Chapter-21: Electric Charge And Electric Field

Topic

1. Coulomb's Law
2. Electric Field and Electric Forces
3. Electric-Field Calculations
4. Electric Dipoles
5. Conceptual Problems : TYU-21.3, TYU-21.4
6. In class Problems : 21.4, 21.9, 21.13
7. Assignment Problems: 21.7, 21.43, 21.51

LEARNING GOALS

- The nature of electric charge, and how we know that electric charge is conserved.
- How objects become electrically charged.
- How to use Coulomb's law to calculate the electric force between charges.
- The distinction between electric force and electric field.
- How to calculate the electric field due to a collection of charges.
- How to use the idea of electric field lines to visualize and interpret electric fields.
- How to calculate the properties of electric dipoles.

Electric Charge

Properties of Electric Charge

1. Two types: positive, negative
2. Like charges repel; opposite attract
3. Charge is conserved
4. Charge is quantized: Charge comes in discrete "bundles" called quanta. All charges are multiples of this fundamental unit. The fundamental charge is the charge of one electron, called e. $e = 1.602 \times 10^{-19}$ C.
5. Ordinary matter is electrically neutral.

Masses of Common Particles

Electron: $m_e = 9.11 \times 10^{-31}$ kg

Proton: $m_p = 1.673 \times 10^{-27}$ kg

Neutron: $m_n = 1.675 \times 10^{-27}$ kg

Insulators, Conductors and Semiconductors

The degree to which a given material conducts electricity depends on the mobility of charge carriers within it.

- Insulators: charge carriers (electrons) very tightly bound to particular atomic nuclei; electrons not free to move around.
  - amber, glass, polystyrene, polyvinyl chloride (PVC), teflon, etc.
  - any excess charge tends to stay where it is; some local area of the material gets charged up.
- Conductors: lots of "free" electrons (outer electrons) not very tightly bound to any particular nuclear site.
  - If you establish some electric field within the material, these electrons readily move, giving rise to a current that flows within the material. As a consequence, any excess charged placed on surface of conductor readily redistributes itself… flows over the surface of the conductor.
  - metals tend to be good conductors: copper, silver, gold, etc.
- Semiconductors: Semiconductors are between good insulators and good conductors in their conductivity.
  - silicon, germanium, e.g.
  - can control conductivity by adding impurities ("doping")

Charging an Object

Two ways:

- Friction ("triboelectric charging"). Outer electrons stripped off of one material, deposited onto other
    - * rub amber rod with fur, amber acquires net negative charge
    - * rub glass rod with silk, glass acquires net positive charge
- Charging by induction: charge up some object without touching it with another charged object. Related to phenomenon of polarization.

Polarization

- Charged object can exert a force on an electrically neutral object.
- Bringing charged object near neutral object polarizes atoms in neutral object: gives these atoms an effectively positive "pole" and an effectively negative pole.

Coulomb's Law

- gives the dependence of the electrostatic force on the charges involved and the distance between them
- Charles Augustin de Coulomb (1736-1806)

The magnitude of the electrostatic force that one point charge exerts on another is directly proportional to the magnitudes of the charges and inversely proportional to the square of the distance between them.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1\ q_2|}{r^2}$$

The force that q_1 exerts on q_2 in vector form is:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \qquad \hat{r} = \text{unit vector in direction from } q_1 \text{ to } q_2$$

$$\varepsilon_0 = \text{Permittivity of air} = 8.85 \text{ x } 10^{-12}\ C^2.N^{-1}.m^2 \quad \text{and} \quad \frac{1}{4\pi\varepsilon_0} = 9 \text{ x } 10^9\ N.m^2\ C^{-2}$$

The directions of the forces the two charges exert on each other are always along the line joining them.

When the charges q_1 and q_2 have the same sign, either both positive or both negative, the forces are repulsive;
When the charges have opposite signs, the forces are attractive.

The two forces obey Newton's third law; they are always equal in magnitude and opposite in direction, even when the charges are not equal in magnitude.

Superposition of Forces (Force due to many charges)

When a charge q_1 has forces exerted on it by more than one other charges (q_2, q_3, q_4 , etc.), the net force on q_1 is the **vector sum** of the individual forces:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$$

Electric field: Let us consider an electric field due to a point charge Q. The electric field, E , at a point is defined as the electric force per unit test charge (q_0) placed at that point.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

The electrostatic force between Q and q_0 is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q\,q_0}{r^2}\hat{r}$$

Thus electric field is

$$\vec{E} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{Q\,q_0}{r^2}\hat{r}}{q_0} \quad \Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

A point charge 'q' produces an electric field at all points in space. The field strength decreases with increasing distance.

- The field produced by a positive point charge points away from the charge.
- The field produced by a negative point charge points toward the charge.

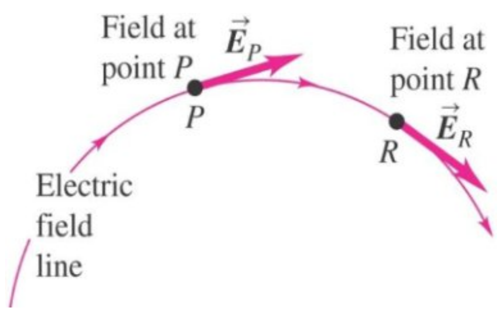
Electric-Field Calculations

Electric field at a point due to many is the **vector sum** of the individual electric field. So

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$$

Electric Field Lines

An electric field line is an imaginary line or curve whose tangent at any point is the direction of the electric field vector at that point.



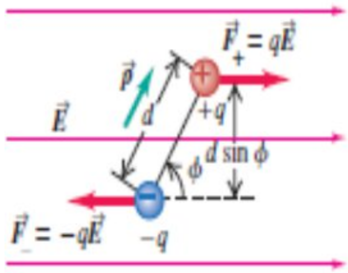
Rules for Drawing E-Field Lines

- Start at positive charges.
- End at negative charges.
- Number of lines leaving a + charge or terminating on a – charge is proportional to the charge.
- Number of lines/unit area perpendicular to the lines is proportional to E.
- No two field lines can ever intersect.

Electric Dipoles

An electric dipole is a pair of electric charges of equal magnitude ‘q’ but opposite sign, separated by a distance ‘d’. .

The magnitude electric dipole moment (p) is given by:



$p = q\,p$

The direction of is from negative toward positive charge.

An electric dipole in an electric field **E** experiences a torque τ given by:

$\tau = p\,E \sin \varphi$

$\Rightarrow \quad \vec{\tau} = \vec{p} \times \vec{E}$

Direction of τ can be determine by the right-hand rule. The situation shown in figure, τ is directed into the page.

The torque is greatest when $\vec{p}$ and $\vec{E}$ are perpendicular and is zero when they are parallel or anti-parallel.

The torque always tends to turn $\vec{p}$ to line it up with $\vec{E}$.

- The position $\varphi = 0$ with $\vec{p}$ parallel to $\vec{E}$ is a position of stable equilibrium,
- The position $\varphi = \pi$ with $\vec{p}$ and $\vec{E}$ anti-parallel, is a position of unstable equilibrium.

Potential Energy of an Electric Dipole

Let **dw** be the work done by a torque τ during an infinitesimal displacement $d\varphi$ is given by

$dw = \tau\, d\varphi$

Because the torque τ is in the direction of decreasing φ, the torque is written as

$\tau = -p\,E \sin \varphi$

Thus, **$dw = \tau\, d\varphi$** => **$dw = (-p\,E \sin \varphi)\, d\varphi$**

The total work done on the dipole is

$$W = \int_{\varphi_1}^{\varphi_2} (-p\,E \sin \varphi)\, d\varphi \quad \Rightarrow \quad W = -p\,E \int_{\varphi_1}^{\varphi_2} \sin \varphi\, d\varphi$$

$$\Rightarrow \quad W = -p\,E\left[-\cos \varphi\right]_{\varphi_1}^{\varphi_2} \quad \Rightarrow \quad W = p\,E\left[\cos \varphi\right]_{\varphi_1}^{\varphi_2}$$

$$\Rightarrow \quad W = p\,E\cos \varphi_2 - p\,E\cos \varphi_1$$

This work done is stored as potential energy. So,

$$U = p\,E\cos \varphi_2 - p\,E\cos \varphi_1$$

The work is the negative of the change of potential energy. So the suitable potential energy function is

$U = -p\,E\cos\varphi$

$U = -\vec{p}\cdot\vec{E}$

Case-I: If $\varphi = 0$, i.e. p and E are in the same direction

then $U = -pE$

This is the minimum (most negative) value of potential energy. Therefore this is the stable equilibrium position.

Case-II: If $\varphi = \pi$, i.e. p and E are anti-parallel
then $U = +pE$
This is the maximum potential energy.

Case-III: If $\varphi = \pi/2$, i.e. p and E are perpendicular to each other
then $U = 0$
This is the minimum (most negative) value of potential energy. This happens at the stable equilibrium position.

The potential energy 'U' for an electric dipole in an electric field depends on the relative orientation of 'p' and **'E'.**

U = -p . E

Test Your Understanding of Section 21.3

Suppose that charge q_2 in Example 21.4 were $-2.0\ \mu C$ In this case, the total electric force on Q would be (i) in the positive x-direction; (ii) in the negative x-direction; (iii) in the positive y-direction; (iv) in the negative y-direction; (v) zero; (vi) none of these.

Answer: (iv)

The force exerted by q_1 on Q is away from q_1 . The force exerted by q_2 on Q toward q_2 at an angle α below the x-axis. Hence the x-components of the two forces cancel while the (negative) y-components add together, and the total electric force is in the negative y-direction.

Test Your Understanding of Section 21.4

(a) A negative point charge moves along a straight-line path directly toward a stationary positive point charge. Which aspect(s) of the electric force on the negative point charge will remain constant as it moves? (i) magnitude; (ii) direction; (iii) both magnitude and direction; (iv) neither magnitude nor direction.

(b) A negative point charge moves along a circular orbit around a positive point charge. Which aspect(s) of the electric force on the negative point charge will remain constant as it moves? (i) magnitude; (ii) direction; (iii) both magnitude and direction; (iv) neither magnitude nor direction.

Answers: (a) (ii), (b) (i)

The electric field E produced by a positive point charge points directly away from the charge and has a magnitude that depends on the distance 'r' from the charge to the field point. Hence a second, negative point charge $q<0$ will feel a force $F = qE$ that points directly toward the positive charge and has a magnitude that depends on the distance between the two charges. If the negative charge moves directly toward the positive charge, the direction of the force remains the same but the force magnitude increases as the distance 'r' decreases. If the negative charge moves in a circle around the positive charge, the force magnitude stays the same (because the distance is constant) but the force direction changes.

In class Problems

Example 21.4: Vector addition of electric forces in a plane

Two equal positive charges $q_1 = q_2 = 2.0\ \mu C$ are located at $x = 0$, $y = 0.30$ m and $x = 0$, $y = -0.30$m, respectively. What are the magnitude and direction of the total electric force that q_1 and q_2 exert on a third charge $Q = 4.0\ \mu C$ at $x = 0.40$ m, $y = 0$?

Solution:

$$F_1 = F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{(9 \times 10^9\ N\,m^2\,C^{-2})(2 \times 10^{-6}\ C)(2 \times 10^{-6}\ C)}{(0.5\ m)^2} = 0.29\ N$$

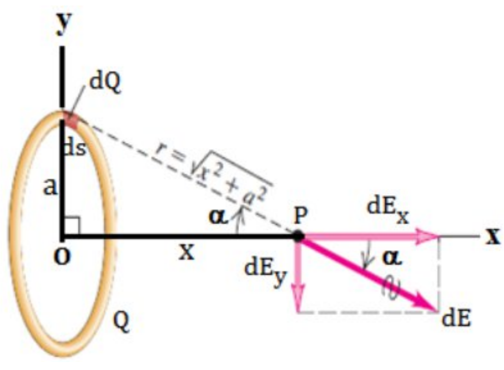
The x-components of the two forces are equal:

$$F_{1x} = F_{2x} = (0.29\ N)\cos\alpha = (0.29\ N)\left(\frac{0.4\ m}{0.5\ m}\right) = 0.23\ N$$

$$F_x = F_{1x} + F_{2x} = 0.23\ N + 0.23\ N = 0.46\ N$$

Example 21.9: Field of a ring of charge

Charge Q is uniformly distributed around a conducting ring of radius (shown in Figure). Find the electric field at a point P on the ring axis at a distance from its center.



Solution:

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2 + a^2}$$

$$dE_x = dE\cos\alpha$$

$$dQ = \lambda\, ds \qquad \text{and} \quad \cos\alpha = \frac{x}{\sqrt{x^2+a^2}}$$

$$dE_x = \left[\frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}\right]\cos\alpha$$

$$dE_x = \left[\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{x^2+a^2}\right]\left[\frac{x}{\sqrt{x^2+a^2}}\right] \quad \Rightarrow \quad dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, x}{\left(x^2+a^2\right)^{3/2}}ds$$

$$E_x = \int_0^{2\pi a}\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, x}{\left(x^2+a^2\right)^{3/2}}ds \quad \Rightarrow \quad E_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, x}{\left(x^2+a^2\right)^{3/2}}\int_0^{2\pi a}ds$$

$$\Rightarrow \quad E_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, x}{\left(x^2+a^2\right)^{3/2}}(2\pi\, a) \quad \Rightarrow \quad E_x = \frac{1}{4\pi\varepsilon_0}\frac{Q\, x}{\left(x^2+a^2\right)^{3/2}}$$

$$\vec{E} = E_x\hat{i} = \frac{1}{4\pi\varepsilon_0}\frac{Q\, x}{\left(x^2+a^2\right)^{3/2}}\hat{i}$$

Directions of the electric dipole moment, electric field, and torque (points out of the page).

Example 21.13: Force and torque on an electric dipole

Figure(a) shows an electric dipole in a uniform electric field of magnitude 5.0×10^5 N/C that is directed parallel to the plane of the figure. The charges are $\pm 1.6 \times 10^{-19}$ C; both lie in the plane and are separated by 0.125 nm = 0.125×10^{-9} m. Find

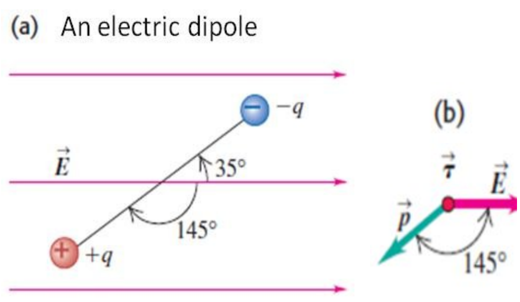
a) the net force exerted by the field on the dipole;

b) the magnitude and direction of the electric dipole moment;

c) the magnitude and direction of the torque;

d) the potential energy of the system in the position shown.

Solution:



(a) The field is uniform, so the forces on the two charges are equal and opposite. Hence the total force on the dipole is zero.

(b) $p = q\, d = (1.6 \times 10^{-19}\ \text{C})\ (0.125 \times 10^{-9}\ \text{m}) = 2.0 \times 10^{-29}$ C. M

The direction of is from the negative to the positive charge, clockwise from the electric-field direction.

(c) $\tau = p\, E \sin\varphi = 5.7 \times 10^{-24}$ N . M and is out of the page

(d) $U = -\, p\, E\, \cos\varphi = 8.2 \times 10^{-34}$ J

Assignment Problems

21.7: Consider an electron – proton pair and compare its electrostatic force with that of the gravitational force. $G = 6.67 \times 10^{-11}\ N.m^2.kg^{-2}$.

Solution:

$m_e = 9.31 \times 10^{-31}$ kg, $m_p = 1.673 \times 10^{-27}$ kg,

Electrostatic force between electron – proton pair is $F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_e q_p}{r^2}$

Gravitational force between electron – proton pair is $F_G = G\frac{m_e m_p}{r^2}$

Now,

$$\frac{F_e}{F_G} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q_e q_p}{r^2}}{G\frac{m_e m_p}{r^2}} \Rightarrow \frac{F_e}{F_G} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_e q_p}{G\, m_e m_p}$$

$$\Rightarrow \frac{F_e}{F_G} = \frac{\left(9 \times 10^9 N.m^2.C^{-2}\right)\left(1.6 \times 10^{-19}C\right)\left(1.6 \times 10^{-19}C\right)}{\left(6.67 \times 10^{-11}N.m^2.kg^{-2}\right)\left(9.31 \times 10^{-31}kg\right)\left(1.673 \times 10^{-27}kg\right)} = 2.23 \times 10^{39}$$

21.43: Two positive point charges ‘q’ are placed on the x-axis, one at x = a and one at x = -a.

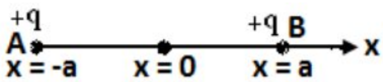
a) Find the magnitude and direction of the electric field at x = 0.

b) Derive an expression for the electric field at points on the x-axis.

Use your result to graph the x -component of the electric field as a function of x, for values of x between -4a and +4a.

Solution:

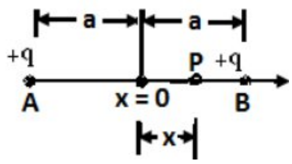
a) $E_{-a} = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$ along +ve x-direction



$E_a = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$ along -ve x-direction

$E = E_{-a} - E_a = 0$

b) **For -a < x < a**



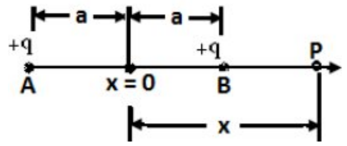
$E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{PB^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(a-x)^2}$ along -ve x-direction

$E_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{AP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(a+x)^2}$ along +ve x-direction

$$E_P = E_B - E_A = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(a\text{-}x)^2} - \frac{q}{(a+x)^2}\right] \quad \text{along -ve x-direction}$$

$$E_P = \frac{1}{4\pi\varepsilon_0}\frac{4axq}{\left(a^2\text{-}x^2\right)^2} \quad \text{along -ve x-direction}$$

For x > a



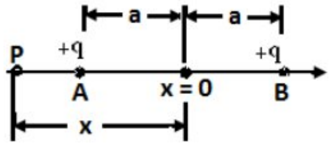
$$E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x\text{-}a)^2} \quad \text{along +ve x-direction}$$

$$E_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{AP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x+a)^2} \quad \text{along +ve x-direction}$$

$$E_P = E_B + E_A = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(x\text{-}a)^2} + \frac{q}{(x+a)^2}\right] \quad \text{along +ve x-direction}$$

$$E_P = \frac{1}{4\pi\varepsilon_0}\frac{2q\left(a^2+x^2\right)}{\left(a^2\text{-}x^2\right)^2} \quad \text{along +ve x-direction}$$

For x < - a

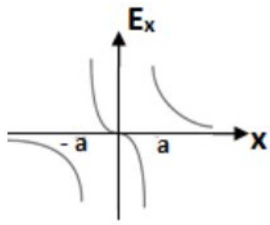


$$E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x+a)^2} \quad \text{along -ve x-direction}$$

$$E_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{AP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x\text{-}a)^2} \quad \text{along -ve x-direction}$$

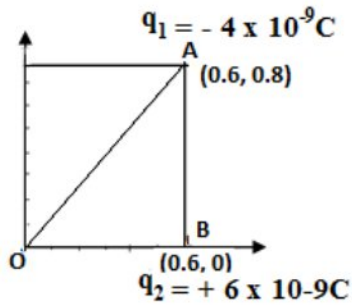
$$E_P = E_B + E_A = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(x\text{-}a)^2} + \frac{1}{(x+a)^2}\right] \quad \text{along -ve x-direction}$$

$$E_P = \frac{1}{4\pi\varepsilon_0}\frac{2q\left(a^2+x^2\right)}{\left(a^2\text{-}x^2\right)^2} \quad \text{along -ve x-direction}$$



Electric field as a function of x is shown in graph

21.51: A point charge $q_1 = -4 \times 10^{-9}$C is at the point x = 0.6 m, y = 0.8 m and a second point charge $q_2 = +6 \times 10^{-9}$C is at the point x = 0.6 m, y = 0. Calculate the magnitude and direction of the net electric field at the origin due to these two point charges.



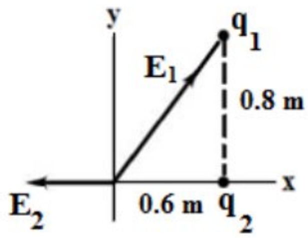
Ans: $q_1 = -4 \times 10^{-9}$C, $q_2 = +6 \times 10^{-9}$C

$$\vec{E}_1 = \frac{q}{4\pi\varepsilon_0}\left(0.6\,\hat{i} + 0.8\,\hat{j}\right) = \left(9 \times 10^9\right)\left(4 \times 10^{-9}\right)\left(0.6\,\hat{i} + 0.8\,\hat{j}\right)$$

$$\Rightarrow \quad \vec{E}_1 = \left(21.6\,\hat{i} + 28.8\,\hat{j}\right)\text{N/C}$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{(0.6\text{ m})^2}\left(-\,\hat{i}\right) = \left(9 \times 10^9\ \text{N.m}^2.\text{C}^{-2}\right)\frac{\left(6 \times 10^{-9}\ \text{C}\right)}{(0.6\text{ m})^2}\left(-\,\hat{i}\right)$$

$$\vec{E}_2 = -\,150.6\,\hat{i}\ \text{N/C}$$



$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E} = \left(21.6\,\hat{i} + 28.8\,\hat{j}\right)\text{N/C} + \left(-\,150.6\,\hat{i}\right)\text{N/C}$$

$$\vec{E} = \left(-128.4\,\hat{i} + 28.8\,\hat{j}\right)\text{N/C}$$

$$\left|\vec{E}\right| = \sqrt{(-128.4)^2 + (28.8)^2}\ \text{N/C} = 131.6\ \text{N/C}$$

The direction of $\vec{E}$ is

$$\tan\theta = \frac{28.8}{128.4} = 0.224 \quad \Rightarrow \theta = \frac{28.8}{128.4} = \tan^{-1}(0.224) = 12.6^0$$

12.6^0 above the $-$ ve x -axis and therefore 167.4° counter-clockwise from the +ve x-axis.