Chapter 1 : Mechanics (Excluding Relativity)

From the book : International Physics Olympiad - Training and Selection, 2nd Edi.

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Problems

1. At a distance of d from a high wall with a height of h, throw a small ball from the ground over the wall, try to find the required minimum initial velocity $v_{0\ \min}$ and the corresponding throwing angle θ.

2. A lamp is hung under the ceiling at h_1, the height of the lamp from the ground is h_2, the bulb suddenly explodes into many fragments, all the fragments fly away in all directions with the same initial velocity v_0. If the collision between the fragments and the ceiling is elastic, the fragments do not bounce back when they hit the ground, and they do not hit the wall, find the radius of the area where the fragments fall.

3. Someone is playing a shooting game while standing at a distance of d from a high wall. The initial velocity of each shot he fires is v_0.

   (a) He shoots multiple bullets to hit the same horizontal line on the wall at a height of h from the ground, find the trajectory of the bullet aiming point on the wall.

   (b) If the bullet hits the wall above this horizontal line, find the range of the aiming point.

4. Fix a baffle perpendicular to the inclined plane at the bottom of the inclined plane with an inclination angle of $\alpha = 30°$, and release a small ball from rest at point A in the space above the inclined plane. The ball falls freely, collides with the slope and the baffle and returns to point A. It is known that the vertical distance from the A point to the inclined plane is $h = 1.0$ m, the collision between the ball and the inclined plane and the baffle is elastic, and the collision between the ball and the baffle does not exceed twice. Find:

   (a) The vertical distance L between the A point and the baffle.

   (b) The elapsed time t from the release of the ball from the point A to the point where it returns to the point A.

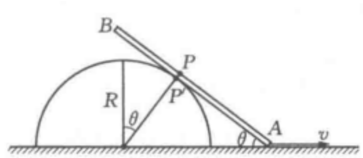
5. An aircraft is flying at a constant speed of v_0 at a height of h from the ground, and drops a bomb on the ground target.

(a) In order for the bomb to hit a target on the ground, what is the horizontal distance L from the ground target at which the aircraft should drop the bomb?

(b) If there is a gun at a distance of d from the ground target, a projectile is fired at the same time as the bomb is dropped by the aircraft, as shown. In order for the projectile to hit the bomb dropped by the aircraft in the air, try to find the smallest initial velocity $v_{\min}$ and corresponding launch angle.

6. A circle of radius R is in constant translation to the right with velocity v_1 in the paper, and the straight line MN is uniformly moving upwards with velocity v_2. When the angle between the line OP connecting the intersection point P of the straight line and the circle and the center O of the circle and the straight line MN is θ, find the speed and acceleration of the position change of point P.

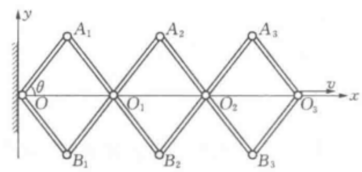
7. A thin rod AB rests obliquely on a fixed semi-cylindrical surface with radius R, and the end of A that is in contact with the ground moves in a straight line at a constant speed v along the horizontal plane, as shown in the figure. When the angle with the ground is θ, try to find:



(a) The velocity v_P of the point P on the rod in contact with the cylindrical surface.

(b) The velocity of the position change $v_{P'}$ of the intersection point P' of the rod and the cylinder.

8. The hinge member is composed of 4 thin rods with a length of $2l$ and a length of l, and each intersection is connected by a hinge, as shown in the figure. Now the hinge point O_3 moves in the direction of x at a constant speed v. When each rod forms an angle θ with the axis of x, find:

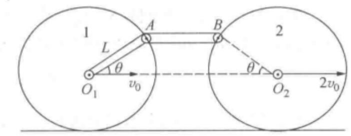


(a) The value of the velocity and acceleration of the hinges A_1 and B_3.

(b) Rotational angular velocity and angular acceleration of thin rod A_3B_2.

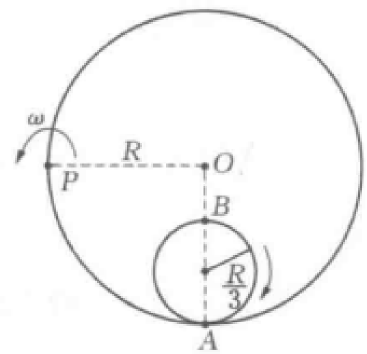
(c) The equation of motion trajectory of each hinge.

9. Two wheels 1 and 2 with a radius of L are placed on the horizontal ground and in the same vertical plane. Two thin rods with a length of L are connected at one end with a hinge A, and the other ends are connected with a hinge at 1 on the axle O_1 of the wheel and on the point B of the wheel side of the 2nd wheel, as shown in the figure. The two wheels roll along the ground at a constant speed, and the speeds of the centers O_1 and O_2 of the two wheels 1 and 2 are v_0 and $2v_0$ respectively. When moving to the position shown in the figure, the AB rod is horizontal, and O_1A and BO_2 are both at angle $\theta = 30°$ in the horizontal direction. At this time, try to find the speed and acceleration of hinge A. Note: Hinge A is not in contact with the edge of wheel 1.

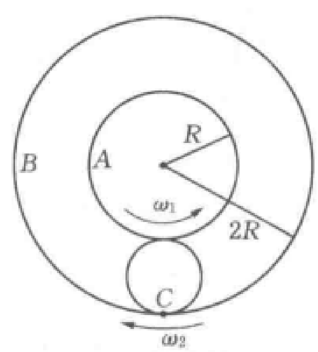


10. A large steel ring of radius R rotates counterclockwise around a point P in the ring plane with a constant angular velocity ω. Another small steel ring with a radius of $R/3$ rolls without sliding along the inside of the large steel ring in the same plane, and the rolling direction is as shown in the figure. When the big steel ring rotates around the P point, the small steel ring rotates exactly two weeks relative to the large steel ring. When the small steel ring moves relative to the large steel

ring to the position shown in the figure, try to find the accelerations of the two points on the small steel ring, A and B, respectively.



11. Between the walls of two coaxial thin-walled cylinders A and B with radii R and $2R$ respectively, there is a small cylinder with a radius just $\frac{1}{2}R$. When the two cylinders A and B rotate uniformly in opposite directions at the angular velocities of ω_1 and ω_2 respectively, the small cylinder also rotates, and when the small cylinder rotates, there is no relative sliding between the contact points of the two cylinders A, B, as shown in the figure. Try to find:



(a) How much time does it take for the small cylinder to move around the ground and relative to the B cylinder?

(b) What is the acceleration of the contact point C on the small cylinder with the B cylinder relative to the ground and relative to the A cylinder?

12. A smooth oblique rod AB of length l forms a fixed angle α with a straight axis passing through its A end, and rotates with a constant angular velocity ω about this straight axis. Now, a small ring of mass m is placed on the rod, and the small ring starts to move from the end of A with an upward initial velocity v_0 relative to the rod. Try to find:

    (a) How big should v_0 be to reach the B end?

    (b) The force acting on the rod when the ring reaches the end of B.

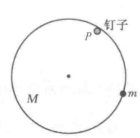
13. A large horizontal disk of radius R rotates with angular velocity Ω around its straight axis passing through the center of the circle at a constant speed. A small disk of radius r and mass m is gently placed on the large disc, the distance between the two disc axes after stabilization is $d(d > r, R > d + r)$. The known friction coefficient between the two discs is μ.

    (a) Try to find the angular velocity ω of the small disc rotating steadily.

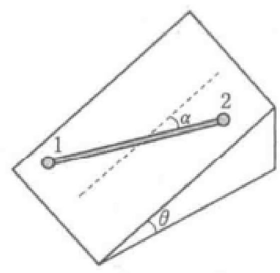
    (b) How much torque must be applied to the big disc to keep the rotational speed of the big disc constant?

14. Two identical balls A and B of mass M and radius R are suspended from the same point O by two ropes of length $l = 2R$, and a mass of m $(m = nM)$ (A small sphere C with r $(r = R/2)$) is placed on the two balls. It is known that the surfaces of the three spheres are smooth. When the system is in equilibrium, try to discuss the relationship between the angle θ between the rope and the vertical direction and n.

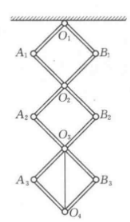
15. A small ball of mass m is fastened to a large ring of mass M. Hang the ring on a rough nail, as shown. What is the minimum coefficient of friction μ between the ring and the nail to keep the ring in balance so that any point on the ring (except where the ball is) hangs on the nail?



16. Two balls 1 and 2 of equal mass are connected by a light rope and placed on an inclined plane with an inclination angle of θ as shown in the figure. The friction coefficients between balls 1 and 2 and the inclined plane are μ_1 and μ_2, respectively. It is known that $\mu_1 < \mu_2$, and $\tan\theta = \sqrt{\mu_1\mu_2}$. Find the maximum value of the angle α between the rope and the line of maximum inclination of the inclined plane when the system is in equilibrium.

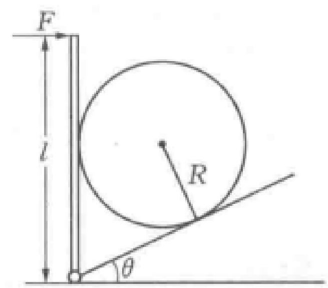


17. The hinge component consists of 4 homogeneous rods with a length of $2l$ and a mass of $2m$ and 4 homogeneous rods with a length of m and a mass of m. There are 10 lightweight smooth hinges in total. O_1 is suspended on the horizontal axis, and a rope is connected between the hinges O_3, O_4, and the rope length is $\sqrt{2}l$. The angle between the two rods is 90°, as shown in the figure. Try to find the tension in the rope at equilibrium.

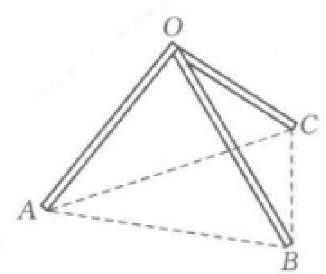


18. Connect a solid straight plate of length l with a smooth hinge at the bottom end of the inclined plane with an inclination angle θ, and sandwich a sphere of radius R between the straight plate and the inclined plane. For a sphere of mass m, a horizontal force F perpendicular to the plate is applied to the top of the plate to keep the sphere and plate in balance, as shown in the figure. The known friction coefficient between the plate and the sphere is μ_1, the friction coefficient between

the sphere and the inclined plane is μ_2, and now F is increasing continuously. How much will the balance of the system be destroyed when F increases?



19. Three identical homogeneous rods of mass m and length l are placed together as shown in the figure, and the line connecting the three rods and the ground contact points forms an equilateral triangle with a length of l on one side. It is known that the coefficient of friction between the three rods and the ground is equal.



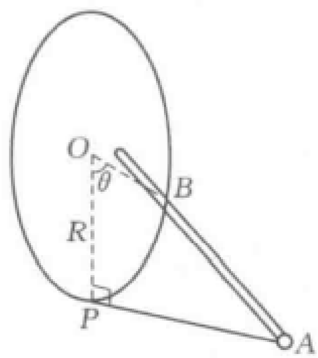
(a) Find the magnitude and direction of the force acting on the vertices of the rod OA.

(b) If a small ball of mass m is fixed at the midpoint of the OA rod, then find the magnitude and direction of the force acting on its top.

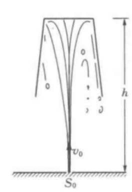
(c) What is the minimum coefficient of friction between the rod and the ground for the system to remain stationary?

20. A ring of radius R stands upright at point P on the ground, AP is a straight line on the ground perpendicular to the plane of the ring,

$\overline{AP} = 3R/2$, one end of a thin rod is connected to point A by a hinge, and the shaft is held at point B. The angle $\theta = 60°$ is between the radius OB passing through the point B and OP, as shown in the figure. In order to make the rod balance at this position, try to find the minimum friction coefficient between the rod and the ring.



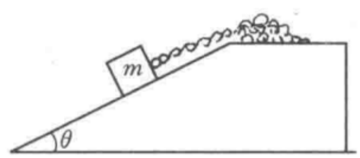
21. The water column on the ground spewed from the spring, upturned a trash can with mass M in the air. It is known that the velocity of the water column when it is sprayed from the fountain hole with area S_0 is v_0, and shot straight into the air. After impacting the inner bottom of the trash can, half of the mass of water was adsorbed on the inner bottom of the trash can, and flowed down the inner wall of the bucket, while the other half of the mass of water is going down at the original speed, as shown in the figure. Try to find the height h where the trash can stays in the air. Set the density of water to be ρ.



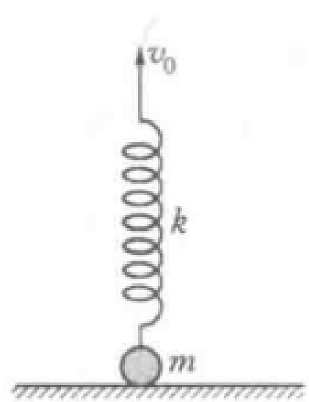
22. A very small spherical raindrop condenses into nuclei in the uniform static cloud layer, starts to fall from the static state, and absorbs all the fog it encounters through it, so that its volume continues to increase, and it still maintains a spherical shape. If the viscous resistance of

raindrops in motion is not considered, prove that raindrops will tend to fall with uniform acceleration, and obtain this acceleration.

23. A block of mass m is attached to a rope of mass linear density λ. Initially, the block is at the top of a slope with an angle of θ, and the rope is coiled around the top of the slope. The slope and the platform are known to be smooth. Now release the block and let it slide down the slope, as shown in the figure. Find the speed of block when it slides a distance x down the slope (x is less than the length of the rope).

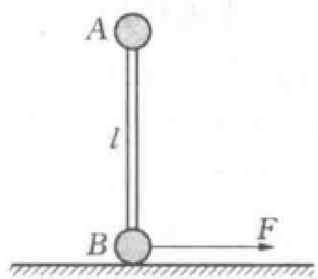


24. There is a small ball with a mass of m on the table, and a vertical light spring with a stiffness coefficient of k is connected to it. At the beginning, the spring is in the original length state, and its upper end is under the action of an external force and moves at a uniform speed of v_0 upwards, as shown in the figure. Find the work done by the force acting on the upper end of the spring from the start of motion at the upper end of the spring until the spring reaches its maximum elongation for the first time.



25. Small balls A and B of mass m are connected by a light rod of length l, standing upright on a smooth horizontal surface, as shown in the figure.

If a horizontal constant force F acts on the lower ball B to make the B ball move a certain distance, then the angle between the rod and the horizontal direction is θ, find the force acting on B at this time.



26. An object of mass m is thrown straight up at a certain initial velocity, and the wind blows horizontally at a constant speed u, the air resistance on the object is proportional to the speed of the object relative to the air, which can be expressed as $f = -kv$. After the time τ the object returns to the ground, the landing point of the object is S away from the throwing point, and the sub-velocity in the vertical direction when landing is less than Δv when it is thrown. Try to find the work done by air resistance in entire motion process.

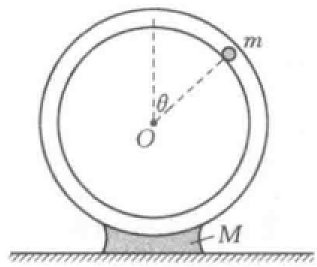
27. A small ring A of mass m is set in a smooth horizontal fixed rod, and connected by a string of length l to a small ball B of mass m. First pull the rope to the horizontal direction, and then release the system from rest, and try to find:

    (a) What is the angle θ between the rope and the horizontal rod, when the speed of the ball is maximum? Find the maximum speed.

    (b) What is the tension in the rope when the ball is at maximum speed?

28. A hollow thin tube is bent into a ring of radius R, the ring is fixed upright on a slider placed on a smooth horizontal plane, and the total mass of the ring and the slider is M. There is a small ball of mass m in the lumen(cavity) of the ring, which can move along the lumen without friction, as shown in the figure. At first, the ball is located at the highest point of the lumen and the ring and the ball are stationary.

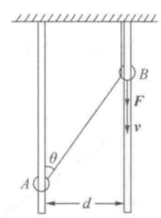
Then the ball slides down the lumen from the right under a slight disturbance, and when it turns around the center O by angle $\theta = \frac{\pi}{4}$ (relative to ring), the speed of the ring moving to the left reaches the maximum. Try to find:



(a) The ratio of the mass of the ring to the ball $\frac{M}{m}$ and the maximum speed of the ring moving to the left.

(b) The radius of curvature of the ball's trajectory here.

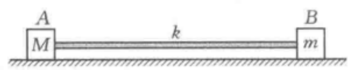
29. Two smooth thin rods are fixed straight to the ceiling, the distance between the two rods is d, a small ring A, B of mass m_1 and m_2 is set on each of the two rods. A rope passes through the B ring, and its one end is connected to the ceiling next to the top of the pole where the B ring is located and the other end is connected to the A ring, as shown in the figure. Under the action of the external force F down, the B ring moves downward with a constant velocity v. Try to find:



(a) The value of F when the angle between the rope between the two rings and the vertical rod is θ.

(b) The work done by the force of F in the process from $\theta_1 = 37°$ to $\theta_2 = 53°$. For this sub-problem, set $m_1 = 2m$, $m_2 = m, v = \sqrt{\frac{12}{11}gd}$.

30. Two blocks A and B with masses $M = 3.60$ kg and $m = 1.80$ kg, respectively, are connected to the two ends of the elastic rope with original length $l_0 = 0.300$ m and the stiffness coefficient $k = 24.0$ N/m, and placed on the horizontal table top, as shown in the figure. It is known that the friction coefficient between the two blocks and the table is $\mu = 0.300$. Now pull the two blocks apart to a distance of $l = 1.200$ m, and release them from rest. Try to find:



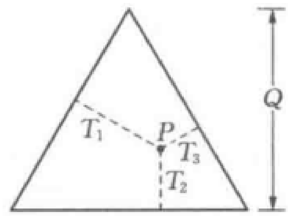
(a) The velocity of two blocks v_A and v_B when they collide.

(b) The time elapsed from release to collision.

31. Two supernovae with masses M and m are separated by d, and make respective circular motions around their immovable center of mass. In the explosion, the mass lost of the star with mass M is ΔM. It is assumed that the explosion is instantaneous and completely spherically symmetric, and the direct effect of the explosive debris on the star with mass m is ignored. What relationship should ΔM satisfy, so that the remaining binaries are still bound and will not move away from each other?

32. After the particle with mass m_1 and velocity v_1 is captured by a stationary nucleus, a particle with mass m_2 is produced, which is ejected in the direction perpendicular to v_1. The non-mechanical energy of Q is converted into mechanical energy, and obtain the kinetic energy of the new particle generated.

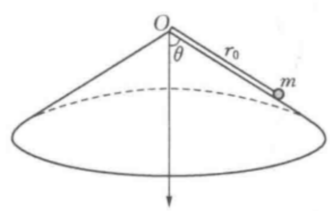
33. A stationary block explodes into three blocks of equal mass 1, 2, and 3. Let the total kinetic energy released in the explosion be a certain value Q, but the kinetic energy of each block is T_1, T_2, T_3 and have many possible values, which can be represented by the length of the perpendicular line drawn by a point P to the 3 sides of an equilateral triangle with a height of Q to represent T_1, T_2, T_3, as shown in the figure, but not a set of kinetic energy values corresponding to each point in the triangle are physically allowed.



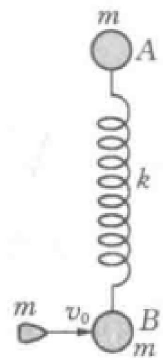
(a) Try to find the boundary of the physical allowable point.

(b) If an electron couple decays into three Γ photons with the energy sum of Q at rest, find the boundary of the range of physically permitted points.

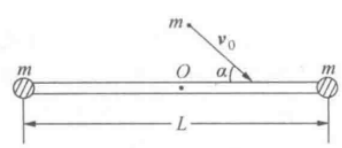
34. On a smooth circular surface with a half-apex angle $\theta = 60°$, a small ball with mass $m = 1.0$ kg is connected by a rope passing through a small hole in the top of the cone, making uniform circular motion at a speed $v_0 = \frac{\sqrt{15}}{4}$ m/s, the length of the rope from the ball to the hole is $r_0 = 0.50$ m, as shown in the figure. Now slowly pull the other end of the rope downward until the ball is separated from the surface, and try to find the work done by the force of the rope in this process.



35. On a smooth horizontal plane, there are two balls A and B of mass m connected by a light spring with a stiffness coefficient of k. The natural length of the spring is a. At the beginning, the two balls are at rest, the spring has no deformation. Now a pellet of mass m is shot into one of the small balls B at a horizontal velocity perpendicular to the direction of the line connecting the two balls, and remains in it, as shown in the figure. It is known that in the subsequent motion process, the maximum length of the spring is $2a$, Find the velocity v_0 of the lead before it hits the ball.



36. A light rod of length L, with a small ball of mass m fixed to each of the two segments, can rotate freely in a straight plane around a fixed horizontal axis at its midpoint. At the beginning, the rod is still in a horizontal position, and in the same vertical plane as the rod rotates, there is a small flying insect with a mass of m flying at a constant speed obliquely downward to the right, and the direction of flight is $\alpha = 45°$ angle with the rod, and lands on the midpoint of the rod. Before and after the insect lands, the speed of the small flying insect along the rod direction remains unchanged. After the small flying insect lands on the rod, it immediately crawls along the rod toward the ball, and the rod sets into rotation before the insect can reach the ball. Take the x axis along the rod direction, the rotation axis as the origin, and take the moment when the small flying insect falls on the rod as the time zero point and ignore the friction at the rotation axis. Try to find:

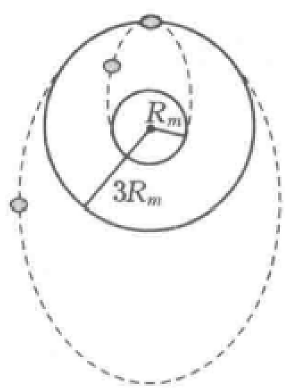


(a) The value of the flying speed of the flying insect v_0.

(b) The relationship between the position x and the time t when the flying insect crawls along the pole,

(c) The angle the rod rotates till the insect reaches the ball.

37. The lunar lander with mass m is connected to the space shuttle with mass M ($M = 2m$), and together make a circular motion around the moon, and the orbital radius is 3 times the radius of the moon R_m. At a certain time, after the space shuttle shot the lunar lander in the opposite direction, the lunar lander still moved in the original direction, and landed on the lunar surface along the elliptical orbit as shown in the figure, and stayed on the lunar surface for a period of time to complete the scientific research work. After a quick start, it still returns to the separation point along the original elliptical orbit to achieve docking with the space shuttle. Try to find the time that the lunar lander can stay on the lunar surface. The known gravitational acceleration $g_m = 1.62\ \text{m/s}^2$ on the lunar surface and the lunar radius is 1.74 x 10^6 m.



38. "Interstellar bullets" are considered to be dense gas masses that pass through low-density interstellar gas clouds like ballistic particles. Somewhere in the universe, there is a spherical low-density uniform gas cloud with a mass M and a radius R, The radius of an interstellar bullet is much smaller than R, the mass $m \ll M$, the angular momentum of the bullet to the center of the gas cloud $L = m\sqrt{\frac{GMR}{32}}$, the total energy of the bullet $E = -\frac{5GMm}{4R}$, ignoring all non-gravitational interactions.

   (a) Is the movement of the bullet always inside the cloud? Always outside the cloud? Or is it sometimes inside the cloud and sometimes outside the cloud?

   (b) Find the distance from the turning point of the bullet's trajectory to the center of the gas cloud.

(c) Discuss the shape of the bullet's trajectory and find its period of motion.

39. There are two options for launching the spacecraft from the earth to the outer solar system. Option 1: Direct launch at a sufficiently large speed (greater than the escape velocity of the solar system). Option 2: Make the spacecraft approach an outer planet in the solar system (such as Mars). Under its action, it changes the direction of motion and then escapes from the solar system. Suppose all the planets run in circular orbits around the sun in the same plane and in the same direction, and the effects of air resistance and the rotation of the earth can be ignored. Try to find:

    (a) According to the scheme 1, the minimum velocity v_a and launch direction relative to the earth required for launching from the ground.

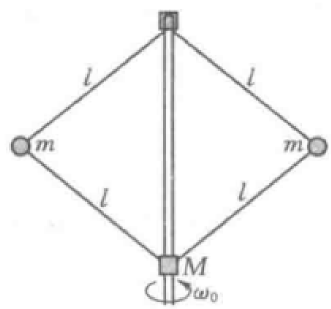
    (b) Launch the spacecraft from the ground relative to the earth in the direction in (a) at a speed v_b, and find the speed of the spacecraft when it passes through the orbit of Mars; (Assume that when the spacecraft passes through the orbit of Mars, it is very far from Mars)

    (c) Let the spacecraft enter the gravitational field of Mars, but it can still fly out of the solar system, and find the speed of launch from the ground; (Assume that the launch direction of the spacecraft relative to the earth is the same as in (a), and set the spacecraft to be separated from the gravitational field of Mars along the tangential direction of the orbit of Mars)

    (d) Estimate the maximum percentage of energy savings for option 2 over option 1.

40. Dust with mass density ρ is uniformly distributed spherically symmetrically in a large area around a star of mass M. Within the dust range, a planet with a mass of m moves around the star in a circular orbit with a radius of r_0 (ignoring the drag effect of the dust on the planet). If the planet is subjected to a small radial perturbation in its motion, which makes it slightly deviated from the circular orbit, try to quantitatively describe the subsequent motion of the planet.

41. The fly-ball governor of a steam engine is composed of two balls of mass m connected by four hinged arms of length l, and the upper and lower sleeves set on the vertical shaft. The upper sleeve is fixed and the mass of the lower sleeve is M, and it can slide up and down along the axis without friction, as shown in the figure. The whole device rotates at a constant angular velocity ω_0 around the straight axis. The mass of the arm and the moment of inertia of the sleeve M are ignored.

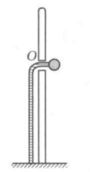


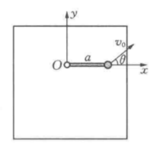
(a) Find the distance between the sleeve M and the upper sleeve when it is balanced.

(b) Find the frequency of the upper and lower vibrations of the sleeve M near the equilibrium position.

(c) If the shaft can rotate freely, and the system rotates stably at the initial angular velocity ω_0, giving the sleeve M a small disturbance up and down, is its small vibration frequency the same as the value in (b)? If not, find this frequency.

42. A small ball of mass m is attached to one end of an elastic rope with a coefficient of stiffness k, the elastic rope passes through a small hole O on a fixed straight plate, and the other end is connected to the ground just below the small hole. When the ball is located at the small hole, the rope is exactly the original length, as shown in figure. Now move the ball against the board along the horizontal (x) direction with a distance a, and it starts to move with the initial velocity $v_0 = 2\sqrt{ga}$, the direction of v_0 is parallel to the board, and forms an angle of $\theta = 60°$ with the direction of x, as shown in Figure 2. Let $ka = mg$, try to find:





(a) How long does it take for the ball to reach the highest position, and find this height.

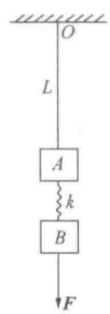
(b) The position, velocity, and time required for the ball to pass the x axis again.

43. A long cylinder of mass M and length l hangs bottom up and mouth down under a spring with stiffness coefficient k. There is a piston with a mass of m and a negligible thickness at the inner bottom of the cylinder. A small spring with a negligible original length is sandwiched between the piston and the bottom of the cylinder. At first, the spring is compressed and locked, and the whole system is in a state of equilibrium. as the picture shows. Now release this lock, so that the cylinder and the piston can gain speed in a very short time, the speed obtained by the cylinder is known $V_0 = \frac{g}{2}\sqrt{\frac{m}{k}}$, in the subsequent motion process, the friction force between the piston and the cylinder is $f = \frac{1}{2}mg$, and set $M = 3m$. Try to find:

(a) The lowest position that the cylinder can reach and the time elapsed during the subsequent movement.

(b) For the cylinder to reach this lowest position, what relation should the cylinder length l satisfy?.

44. Between two blocks A and B with a mass of m, a light spring with a stiffness coefficient of k is used to connect it, and it is suspended from the ceiling at point O by a rope attached to the block A, such as as shown in the figure. A downward constant force $F = nmg$ is applied to the block B, and the F force is removed after reaching equilibrium, after which the spring contracts, causing the system to move upward. When the block A collides with the ceiling, the spring reaches the maximum extension for the first time, and the extension amount is 5mg/k, try to find:

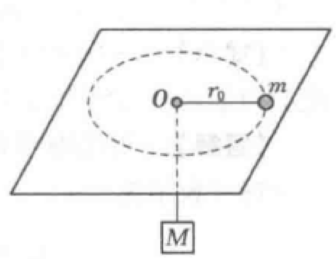


(a) The value of n.

(b) A's velocity v_A when hitting the ceiling.

(c) Length L of rope OA.

45. On a smooth horizontal tabletop, a ball of mass m is connected to a string passing through a hole O on the tabletop, and a block of mass M is suspended from the other end of the string. The tabletop makes a uniform circular motion around the small hole O with an angular velocity ω_0, and the block under the tabletop remains stationary, as shown in the figure. Now give the block a slight disturbance in a straight direction, and try to prove that the block will make a simple harmonic vibration and find its period of vibration.



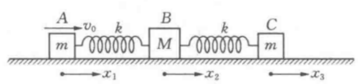
46. When the wind blows from a distance, if the wind speed has a gradient in the vertical direction perpendicular to the ground, people will occasionally hear the sound from a distant sound source, sometimes very clearly. Assuming that the distribution of wind speed in the direction of vertical (y) can be expressed as $v = v_0 + ky^2$(k is a very small positive constant), it is known that the distant sound source ($x = 0$) emits

a sound wave, and the angle between the emission direction and the vertical direction is θ_0, and the speed of sound at the ground ($y = 0$) is V_S, assuming that within the range of the sound wave propagation path, $\frac{ky^2}{V_S} \ll 1$. Try to find:

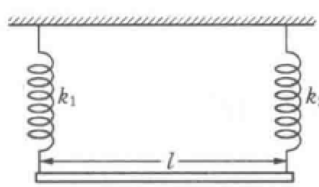
(a) The trajectory of the beam of sound waves propagating in space.

(b) The distance between the point on the ground where the sound can be clearly heard and the source of the sound.

47. The three blocks A, B, C are connected by two light springs with a stiffness coefficient of k, and rest on a smooth horizontal surface, as shown in the figure. It is known that the masses of A, C are m, the mass of B is M. Use x_1, x_2, x_3 to represent the displacements of the three blocks of A, B, C taking the illustrated equilibrium position as the origin. Now give A in a very short time to obtain the initial velocity v_0, the direction points to B, and try to find the relationship between the displacements x_1, x_2, x_3 of the three blocks and the time t.



48. A uniform rod of length l and mass m is suspended at both ends by two light springs with stiffness coefficients k_1 and k_2 respectively, and the rod is horizontal at equilibrium, as shown in the figure. Assuming that the spring can only move in the straight direction, the rod can make a small vibration near the equilibrium position.



(a) Take $k_1 = k_2$, find the eigenfrequency of the normal mode, and describe the corresponding normal mode motion.

(b) Take $k_1 \neq k_2$ to find the eigenfrequency of the normal mode.

49. There is a small hemispherical nozzle on the ground, and there are many small holes distributed on the hemispherical surface of the nozzle. When water is sprayed, the water is sprayed from these small holes in different directions at the same speed v_0. Compared with the spraying area, the nozzle can be seen as a point.

    (a) The water sprayed from the nozzle draws a three-dimensional figure in the air, please describe this figure quantitatively.

    (b) How to distribute the number density of the small holes on the hemisphere of the nozzle to make the water splash evenly on the ground?

50. A planet of mass M is composed of an incompressible fluid with a density ρ, and the planet rotates slowly at a constant angular velocity ω. Due to the rotation, its equator is slightly more convex than its poles. The average radius of the planet is known to be R. As a first-order approximation, find:

    (a) Find the pressure at a depth h (smaller than radius) near the surface at the polar angle θ.

    (b) Let the radii at the poles and the equator be R_{p} and R_{e} respectively, try to find $R_{\mathrm{e}} - R_{\mathrm{p}}$ value.

51. A satellite of mass m moves around a planet of mass M in a circular orbit of radius r, with an angular velocity of ω, while the planet revolves around its center of mass with an angular velocity of Ω, and the directions of the two angular velocities are the same. It is known that the rotational inertia of the planet around the axis of rotation is I.

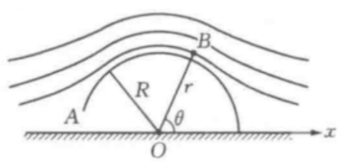
    (a) Generally, ω and Ω are not equal. Due to tidal friction, the angular velocity of planetary rotation and the angular velocity of satellite orbit will change. Try to find the relationship between these two angular velocity changes.

    (b) Tidal friction will reduce the mechanical energy of the planetary and satellite system. Under what conditions can the system finally reach a stable state, that is, a state in which the mechanical energy remains unchanged?

52. The semi-cylindrical hangar has a length of l and a radius of R. It is exposed to the wind. The direction of the wind speed is perpendicular to its axis. The streamline is shown in the figure. The wind speed at any point around the shed can be determined by the "velocity potential" ϕ:

$$\phi = -v_0 \left(r + \frac{R^2}{r} \right) \cos\theta$$

Where r is the distance from a point B near the roof to the axis O of the semi-cylindrical, θ is the included angle between $\overline{OB}$ and the x axis. The known velocity and the relationship of velocity potential is similar to that of conservative force and potential energy. If the hangar door A is open, find the force exerted by the wind on the hangar. Let $l = 70$ m, $R = 10$ m, $v_0 = 20$ m/s, $\rho = 1.20$ kg/m^3, Ignore pressure differences due to altitude (take two significant figures).

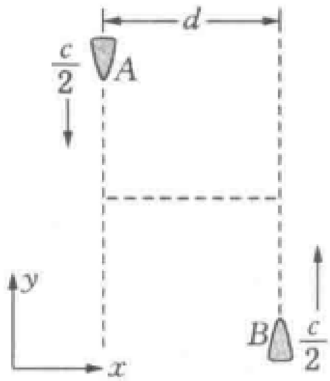


53. A uniform rod AB of mass m and length $2a$, the end A of which is connected by a light smooth chain to a horizontal light rod OA of length b. The AB rod can rotate freely around the hinge in the vertical plane, and now the two rods rotate together around the vertical axis of O at a constant angular velocity ω. Let θ be the angle between the AB rod and the solid line, as shown.

   (a) Try to find the relation satisfied by the possible θ values when the AB rod is balanced relative to the OA rod during rotation.

   (b) Point out several possible equilibrium positions for θ between θ and 2π, and discuss the stability of the equilibrium.

54. In an inertial coordinate system S, it is observed that two spaceships A, B are flying parallel to each other along a straight line, and the orbital distance is d, as shown in the figure. The speed of the spaceships is $\frac{c}{2}$. When the two spaceships reach the closest point to each other (indicated by the dotted line in the figure), the spaceship A throws a small packet at the rate of $\frac{3}{4}c$ (observed in the S system).



(a) Assuming that there is a coordinate system S' parallel to the coordinate axis of the S system on the spacecraft A, the motion direction of the S' system relative to the S system is parallel to the y axis. In the S' system, in order for the packet to be received by the spaceship B, at what angle should the packet be thrown?

(b) To an observer in the S' system, what is the rate at which packets are thrown?