\varphi} = \frac{\mu_0}{4\pi}\frac{I\,x\sin\varphi\,d\varphi}{x^2}$$

$$dB = \frac{\mu_0 I}{4\pi x}\sin\varphi\,d\varphi$$

$$B = \int_{-a}^{a}\frac{\mu_0\,I}{4\,\pi\,x}\sin\varphi\,d\varphi = \frac{\mu_0\,I}{4\,\pi\,x}\int_{a}^{-a}\sin\varphi\,d\varphi$$

$$B = \frac{\mu_0\,I}{4\,\pi\,x}\left[-\cos\varphi\right]_{-a}^{a} = \frac{\mu_0\,I}{4\,\pi\,x}\left[\frac{y}{\sqrt{x^2+y^2}}\right]_{-a}^{a}$$

$$B = \frac{\mu_0\,I}{4\,\pi\,x}\left[\frac{a}{\sqrt{x^2+a^2}} - \frac{-a}{\sqrt{x^2+a^2}}\right]$$

$$B = \frac{\mu_0\,I}{4\,\pi}\frac{2a}{x\sqrt{x^2+a^2}}$$

When the length of the conductor is very great in comparison to its distance x from point P, we can consider it to be infinitely long.

Then, $\sqrt{x^2+a^2} \approx a$

So, $B = \dfrac{\mu_0 I}{2\,\pi\,x}$

## Magnetic Field of a Circular Current Loop

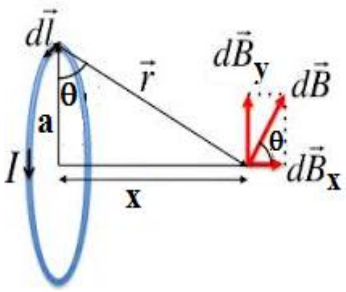



Figure in the right shows a circular conductor with radius **a**. A current **I** is flowing through the coil.

Lret us take a point P on the axis of the loop, at a distance x from the center.

From Biot and Savart's law, the magnetic field at P due to the current element dl is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,d\ell}{r^2} \qquad ------(1)$$

The components of the vector dB are

$$dB_x = dB\cos\theta = \frac{\mu_0}{4\pi}\frac{I\,d\ell\cos\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,d\ell}{r^2}\frac{a}{r} = \frac{\mu_0}{4\pi}\frac{I\,a\,d\ell}{r^3} \qquad ------(2)$$

$$dB_y = dB\sin\theta = \frac{\mu_0}{4\pi}\frac{I\,d\ell}{r^2}\sin\theta = \frac{\mu_0}{4\pi}\frac{I\,d\ell}{r^2}\frac{x}{r} = \frac{\mu_0}{4\pi}\frac{I\,x\,d\ell}{r^3} \qquad ------(3)$$

The total magnetic field at P has only an x-component (it is perpendicular to the plane of the loop).

For every element dl there is a corresponding element on the opposite side of the loop, with opposite direction. All the perpendicular components cancel and only the x-components survive.

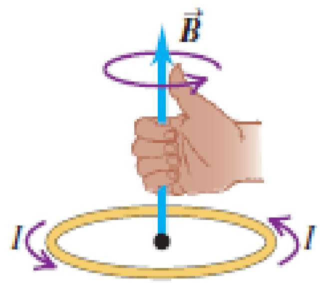
The x-component of the total field B, is obtained by integrating Eq. (1), including all the dl ’s around the loop.

$$B_x = \int dB_x \quad \Rightarrow \quad B_x = \int \frac{\mu_0}{4\pi}\frac{I\,a\,d\ell}{r^3}$$

$$\Rightarrow \quad B_x = \frac{\mu_0}{4\pi}\frac{I\,a}{r^3}\int d\ell \quad \Rightarrow \quad B_x = \frac{\mu_0}{4\pi}\frac{I\,a}{r^3}\,2\pi a$$

$$\Rightarrow \quad B_x = \frac{\mu_0}{2}\frac{I\,a^2}{\left(\sqrt{x^2+y^2}\right)^3}$$

$$\Rightarrow \quad B_x = \frac{\mu_0}{2}\frac{I\,a^2}{\left(x^2+y^2\right)^{3/2}} \text{ ------ (4)}$$




Eqn (4) represents the total field B on the axis of a circular loop.

If a coil consists of N loops, then the total field is

$$\Rightarrow \quad B_x = \frac{\mu_0}{2}\frac{N\,I\,a^2}{\left(x^2+a^2\right)^{3/2}} \text{ ------ (5)}$$

The direction of the magnetic field on the axis of a current-carrying loop is given by a right-hand rule. If the fingers of the right is curl in the direction of the current then right thumb points in the direction of the field.

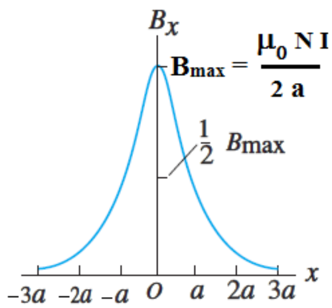



Figure in the right shows a graph of $B_x$ as a function of x. The maximum value of the field is at x = 0, (the center of the loop or coil)

At the center of the coil x = 0

So, from eqn (5) we have

$$B_x = \frac{\mu_0 N I}{2a} \text{ ------ (6)}$$

As we go out along the axis, the field decreases in magnitude.

If μ = magnetic moment of a current-carrying loop

Then, μ = N I A

⇨ μ = N I A

⇨ $\mu = N\,I\,\pi\,a^2$

Where A = cross-sectional area of the loop.

Now equation (5) become

$$\Rightarrow \quad B_x = \frac{\mu_0\,\mu}{2\pi\left(x^2+a^2\right)^{3/2}} \qquad ------(6)$$

Eq. (6) describes the magnetic field produced by a magnetic dipole for points along the dipole axis.

This field is directly proportional to the magnetic dipole moment $\mu$.

Here that the field along the x-axis is in the same direction as the vector magnetic moment $\vec{\mu}$.

**Ampere's Law:**

The line integral of $\vec{B}$ qual to $\mu_0$ times the current passing through the area bounded by the surface.

$$\oint \vec{B}.\vec{d\ell} = \mu_0\, I_{encl}$$

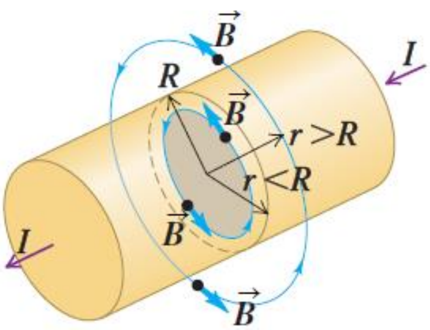
The line integral doesn't depend on the shape of the path or on the position of the wire inside it.

- If the current in a wire is opposite, the integral has the opposite sign.
- If the path doesn't enclose the wire, then the net change in $\theta$ during the trip around the integration path is zero;
- If $\oint \vec{B}\,.\,\vec{d\ell} = 0$ then, it doesn't mean that $\vec{B} = 0$ everywhere along the path, only that the total current through an area bounded by the path is zero.

## Applications of Ampere's Law

### Field of a long cylindrical conductor

A cylindrical conductor with radius R carries a current I (shown in the figure). The current is uniformly distributed over the cross-sectional area of the conductor. Find the magnetic field as a function of the distance **r** from the conductor axis for points both inside (r < R) and outside (r > R) the conductor.




The field $\vec{B}$ has the same magnitude at every point on the circular integration path and is tangent to the path.

<u>Magnetic field at radius r < R (inside the conductor)</u>

Let us apply Ampere's law to the enclosing area of circle of radius r.

Let $I'$ = current

As the current is uniformly distributed over the cross-sectional area of the conductor, we have,

Current density = constant

$\Rightarrow \quad J = J'$

$$\Rightarrow \quad \frac{I}{A} = \frac{I'}{A'}$$

$$\Rightarrow \quad \frac{I}{\pi R^2} = \frac{I'}{\pi r^2}$$

$$\Rightarrow \quad I' = \left(\frac{r^2}{R^2}\right) I$$

Hence Amper Law gives, $\oint B\,dA = \mu_0 I' \quad \Rightarrow \quad B(2\pi r) = \mu_0 \left(\frac{r^2}{R^2}\right) I \quad \Rightarrow \quad B = \frac{\mu_0 I r}{2\pi R^2}$

Magnetic field at radius r < R (outside the conductor)

Let us apply Ampere's law to the enclosing area of circle of radius r.

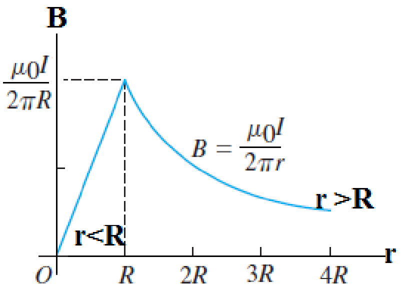
Here, I = current

Hence Amper Law gives, $\oint B\,dA = \mu_0 I \quad \Rightarrow \quad B(2\pi r) = \mu_0 I \quad \Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$

Outside the conductor, the magnetic field is the same as that of a long, straight conductor carrying current I. It is independent of the radius R over which the current is distributed.

Indeed, the magnetic field outside any cylindrically symmetric current distribution is the same as if the entire current were concentrated along the axis of the distribution.

Magnitude of the magnetic field inside and outside a long, straight cylindrical conductor with radius R carrying a current I is plotted in the figure.



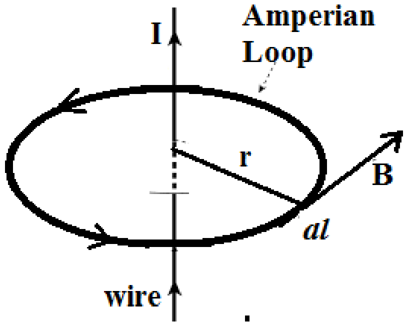

## Field of a long, straight, current-carrying conductor

Using Ampere's law we get

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl}$$

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$




Where, r = radius of the Amerian circle.

## Some other result with the application of Amper's Law

- Magnetic field inside an ideal solenoid: $B = \mu_0 n I$
- Magnetic field of a toroid: $B = \frac{\mu_0 N I}{2\pi r}$

## Conceptual Problems: TYU 28.2, 28.5, 28.6

### Test Your Understanding of Section 28.2

An infinitesimal current element located at the origin ($x = y = z = 0$) carries current I in the positive $y$-direction. Rank the following locations in order of the strength of the magnetic field that the current element produces at that location, from largest to smallest value.

i) $x = L, y = 0, z = 0$; ii) $x = 0, y = L, z = 0$;

iii) $x = 0, y = 0, z = L$; iv) $x = L/\sqrt{2}, y = L/\sqrt{2}, z = 0$.

**Answer:** (i) and (iii) (tie), (iv), (ii)

We know that the mag-nitude of the field dB due to a current element of length dl carrying current I is

$$\mathbf{dB} = \frac{\boldsymbol{\mu}}{\mathbf{4\pi}}\,\frac{\mathbf{I\,dl\,\sin\theta\, i}}{\mathbf{r^2}}$$

Here, r = distance from the element to the field point, and

θ = angle between the direction of the current and a vector from the current element to the field point.

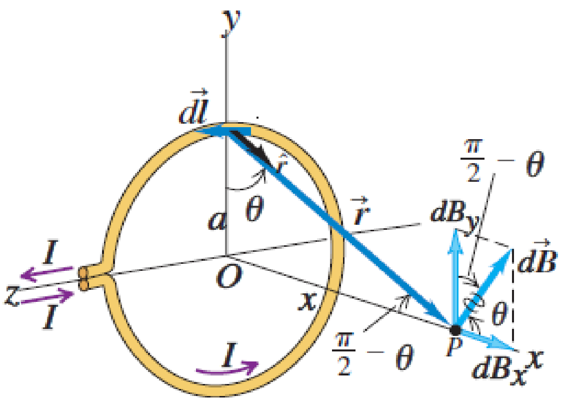
All four points are the same distance r = L, from the current element,

So the value of dB is proportional to the value of sin θ.

(i) $\theta = 90°$, $\sin\theta = 1$ (ii) $\theta = 0°$, $\sin\theta = 0$

(ii) $\theta = 90^0$, $\sin\theta = 1$ (iv) $\theta = 45^0$, $\sin\theta = \frac{1}{\sqrt{2}}$

### Test Your Understanding of Section 28.5




The figure at right shows the magnetic field dB produced at point P by a segment d l that lies on the positive y-axis (at the top of the loop). This field has components $dB_x > 0$, $dB_y > 0$, $dB_z = 0$

a) What are the signs of the components of the field dB produced at P by a segment dl on the negative y-axis (at the bottom of the loop)?

i) $dB_x > 0, dB_y > 0, dB_z = 0$ ii) $dB_x > 0, dB_y < 0, dB_z = 0$

iii) $dB_x < 0, dB_y > 0, dB_z = 0$ iv) $dB_x < 0, dB_y < 0, dB_z = 0$

(v) none of these.

b) What are the signs of the components of the field dB produced at P by a segment dl on the negative z-axis (at the right hand side of the loop)?

i) $dB_x > 0, dB_y > 0, dB_z = 0$ ii) $dB_x > 0, dB_y < 0, dB_z = 0$

iii) $dB_x < 0, dB_y > 0, dB_z = 0$ iv) $dB_x < 0, dB_y < 0, dB_z = 0$

(v) none of these.

**Answers:** (a) (ii), (b) (v)

The vector dB is in the direction of . $d\vec{\ell} \times \vec{r}$

For a segment on the negative y-axis, $d\vec{\ell} = -\hat{k}\,.\,d\ell$ points in the negative z-direction and

$\vec{r} = x\,\hat{i} + y\,\hat{j}$

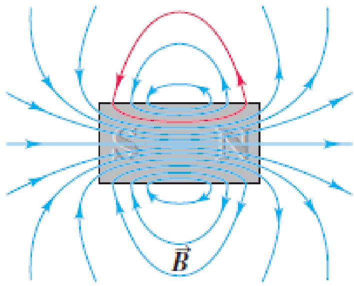
So, $d\vec{\ell} \times \vec{r} = (a\,d\ell)\,\hat{i} - (a\,d\ell)\,\hat{j}$

which has a positive x-component, a negative y-component , and zero z-component. For a segment on the negative z-axis, $d\vec{\ell} = \hat{j}\,.\,d\ell$ points in the positive y-direction and

$\vec{r} = x\,\hat{i} + a\,\hat{j}$

Hence, $d\vec{\ell} \times \vec{r} = 1(a\,d\ell)\,\hat{i} - 1(x\,d\ell)\,\hat{k}$ which has a positive x-component, zero y-component, and a negative z - component

## Test Your Understanding of Section 28.6




The figure at right shows magnetic field lines through the center of a permanent magnet. The magnet is not connected to a source of emf. One of the field lines is colored red. What can you conclude about the currents inside the permanent magnet within the region enclosed by this field line?

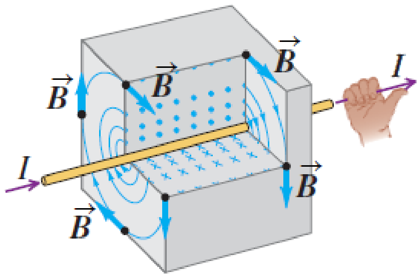
(i) There are no currents inside the magnet;
(ii) there are currents directed out of the plane of the page;
(iii) there are currents directed into the plane of the page;
(iv) not enough informa-tion is given to decide.

**Answer: (ii)**

Imagine carrying out the integral $\oint \vec{B}\,.\,\vec{dl}$ along an integration path that goes counterclockwise around the red magnetic field line. At each point along the path the magnetic field $\vec{B}$ and the infinitesimal segment $\vec{dl}$ are both tangent to the path, so $\vec{B}\,.\,\vec{dl}$ is positive at each point and the integral $\oint \vec{B}\,.\,\vec{dl}$ is likewise positive. It follows from Ampere's law $\oint \vec{B}\,.\,\vec{dl} = \mu_0\,I$ and the right-hand rule that the integration path encloses a current directed out of the plane of the page. There are no currents in the empty space outside the magnet, so there must be currents inside the magnet.

# In Class Problems:

## Example 28.3 Magnetic field of a single wire




A long, straight conductor carries a 1.0A current. At what distance from the axis of the conductor does the resulting magnetic field have magnitude B = 0.5 x $10^{-4}$ T (about that of the earth's mag-netic field in Pittsburgh)?

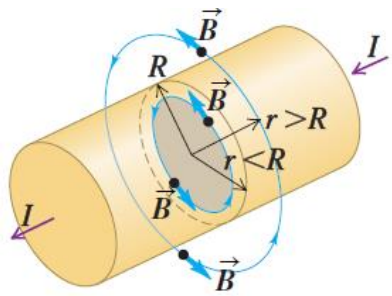
**Solution:**

The figure is shown in the right.

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow \quad r = \frac{\mu_0 I}{2\pi B} = \frac{\mu_0 (4\pi \times 10^{-7}\ T.m/A)(1.0\ A)}{(2\pi)(0.5 \times 10^{-4}\ T)} = 4 \times 10^{-3} mm$$

## Example 28.8 Field of a long cylindrical conductor




A cylindrical conductor with radius R carries a current I (shown in the figure). The current is uniformly distributed over the cross-sectional area of the conductor. Find the magnetic field as a function of the distance **r** from the conductor axis for points both inside (r < R) and outside (r > R) the conductor.

**Solution:**

The field $\vec{B}$ has the same magnitude at every point on the circular integration path and is tangent to the path.

<u>Magnetic field at radius r < R (inside the conductor)</u>

Let us apply Ampere's law to the enclosing area of circle of radius r.

Let I′ = current

As the current is uniformly distributed over the cross-sectional area of the conductor, we have,

Current density = constant

$$\Rightarrow \quad J = J'$$

$$\Rightarrow \quad \frac{I}{A} = \frac{I'}{A'}$$

$$\Rightarrow \quad \frac{I}{\pi R^2} = \frac{I'}{\pi r^2}$$

$$\Rightarrow \quad I' = \left(\frac{r^2}{R^2}\right) I$$

Hence Amper Law gives, $\oint B\, dA = \mu_0 I' \quad \Rightarrow \quad B(2\pi r) = \mu_0 \left(\frac{r^2}{R^2}\right) I \quad \Rightarrow \quad B = \frac{\mu_0 I r}{2\pi R^2}$

Magnetic field at radius r < R (outside the conductor)

Let us apply Ampere's law to the enclosing area of circle of radius r.

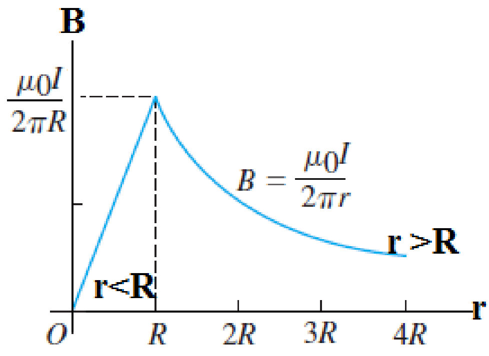
Here, I = current

Hence Amper Law gives, $\oiint B\, dA = \mu_0 I \Rightarrow B(2\pi r) = \mu_0 I \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}$

Outside the conductor, the magnetic field is the same as that of a long, straight conductor carrying current I. It is independent of the radius R over which the current is distributed.

Indeed, the magnetic field outside any cylindrically symmetric current distribution is the same as if the entire current were concentrated along the axis of the distribution.

Magnitude of the magnetic field inside and outside a long, straight cylindrical conductor with radius R carrying a current I is plotted in the figure.




## Assignment Problems

28.33: Two long, parallel wires are separated by a distance of 2.50 cm. The force per unit length that each wire exerts on the other is 4.0 x $10^{-5}$ N/m, and the wires repel each other. The current in one wire is 0.6 A.

a) What is the current in the second wire?
b) Are the two currents in the same direction or in opposite directions?

Ans: a) $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r} \Rightarrow I_2 = \left(\frac{F}{L}\right)\left(\frac{2\pi r}{\mu_0 I_1}\right) = (4 \times 10^{-5}\ \text{N/m})\frac{2\pi\,(0.025\ \text{m})}{\mu_0\,(0.6\ \text{A})} = 8.33\text{A}$

(b) The two wires repel so the currents are in opposite directions.

28.44: A closed curve encircles several conductors. The line integral around this curve is 3.83 x $10^{-4}$ T m.

a) What is the net current in the conductors?
b) If you were to integrate around the curve in the opposite direction, what would be the value of the line integral? Explain.

Ans: a) From Ampere's law we have

$$\oint \vec{B}\, d\vec{l} = \mu_0 I$$

$$\Rightarrow 3.83 \times 10^{-4}\text{T m} = \mu_0 I$$

$$\Rightarrow I = \frac{3.83 \times 10^{-4}\text{T m}}{\mu_0} = \frac{3.83 \times 10^{-4}\text{T m}}{\mu_0} = \frac{3.83 \times 10^{-4}}{4\pi \times 10^{-7}} = 305\ \text{A}$$

b) Since at each point on the curve the direction of ***dl*** is reversed the line integral will be −3.83.10−4 T.m