Energy Conversion and Management 50 (2009) 2593–2599

Contents lists available at ScienceDirect

Energy Conversion and Management

journal homepage: www.elsevier.com/locate/enconman

Portfolio management of hydropower producer via stochastic programming

Hongling Liu *, Chuanwen Jiang, Yan Zhang

Department of Electrical Engineering, Shanghai Jiaotong University, Huashan Road 1954, Shanghai 200030, PR China

ARTICLE INFO

Article history:
Received 10 November 2008
Accepted 8 June 2009
Available online 12 July 2009

Keywords:
Hydropower scheduling
Stochastic programming
Portfolio management
Value of stochastic solution

ABSTRACT

This paper presents a stochastic linear programming framework for the hydropower portfolio management problem with uncertainty in market prices and inflows on medium term. The uncertainty is modeled as a scenario tree using the Monte Carlo simulation method, and the objective is to maximize the expected revenue over the entire scenario tree. The portfolio decisions of the stochastic model are formulated as a tradeoff involving different scenarios. Numerical results illustrate the impact of uncertainty on the portfolio management decisions, and indicate the significant value of stochastic solution.

1. Introduction

With the restructuring of the power industry, the optimal management of hydropower resources has changed to participate in the market with the sole objective of maximizing profits. The economic impacts due to uncertainties are concerned accordingly, for large volatility in market prices and inflows means a large variability in profit. Meanwhile, hydropower producers are required to devise their own strategies on how to allocate generation proportions for exchanges in bilateral contract and spot market on medium term. Thus, it is necessary and important for hydropower producers to incorporate the uncertainty of market prices and inflows into the portfolio management problem.

There are various approaches to deal with the stochasticity in hydropower scheduling problem. The most general way of solving the stochastic scheduling problem is to use stochastic dynamic programming (SDP) to find an optimal operation policy, which usually requires discretization of the state variables [1]. However, SDP becomes intractable for any system with more than a few state variables. Therefore, effective approximation methods have been developed in the literature [2]. Most of these approaches are based on SDP, which are meant to reduce the number of state variables and retain the stochastic structure [3,4].

Another way of dealing with stochasticity is to simplify the representation of the stochastic inflow process through the multiple-scenario methods. Stochastic dual dynamic programming approach (SDDP) further developed this method to model inflow uncertainty, via a combination of sampling and decomposition [5,6]. However, when market price uncertainty is also considered as state variable due to the strong autocorrelation, the objective function is made nonconvex and SDDP can not be used directly. But for each discrete price value in the price model, SDDP is available since the price is fixed. Accordingly, the approach with the combination of SDDP and SDP is proposed [7].

Alternatively, many researchers have come to emphasize the need for applying stochastic programming to hydropower scheduling problem [8–11]. In deregulated power market, portfolio management is coupled with the hydropower scheduling problem. The portfolio exchange of hydropower producer can be regarded as a dynamic problem, for the tradeoff is gained through diversification in bilateral contract market and spot market. Moreover, hydropower scheduling is also a dynamic problem, where a scheduling decision in a given stage is linked to the future consequences of this decision. Thus, for hydropower producer, a stochastic programming approach is appropriate to support the portfolio management. Two-stage stochastic linear programming with recourse has been recognized as a natural way of modeling hydropower operation management problem under uncertainty [12]. When uncertainty is involved, the selected decisions may not be feasible after the realization of random variables. Thus first-stage decisions have to be made without any information on realization of random variables. Next, second-stage decisions, called recourse actions, are made to restore feasibility. In the above framework, the stochastic linear program with recourse model was proposed to formulate the hydropower portfolio management problem.

The present paper does not focus on how to make decision on bilateral contracts [10]. Rather, the key contribution of this paper is to simulate the impact of uncertainty on the electric energy allocation between bilateral contract and spot market, and identify the advantage of proposed stochastic model through the calculation of the value of stochastic solution. The uncertainty in market prices

* Corresponding author. Tel.: +86 21 34204296; fax: +86 21 54747501.
E-mail address: lbliuhl@hotmail.com (H. Liu).

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doi:10.1016/j.enconman.2009.06.010

Nomenclature

t	time period
n	number of periods
V_t	water reservoir level
SP_t	spillage in period t
Q_t	generation in period t
I_t	expected inflow in period t
P_{st}	expected spot price in period t
Q_{bt}	bilateral contract signed for period t
P_{bt}	price for bilateral contract in period t
R_t	average revenue in each period
I_t^s	reservoir inflow associated with scenario s
P_{st}^s	spot price associated with scenario s
Q_t^s	corrective recourse action taken for generation in each period for scenario s
SP_t^s	recourse action taken for spillage in each period for scenario s
V_t^s	recourse action taken for reservoir level in each period for scenario s

and inflows is modeled as scenario tree, and the objective is to maximize the weighted–average profit over the entire scenario set. In practice, stochastic linear programming with discrete random variables leads to deterministic equivalents, which can be dealt with large-scale linear programming method.

The paper is organized as follows. Section 2 provides the stochastic formulation of the uncertainty of market prices and inflows. In Section 3 the medium term portfolio management problem is formulated based on stochastic program with recourse model. Numerical results are shown and discussed in Section 4. Finally, the conclusion is presented in Section 5.

2. Stochastic formulation

Without specifying the probability distribution of uncertainties, we even cannot formulate the portfolio management problem mathematically. To solve the corresponding stochastic programming problem, we generally resort to constructing scenarios. Each possible discrete outcome of market prices and inflows is called a scenario, and the generated set of scenarios, with the corresponding probabilities, can be viewed as a representation of the underlying probability distribution.

The generation of scenarios involves considerable effort in stochastic programming models. It is important to note that the decision made now will be based solely on the information available at the point of decision. Thus, the stochastic input model is needed to represent a well-defined problem, producing a nonanticipative solution that is in some sense optimal. For this purpose, a scenario tree which can represent the dynamic information about uncertainty in market prices and inflows should be considered. Fig. 1 shows the process how the information about uncertainty factor is modeled via a five-stage scenario tree, in which two branches leave each node, resulting in 16 scenarios.

The root node represents the decision today and hence is considered deterministic. The nodes further down represent conditional decisions at later stages. The arcs linking the nodes represent realization of the uncertain variables. In this way a scenario tree captures the dynamic of decision making since the decisions are adjusted to the currently available information.

For scenario construction, a survey of methods of generating sets of scenarios that form an approximation of the underlying random data process is given in [13]. For a decision maker choosing to incorporate the uncertainty into stochastic programming model, the uncertainty parameters must be specified. Sampling from historical time series or from statistical models is the most popular method for generating scenarios [14]. However, precise statistical characterization of the spot price and the inflow is not the main emphasis of this paper. In this study, the market price and inflow in each period under consideration are discretized based on the forecasted values of the expected price and inflow and their standard deviations, which are assumed to be calculated by applying techniques such as time series and artificial method network [15]. In this paper, the Monte Carlo simulation method is applied to generate scenarios [16].

Note that different scenario set would have impact on the portfolio decisions, which needs to be simulated. As can be known, the variation of inflows of historical years can be categorized as wet, normal and dry representative year, respectively. The inflows of years can be regarded as the cyclical among these three representative years. Thus, we can generate scenario set based on the specified statistical properties of representative year inflows. On the other hand, decisions made only on forecast value of uncertainty might be a bias, if the actual state of nature turns out to be different from the specification of the model input. From this prospect, we can generate scenario set based on extensive distribution properties of uncertainty, which is derived from historical inflows data. This approach would incorporate the forecast variation into the scenarios. Accordingly, the resulting market price and inflow scenarios are used as the input for the stochastic linear program with recourse model which determines an optimal strategy.

3. Portfolio management problem formulation based on stochastic program with recourse model

The philosophy of stochastic programming with recourse is that decisions and observations alternate, but that some decisions have to be taken that do not anticipate future data of the model. At the time these data are known, second-stage decisions follow which exploit the additional information and depend on the variables that were fixed in the first-stage.

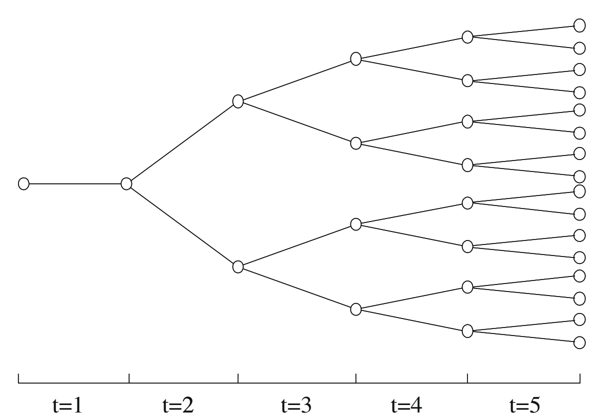

Fig. 1. Scenario tree example.

3.1. Objective Function

For a hydropower producer seeking to maximize the revenue while taking into consideration the expected recourse, the scenario aggregation method is formulated as follows:

$$\max \sum_{t=1}^{T} R_t = \sum_{t=1}^{T} [P_{st} * (k * Q_t - Q_{bt}) + P_{bt} * Q_{bt}] + \sum_{s \in S} \text{Prob}^s \sum_{t=1}^{T} P_{st}^s * k * Q_t^s \tag{1}$$

The objective function (1) is composed of portfolio decisions in the first-stage and the corrective recourse actions in the second-stage. The concept of utilizing scenarios adds another dimension to the medium term hydropower portfolio solution.

3.2. Constraints

(1) Water reservoir balance
For each scenario, water reservoir level at the end of a period depends on the water reservoir level at the beginning of the period, inflow during the period, and discharge for generation in the period and the spillage in the period.

$$Q_t + Q_t^s + SP_t^s - V_{t-1}^s + V_t^s = I_t^s \tag{2}$$

(2) Generation upper and lower limit

$$Q_{\min} \leqslant Q_t \leqslant Q_{\max} \tag{3}$$

(3) Stochastic generation balance

$$Q_{\min} \leqslant Q_t + Q_t^s \leqslant Q_{\max} \tag{4}$$

(4) Spillage upper and lower limit

$$0 \leqslant SP_t^s \leqslant V_{\max} \tag{5}$$

(5) Reservoir upper and lower limit

$$V_{\min} \leqslant V_t^s \leqslant V_{\max} \tag{6}$$

(6) Contract upper and lower limit

$$0 \leqslant Q_{bt} \leqslant k * (Q_t^{mean} + Q_t) \tag{7}$$

It is assumed that the amount of electric energy allocated in the bilateral market is limited to the generation associated with the scenario, which is corresponding to the mean value of uncertainty.

(7) Nonanticipativity constraints

The nonanticipativity principle is stated as following: if two different scenarios S and S' are identical to stage t on the basis of the information available about them at stage t, then the values for the decision variables must be identical up to stage t [17]. This can be represented as:

$$\begin{aligned} Q_t^s &= Q_t^{s'} \\ SP_t^s &= SP_t^{s'} \\ V_t^s &= V_t^{s'} \end{aligned} \tag{8}$$

In th stochastic programming model, constraints are formulated for each node in the scenario tree, and the nonanticipativity constraints are enforced implicitly accordingly. Note that our mathematical model is a generalization of medium term hydropower scheduling problem based on the simulated market price and inflow scenarios. Accordingly, our scenario set can include the most-wet scenario, the worst–dry scenario, and other possible scenarios. In this way, the mathematical formulation of stochastic linear program with recourse model with discrete random variables lead to a deterministic equivalent of large size which can be solved by commercially available software.

4. Numerical examples

We applied our approach to a model based on the generating and trading system of an electric utility in the regional power market of China. A power plant is considered to be participating in the market for a midterm period of 12 months, with the specification as shown in Table 1, which is referred to as power plant X hereafter. Assuming the producer considered in this paper is a price-taker, the specification of market prices could be the same for different case study, which is shown in Table 2.

Here, the study horizon was split into six periods each of two months and 243 scenarios are considered. We created scenarios based on historical data and each possible scenario is considered with a weight of 1/243. To create scenario bundles, we assumed that the uncertainty of inflows and market prices over first two months for all scenarios are the same as that of the forecasted value. We assumed that market prices and inflows have a normal distribution, however, other distribution properties could be considered. Then, Monte Carlo simulation is executed to generate 243 scenarios for prices and inflows based on the discussed

Table 1
Specifications of hydropower plant X.

Specification	Values
Minimum generation (m^3s^{-1})	192
Maximum generation (m^3s^{-1})	2226
Average energy equivalent (MW/m^3s^{-1})	1.48
Minimum reservoir level (10^8 m^3)	24.3
Maximum reservoir level (10^8 m^3)	57.93
Initial reservoir level (10^8 m^3)	57.9

Table 2
Details of price forecast and variations.

Period	Forecasted price (RMB/MWh)	Maximum (RMB/MWh)	Minimum (RMB/MWh)	Standard deviation (RMB/MWh)
1	285.3	–	–	–
2	277.3	–	–	–
3	272.4	450	200	25.80
4	285.5	450	200	40.47
5	333.5	450	200	29.08
6	365.1	450	200	36.25
7	425.0	450	200	24.53
8	404.5	450	200	40.94
9	405.4	450	200	31.35
10	381.8	450	200	45.80
11	361.2	450	200	45.61
12	324.1	450	200	37.81

Table 3
Details of Inflow forecast and variations of wet represent year.

Period	Forecasted inflow (m^3s^{-1})	Maximum (m^3s^{-1})	Minimum (m^3s^{-1})	Standard deviation (m^3s^{-1})
1	532	–	–	–
2	467	–	–	–
3	453	534	383	46.04
4	564	858	473	100.82
5	846	1180	557	164.44
6	2220	3630	973	746.46
7	4239	6110	2540	1106.18
8	4979	7070	3410	1069.92
9	4449	5710	2990	748
10	2889	3610	1900	510.64
11	1329	1620	1050	161.43
12	805	944	682	70.46

Table 4
Details of inflow forecast and variations of normal representative year.

Period	Forecasted inflow (m^3s^{-1})	Maximum (m^3s^{-1})	Minimum (m^3s^{-1})	Standard deviation (m^3s^{-1})
1	511	–	–	–
2	445	–	–	–
3	431	505	384	42.65
4	532	683	433	75.44
5	809	1160	544	180.03
6	1948	2820	1200	550
7	3863	5620	2360	1005.58
8	3196	4300	2610	592.52
9	3582	4370	2610	497.75
10	2418	3730	1670	527.29
11	1162	1520	925	145.02
12	718	866	618	62.31

Table 5
Details of inflow forecast and variations of dry representative year.

Period	Forecasted inflow (m^3s^{-1})	Maximum (m^3s^{-1})	Minimum (m^3s^{-1})	Standard deviation (m^3s^{-1})
1	520	–	–	–
2	455	–	–	–
3	451	496	378	32.69
4	537	711	458	72.01
5	836	1040	455	154.12
6	1749	2470	932	458.73
7	2884	4130	1970	662.36
8	2571	3380	1790	400.08
9	2699	4270	1940	670.24
10	1865	2830	1350	348.58
11	992	1170	831	101.84
12	617	707	558	46.19

Table 6
Stochastic versus expected value policy for three representative year scenario sets.

Scenario set of representative year	Stochastic (RMB)	Expected (RMB)	$E(\Delta)$ (RMB)	$E(\Delta)$ (%)
Wet	105,92,990	103,76,223	216,767	2.05
Normal	100,90,026	98,60,289	229,737	2.28
Dry	98,87,477	96,29,650	257,827	2.61

methods in Section 2. The corresponding portfolio optimization problem was solved using the large-scale linear programming algorithm in Matlab optimization toolbox ("linprog" function) [18]. Furthermore, to identify the value of the proposed stochastic program model, the impact of uncertainty on portfolio management decisions and the value of stochastic solution are discussed.

4.1. Results of case study 1: three representative year scenario sets

From the experienced inflow data, three representative years can be categorized. The details of inflow forecast and variation of three representative years are shown in Tables 3–5.

4.1.1. Calculation of value of stochastic solution

Expected value policy was computed with the forecast value of price and inflow. For each model, we apply the expected value policy to all available scenarios and compute the expected value of stochastic program model. We compare the stochastic policy and the expected value policy in Table 6. The difference between the "Expected" value of column and the "Stochastic" column is called the value of the stochastic solution and is denoted by $E(\Delta)$ [19]. We also provide the value of stochastic solution as a percentage of the generation revenue of the stochastic policy in the last column of Table 6.

The value of stochastic solution assesses the possible gain from solving the stochastic program model. From the table, clearly, the stochastic program model provides gaining of approximately 2% over the expected value policy.

4.1.2. Impact of uncertainty on portfolio management decisions

Table 7 shows the different portfolio decisions of the expected value model and the stochastic programming model, which corresponds to three representative scenario sets. The bilateral contract prices are assumed equal to the average of all expected prices, i.e., 343.43 RMB/MWh, and negative and positive trade results of spot market represent buying and selling in the spot market, respectively. The column labeled "$k * Q_t - Q_{bt}$" provides the portfolio exchange in spot market.

Table 7 indicates that when uncertainty is considered, the electric energy would be more traded in contract and spot market. This is because that the portfolio decisions should be made in advance as the first-stage decisions in the stochastic case. To guarantee revenue, the producer would sign more stable bilateral contracts, which are even so many that the shortage beyond its generation planning will be supplied in the spot market. Compared to the expected value case, the stochastic case performs higher expected revenue, as the portfolio decisions are balanced with reference to each probable scenario. It also can be seen that variation for the portfolio decisions suggested by stochastic case results in higher variance.

Table 7
Portfolio decisions comparison for stochastic program model and expected value model for three representative year scenario sets.

Period	Wet				Normal				Dry			
	Expected		Stochastic		Expected		Stochastic		Expected		Stochastic	
	Q_{bt}	$k * Q_t - Q_{bt}$	Q_{bt}	$k * Q_t - Q_{bt}$	Q_{bt}	$k * Q_t - Q_{bt}$	Q_{bt}	$k * Q_t - Q_{bt}$	Q_{bt}	$k * Q_t - Q_{bt}$	Q_{bt}	$k * Q_t - Q_{bt}$
3	1162	1054	2595	−2311	1086	714	2561	−2277	1040	552	2581	−2297
4	284	3010	825	−541	284	3010	772	−488	284	3010	913	−629
5	2215	1079	1340	−1056	1799	1495	1396	−1112	1593	1701	1285	−1001
6	0	3294	0	284	0	3294	0	284	0	3294	0	284
7	0	3294	0	284	0	3294	0	284	0	3294	0	284
8	0	3294	0	284	0	3294	0	284	0	3294	0	284
9	0	3294	0	284	0	3294	0	284	0	3294	0	284
10	0	3294	0	284	0	3294	0	284	0	3294	0	284
11	0	3294	0	284	0	3294	0	284	0	2853	0	284
12	1784	0	3115	−2831	1408	0	2997	−2713	914	0	2791	−2507
Exp (RMB)	101,52,218		105,92,990		98,24,459		100,90,026		94,28,467		98,87,477	
Std (RMB)	270,784		354,397		270,763		388,446		260,930		451,056	

Note that the expected revenue of normal and dry scenario set is less than the revenue reported in wet scenario set. The reason is that ample inflows guarantee more revenue obtained through trading more in bilateral contract and spot market.

Figs. 2–4 show more clearly the variation of revenue of the expected value model and the stochastic programming model. For the expected value model, the variance is low as well as the expected revenue, which represents a lower variability in revenue. In contrast, the variation of portfolio decisions suggested by stochastic program model results in higher total expected revenue as well as higher variability in revenue.

4.2. Results of case study 2: extensive scenario set

To further test the performance of our approach as a function of scenario set, in this case, extensive scenario set is generated based on the analysis of the total historical inflow data. The details of inflow forecast and variations are shown in Table 8.

4.2.1. Calculation of value of stochastic solution

Table 9 shows the comparison between the expected value policy and the stochastic policy. The stochastic policy provides savings of approximately 3% over the expected value policy. Compared with the results in case 1, the value of knowing and using distribution on future outcomes is more significant. As a matter of fact, the stochastic program approach is appropriate.

4.2.2. Impact of uncertainty on portfolio management decisions

Table 10 shows the difference between the expected value model and the stochastic program model, which corresponds to the extensive scenario set. The comparison of Tables 7 and 10 show that the performance of stochastic case is consistent to different

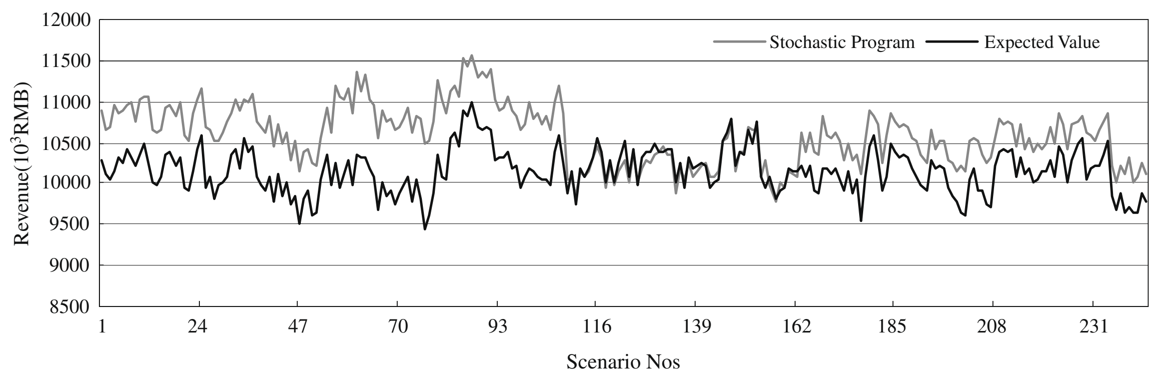

Fig. 2. Revenue comparison for stochastic program model and expected value model for wet representative scenario set.

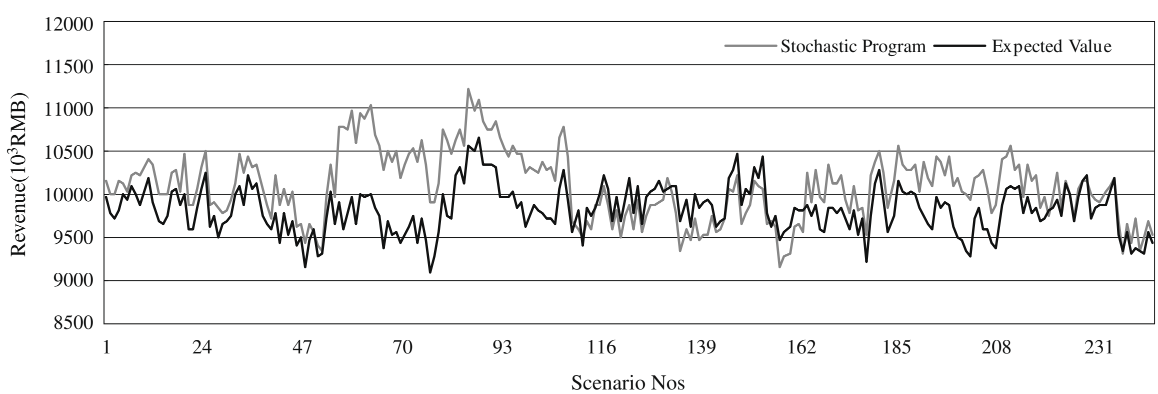

Fig. 3. Revenue comparison for stochastic program model and expected value model for normal representative scenario set.

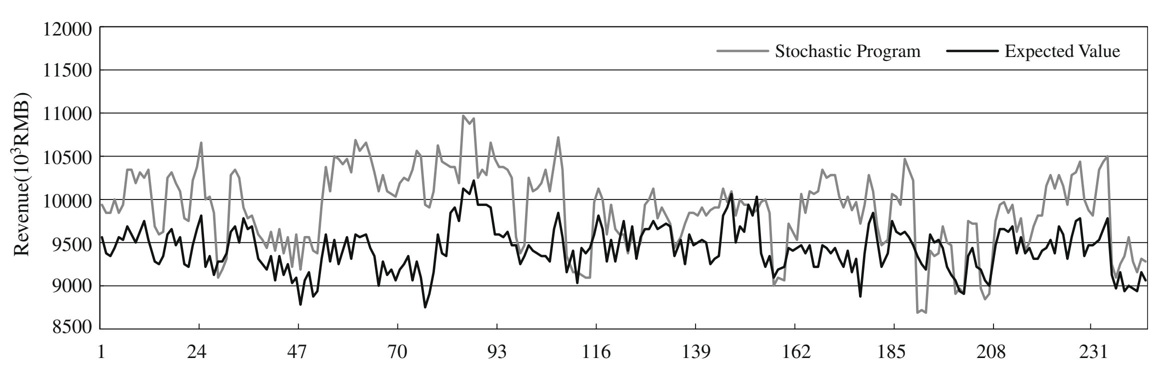

Fig. 4. Revenue comparison for stochastic program model and expected value model for dry representative scenario set.

Table 8
Details of inflow forecast and variations of extensive scenario set.

Period	Forecasted inflow (m^3s^{-1})	Maximum (m^3s^{-1})	Minimum (m^3s^{-1})	Standard deviation (m^3s^{-1})
1	520	–	–	–
2	456	–	–	–
3	444	534	378	40.58
4	541	858	433	82.58
5	836	1180	455	162.31
6	2057	3630	932	611.00
7	3716	6110	1970	1084.58
8	3588	7070	1790	1263.22
9	3608	5710	1940	990.60
10	2310	3730	1350	625.31
11	1148	1620	831	194.68
12	708	944	558	98.72

Table 9
Stochastic versus expected value policy for extensive scenario set.

Scenarios	Stochastic (RMB)	Expected (RMB)	$E(\Delta)$ (RMB)	$E(\Delta)$ (%)
243	102,30,764	99,00,130	330,634	3.23

Table 10
Portfolio decisions comparison for stochastic program model and expected value model for extensive scenario set.

Period	Expected Value		Stochastic Program	
	Q_{bt}	$k * Q_t - Q_{bt}$	Q_{bt}	$k * Q_t - Q_{bt}$
3	1114	878	2588	−2304
4	284	3010	741	−457
5	1992	1302	1157	−873
6	0	3294	0	284
7	0	3294	0	284
8	0	3294	0	284
9	0	3294	0	284
10	0	3294	0	284
11	0	3294	0	284
12	1373	0	2911	−2627
Exp (RMB)	99,00,130		102,30,764	
Std (RMB)	270,784		374,598	

scenario set, for the portfolio decisions should be balanced among scenarios. As expected, the stochastic case performs better than the expected value case as the uncertainty is represented. The results further identify the effective of the proposed stochastic model.

Fig. 5 shows more clearly the variance of the revenue, which is corresponding to the case depicted in Table 10. As a matter of fact, the total expected revenue is higher for stochastic program model but so is the variance.

5. Conclusions

This paper presents a stochastic program with recourse model for portfolio management problem on medium term. The formulation of the stochastic inflows and market prices as scenarios in the model constitutes a flexible and practical way of handing stochasticity. The optimal electric energy allocation between bilateral contract and spot market is decided in advance as the first-stage decisions, which is the balance with reference to each possible scenario. Numerical results illustrate that it is necessary to consider the uncertainty in inflows and market prices and incorporate the impact of uncertainties on the portfolio management problem.

The coupling relation among consecutive inflows and market prices could be further considered in the formulation of portfolio management problem. This could be implemented by different uncertainty simulation approaches. Note that, however, these approaches would only impact the input to the stochastic model. Furthermore, as the results have shown, the stochastic model is dedicated a higher expected revenue, but not to a more stable revenue. This indicates that hydropower producer could be exposed to a significant risk level for the variability of uncertainties. Thus, the objective of maximizing expected revenue at some acceptable risk level could provide more insight for hydropower producers.

Acknowledgements

The authors gratefully acknowledge the support of Yalong River joint research foundation of National Natural Science Foundation of China (NSFC) and Ertan Hydropower Development Company. LTD (No.: 50539140).

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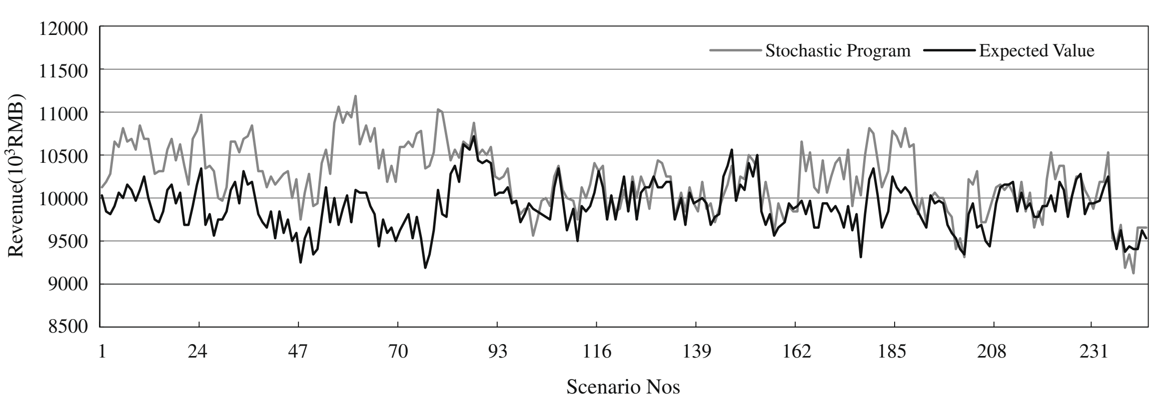

Fig. 5. Revenue comparison for stochastic program model and expected value model for extensive scenario set.

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