Mid-semester examination: Fields & Waves (ECE230), Winter 2020

Total: 25 points

Deadline: March 7th, 11:59pm [Email your answer scripts to the instructor]

Any case of unfair means will be dealt as per rules laid down by the institute.

Q1. The electrostatic potential of some charge-distribution is given by:

$$V(r) = A\frac{e^{-\lambda r}}{r}$$

where, A and λ are constants. Find the electric field $\vec{E}(r)$ and the charge density $\rho(r)$. [$3 + 4 = 7$ points]

Q2. Let's try building a one-dimensional crystal lattice from the scratch. Consider an atomic nucleus carrying Z number of protons. Assume that the Coulomb's law holds in this situation. We take many of such nuclei and start placing them at a distance a from each other along a straight line. So, we end up constructing a very long (perhaps, infinite if we could actually have an infinite number of nuclei) straight chain of equi-spaced nuclei. Can you approximately draw the potential energy seen by a free electron in this lattice? A free electron is the one that is free to roam around the lattice, just like the ones in a metal. A good way to attack this problem would be to start by drawing the potential energy of the electron when there was just one nucleus present. After this, you can think that when you bring the nuclei close, they start sensing each others' presence through the electric field.

10 points (there are partial points depending on your approach, so think harder!)

Q3. You must have heard that lightning prefers to strike the sharper metals more readily. The reason is, the electric fields are more intense near the sharp parts of a metallic object. So, the air molecules near the sharper parts become ionized more readily by the intense electric field existing between the object and the clouds. This ionized path provides a conducting channel for the current to flow between the sharp part of the metallic object and the cloud. This is what you see as a lightning-strike. But what causes the electric field to be more intense near the sharp parts of the metal? Walk through the following steps to answer this:

(a) Imagine two charged metal spheres, one bigger than the other. Also, they are placed at sufficiently large distance so that the charge distribution on one sphere has negligible effect on the charge distribution of the other. The two are connected through a metal wire. If all the metals are perfect conductors and the potential of the wire is V, compare the total charges residing on each sphere.

(b) Compare the surface charge densities on the spheres.

(c) Apply Gauss's law to qualitatively compare the electric fields on each sphere.

(d) Now, try to reason why the electric field is more intense near the sharper parts of an arbitrarily shaped object.

$4 \times 2 = 8$ points