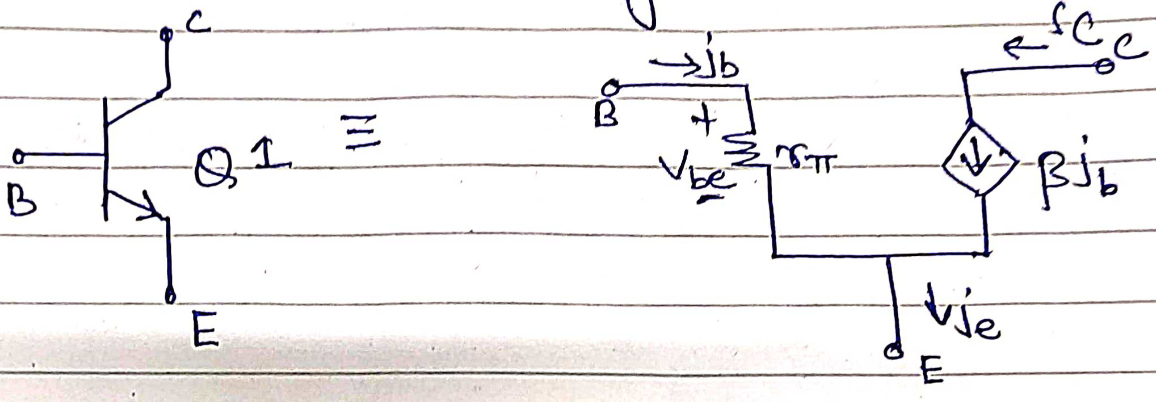
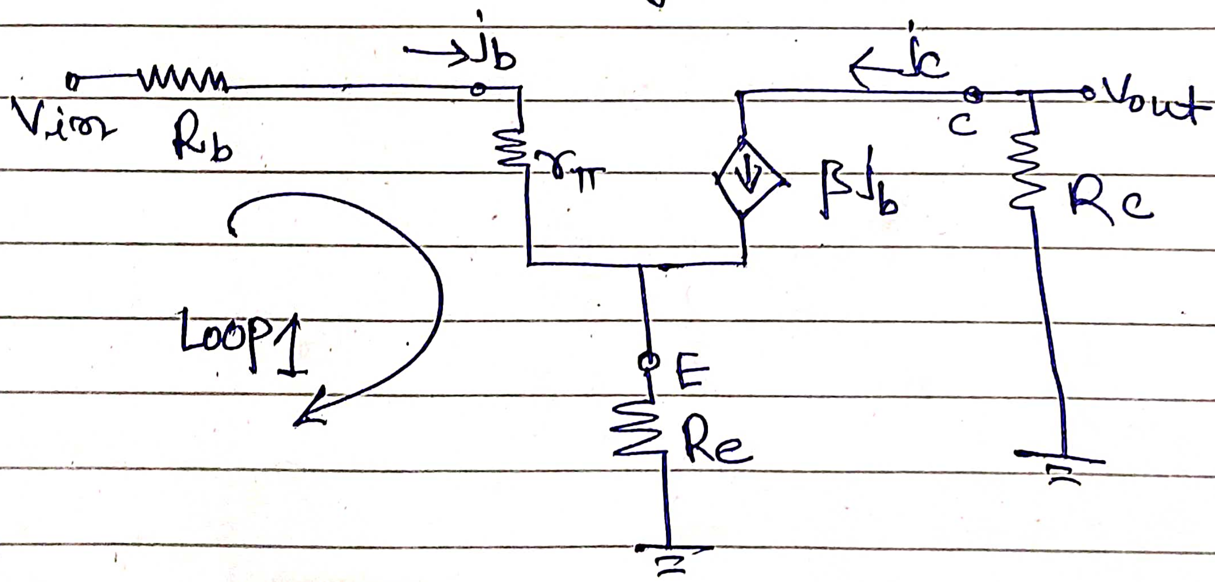
IE-Winter 2022

Mid-Sem Sol^{n}

Q.1. a) Both the circuit given in fig1 and fig2 can be converted into small signal AC equivalent circuit for gain calculation.



$\therefore$ Analysing the circuit in fig 1



$$V_{out} = -i_c R_c = -\beta i_b R_c \quad \text{--- (1)}$$

for loop1 applying KVL

$$-V_{in} + R_b i_b + i_b r_\pi + (1+\beta) i_b R_e = 0$$

$$\because \beta = g_m \cdot r_\pi \Rightarrow r_\pi = \beta / g_m$$

$$\therefore V_{in} = \left[R_b + \frac{\beta}{g_m} + (1+\beta)R_e\right] i_b \quad \text{—} ②$$

$\therefore$ by ① & ②

Voltage gain

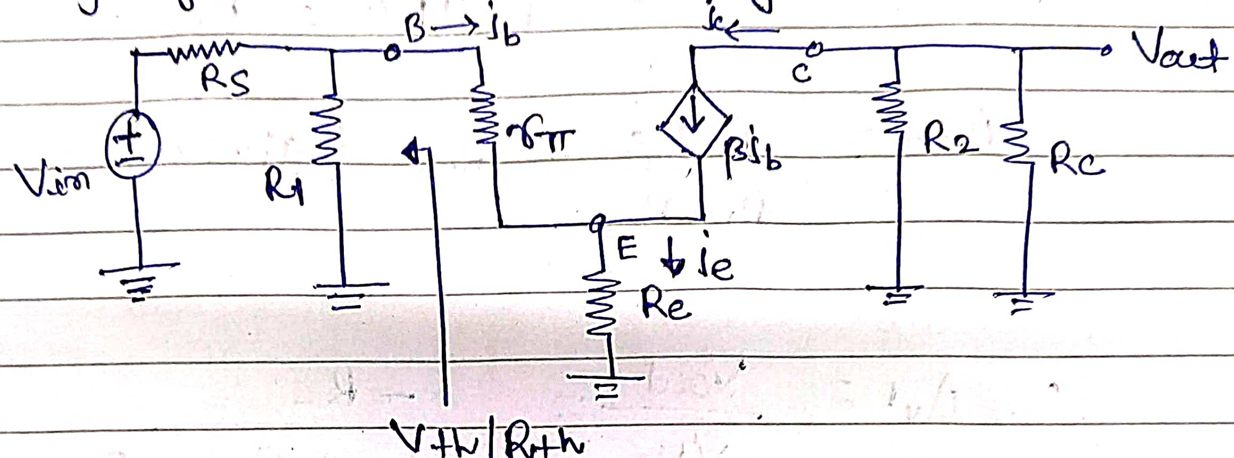
$$A_v = \frac{V_{out}}{V_{in}} = \frac{-\beta R_c}{R_b + \beta/g_m + (1+\beta)R_e}$$

$$A_v = \frac{-R_c}{\frac{1}{g_m} + \frac{R_b}{\beta} + \frac{1+\beta}{\beta} R_e}$$

$\because$ β is large $\Longrightarrow$ $\beta + 1 \approx \beta$.

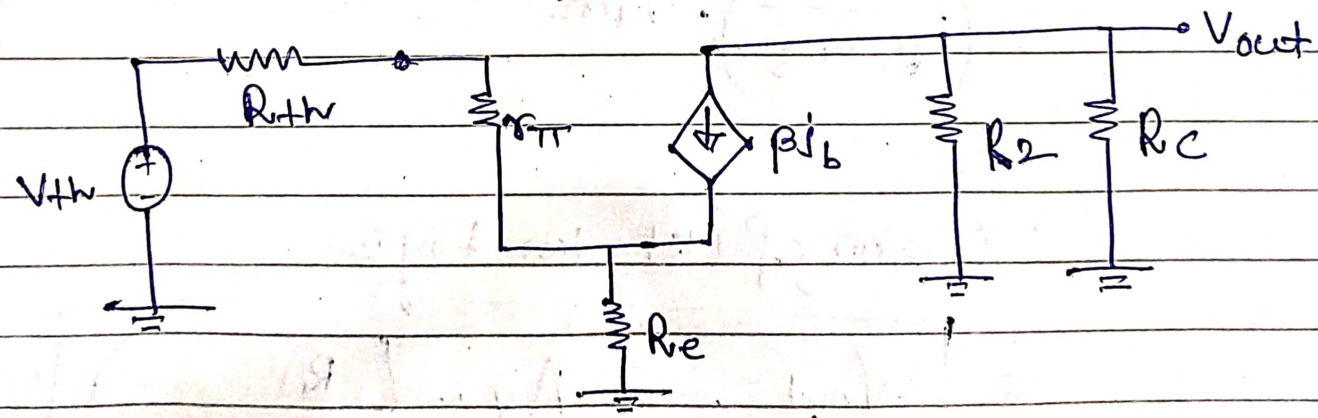
$$\boxed{A_v \approx \frac{-R_c}{\frac{1}{g_m} + R_e + \frac{R_b}{\beta+1}}}$$

Now analysing the circuit in Fig2



$$V_{th} = \frac{R_1}{R_1+R_S} \cdot V_{in}, \qquad R_{th} = R_1 \| R_S$$

∴ The Thevenin eq. ckt will look like



∴ As per the previous analysis of Fig1 gain of the above ckt can be given as

$$A_v' = \frac{V_{out}}{V_{th}} \approx \frac{-R_C \| R_2}{\frac{1}{g_m} + R_e + \frac{R_{th}}{\beta+1}}$$

given that $R_2 >> R_C$

$\therefore R_2 \| R_C = R_C$

$$R_{th} = \frac{R_1 \cdot R_s}{R_1 + R_s} = R_b \text{ (given)}$$

$$\therefore A_v' = \frac{V_{out}}{V_{th}} = \frac{-R_c}{\frac{1}{g_m} + R_e + \frac{R_b}{\beta+1}} = A_v$$

$$\therefore \frac{V_{out}}{\left(\frac{R_1}{R_1+R_s}\right) \cdot V_{in}} = A_v \cdot \frac{R_1}{R_1+R_s}$$

$\therefore$ Gain of ckt in Fig 2

$$= \boxed{\frac{V_{out}}{V_{in}} = A_v \cdot \left(\frac{R_1}{R_1+R_s}\right)}.$$

Q.1.(b) Gain equation obtained in Fig2 is

$$\frac{V_{out}}{V_{in}} = \frac{-R_C}{\frac{1}{g_m} + R_e + \frac{R_b}{\beta+1}} \cdot \left(\frac{R_i}{R_i + R_s}\right)$$

From, the above equation, the effect of β can be nullified if the term

$\frac{R_b}{\beta+1}$ can be made negligible in comparision with the other terms in denominator.

Case 1: → Let us say R_b is very small, however this is not a practical solution, as R_b is modeling input source impedance and it will always pose a significant non-zero value.

Case 2: → Very large β ! It has other Consequences. Too high beta will vary the operating point very significantly for a small change in I_b and the output will be very sensitive to temperature variations.

Case 3: → $1/g_m$ factor very high: →

This is also not a very practical sol^{n} wrt the given ckt, as we know

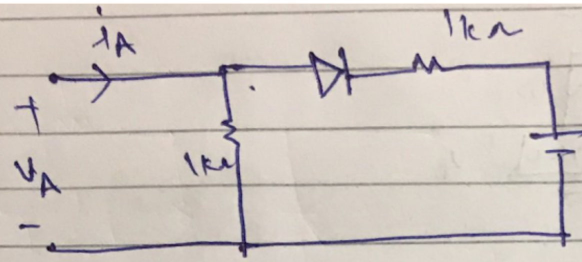
$$g_m = \frac{I_C}{V_T}$$

V_T is constant at a fixed temp. (say ambient temperate) and hence, we can not make I_C too small, forcing a very small I_C bias will make BJT go in non-linear region too often.

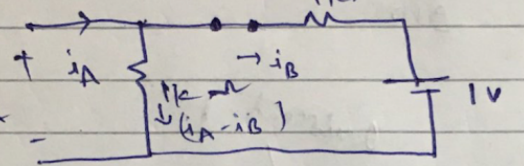
Case 4: → R_E very large:

with a suitable biasing arrangement we can choose a large R_E. However, large R_E will result in a very significant ~~also~~ decrease in gain. Hence there is a trade off, and we need to select an optimum value.

Ans 2 a)



Diode ON for $v_A > 1V$



$v_A = (i_A - i_B)1k \Rightarrow i_B = i_A - v_A$ — i)

$-1 - i_B + v_A = 0$

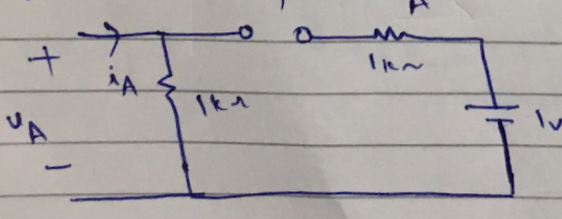
$\Rightarrow v_A = 1 + i_B$ — ii)

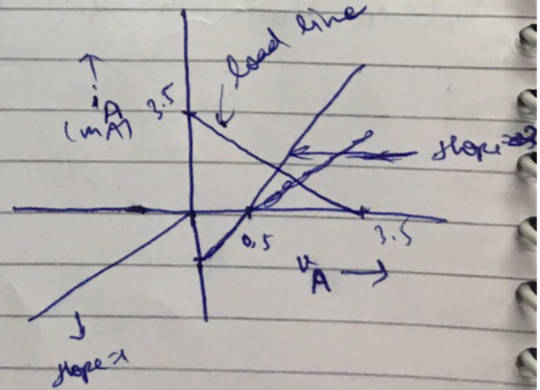
$\Rightarrow v_A = 1 + i_A - v_A$

$\Rightarrow 2v_A = 1 + i_A \Rightarrow \boxed{i_A = 2v_A - 1}$ — iii)

$\cancel{\Rightarrow v_A = \frac{1+i_A}{2} = 0.5 + 0.5 i_A \Rightarrow i_A = 2v_A + 1}$

Diode Off for $v_A < 1V$

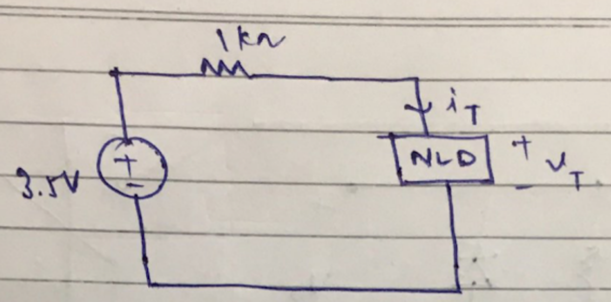




$v_A = i_A$

$\Rightarrow i_A = v_A$

Ans 2 b)



$$-3.5 + i_T + v_T = 0$$

$$\Rightarrow \boxed{i_T = (3.5 - v_T)\ mA}$$

From the figure, $v_T = v_A$, $i_T = i_A$

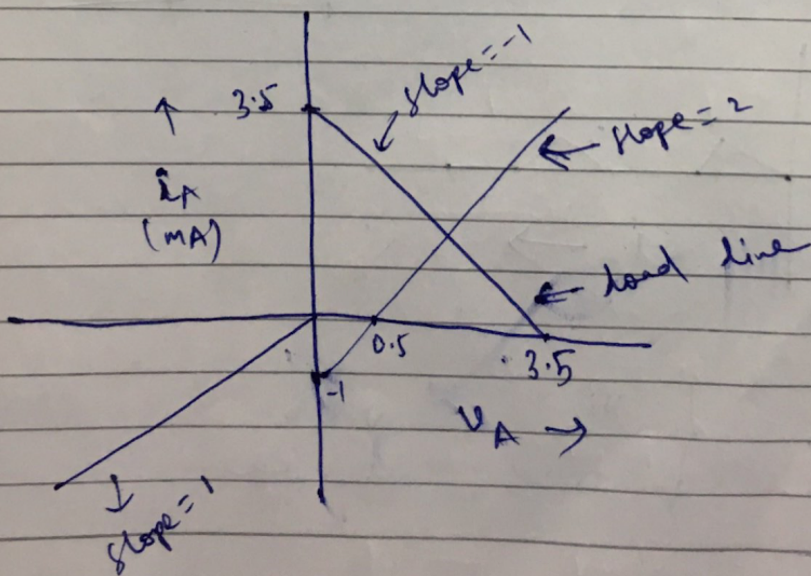
$$3.5 - v_A = 2v_A - 1$$

$$\Rightarrow 3v_A = 4.5$$

$$v_A = 1.5V = v_T$$

$$i_T = 3.5 - 1.5$$

$$i_T = 2mA$$



Q.3) a.) $V_B = 0.7V$

Since, the MOSFET (M) drain and gate are shorted, it will always be in saturation.

$$\therefore I_D = \frac{1}{2} \cdot \mu_n C_{ox} \cdot \left(\frac{W}{L}\right) \cdot \left(V_{GS} - V_T\right)^2$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{1}{2} (\mu_n C_{ox}) \cdot \left(\frac{W}{L}\right) \cdot 2 \cdot (V_{GS} - V_T)$$

$\because V_G = V_B \therefore V_{GS} = V_{BS} = V_B - V_S = 0.7V$

$$\therefore g_m = \frac{1}{\cancel{2}} \times 100\left(\frac{\mu A}{V}\right) \cdot 10 \cdot \cancel{2} \cdot (0.7 - 0.2)$$

$$\therefore g_m = 10^3 \times 10^{-6} \times 0.5 = 0.5 \times 10^{-3}$$

$$\therefore \boxed{g_m = 0.5 \frac{mA}{V}}$$

Q.3 b.) Here, $V_o = V_B$

$\therefore$ Small signal resistance (r_o)

$$= \frac{\partial V_o}{\partial I_D} = \frac{\partial V_B}{\partial I_D}$$

$\frac{1}{2}$

Since, MOSFET is in saturation

$$\therefore I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_T)^2$$

$$V_G = V_B = V_{GS}$$

$$\therefore I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_B - V_T)^2$$

$$\therefore (V_B - V_T) = \sqrt{\frac{2 \cdot I_D}{\mu_n C_{ox} (W/L)}}$$

$$\therefore V_B = \sqrt{\frac{2 I_D}{\mu_n C_{ox} (W/L)}} + V_T$$

$$\therefore \frac{\partial V_B}{\partial I_D} = \frac{1}{\cancel{2} \sqrt{\frac{2 I_D}{\mu_n C_{ox} (W/L)}}} \cdot \frac{\cancel{2}}{\mu_n C_{ox} (W/L)}$$

$$= \frac{1}{\sqrt{2 I_D \mu_n C_{ox} (W/L)}}$$

$$\therefore \quad r_o = \frac{1}{\sqrt{2 \times I_D \times 100\times10^{-6} \times 10}}$$

$$\because \quad I_D = \frac{1}{2} \times 100 \times 10^{-6} \times 10 \,(0.5)^2 = 0.125\, mA$$

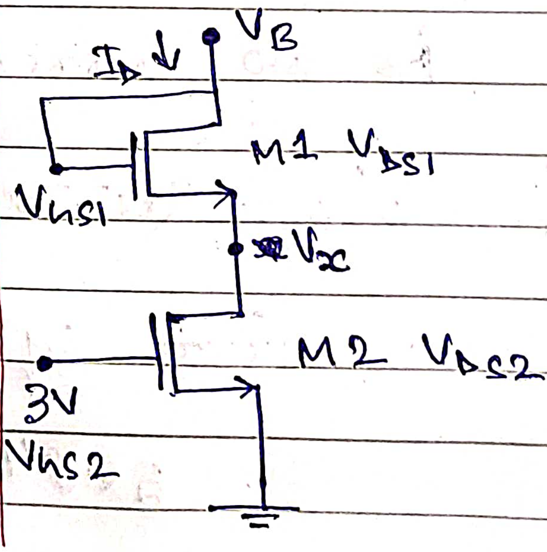
$$\therefore \quad r_o = \frac{1}{\left[2\times0.125\times10^{-3}\times100\times10^{-6}\times10\right]^{1/2}}$$

$$= \frac{1}{\left[2\times12.5\times10^{-8}\right]^{1/2}} = \frac{1}{5\times10^{-4}}$$

$$\therefore \quad r_o = 0.2\times10^{+4} \Rightarrow \boxed{r_o = 2\,k\Omega}$$

Q. 3> c>

M1 is operating in saturation region



$$\therefore \quad I_{D1} = \frac{1}{2} \mu_n C_{ox} \cdot \frac{W}{L} (V_{GS1} - V_T)^2$$

$$I_{D1} = \frac{1}{\cancel{2}} \times 100\times10^{-6} \times \cancel{10}^{\,5} \,(V_B - V_x - 0.2)^2$$

$$I_{D1} = 5\times10^{-4} (V_B - V_x - 0.2)^2 \quad (1)$$

Now for M2 to operate in sat. region

$V_{DS2} \geq V_{GS2} - V_T \geq (3 - 0.2) \geq 2.8$

$\therefore V_{DS2} = V_x \geq 2.8\,V$ ——— (2)

Now, if M2 will operate in saturation then,

$$I_{D2} = \frac{1}{2} \cdot \mu_n C_{ox} \cdot \frac{W}{L} (V_{GS} - V_T)^2$$

$$= \frac{1}{2} \cdot 100 \times 10^{-6} \times 10 \times (2.8)^2$$

$$\therefore I_{D2} = 3.92 \times 10^{-3}\,A$$

As no current will go in gate

$\therefore I_{D1} = I_{D2}$

From eqn (1)

$$5 \times 10^{-4} (V_B - V_x - 0.2)^2 = 3.92 \times 10^{-3}$$

$$(V_B - V_x - 0.2)^2 = 0.784 \times 10^1 = 7.84$$

$$\therefore V_B - V_x - 0.2 = 2.8$$

$$V_B - 0.2 - 2.8 = V_x$$

or, $V_x = V_B - 3$

$\therefore$ For M2 to operate in saturation

$V_x \geq 2.8V$ (by ②)

$$\therefore V_B - 3 \geq 2.8V$$

$$\therefore V_B \geq 5.8V$$

$$\therefore \boxed{V_{B\,min} = 5.8V}$$

Q.4> a>

$V_{ov} = 0.3V$, $I_D = 0.3mA$

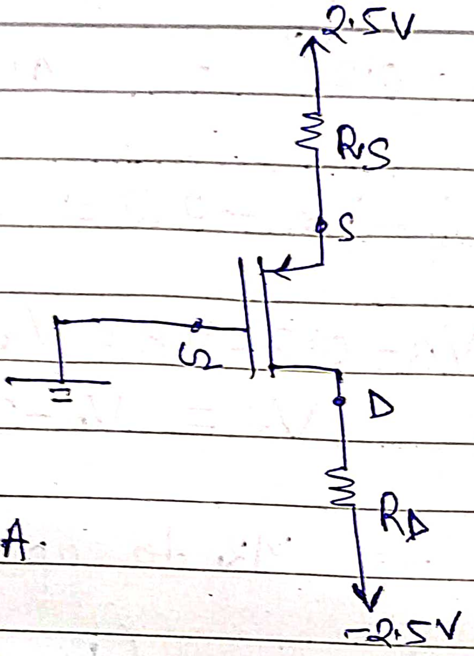
$$\therefore V_{ov} = V_{SG} - |V_T| = 0.3$$

$$\therefore V_{SG} = 0.3 + |V_T| = 0.3 + 0.8 = 1.1V$$

$\because V_{SH} = 1.1V$

$\therefore V_S = 1.1V$

$\therefore I_D = \frac{2.5 - 1.1}{R_S}$



$\frac{1.4}{R_S} = 0.3mA$

Eq. ckt for DC analysis

$\therefore$ $\boxed{R_S = \frac{1.4}{0.3} \times 10^3 = \text{4.67 K ohm}}$

Q.4)b) Overall voltage gain $(G_V) = -10(V/V)$

Since, overall voltage gain for the given ckt will be

$= -g_m R_D$ → This can be derived from small signal model.

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.3}{0.3} = 2\,mA/V$$

$\therefore \quad 10 = 2 \cdot R_D \Rightarrow \boxed{R_D = 5k\Omega}$ Ans.