Q2 $\vec{A}' = (\vec{A} + \vec{\nabla}\lambda)$ and $\phi \to \phi'$ —— (1)

$(\vec{\nabla} \times \vec{A}') = (\vec{\nabla} \times \vec{A})$

However,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t} \quad \text{—— (2)}$$

Let, under the transformation (1), $\vec{E}$ transforms to $\vec{E}'$.

$$\therefore \vec{E}' = -\vec{\nabla}\phi' - \frac{\partial \vec{A}'}{\partial t}$$

$$= -\vec{\nabla}\phi' - \frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\left(\frac{\partial \lambda}{\partial t}\right)$$

$$= -\vec{\nabla}\left(\phi' + \frac{\partial \lambda}{\partial t}\right) - \left(\frac{\partial \vec{A}}{\partial t}\right) \quad \text{—— (3)}$$

For $\vec{E}$ to remain unchanged,

$\vec{E} = \vec{E}'$

$\Rightarrow \phi = \phi' + \left(\frac{\partial \lambda}{\partial t}\right)$ [Equating (2) and (3)]

$\Rightarrow \boxed{\phi' = \left(\phi - \frac{\partial \lambda}{\partial t}\right)}$ $\leftarrow$ Transformation rule for ϕ so that $\vec{E}$ remains unchanged.

To sumarize,

$$\boxed{\begin{aligned} \vec{A}' &= \left(\vec{A} + \vec{\nabla}\lambda\right) \\ \phi' &= \phi - \frac{\partial \lambda}{\partial t} \end{aligned}}$$

← Leaves both $\vec{E}$ and $\vec{B}$ unchanged.

Q1. $r = 1\,m$.

$\therefore$ volume of the sphere $= \left(\frac{4}{3}\pi\right)$

$$\therefore \rho_0 = \frac{4\pi}{\frac{4}{3}\pi}\, c/m^3 = 3\,c/m^3$$

$$\rho = \rho_0 e^{-\frac{\sigma}{\varepsilon}t}$$

$$= \rho_0 e^{-\frac{100}{20}t}$$

$$= 3e^{-5t}$$

(a) $\tau = \frac{1}{5}$ sec.

(b) surface area $= 4\pi\, m^2$

volume $= \frac{4}{3}\pi\, m^3$

Total charge is same at $t = \tau$ and $t = 0$:

$$4\pi\sigma_s + \frac{4\pi}{3}\left(\rho_0 e^{-1}\right) = \frac{4\pi}{3}\rho_0$$

$$\Rightarrow \sigma_s = \frac{1}{3}\rho_0\left(1 - e^{-1}\right)$$

$$= \frac{1}{\cancel{3}} \times \cancel{3}\left(1 - e^{-1}\right)$$

$$= \left(1 - e^{-1}\right)$$