# Appendix Suggested code structure and experiments for a simple shallow-fluid model

Many NWP courses involve the coding by students of one- or two-dimensional shallow-fluid models, and the use of these models in experiments to evaluate the influence of different numerical methods on model solutions (described in Chapter 3). This allows students to become familiar with the structural components of models, to gain experience in debugging model code, and to conduct experiments to confirm concepts discussed in the text.

This appendix suggests an overall framework for coding the shallow-fluid equations that are described in Section 2.3.3, as well as some experiments that can be part of a laboratory component of an NWP course. Because the specific programming language used will determine the details of the model code, only a high-level outline will be provided here. The best approach is to start with the development of a one-dimensional model. Figure A.1 shows a schematic of the procedure for solving such a system, using an advection equation as a simple example. The abscissa is the space dimension and the ordinate is time. A predictive equation would of course be required for $u$, unless a constant mean speed is employed.

Components, or subroutines, of the model could be organized as follows.

- *Set parameters* – Define physical constants and quantities that establish the structure of the model. These would include the gravitational constant ($g$), the Coriolis parameter ($f$), the grid increment ($\Delta x$), the time step ($\Delta t$), the length of the simulation (timemax), the dimension of the computational array in terms of the number of grid points (*idim*), the output frequency, etc. Additional quantities may need to be defined here, depending on the specific form of the equations being used.
- *Initialization* – Define the initial value of the model dependent variables; $u(1 \rightarrow idim)$, $v(1 \rightarrow idim)$, and $h(1 \rightarrow idim)$.
- *Tendency calculation* – For example,

$$UTEND_i^{\tau} = u_i^{\tau}\frac{u_{i+1}^{\tau} - u_{i-1}^{\tau}}{2\Delta x} + fv_i^{\tau} - g\frac{h_{i+1}^{\tau} - h_{i-1}^{\tau}}{2\Delta x}\text{, for grid points } i = 2 \rightarrow idim - 1.$$

- *Extrapolation* – For example,

$$u_i^{\tau+1} = u_i^{\tau-1} + 2\Delta t\, UTEND_i^{\tau}\text{, for grid points } i = 2 \rightarrow idim - 1. \qquad \text{(A.1)}$$

- *Define lateral-boundary conditions (LBCs)* – constant, periodic, etc., for $i$ equal 1 and *idim*.

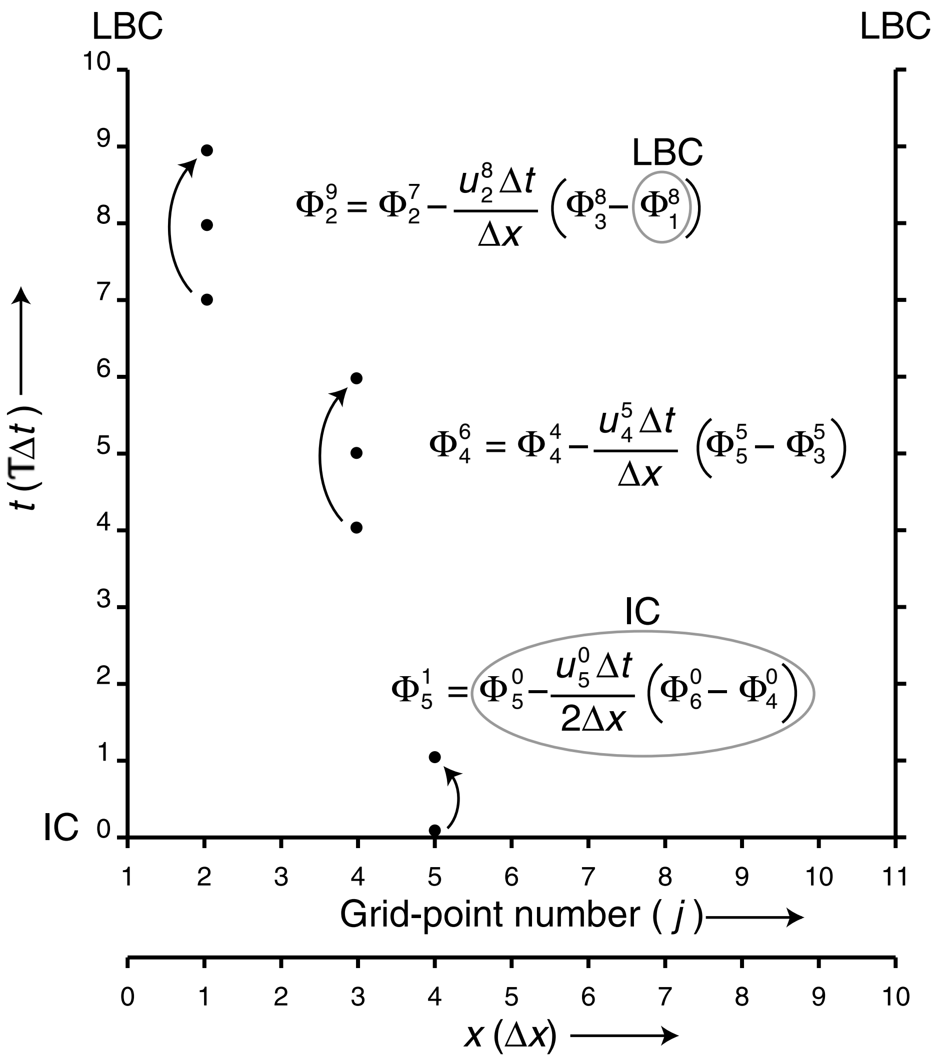


Fig. A.1 Schematic showing the method of solving a one-dimensional shallow-fluid model that is based on an advection equation for a variable $\Phi$ . The subscript is the grid-point number (abscissa) and the superscript is the time-step number (ordinate).

- *Output plotting* – graphical display of dependent variables.
- *Digital save* – archive for analysis, restart. The restart file contains all the information needed to seamlessly start the model in the middle of a simulation in the event of a hardware- or software-related failure. Without this file, the simulation would need to be restarted from the beginning, wasting computing resources. For simple experiments, this capability is typically unnecessary.

Figure A.2 shows a standard sequence in which these operations take place in a model integration.

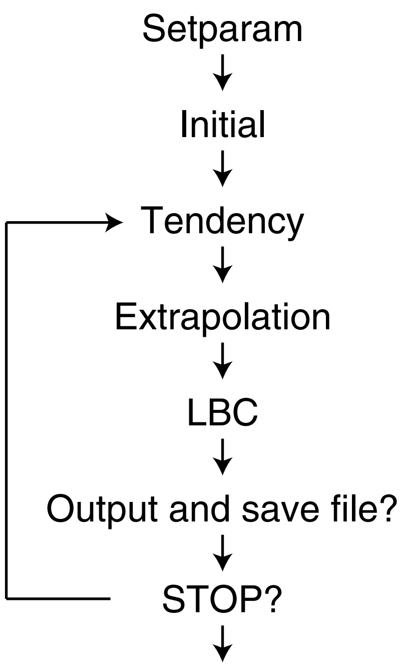


**Fig. A.2** A standard sequence (flow chart) for executing components of a simple model.

The extrapolation step can be problematic to program without the use of temporary arrays. That is, in Eq. A.1 the grid-point number ($i$) can be used as the index of the variable array, but the time-step number ($\tau$) should not also be employed as an array index. This is because the value of a variable does not need to be saved for every time step, and to do so would require the use of a prohibitive amount of storage. Thus temporary arrays can be used in the following way. For each time step, solve

$$ua_i = ub_i + 2\Delta t UTEND_i^{\tau}\text{, for grid points } i = 2 \rightarrow idim - 1\,,$$

where *ua* represents an "advance" value of *u,* and *ub* is a "back" value. After $ua_i$ is calculated for each interior grid point, the following exchange is made:

$$u_i \rightarrow ub_i \quad \text{and}$$

$$ua_i \rightarrow u_i\,.$$

This process is repeated every time step.

Some configuration suggestions follow, for initial experiments.

- The total number of grid points (*idim*) may be 100.
- Let $\Delta x = 10$ km.
- The LBCs should be periodic (cyclic). See Fig. 3.49 for an illustration of how information is exchanged. The boundary values on each edge of the grid are defined based on

the penultimate points on the opposite end. What leaves the grid on one edge comes back in on the other, where essentially the computational domain is wrapped around on itself.

- Choose a midlatitude value for the Coriolis parameter.
- Define the time step to be 80% of the value calculated using the CFL criterion.

Lastly, the following list is of experiments that can be performed with the shallow-fluid model. Note that the model-check-out process extends throughout virtually all of the experiments.

- *Linear advection tests* – The initial experiment consists of the advection of a perturbation in the height field. Use three-point time and space differencing, with no explicit diffusion. Other terms in the equations should not be included, and a constant $u$ should be employed. That is, there is no predictive equation for $u$. Different perturbation shapes can be employed, such as a Gaussian function, a square wave, and a triangular wave. The shapes with first-order discontinuities have more short-wavelength energy, and thus numerical dispersion should be more evident.
- *CFL violation test* – Choose a time step that violates the linear stability criterion, perhaps using a Courant number of 1.1, and print the model solution each time step to observe the instability.
- *Effect of Courant number on the solution* – Using the equation for the linear-advection of $h$, vary the time step over a range of stable values to evaluate how the use of different Courant numbers affects the model solution. Test Courant numbers ranging from 0.1 to 0.99.
- *Diffusion-term tests* – Add an explicit second-order diffusion term to the equation for the linear advection of $h$, and show its effect on the model solution for different stable diffusion coefficients. Repeat with higher-order forms of the diffusion term.
- *Gravity-wave tests* – Use the complete forms of all three predictive equations, and simulate a gravity wave. Define the initial wind components to be zero, and the initial height field as a smooth perturbation (maximum) in the middle of the grid, superimposed on the mean value. The model solution will show the mass being transported in both directions by gravity waves. Repeat the experiment with different mean depths for the fluid.
- *Horizontal-resolution tests* – For the same initial conditions (perturbation wavelength and amplitude), evaluate the effect of horizontal resolution on the model solution. Begin with a grid increment that resolves the wave very well, and progressively increase the grid increment in subsequent experiments.
- *Geostrophic adjustment experiments* – Establish in the initial conditions a geostrophic balance between a perturbation in $h$ (e.g., Gaussian) and the $v$ component of the wind. Run the model to determine if the balance is correct. Then run the model for 48 h with the $h$ perturbation but with an initial $v$ of zero, and observe the adjustment. Perform the analogous experiment with the initial $v$ that is in geostrophic balance with the original $h$ perturbation, but do not include the $h$ perturbation. Again run the simulation for 48 h and observe the adjustment. Perform the experiments for both synoptic-scale and mesoscale perturbations, and observe the differences in the adjustment process.
- *Advanced experiment* – Program a spectral version of the shallow-fluid equations, and compare the solution with the equivalent grid-point model.

Table A.1 summarizes these experiments.

**Table A.1** Suggested experiments with the one-dimensional shallow-fluid model

| Experiment | Initial conditions | Equation set | Comments |
|---|---|---|---|
| **1. Linear advection tests** | | | |
| 1a Wave shape | Gaussian wave | Linear advection of $h$, second-order time and space differencing | Courant number = 0.8 |
| 1b Wave shape | Square wave | same | same |
| 1c Wave shape | Triangular wave | same | same |
| 1d CFL violation test | Gaussian wave | same | Courant number = 1.1 |
| 1e Courant number effect | Gaussian wave | same | Courant number from 0.1 to 0.99 |
| 1f Horizontal resolution tests | Gaussian wave | same | Vary $\Delta x$ so that $L = 4\,\Delta x$ to $L = 20\,\Delta x$ |
| 1g Higher-order differencing | Gaussian wave | Linear advection, higher-order space differencing, and multi-step methods | |
| **2. Diffusion tests** | | | |
| 2a Stable diffusion | Gaussian wave | same + 2nd-order diffusion | Use $K$ that is stable and damps the same fraction of the $2\Delta x$ wave |
| 2b Stable diffusion | Gaussian wave | same + 4th-order diffusion | |
| 2c Stable diffusion | Gaussian wave | same + 6th-order diffusion | |
| 2d Unstable diffusion | Gaussian wave | same + 6th-order diffusion | Use slightly unstable $\Delta t$ |
| **3. Gravity wave tests** | | | |
| 3a Standard depth | Gaussian $h$, $u = v = 0$, $H = 8$ km | Complete, with diffusion | |
| 3b Reduced depth | same, $H = 2$ km | same | |
| **4. Geostrophic-adjustment experiments** | | | |
| Balanced ICs | Gaussian $h$, $u = 0$, $v = v_g$ | Complete, with diffusion | |
| Unbalanced ICs | Gaussian $h$, $u = v = 0$ | Complete, with diffusion | Perform for both synoptic-scale and mesoscale perturbations |
| Unbalanced ICs | $h = u = 0$, $v = v_g$ for Gaussian $h$ | Complete, with diffusion | same |