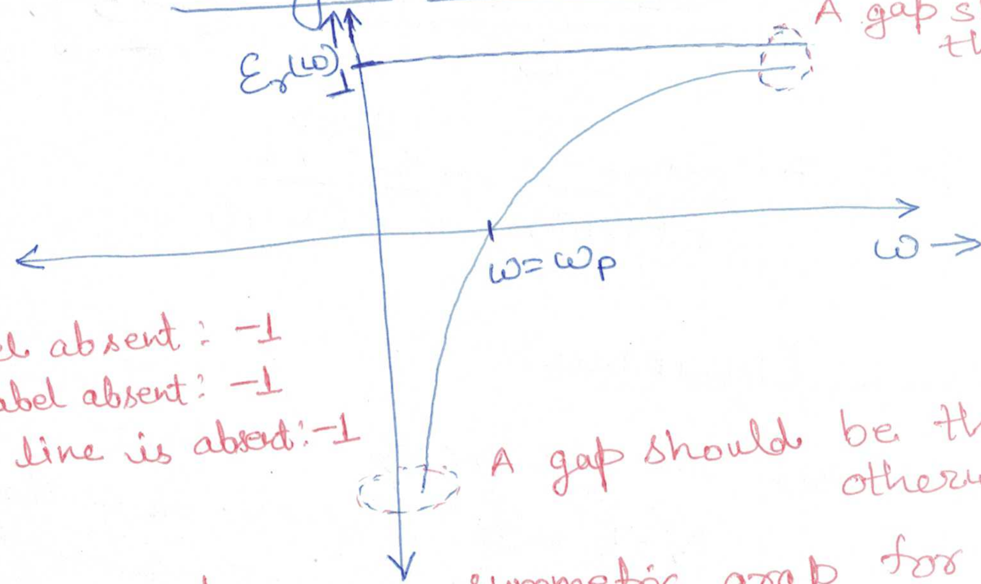
Quiz 3 RUBRIC

A.1)

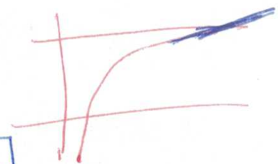


A gap should be there otherwise −1

Axis label absent : −1
$\omega=\omega_p$ label absent : −1
$\varepsilon_r(\omega)=1$ line is absent : −1

A gap should be there otherwise −1.

*(Some may draw a symmetric graph for $\omega<0$ also: don't deduct marks)

If someone draws like:  NO Marks.

$$\vec{E} = 4\hat{x}\exp[i(5\times10^6 t - kz)]$$

A.2) (a) Polarization along x-axis (1)

(b) Travelling along z-direction. (1)

(c) $\omega = 5\times10^6$

$c = 3\times10^8$

$k = \frac{\omega}{c} = \frac{5}{3}\times10^{-2} = 0.0167\ m^{-1}$ (1)

No part marking

(d) $\lambda = \frac{2\pi}{k}$ (1)

$$= \frac{2\pi}{\frac{5}{3}\times10^{-2}} = \frac{\cancel{600}^{120}\pi}{\cancel{5}} = 120\pi = 376.8\ m$$

A.3) For an inhomogeneous medium, $\varepsilon = \varepsilon(x,y,z)$

Gauss Law is:

$$\vec{\nabla}\cdot\vec{D} = \rho_{free} \qquad \vec{D} = \varepsilon\vec{E} \Rightarrow \vec{\nabla}\cdot(\varepsilon\vec{E}) = \rho_{free}$$

Using the identity of a product of a scalar and vector field:

$$\vec{\nabla}\cdot(\varepsilon\vec{E}) = \varepsilon(\vec{\nabla}\cdot\vec{E}) + \vec{E}\cdot\vec{\nabla}\varepsilon \leftarrow \boxed{1.5}$$

If correct till here.

$$\Rightarrow \rho_{free} = \varepsilon(\vec{\nabla}\cdot\vec{E}) + \vec{E}\cdot\vec{\nabla}\varepsilon$$

$$\Rightarrow \vec{\nabla}\cdot\vec{E} = \frac{\rho_{free} - \vec{E}\cdot\nabla\varepsilon}{\varepsilon} \leftarrow \boxed{3}$$

$$\Rightarrow \vec{\nabla}\cdot\vec{E} = \frac{\rho_{free}}{\varepsilon(x,y,z)} - \frac{\vec{E}\cdot\vec{\nabla}\varepsilon}{\varepsilon(x,y,z)}$$

A.4) Maxwell's Equations:

(a) Gauss's Law for electricity

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_e}{\varepsilon_0}$$ where ρ_e is electric charge density

2.5

(b) Gauss Law for magnetism.

$$\vec{\nabla}\cdot\vec{B} = 0$$ (when magnetic monopole doesn't exist).

As per question, if a magnetic monopole exists it will be modified as -

$$\vec{\nabla}\cdot\vec{B} = \mu_0\rho_m$$ 2.5

where ρ_m is magnetic charge density.

(c) Faraday's Law

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$ (when magnetic monopole doesn't exists.)

Modified:

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} - \vec{J}_m$$ 2.5

where, J_m is magnetic current density.

(d) Ampere-Maxwell Law

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}_e + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$ 2.5

where $\vec{J}_e$ is electric current density.