

ASSIGNMENT-2

Sub-CALCULUS B(MTH2101)

Answer All the Questions (GATE)

Q.1 If $Z = e^{ax+by} F(ax-by)$ then find the value of $b \cdot \frac{\partial Z}{\partial x} + a \cdot \frac{\partial Z}{\partial y}$

Q.2 For a scalar function $f(x, y, z) = x^2 + 3y^2 + 2z^2$, Find the gradient at the point $P(1, 2, -1)$.

Q.3 For the scalar field $u = \frac{x^2}{2} + \frac{y^2}{3}$, find the magnitude of the gradient at the point $(1, 3)$.

Q.4 Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point $P(2, 1, 3)$ in the direction of the vector $a = i - 2k$.

Q.5 Let $f = y^x$, What is $\frac{\partial^2 f}{\partial x \partial y}$ at $x = 2, y = 1$?

Q.6 Find the directional derivative of the scalar function $f(x, y, z) = x^2 + 2y^2 + z$ at the point $P = (1, 1, 2)$ in the direction of the vector $\vec{a} = 3\hat{i} - 4\hat{j}$.

Q.7 Find the total derivative of function xy .

Q.8 For a scalar function $f(x, y, z) = x^2 + 3y^2 + 2z^2$, find the directional derivative at the point $P(1, 2, -1)$ in the direction of a vector $\vec{i} - \vec{j} + 2\vec{k}$.

Answer the following questions

Q.9 If $z = u\sqrt{v-w}$ then find $\frac{\partial^3 u}{\partial u \partial v \partial w}$

Q.10 Find an equation of the tangent plane to the given surface at the specified point.

$$z = 3y^2 - 2x^2 + x, \quad (2, -1, -3)$$

Q.11 find the linearization $L(x, y)$ of the function at that point.

$$f(x, y) = \sqrt{x + e^{4y}}, (3, 0)$$

Q.12 If $z = x^2 - xy + 3y^2$ and (x, y) changes from $(3, -1)$ to $(2.96, 0.95)$

Compare the value of Δz and dz .

Q.13 Let $F(x, y, z) = \sqrt{x} + \sqrt{y} + \sqrt{z} + \ln(4 - x^2 - y^2 - z^2)$

- a) Evaluate $f(1,1,1)$.
- b) Find and describe the domain of f .

Q.14 Find the limit if it exists, or show that the limit does not exist.

$$\text{i) } \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + \sin^2 y}{2x^2 + y^2} \quad \text{ii) } \lim_{(x,y) \rightarrow (0,0)} \frac{xy \cos y}{3x^2 + y^2}$$

Q.15 Determine the set of points at which the function is continuous.

$$f(x, y) = \begin{cases} \frac{x^2 y^3}{2x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 1 & \text{if } (x, y) = (0, 0) \end{cases}$$

Q.16 Find equation of the tangent plane to the given surface at specified point

i) $z = 3y^2 - 2x^2 + x$, $(2, -1, -3)$

ii) $z = x e^{xy}$, $(2, 0, 2)$

iii) $z = \ln(x - 2y)$, $(3, 1, 0)$

Q.17 Find the directional derivative of the function at the given point in the direction of the vector v .

i) $f(x, y) = e^x \sin y$, $(0, \pi/3)$, $v = \langle -6, 8 \rangle$

ii) $g(r, s) = \tan^{-1}(rs)$, $(1, 2)$, $v = 5i + 10j$

iii) $f(x, y, z) = x e^x + y e^z + z e^x$, $(0, 0, 0)$, $v = \langle 5, 1, -2 \rangle$

Q.18 If $f(x, y) = y^2/x$, $P(1, 2)$, $u = \frac{1}{3}(2i + \sqrt{5}j)$ then

- a) Find the gradient of f .
- b) Evaluate the gradient at the point P .
- c) Find the rate of change of f at P in direction of the vector u .

Q.19 If $e^y \sin x = x + x y$, find dy/dx

Q.20 Find three positive numbers whose sum is 100 and whose product is a maximum.

The End