# Quiz 1: Fields & Waves (ECE230), Winter 2025

Duration: 1hr 15min, Total: 25 points (Attempt all questions)

Feb 2, 2025

**Vector quantities must be represented with an overhead arrow, vector calculus operators must be written correctly, dot and cross products, closed and open integrals must be written properly. In case of any of these mistake, 1 will be awarded in each question with such mistake.

NO credit will be given to the answers that do not accompany proper explanation.

Any case of copying/cheating will be dealt as per institute guidelines.

Q1. The electrostatic potential of some charge-distribution is given by: $\nabla^2 \cdot V(r) = -\rho/\epsilon_0$

$$V(r) = A\frac{e^{-\lambda r}}{r}$$

where, $A$ and $\lambda$ are constants. Find the electric field $\vec{E}(r)$ and the charge density $\rho(r)$. [2 + 3 = 5]

Q2. Calculate curl of the function $\vec{A}(x, y, z) = \hat{x}yz + \hat{y}4xy + \hat{z}y$. [3]

Q3. Consider an infinitely long, thin wire, oriented along $z-$direction. Assume that it carries a uniform line-charge density $\lambda$.

a) Calculate the electric field at a point $P$ with cylindrical coordinates $(\rho, \phi, z)$.

b) How much work would you need to do if you want to move a charge $+q$ from $\rho = a$ to $\rho = b$? [3 + 3 = 6]

Q4. An infinite plane carries a surface charge density $\sigma$.

(a) Draw the geometry and direction of $\vec{E}$.

(b) Calculate the electric field. [1 + 3 = 4]

Q5. A total charge $Q$ is distributed uniformly over a sphere of radius $R$. How much work would you need to do to move a charge $q$ from $r = a$ to $r = b$? Here, $a, b < R$. [4]

Q6. Consider two infinite parallel plates kept at $x = 0$ and $x = d$, respectively. The first plate is connected to $5V$ and the second one to $-5V$. Calculate the potential distribution between them. [3]

# Quiz 2: Fields & Waves (ECE230), Winter 2025

Duration: 1hr 10 mins, Total: 20 points (Attempt all questions)

Feb 17, 2025

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Any case of copying, cheating will be dealt as per institute guidelines.

Q1. Consider an electrostatic potential $\phi(r) = \frac{k}{r^4}$ over free space. Calculate the energy density of the field. [3 points]

Q2. Evaluate the following integrals:

(a) $\int_{-1}^{+1} (r^2 + 2)\, \vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right) dv$

(b) $\int_{1}^{\infty} (r^2 + 2)\, \vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right) dv$ [1.5 + 1.5 = 3 points]

Q3. Choose correct options (if multiple options are correct you must choose all of them to get any credit):

(a) Which expression of divergence theorem is correct: (i) $\oint_V (\vec{\nabla}.\vec{E})\mathrm{dv} = \oint_S \vec{E}.d\vec{S}$, (ii) $\oint_V (\vec{\nabla}.\vec{E})\mathrm{dv} = \int_S \vec{E}.d\vec{S}$, (iii) $\int_V (\vec{\nabla}.\vec{E})\mathrm{dv} = \oint_S \vec{E}.d\vec{S}$, (iv) $\int_V (\vec{\nabla}.\vec{E})\mathrm{dv} = \int_S \vec{E}.d\vec{S}$ [1 point]

(b) Consider a charge-free interface between two media with permittivities $\epsilon_1$ and $\epsilon_2$. $E_{n1}$ and $E_{n2}$ are the normal components of the electric fields and $E_{t1}$ and $E_{t2}$ are the tangential components at the interface: (i) $E_{t1} = E_{t2}$, (ii) $E_{n1} = E_{n2}$, (iii) $\epsilon_1 E_{n1} = \epsilon_2 E_{n2}$, (iv) $\epsilon_1 E_{t1} = \epsilon_2 E_{t2}$. [1 point]

Q4. A thin uniformly charged spherical shell of radius $R$ carries a charge $Q$. Calculate:

(a) Energy density inside and outside the shell. [1 + 1 = 2 points]

(b) Total energy. [3 points]

Q5. An infinite plane carries a uniform surface charge $\sigma$. Find its electric field. [3 points]

Q6. A primitive model for an atom consists of a point nucleus $+q$ surrounded by a uniformly charged spherical cloud $-q$ of radius $a$. Calculate the atomic polarizability of such an atom (recall: $\vec{p} = \alpha\vec{E}$, $\alpha$ being the polarizability). You can assume that the spherical shape of the electron cloud remains undistorted. [4 points]

# Quiz 3: Fields & Waves (ECE230), Winter 2025

***Total marks:* 20. *Duration 1hr 15mins***

***Plagiarism policy: ZERO tolerance towards copying assignments from others/ plagiarism from any other sources. Such cases will be dealt strictly according to the institute policy.***

Q1. Under certain condition, the relative permittivity of a material can be written as: $\epsilon_r(\omega) = \left(1 - \frac{\omega_p^2}{\omega^2}\right)$.
Plot $\epsilon_r(\omega)$ as a function of frequency. You need to clearly show all the critical values in this plot. [3] points

Q2. Consider the expression of the electric field of a uniform plane wave traveling in vacuum:
$\vec{E} = 4\hat{x}\exp\left[i\left(5 \times 10^6 t - kz\right)\right]$
(a) What is the direction of polarization of this EM wave?
(b) Which direction is the wave traveling?
(c) How much is the value of $k$?
(d) What is the wavelength?

[1 + 1 + 1 + 1 = 4] points

Q3 You have studied Gauss's law already and this is one of the Maxwell's equations. Consider an inhomogeneous medium i.e. a medium where the permittivity is a function of $(x, y, z)$. How much is $\vec{\nabla}.\vec{E}$ for such a medium? [3] points

Q4. Suppose, you wake up one fine morning and find a magnetic monopole on your breakfast table. As a student of F&W, it is your responsibility to save EM theory by modifying Maxwell's equations. How are you going to modify 'em? What is your thought process behind those modifications? *[10] points*

# Quiz 4: Fields & Waves (ECE230), Winter 2025

***Total marks:*** 15. ***Duration*** 50 ***mins.***

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Q1. $(a)$ What is the unit of the ratio of $E$ to $H$ in a medium? Explain your answer (without explanation there's no marks).

$(b)$ Write down the above ratio for a medium with a finite conductivity $\sigma$.

$(c)$ Will $E$ and $H$ be in the same phase for the medium in part $(b)$?

$(d)$ Suppose $\sigma$ is very high within certain frequency range of interest. What would be the phase angle between $E$ and $H$?

$[1 + 2 + 1 + 3] = 7$ points

Q2. In free space:

$\vec{E} = 4\hat{x} \exp\left[i\left(5 \times 10^6 t - kz\right)\right]$

Calculate the displacement current. 2 points

Q3. Consider two uniform plane electromagnetic waves, of unequal amplitudes and same frequencies, propagating in opposite directions ($+z$ and $-z$). Obtain an expression for the resultant electric field and plot the amplitude of the total e-field as a function of $z$. [Hint: you may want to forget about the time variation part here and consider only the frequency domain field. Then try to put the total field in a form: $re^{i\theta}$] $[4 + 2] = 6$ points

# Quiz 5: Fields & Waves (ECE230), Winter 2025

***Total marks: 8. Duration 30mins***

***Plagiarism policy: ZERO tolerance towards copying assignments from others/ plagiarism from any other sources. Such cases will be dealt strictly according to the institute policy.***

Q1. What would happen if a solid metal disk rotates in a uniform, DC magnetic field? You need to provide reasoning behind your answer.
[3] points

Q2. Recall, Maxwell's equations in source-free region. Show that potentials in electrodynamics satisfies Helmholtz equation if $\vec{\nabla}.\vec{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0$.
[5] points

Mid-semester examination: Fields & Waves (ECE230), Winter 2025

Total: 20 points

Time: 1hr

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$0.256 \times 10^{-3}$

Q1. A certain material has a conductivity of $5 \times 10^6 s/m$ and a permittivity of $5F/m$. A total charge of $1C$ is distributed uniformly throughout the volume of a sphere made of this material. The radius of the sphere is $5m$. How much would be the surface charge density after $2\mu s$? ($1\mu s = 10^{-6}S$) [5 points]

Q2. An electron has a charge-to-mass ratio of $1.759 \times 10^{11} C/kg$. If the electron is moving with a speed of $3.518 \times 10^{3} m/s$ perpendicular to a uniform magnetic field of 1 Tesla, what is the radius of the cyclotron motion? [4 points]

Q3. Consider two infinite parallel plates kept at $x = 0$ and $x = d$, respectively. The first plate is connected to $5V$ and the second one to $-5V$. Calculate the potential distribution between them. [3 points]

Q4. An infinite plane carries a uniform surface charge $\sigma$. Find its electric field. [3 points]

Q5. A charge distribution has volume density $\rho = \frac{\rho_0 r}{R}$ for $0 \leq r \leq R$ and $0$ for $r > R$. Find the electric field for both regions. [2.5 + 2.5 = 5 points]

$\frac{r^2 \rho_0}{3R\epsilon_0}$ $\frac{\rho_0 R^2}{3r\epsilon_0}$

# End-sem: Fields & Waves (ECE230), Winter 2025

Total marks: 40, Time: 2hrs

**Vector quantities must be represented with an overhead arrow, vector calculus operators must be written correctly, dot and cross products must be written properly. In case of any of these mistake -1 will be awarded.

Q1. Derive the equation of continuity. What physical law does this equation represent? [2 + 1 = 3 points]

Q2. Consider a lossless transmission line has a distributed inductance of $227nH/m$ and distributed capacitance of $90.9pF/m$ and has been connected to a signal generator of frequency $14\pi \times 10^8$ Hz.

(a) How much is the characteristic impedance of the line?

(b) What would be the speed of the voltage wave propagating in this line?

(c) What would be the propagation constant of the wave?

(d) If the transmission line is terminated with a load $Z_L = 30\Omega$, how much would be voltage reflection coefficient at the load?

(e) Under the same condition as part $(d)$, what is the VSWR?

[2 + 2 + 2 + 2 + 2] = 10 points

Q3. Consider a uniform, $x-$polarized plane wave of frequency $3GHz$ is propagating along $z-$direction in a nonmagnetic medium ($\mu_0 = 4\pi \times 10^{-7} H/m$) with conductivity $7.2 \times 10^4 S/m$ and permittivity $24 \times 10^{-12} F/m$.

(a) Write down the effective permittivity of the medium.

(b) How much is the ratio $\frac{E}{H}$ for this wave?

(c) How much is the propagation constant of the EM wave?

[2 + 2 + 2 = 6 points]

Q4. Suppose, you are shining red light from air to glass slab (refractive index= 1.5). [3 points]

$(a)$ Calculate the reflection coefficient at the interface.

$(b)$ Does the electric field flips sign on reflection?

$(c)$ How much is the transmission coefficient is?

Q5. Choose correct options (if multiple options are correct you must choose all of them to get any credit)

(a) Consider an interface between two dielectric media with permeabilities $\mu_1$ and $\mu_2$. $B_{n1}$ and $B_{n2}$ are the normal components of the magnetic fields and $B_{t1}$ and $B_{t2}$ are the tangential components at the interface: $(i)$ $B_{t1} = B_{t2}$, $(ii)$ $\mu_1 B_{n1} = \mu B_{2n2}$, $(iii)$ $H_{n1} = H_{n2}$, $(iv)$ $H_{t1} = H_{t2}$. [1 point]

(b) Magnetostatic field is: $(i)$ conservative, $(ii)$ irrotational, $(iii)$ solenoidal, $(iv)$ all of the previous options. [1 point]

(c) Which expression of divergence theorem is correct: $(i)$ $\oint_V(\vec{\nabla}.\vec{E})dv = \oint_S \vec{E}.d\vec{S}$, $(ii)$ $\oint_V(\vec{\nabla}.\vec{E})dv = \int_S \vec{E}.d\vec{S}$, $(iii)$ $\int_V(\vec{\nabla}.\vec{E})dv = \oint_S \vec{E}.d\vec{S}$, $(iv)$ $\int_V(\vec{\nabla}.\vec{E})dv = \int_S \vec{E}.d\vec{S}$ [1 point]

(d) Consider a charge-free interface between two media with permittivities $\epsilon_1$ and $\epsilon_2$. $E_{n1}$ and $E_{n2}$ are the normal components of the electric fields and $E_{t1}$ and $E_{t2}$ are the tangential components at the interface: $(i)$ $E_{t1} = E_{t2}$, $(ii)$ $E_{n1} = E_{n2}$, $(iii)$ $\epsilon_1 E_{n1} = \epsilon_2 E_{n2}$, $(iv)$ $\epsilon_1 E_{t1} = \epsilon_2 E_{t2}$. [1 point]

Q6. For an electromagnetic field, the scalar potential $(\phi)$ and vector potential $(A)$ are given by:

$$\phi = (1/r^2)\exp(i\omega t)$$

$$\vec{A} = (\hat{r}/r)\exp(i\omega t)$$

Calculate the electric field. [4 points]

Q7. The voltage in a transmission line is given by: $\vec{V} = [6\exp(ikz) + 3\exp(-ikz)]$. Calculate voltage and current SWR. [2 points]

Q8. Consider an electrostatic potential $\phi = \frac{1}{(x^2+y^2+z^2)^{1/4}}$. Calculate:

$(i)$ the electric field $(\vec{E})$

$(ii)$ the volume charge density $\rho$. [3 + 3 = 6 points]

Q9. Calculate curl of the function $\vec{A}(x, y, z) = \hat{x}yz + \hat{y}4xy + \hat{z}y$. [2 points]

2/2.5