Chapter – 25: Current, Resistance, And Electromotive Force

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LEARNING GOAL

- To understand current and how charges move in a conductor
- To understand resistivity and conductivity
- To calculate the resistance of a conductor
- To learn how an emf causes current in a circuit
- To calculate energy and power in circuits

Current (I) through a conductor

Current is due to the motion of the free electrons, present in the conductor, from one region to another. The magnitude of the current is $I = \frac{dq}{dt}$ The current density is $J = \frac{I}{A}$

No electric field applied:

- In a metal such as copper or aluminum, some of the electrons are free to move within the conducting material. These free electrons move randomly in all directions with much greater speeds, of the order of 10^6m/s.

  But these electrons do not escape from the conducting material, because they are being attracted to the positive ions of the material.

  The motion of the electrons is random, and the average velocity is zero, so there is no net flow of charge in any direction and hence no current.
- If 'n' electrons move with velocities v_1, v_2, v_3, …… then the average velocity of electrons is
- $v_{av} = \frac{v_1 + v_2 + v_3 + ...}{n} = 0$

Electric field applied:

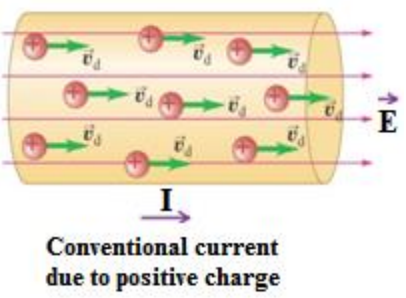
- With the application of electric field $\vec{E}$ each electron experience force given by $\vec{F} = q\,\vec{E}$
- This steady force ($\vec{F}$) causes a steady acceleration in opposite direction (electron being negatively charged) of $\vec{E}$.
- But a charged particle moving in a conductor undergoes frequent collisions with the massive, nearly stationary ions of the material.
- The net effect of $\vec{E}$ is that there is a very slow net motion or drift of the moving charged particles in the direction of $\vec{F}$.
- The velocity acquired by the electron is called the drift velocity (v_d). As result, there is a net current in the conductor. The drift speed is very slow, of the order of 10^{-4}m/s.
- Due to collision the particle undergo a very small displacement $\Delta x = v_d\,\Delta t$, in direction of $\vec{F}$.

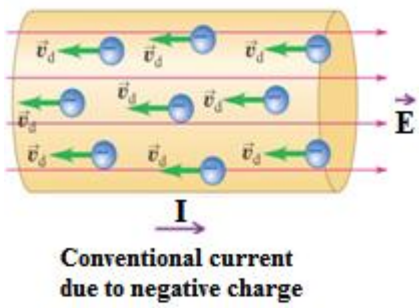
  (Given that the electrons move so slowly, you may wonder why the light comes on immediately when you turn on the switch of a flashlight.

  The reason is that the electric field is set up in the wire with a speed approaching the speed of light, and electrons start to move all along the wire at very nearly the same time. The time that it takes any individual electron to get from the switch to the light bulb isn't really relevant.)

Direction of current flow

- A current can be produced by positive or negative charge flow. But the conventional current flows along the direction of $\vec{E}$.
- In metals the conventional current is due to the electrons





Current, drift velocity, and current density

Let **n = electron density =** number of electron per unit volume

$$n = \frac{\text{No. of electrons}}{\text{Volume}} \quad \Rightarrow \quad \text{No. of electrons} = n\,(\text{Volume}) = n\,(\text{A x displacement})$$

$$\text{No. of electrons} = n\,A\,(v_d\,dt)$$

Where, $\mathbf{v_d}$ = move w/ drift velocity, **A** = cross-sectional area

The charge dQ that flows out of the end of the cylinder during time dt is

dQ = (number of electron) e where, e = charge of an electron

$dQ = n\,A\,v_d\,dt\,e$

We know that the current is

$$I = \frac{dQ}{dt} \quad \Rightarrow \quad I = \frac{n\,A\,v_d\,e\,dt}{dt} \quad \Rightarrow \quad I = e\,v_d\,A\,n$$

Now the current density J is

$$J = \frac{I}{A} \quad \Rightarrow \quad J = \frac{A\;v_d\,e\,n}{A} \quad \Rightarrow \quad J = v_d\;e\;n$$

Current density is a vector quantity. So this can be written as

$$\vec{J} = \vec{v}_d\;e\;n$$

The direction of $\vec{J}$ depends on the direction of $\vec{E}$

If charge is positive, $\vec{v}_d$ is in the same direction as $\vec{E}$

If charge is negative, $\vec{v}_d$ is opposite to $\vec{E}$

In either case, $\vec{J}$ is in the same direction as $\vec{E}$

Difference between current and current density

Current density vs. current

Current (**I**)	Current density $(\vec{J})$
The current is a scalar.	Current density is a vector.
The current describes how charges flow through an extended object such as a wire.	Current density describes how charges flow at a certain point and also about the direction of the flow at that point.
Current(I) have the same value at all points in the circuit.	Current density does not have the same value at all points in the circuit, because direction of $\vec{J}$ (same as $\vec{E}$) is not same at all the points. Even the magnitude of $\vec{J}$ can also vary around a circuit because the area of cross-section of the wires may not same as the battery

Resistivity

- Resistivity (ρ) of material is the ratio of the magnitude of electric field ($\vec{E}$) to the current density $(\vec{J})$.

  $\rho = \frac{E}{J}$ (This is called the Ohm's Law)

  $$\Rightarrow \quad \rho = \frac{V/\ell}{I/A} \quad \Rightarrow \quad \rho = \frac{AE}{I}$$
- Large 'r' implies a large E field is needed to generate small current.
- Conductivity (σ) is the reciprocal of the resistivity (ρ).
- High conductivity means a large current result from a small applied E field.

A material that obeys Ohm's law reasonably well is called an ohmic conductor or a linear conductor. For such materials, at a given temperature, ρ is a constant and does not depend on the value of $\vec{E}$.

Many materials show substantial deviation from Ohm's-law behavior; they are non-ohmic, or nonlinear. In these materials, $\vec{J}$ depends on $\vec{E}$ in a more complicated manner.

Resistivity and temperature

The resistivity of a metallic conductor varies with the variation of the temperature, as shown in the figure. As temperature increases, the ions of the conductor vibrate with greater amplitude. So the probability of collision of the electron with ions increases. This impedes the drift of electrons through the conductor and hence reduces the current.

Over a small temperature range (up to $100C^0$), the resistivity of a metal can be represented approximately by the equation

$\rho(T) = \rho_0[1+ \alpha(T-T_0)]$

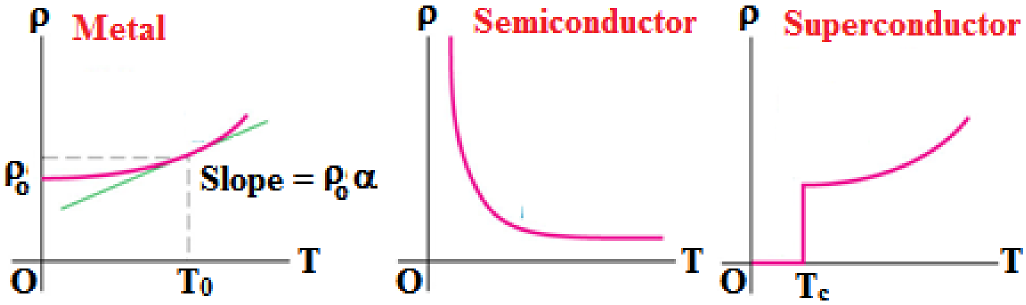
Where ρ_0 = resistivity at a reference temperature.

$\rho(T)$ = resistivity at temperature T , which may be higher or lower than T_0.

α = **temperature coefficient of resistivity.**

The resistivity of the alloy manganin is practically independent of temperature.

The variation of $\rho(T)$ with the temperature T for metal, semiconductors and superconductors are shown below.

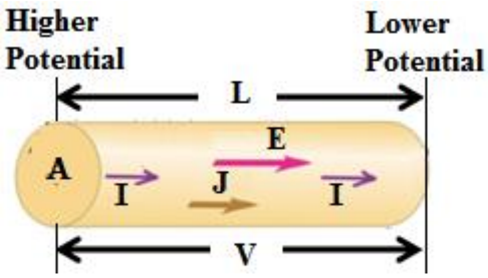


The resistivity of semiconductor materials (also graphite) decreases with increasing of temperature. Here, α is negative. This principle is used in the construction thermistor (a type of thermometer).

In superconductivity materials, as the temperature decreases, the resistivity first decreases smoothly then at a certain critical temperature (T_c) the resistivity suddenly drops to zero. Once a current has been established in a superconducting ring, it continues indefinitely without the presence of any driving field.

Resistance

Let us consider a conductor with uniform cross-sectional area such that



A = area of cross-sectional

L = length of the cross-sectional area

V = potential difference between the higher-potential and lower-potential ends of the conductor

The direction of the current is always from the higher-potential end to the lower potential end.

As the current flows through the potential difference, electric potential energy is lost; this energy is transferred to the ions of the conducting material during collisions.

If the magnitudes of the current density $\vec{J}$ and the electric field $\vec{E}$ are uniform throughout the conductor, the total current I is given by

$$J = \frac{I}{A}$$

And the potential difference between the ends is given by

$V = E\,L$

We know that

$$E = \rho\, J \quad \Rightarrow \quad \frac{V}{L} = \rho \frac{I}{A} \quad \Rightarrow \quad V = I\left(\frac{\rho L}{A}\right) \quad \Rightarrow \quad V = IR$$ This is called Ohm's law.

Where, $R = \dfrac{\rho L}{A}$ and is called the Resistance of the material of the conductor

For ohmic material '**ρ**' is constant. So, R is constant for ohmic material

From the above equation it is clear that for ohmic material the resistance conductor of uniform cross section is directly proportional to its length and inversely proportional to its cross-sectional area. It is also proportional to the resistivity of the material of which the conductor is made.

The resistance of a metallic conductor varies with the variation of the temperature,

The resistance of a metal at a temperature can be represented as

$R_T = R_0[1+ \alpha(T-T_0)]$

Where R_0 = resistance at a reference temperature.

R_T = resistance at temperature T , which may be higher or lower than T_0.

α = **temperature coefficient of resistance.**

$R_T = R_0[1+ \alpha\, \Delta T]$ where $\Delta T = T-T_0$ = change in temperature

=> $R_T = R_0 + R_0 \alpha\, \Delta T$

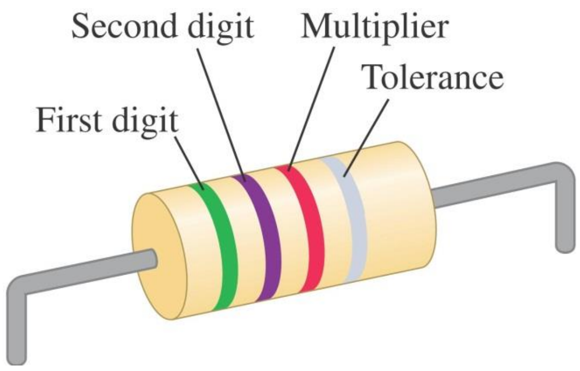
=> $R_T - R_0 = R_0\, \alpha\, \Delta T$

Thus, the change in resistance resulting from a temperature change ΔT is $R_0\, \alpha\, \Delta T$

Resistors are color-coded for easy identification

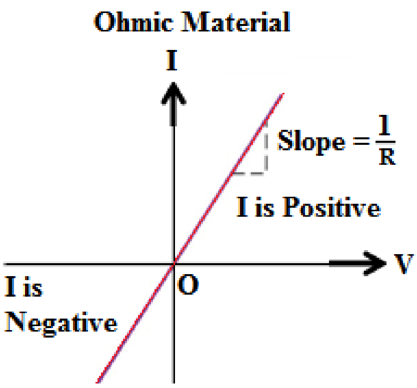
Color Codes for Resistors

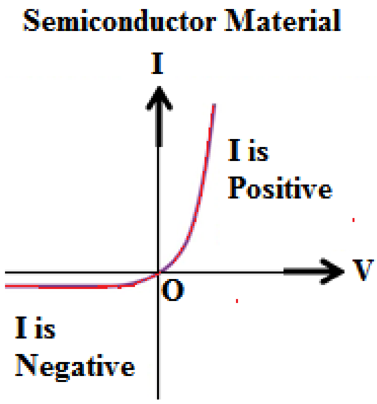
Color	**Value as Digit**	**Value as Multiplier**
Black	0	10^0
Brown	1	10^1
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9



Ohmic and non-ohmic resistors

Ohmic resistors	**Non-ohmic resistors**
Ohmic resistor obeys Ohm's law	Non-Ohmic resistor do not obeys Ohm's law
I α V (I –V is a straight line passing through origin)	I –V is a straight line passing through origin
If the sign of V changes, so does the sign of I produced. So E, J and I all reverse direction.	For +I, I increases exponentially with increasing V, For –V, the current is extremely small. Thus a positive causes a current to flow in the positive direction, but a potential difference of the other sign causes little or no current.

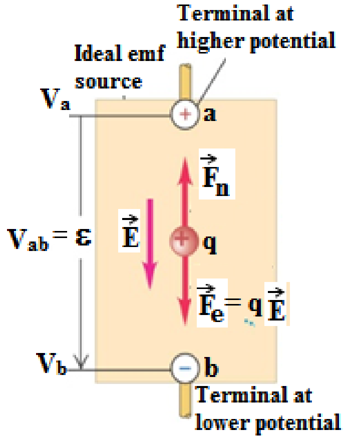




Electromotive force and circuit

The influence that makes current flow from lower to higher potential is called electromotive force.

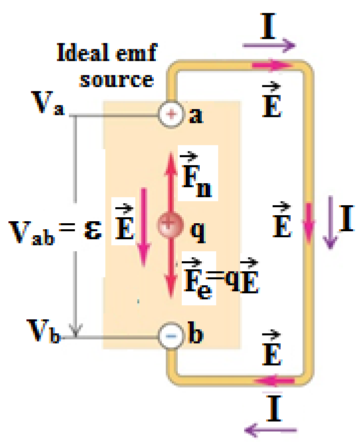
The SI unit of emf is the same as that for potential, the volt



Mechanism:

For ideal source of emf:

- Terminal **a** is maintained at higher potential and terminal **b** is maintained at lower potential.
- Due to potential difference an electric field E is created in the circuit (both inside and outside the source).
- The electric field inside the device is directed from **a** to **b** ,
- A charge q within the source experiences an electric force F = q E
- Another force (F_n) exist inside is of non-electrostatic type.
- F_n maintains the potential difference between the terminals. If F_n were not present, charge would flow between the terminals until the potential difference was zero. (The origin of F_n depends on the kind of source. In a generator it results from magnetic-field forces on moving charges. In a battery or fuel cell it is associated with diffusion processes and varying electrolyte concentrations resulting from chemical reactions. In an electrostatic machine such as a Van de Graaff generator , an actual mechanical force is applied by a moving belt or wheel.)



- F_n does a positive work on the charge to move from **b** to **a** inside the source. It is given by $W = q\varepsilon$

- Due to this work done the potential energy associated with the charge **q** increases and is given by $U = q V_{ab}$ (Where, $V_{ab} = V_a - V_b$)

- When the emf source is not part of a closed circuit, $F_n = F_e$ and there is no net motion of charge between the terminals.

  Thus $q\varepsilon = q V_{ab}$

  $V_{ab} = \varepsilon$ (for ideal source of emf)

- The potential difference between the ends of the wire is given by $V_{ab} = IR$

  So, $\varepsilon = V_{ab} = IR$ (for ideal source of emf)

Internal resistance

When a charge **q** move through a material of any real source, encounters a resistance (r). This is known as internal resistance.

As the current (I) moves through r , it experiences potential drop equal to Ir .

Thus, the terminal voltage of an emf source is given by

$V_{ab} = \varepsilon - Ir$.

$$\Rightarrow \quad IR = \varepsilon - Ir$$

$$\Rightarrow \quad \varepsilon = IR + Ir \; = I(R + r)$$

$$\Rightarrow \quad \boxed{I = \frac{\varepsilon}{(R + r)}}$$

Ammeters

- Ammeters measure flow of current PAST a point.
- Ideally, they should NOT influence the current
- Ideally, $R_{(ammeter)} = 0$
- Put them IN SERIES with circuit “legs”

Voltmeters

- Voltmeters measure pressure difference across (or between) points in the circuit.
- Ideally, they should NOT influence the current
- Ideally, R(voltmeter) = ∞
- Put them in parallel

Energy and power in electric circuits

Rate at which energy is delivered to or extracted from a circuit element is called the power. It is given by $P = V_{ab}I$.

Power delivered to pure resistor: $P = I^2R = V_{ab}^2/R$.

Conceptual Problems:

Test Your Understanding of Section 25.1

Suppose a electrical circuit uses 18 gauge wire and the drift velocity of electron through it was $v_d = 0.15\text{mm/s}$. Now the wire has been replaced by a 12 gauge copper wire which has twice the diameter of 18-gauge wire. If the current remains the same, what effect would this have on the magnitude of the drift velocity v_d

i) none— v_d would be unchanged;
ii) v_d would be twice as great;
iii) v_d would be four times greater;
iv) v_d would be half as great;
v) v_d would be one-fourth as great.

Answer: (v)

We know that

$$v_d = \frac{J}{n\,q} = \frac{I/A}{n\,q} = \frac{I}{n\,qA} = \frac{I}{n\,q\,\pi\,r^2} = \frac{4\,I}{n\,q\,\pi\,D^2} \quad \left(\because J = \frac{I}{A} \text{ and } A = \pi\,r^2 = \frac{\pi\,D^2}{4} \right)$$

$$v_d \;\alpha\; \frac{1}{D^2}$$

Thus when the diameter is doubled, v_d decreased by 2^2 times i.e decreased by 4 times.

Hence the drift velocity will reduced by a factor of $\frac{1}{4}$

The new magnitude is $v_d = \dfrac{0.15\text{ mm/s}}{4} = 0.038\text{ mm/s}$.

Test Your Understanding of Section 25.3

Suppose you increase the voltage across a copper wire. The increased voltage causes more current to flow, which makes the temperature of the wire increase. (The same thing happens to the coils of an electric oven or a toaster when a voltage is applied to them.) If you double the voltage across the wire, the current in the wire increases. By what factor does it increase?

(i) 2; (ii) greater than 2; (iii) less than 2.

Answer: (iii)

We know that the current is $I = \dfrac{V}{R}$

If the resistance R of the wire remained the same, then by doubling the voltage V, the current I will be doubled..

However, the resistance is not constant. As the current increases and the temperature increases. So, R increases.

Thus doubling the voltage produces a current that is *less* than double the original current.

An ohmic conductor is one for which $R = \frac{V}{I}$ has the same value no matter what the voltage, so the wire is *nonohmic*. (In many practical problems the temperature change of the wire is so small that it can be ignored, so we can safely regard the wire as being ohmic).

Test Your Understanding of Section -25.4:

Rank the following circuits

in order from highest to lowest current.

i) A 1.4 Ω resistor connected to a1.5 V battery that has an internal resistance of 0.10 Ω

ii) A 1.8 Ω resistor connected to a 4.0 V battery that has a terminal voltage of 3.6 V but an unknown internal resistance.

iii) An unknown resistor connected to a 12.0 V battery that has an internal resistance of 0.2 Ω and a terminal voltage of 11.0 V.

Answer: (iii), (ii), (i)

i) emf of the battery = 1.5 V, External resistance = 1.4 Ω, Internal resistance = 0.1 Ω

$$\text{So, } I = \frac{\varepsilon}{(R+r)} = \frac{1.5\text{ V}}{(1.4\,\Omega + 0.10\Omega)} = \frac{1.5\text{ V}}{1.5\,\Omega} = 1\text{ A}$$

ii) Terminal voltage V_{ab} = 3.6 V, R = 1.8 Ω.

$$I = \frac{V_{ab}}{R} = \frac{3.6\text{ V}}{1.8\,\Omega} = 2\text{ A}$$

iii) emf of the battery = 12.0 V, Terminal voltage V_{ab} = 11.0 V, Internal resistance = 0.2 Ω.

$$V_{ab} = \varepsilon - Ir \quad \Rightarrow \quad I = \frac{\varepsilon - V_{ab}}{r} = \frac{12.0\text{ V} - 11.0\text{ V}}{0.20\,\Omega} = 5.0\text{ A}$$

In class Problem

Example 25.4 A source in an open circuit

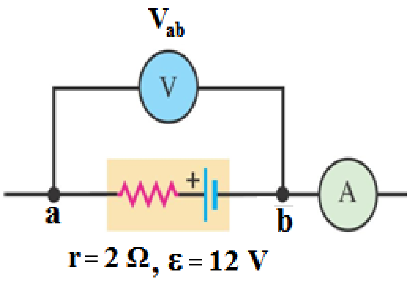


Figure shows a source (a battery) with emf ε = 12V and internal resistance r = 2Ω. The wires to the left of **a** and to the right of the ammeter A are not connected to anything. Determine the respective readings V_{ab} and I of the idealized voltmeter V and the idealized ammeter A.

Solution:

The idealized voltmeter has an infinitely large resistance, no current flows through it.

So, I = 0 because there is no complete circuit.

Hence the ammeter reads I = 0.

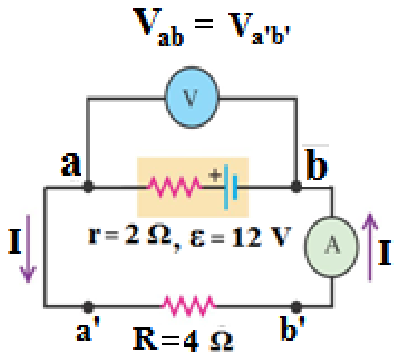
Because there is no current through the battery, there is no potential difference across its internal resistance.

We know that $V_{ab} = \varepsilon - Ir \quad \Rightarrow V_{ab} = \varepsilon - (0)r \quad \Rightarrow V_{ab} = \varepsilon$

So the voltmeter reads $V_{ab} = \varepsilon = 12V$

So, the terminal voltage of a real, non-ideal source equals the emf only if there is no current flow through the source.

Example 25.5 A source in a complete circuit



What are the voltmeter and ammeter readings V_{ab} and I in the circuit shown?

Solution:

The ideal ammeter has zero resistance, so the total resistance external to the source is $R = 4\Omega$

We know that $I = \dfrac{\varepsilon}{R+r} = \dfrac{12.0\text{ V}}{4\,\Omega + 2\,\Omega} = 2\text{ A}$

The resistance of the idealized conducting wires = 0

The resistance of the idealized ammeter = 0

So, potential difference between points a and a’ or between points b and b’ = 0

$V_{ab} = V_{a'b'}$

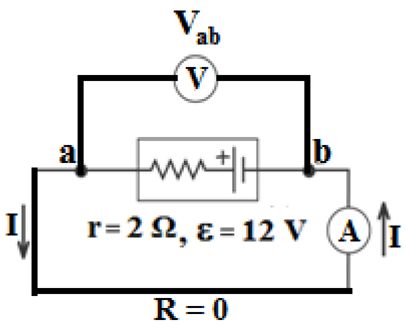
Now, $V_{a'b'}$ = IR = (2A) (4 Ω) = 8V

Alternatively we can calculate the potential difference as:

$$V_{ab} = \varepsilon - Ir \quad \Rightarrow V_{ab} = 12V - (2A)(2\Omega) \quad \Rightarrow V_{ab} = 8V$$

So, the voltmeter reading is 8 V.

Example 25.7 A source with a short circuit



What are the voltmeter and ammeter readings V_{ab} and I in the circuit shown?

Solution:

The terminals of the battery are connected directly to each other.

=> The external-circuit resistance is zero

=> This is called the short circuit and R = 0

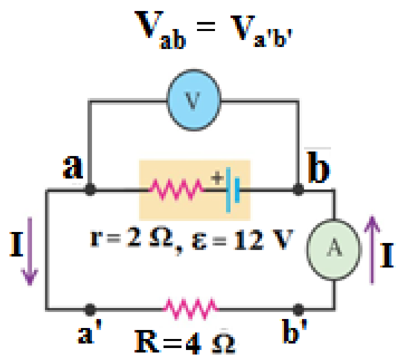
V_{ab} = IR = I (0 Ω) = 0V

Again, $V_{ab} = \varepsilon - Ir \quad \Rightarrow 0 = 12V - I(2\Omega) \quad \Rightarrow I(2\Omega) = 12V \quad \Rightarrow \quad I = 6A$

So, the voltmeter reading is 0 V, and the ammeter reading is 6A

Warning: An automobile battery or a household power line has very small internal resistance (much less than 2 Ω), and the short-circuit current can be great enough to melt a small wire or cause a storage battery to explode. Therefore, short circuits can be dangerous.

Example 25.8: Power input and output in a complete circuit



Find the rates of energy conversion (chemical to electrical) and energy dissipation in the battery, the rate of energy dissipation in the 4 Ω resistor, and the battery's net power output.

Solution:

The current in the circuit is: $I = \dfrac{\varepsilon}{R + r} = \dfrac{12.0\ V}{4\,\Omega + 2\,\Omega} = 2\ A$

Potential difference between 'a' and 'b' is

$$V_{ab} = \varepsilon - Ir \quad \Rightarrow V_{ab} = 12V - (2A)(2\Omega) \quad \Rightarrow V_{ab} = 8V$$

The rates of energy conversion = $P_c = \varepsilon I$ = (12V)(2A) = 24 W

The rate of dissipation of energy in the battery is

$P_d = I^2r = (2A)^2\ (2\ \Omega) = 8\ W$

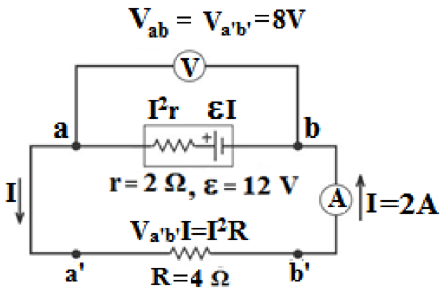
The net electrical power output of the battery is

$P = \varepsilon I - I^2r = 24W - 8W = 16W$

The electrical power input to the 4 Ω resistor is

$V_{a'b'}\ I = (8V)(2A) = 16\ W$

The rate of dissipation of electrical energy in the 4 Ω resistor = $I^2R = (2A)^2\ (4\ \Omega) = 16\ W$

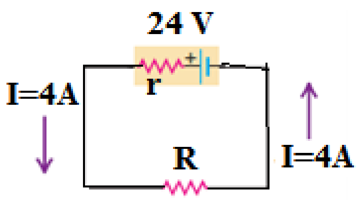


Assignment Problems

25.28: Consider the circuit shown in the figure. The terminal voltage of the 24 V battery is 21.2 V. What are

a) the internal resistance r of the battery and

b) the resistance R of the circuit resistor?



Ans: $\varepsilon = 24$ V, $I = 4$ A , $V_{ab} = 21.2$ V

a) We know that

$V_{ab} = \varepsilon - Ir.$

$$\Rightarrow\ I\,r = \varepsilon - V_{ab} \quad \Rightarrow \quad r = \frac{\varepsilon - V_{ab}}{I} \quad \Rightarrow \quad r = \frac{(24\ V) - (21.2\ V)}{4\ A} = 0.7\ \Omega$$

b) Again, $V_{ab} = IR$, the potential across the circuit resistor

$$. \Rightarrow\ R = \frac{V_{ab}}{I} \quad \Rightarrow \quad R = \frac{21.2\ V}{4\ A} = 5.3\ \Omega$$

25.42: A radio receiver operating at 6 V draws a current of 0.1 A. How much electric energy will it consume in two hours?

Ans: V = 6 V, I = 0.1 A, t = 2 hour = (2 x 3600) s = 7200 s

Electrical energy (E) is

E = V I t = (6 V) (0.1 A) (7200 s) = 4320 J