ECE250: Signals & Systems
Monsoon 2024
Mid-Semester Examination

Date: **4/10/2024** Duration: **1.30 Hours** Total Marks: **36+6 Marks**

Note:

(1) Please provide proper mathematical justifications with your answers. No marks will be awarded without a valid justification.

(2) Do not use any property without proving it mathematically in the paper. No shortcuts or statements are allowed. This will fetch you zero marks.

(3) Institute Plagiarism policy are strictly applicable.

[CO1, CO2] Q1: [6+3 Marks] Given that

$$x(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & otherwise \end{cases}$$

and $h(t) = \beta x(t/\alpha)$, where $0 < \alpha \leq 1$

(a) Calculate and sketch $y(t) = x(t) * h(t)$.

(b) If $\frac{d}{dt}y(t)$ contains only three discontinuities, then what is the value of α and β.

[CO1, CO2] Q2: [4+6 Marks] The cascade of the following two systems S_1 and S_2 is depicted in Figure-1.

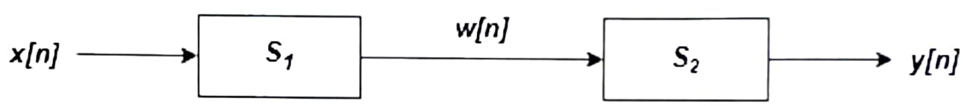

Figure 1

S_1: Causal LTI; $w[n] = \frac{1}{2}w[n-1] + x[n]$

S_2: Causal LTI; $y[n] = \alpha y[n-1] + \beta w[n]$

The systems are initially at rest and the difference equation relating $x[n]$ and $y[n]$ is:

$$y[n] = -\frac{1}{8}y[n-2] + \frac{3}{4}y[n-1] + x[n]$$

(a) Determine $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

(b) Show the impulse response of the cascade connection of S_1 and S_2.

[CO1, CO2, CO3] Q3: [6 Marks] Determine the Fourier series representation for the periodic signal $x(t)$ with time period **4**

$$x(t) = \begin{cases} \sin \pi t, & 0 \leq t \leq 2 \\ 0, & 2 < t \leq 4 \end{cases}$$

[CO1, CO2, CO3] Q4: [2+6+2+1 Marks] Given an impulse train $x[n]$,

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$$

(a) Find the Fourier Series of $x[n]$ and plot the line spectrum.

(b) This signal is applied as an input to a particular LTI system with frequency response $H(e^{jw})$, the output of the system is found to be

$$y[n] = \cos\left(\frac{5\pi}{2}n + \frac{\pi}{4}\right)$$

Determine the values of $H\left(e^{jk\pi/2}\right)$ for $k = 0, 1, 2, and\ 3$.

(c) Plot the line spectrum of $y[n]$.

(d) Write your inference comparing the line spectrum of $x[n]$ and $y[n]$.

[CO1, CO2] Q5[Bonus Question]: [6 Marks] Consider the signal

$$x[n] = \alpha^n u[n]$$

$0 \le \alpha \le 1$

(a) Sketch the signal $g[n] = x[n] - \alpha x[n-1]$

(b) Use the result of part (a) to determine a sequence $h[n]$ such that

$$x[n] * h[n] = \left(\frac{1}{2}\right)^n \{u[n+2] - u[n-2]\}$$

ECE250: Signals & Systems

Monsoon 2024

End-Semester Examination + Quiz-5

Date: **5/12/2024** Duration: **2.30 Hours** Total Marks: **35+4 Marks**

Note:

(1) For End-Semester Exam (from Q1 to Q4): Attempt any 2 questions from Q1 to Q3, whereas Q4 is compulsory. Attempt Q5 for Quiz-5.

(2) Do not use any property without proving it mathematically in the paper. No shortcuts or statements are allowed. This will fetch you zero marks.

(3) Institute Plagiarism policy is strictly applicable.

X **[CO1, CO2, CO3] Q1(a): [7 Marks]** Find the Fourier Series and draw the line spectrum of the following signals:

i. $x(t) = 2 + cos\left(\frac{2\pi}{3}t\right) + 4\,sin\left(\frac{5\pi}{3}t\right)$

ii. $x[n] = 3 + sin\left(\frac{2\pi}{5}n\right) + 2cos\left(\frac{4\pi}{5}n\right)$

[CO1, CO2, CO3] Q1(b): [7 Marks] Compute the output of the filter shown in ***Figure-1*** for input signal $x[n]$.

$$x[n] = 2\cos\left(\frac{3}{8}\pi n\right) + 3\sin\left(\frac{3}{4}\pi n\right)$$

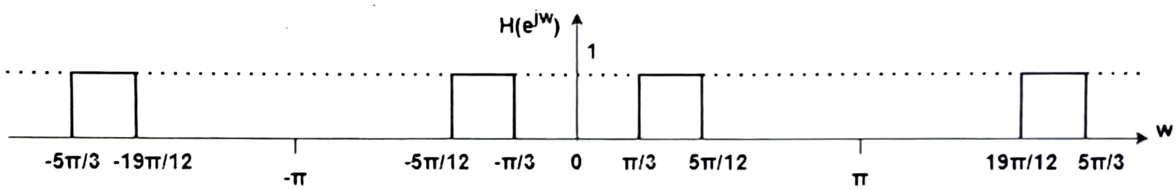

Figure 1

[CO1, CO2, CO4] Q2(a) : [5 Marks] Find the Laplace transform of the signal

$$f(t) = e^{r(t)}, \text{ where } r(t) = tu(t). \rightarrow 1/s$$

$-\frac{e^s + se^s - s}{s(1-s)}$

$t\delta(t)$ $e^{tu(t)}$

[CO1, CO2, CO4] Q2(b): [9 Marks] How many signals have a Laplace transform that may be expressed as

$$F(s) = \frac{(s^2+2s+5)}{(s+3)(s+5)^2}$$

Property Proof

in its region of convergence? Justify your answer. Find the Inverse Laplace transform for each case.

[CO1, CO2, CO3] Q3: [1.5+6+4.5+2 Marks] A signal $x(t)$ (given in ***Figure-2(a)***) that undergoes impulse train sampling, where

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$

$sF(s) = \frac{1}{s}e^{1/s}$

$F(s) = \frac{1}{s^2}e^{1/s}$

Signal $\boldsymbol{x_p(t)}$ is passed through a LPF (given in ***Figure-2(b)***).

Consider the following cases:

(a) $w_m = 2\, rad/sec, w_s = 3\, rad/sec$ and $w_c = 1.5\, rad/sec$
(b) $w_m = 1\, rad/sec, w_s = 1\, rad/sec$ and $w_c = 1\, rad/sec$
(c) $w_m = 1.5\, rad/sec, w_s = 4\, rad/sec$ and $w_c = 2\, rad/sec$

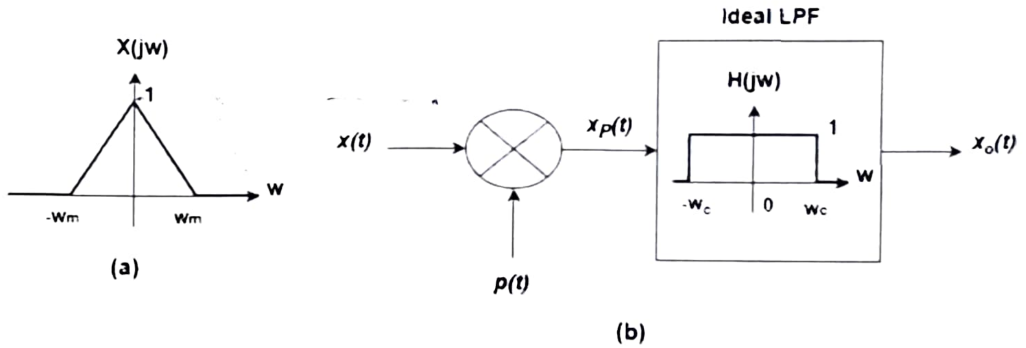

Figure 2

For each of the above cases (a) to (c):

1) Compute and plot the frequency spectrum $(P(jw))$ of $p(t)$.
2) Write the mathematical expression of $x_p(t)$. Compute and plot the frequency spectrum $(X_p(jw))$ of $x_p(t)$.
3) Compute and plot the frequency spectrum $(X_o(jw))$ of $x_o(t)$.
4) Write your inferences based on $X_o(jw)$.

[CO1, CO2, CO5] Q4: [7 Marks] We are given the following five facts about a discrete-time signal $x[n]$ with Z-transform $X(z)$:

1) $x[n]$ is real and right-sided.
2) $X(z)$ has exactly two poles.
3) $X(z)$ has two zeros at the origin.
4) $X(z)$ has a pole at $z = \frac{1}{2}e^{j\left(\frac{\pi}{3}\right)}$
5) $X(1) = \frac{8}{3}$

Determine $X(z)$ and specify its region of convergence.

QUIZ-5 Question

[CO1, CO2, CO4, CO5] Q5: [4 Marks] Consider a discrete-time signal $x[n]$ given by

$$x[n] = \{-1, 0, 1, \underset{\uparrow}{2}, 1, 0, 1, 2, 1, 0, -1\}$$

where arrow indicates the position of n=0. Find the value of the following:

c (a) $X(e^{j0})$
(b) $\int_{-\pi}^{\pi} X(e^{jw})dw$
(c) $\int_{-\pi}^{\pi} |X(e^{jw})|^2 dw$
(d) $X(e^{j\pi})$