Worksheet - 1

Course Name: Math-III (Section - B)

Total marks = 3

Date: 22/08/2024

Question - The length *a*, *b*, and *c* of the edges of a rectangular box are changing with time. At the instant in question, $a = 1$m, $b = 2$m, $c = 3$m, $da/dt = db/dt = 1$m/sec, and $dc/dt = -3$m/sec. At what rates are the box's volume *V* and surface area *S* changing at the instant? Are the box's interior diagonals increasing in length or decreasing? (3)

$a = 1m \quad b = 2m, \quad c = 3m$

Worksheet - 2

Course Name: Math-III (Section - B)

Total marks = 3

Date: 29/08/2024

Question - The derivative of ***f(x, y)*** at $P_0(1,2)$ in the direction of ***i + j*** is $2\sqrt{2}$ and in the direction of ***-2j*** is -3. What is the derivative of ***f*** in the direction of ***-i - 2j***? (3)

Worksheet - 3

Course Name: Math-III (Section - B)

Total marks = 3

Date: 05/09/2024

Question - Find the point on the plane $x + 2y + 3z = 13$ closest to the point (1, 1, 1). (3)

Worksheet - 4

Course Name: Math-III (Section - B)

Total marks = 3

Date: 12/09/2024

In this Question, Sketch the region of Integration, reverse the order of integration, and evaluate the original.

Question - $\iint\limits_{R} xy\, dA$ where R is the region bounded by the lines $y = x,\ y = 2x$, and $x + y = 2$.

Worksheet - 6

Course Name: Math-III (Section - B)

Total marks = 3

Date: 26/09/2024

Question - Solve the initial value problem for the following Differential equation and Initial conditions for *r* as a vector function of *t*.

Differential equation: $$\frac{d^2\mathbf{r}}{dt^2} = -(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

Initial conditions:

$$\mathbf{r}(0) = 10\mathbf{i} + 10\mathbf{j} + 10\mathbf{k} \quad \text{and}$$

$$\left.\frac{d\mathbf{r}}{dt}\right|_{t=0} = \mathbf{0}$$

Worksheet - 7

Course Name: Math-III (Section - B)

Total marks = 3

Date: 17/10/2024

Question - Find $T,\ N,\ and\ \kappa\ (curvature)$ for the following space curve - $r(t)\ =\ (cos^3 t)i\ +\ (sin^3 t)j,\ 0 < t < \pi/2$

MTH203 - Multivariate Calculus
Mid Semester Exam

Section - B
Total Marks - 30

11th October 2024

Problem - 1

A solid cube, 2 units on a side, is bounded by the planes $x = \pm 1$, $z = \pm 1$, $y = 3$, and $y = 5$. Find the moments of inertia about the coordinate axes.

$\delta = 1$

[6 marks]

Problem - 2

Given nonzero vectors **u**, **v**, and **w**, use dot product and cross product notation, as appropriate, to describe the following:

a. The vector projection of **u** onto **v**

b. A vector orthogonal to both $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$

c. The area of the parallelogram determined by **u** and **w**

d. The volume of the parallelepiped determined by **u**, **v**, and **w**

[4 marks]

Problem - 3

The temperature at a point (x, y) on a metal plate is $T(x, y) = 4x^2 - 4xy + y^2$. An ant on the plate walks around the circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the ant? **(Do it using Lagrange Multipliers)**

[8 marks]

Problem - 4

Solve the following questions:

a. Show that the curve $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + (1 - \cos t)\mathbf{k}$, $0 \le t \le 2\pi$, is an ellipse by showing that it is the intersection of a right circular cylinder and a plane. Find equations for the cylinder and plane.

[3 marks]

b. Sketch the ellipse on the cylinder. Add to your sketch the unit tangent vectors at $t = 0$, $\pi/2$, π, and $3\pi/2$.

[3 marks]

c. Show that the acceleration vector always lies parallel to the plane (orthogonal to a vector normal to the plane).

[2 marks]

Problem - 5

Let D be the region bounded below by the plane $z = 0$, above by the sphere $x^2 + y^2 + z^2 = 4$, and on the sides by the cylinder $x^2 + y^2 = 1$. Sketch the figure and Set up the triple integrals in cylindrical coordinates that give the volume of D using the following order of integration: $dz\,dr\,d\theta$

[4 marks]

MTH203 - Multivariate Calculus
End Semester Exam

Section - B
Total Marks - 50 (Including 10 Bonus)

12th December 2024

Instructions

1. The exam duration is **2 hours**.
2. This end-semester exam accounts for **40% of the total course grade.**
3. The question paper contains **5 questions**. Certain questions have sub-parts marked with **, indicating bonus marks.
4. To be eligible for bonus marks, you must first attempt all the mandatory parts of the corresponding question.
5. **Bonus parts worth up to 10 additional marks.**
6. You are allowed to bring **two A4-sized cheat sheets**, which may only include formulas.
   - **Important:** If solutions are found on your cheat sheets, it will be considered academic dishonesty and may result in strict disciplinary action.

Problem - 1

$$u = x^3 - 3xy^2$$

a. Show that the given function is harmonic. [5 marks]

b. Find the harmonic conjugate of $u(x, y)$. [3 marks]

c. ** Find the analytic function $f(z) = u(x, y) + iv(x, y)$ where v is harmonic conjugate of u in terms of z instead of x. and y [2 marks]

Problem - 2

a. Show that the curvature of a smooth curve $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$ defined by twice-differentiable functions $x = f(t)$ and $y = g(t)$ is given by the formula:

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2 + \dot{y}^2)^{3/2}}.$$

[7 marks]

b. ** Apply the formula to find the curvatures of the curve: $\mathbf{r}(t) = [\tan^{-1}(\sinh t)]\mathbf{i} + (\ln \cosh t)\mathbf{j}$. [3 marks]

Problem - 3

Along all rectangular solids defined by the inequalities

$$0 \leq x \leq a, \quad 0 \leq y \leq b, \quad 0 \leq z \leq 1,$$

find the values of a and b for which the total flux of

$$\mathbf{F} = \left(-x^2 - 4xy\right)\mathbf{i} - 6yz\mathbf{j} + 12z\mathbf{k}$$

outward through the six sides is greatest. Also, what is the greatest flux?

[10 marks]

Problem - 4

a. Solve the system

$$u = x - y,$$
$$v = 2x + y,$$

for x and y in terms of u and v. Then, find the value of the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$.

[3 marks]

b. Find the image under the transformation $u = x - y$, $v = 2x + y$ of the triangular region with vertices $(0,0)$, $(1,1)$, and $(1,-2)$ in the xy-plane. Sketch the transformed region in the uv-plane.

[4 marks]

c. ** Use the above transformation to evaluate the integral

$$\iint_R (2x^2 - xy - y^2)\, dx\, dy$$

for the region R in the first quadrant bounded by the lines $y = -2x + 4$, $y = -2x + 7$, $y = x - 2$, and $y = x + 1$. Sketch the figure and label the sides.

[5 marks]

Problem - 5

Find the point on the plane $x + 2y + 3z = 13$ closest to the point $(1,\ 1,\ 1)$. (Do it using Lagrange multiplier)

[8 marks]