Q.1) $I = -\frac{dQ}{dt}$

$\Rightarrow \oint \vec{J} \cdot \vec{ds} = -\frac{d}{dt} \int \rho \, dV$

If this circle isn't present -0.5

$\Rightarrow \int (\vec{\nabla} \cdot \vec{J}) \, dV = -\int \left(\frac{\partial \rho}{\partial t}\right) dV$

$\Rightarrow \vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$ 2

It represents law of conservation of charge 1

Q.2 (a) Characteristic impedance $(Z_0) = \sqrt{\frac{L}{C}}$

No part marking

$$= \sqrt{\frac{227\times10^{-9}}{90.9\times10^{-12}}}\ \Omega$$

$$= 49.97\ \Omega$$ (2)

(b) speed $= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{227\times10^{-9}\times90.9\times10^{-12}}}$ m/s (2)

$$= 2.2\times10^{8}\ \text{m/s}$$

(c) $v = \frac{\omega}{\beta} = \frac{2\pi f}{\beta}$

$$\Rightarrow \beta = \frac{2\pi f}{v} = \frac{2\pi\times14\pi\times10^{8}}{2.2\times10^{8}}\ \text{rad/s}$$

$$= \frac{28\pi^{2}}{2.2} = 62.81\ \text{rad/s}$$ (2)

(d) $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30-49.97}{30+49.97} = -0.25$ (2)

(e) VSWR $= \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+0.25}{1-0.25} = 1.67$ (2)

Q.3) (a) $f = 3\,GHz = 3\times10^{9}\,Hz$

$\omega = 2\pi f = 6\pi\times10^{9}\,Hz$

$\sigma = 7.2\times10^{4}\,S/m$

$\varepsilon = 24\times10^{-12}\,F/m$

$$\varepsilon_{eff} = \varepsilon\left(1 - \frac{i\sigma}{\omega\varepsilon}\right)$$

$$= \varepsilon - \frac{i\sigma}{\omega}$$

$$= 24\times10^{-12} - i\,\frac{7.2\times10^{4}}{6\pi\times10^{9}}$$

$$= 24\times10^{-12} - i\,0.382\times10^{-5}$$

Award (-1) if this part of the conversion isn't done. Also, this/any other mistake here will make the answers of part (b) and (c) wrong. So, leads to '0' in subsequent parts.

(b) $$\frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_{eff}}} = \sqrt{\frac{4\pi\times10^{-7}}{24\times10^{-12} - i\,0.382\times10^{-5}}}$$

$$= \sqrt{\frac{4\pi\times10^{-7}\,(24\times10^{-12} + i\,0.382\times10^{-5})}{(24\times10^{-12} - i\,0.382\times10^{-5})(24\times10^{-12} + i\,0.382\times10^{-5})}}$$

$$= \sqrt{\frac{4\pi\times10^{-7}\,(24\times10^{-12} + i\,0.382\times10^{-5})}{576\times10^{-24} + 14.52\times10^{-12}}}$$

↳ neglecting this term as it is very small.

$$= \sqrt{8.65\times10^{4}\,(24\times10^{-12} + i\,3.82\times10^{-6})}$$

$$= \sqrt{207.6\times10^{-8} + i\,32.96\times10^{-2}}$$

[Some may simplify further by neglecting real part as it is small comparison to imaginary part].

(c) $\beta = \omega\sqrt{\mu_0 \epsilon_{eff}}$

$= 6\pi \times 10^9 \sqrt{4\pi \times 10^{-7}(24 \times 10^{-12} - i0.382 \times 10^{-5})}$

$= 6\pi \times 10^9 \sqrt{301.44 \times 10^{-19} - i4.797 \times 10^{-12}}$

[Again, some may simplify further by ~~using~~ neglecting real part.]

Q.4) (a) $n_1 = 1$, $n_2 = 1.5$

No part marking

$\Gamma = \dfrac{n_1 - n_2}{n_1 + n_2} = \dfrac{1 - 1.5}{1 + 1.5} = \dfrac{-0.5}{2.5} = -\dfrac{1}{5} = -0.2$

(b) $\because \Gamma < 0$, Yes electric field flip sign.

[No marks if part (a) is wrong. No marks if $\Gamma < 0$ isn't specified as reason]

(c) $T = \dfrac{2n_1}{n_1 + n_2} = \dfrac{2}{2.5} = \dfrac{4}{5} = 0.8$

Alternate solution:

$T = 1 + \Gamma$

$= 1 - 0.2 = 0.8$.

Q.5) (a) (iv) $\rightarrow H_{t1} = H_{t2}$

No part marking

(b) (iii) $\rightarrow$ Solenoidal

(c) (iii) $\rightarrow \int (\vec{\nabla} \cdot \vec{E}) dv = \oint \vec{E} \cdot \vec{dS}$

(d) (i) & (iii)

Q.6) $\phi = \frac{1}{r^2} e^{i\omega t}$

$\vec{A} = \frac{\hat{r}}{r} e^{i\omega t}$

$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$ ← 1 mark

- $\vec{\nabla}\phi = \frac{\partial \phi}{\partial r}\hat{r} = \frac{\partial}{\partial r}\left(\frac{1}{r^2} e^{i\omega t}\right)\hat{r}$

$= -\frac{2}{r^3} e^{i\omega t}\hat{r}$

- $\frac{\partial \vec{A}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\hat{r}}{r} e^{i\omega t}\right)$

$= i\omega \cdot \frac{\hat{r}}{r} e^{i\omega t}$

No part marking other than stated.

$\Rightarrow \vec{E} = -\left(-\frac{2}{r^3} e^{i\omega t}\hat{r}\right) - \left(i\omega \cdot \frac{1}{r} e^{i\omega t}\hat{r}\right)$

$= \frac{2}{r^3} e^{i\omega t}\hat{r} - \frac{i\omega}{r} e^{i\omega t}\hat{r}$

3 marks

~~marks~~

$= \left(\frac{2}{r^2} - i\omega\right)\frac{e^{i\omega t}}{r}\hat{r}$

Ques 7 $\Rightarrow$ Voltage in a transmission line is given by

$$\vec{V} = [6 \exp(ikz) + 3 \exp(-ikz)].$$

Calculate Voltage and Current SWR.

Solution

$$CSWR = -VSWR$$

$$\Gamma = \frac{6e^{ikz}}{3e^{-ikz}} = 2e^{2ikz}$$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+2}{1-2} = -3$$ — Mark ①

$$CSWR = +VSWR = -3$$ — Mark ①

Ques 8 $\Rightarrow$ Electrostatic potential $\phi = \frac{1}{(x^2+y^2+z^2)^{1/4}}$.

(i) The electric field $(\vec{E})$

$$\vec{E} = -\nabla\phi$$

or in Cartesian Coordinates: $\Rightarrow$

$$\vec{E} = -\left(\frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}\right)$$

Let $R = x^2+y^2+z^2$

so $\phi = R^{-1/4}$

and $$\frac{\partial\phi}{\partial x} = \frac{\partial\phi}{\partial R}\cdot\frac{\partial R}{\partial x} = \frac{\partial(R^{-1/4})}{\partial R}\cdot\frac{\partial(x^2+y^2+z^2)}{\partial x}$$

$$= -\frac{1}{4}R^{-5/4}\cdot 2x = -\frac{1}{2}\frac{x}{(x^2+y^2+z^2)^{5/4}}$$

— ① Mark upto this point

Similarly for y and z $\Rightarrow$

$$\frac{\partial \phi}{\partial y} = \frac{-1}{2} \frac{y}{(x^2+y^2+z^2)^{5/4}} \text{ and } \frac{\partial \phi}{\partial z} = \frac{-1}{2} \frac{z}{(x^2+y^2+z^2)^{5/4}}$$ full

So, finally $\Rightarrow$ $\vec{E} = -\nabla\phi = \frac{1}{2} \frac{x\hat{i}+y\hat{j}+z\hat{k}}{(x^2+y^2+z^2)^{5/4}}$ — 3 Marks

(b) Charge density $\Rightarrow$ $-\nabla^2\phi = \frac{\rho}{\varepsilon_0}$

or $\rho = -\varepsilon_0 \nabla^2\phi$

In Cartesian Coordinate $\Rightarrow$ $\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$

As we already calculated $\frac{\partial\phi}{\partial x} = \frac{-1}{2} \frac{x}{(x^2+y^2+z^2)^{5/4}} = \frac{-1}{2}\frac{x}{R^{5/4}}$

Differentiating again will give $\Rightarrow$ $\frac{\partial^2\phi}{\partial x^2} = \frac{\partial}{\partial R}\left(\frac{\partial\phi}{\partial x}\right) \cdot \frac{\partial^2 R}{\partial x^2}$

$=$

and

$$\frac{\partial^2\phi}{\partial x^2} = \frac{-1}{2}\left[\frac{1}{R^{5/4}} - \frac{5x^2}{2R^{9/4}}\right]$$ — 1 Mark upto this point

Similar terms will appear for y and z $\Rightarrow$

and finally $\Rightarrow$ $\nabla^2\phi = -\frac{1}{2}\left[\left(\frac{1}{R^{5/4}} - \frac{5x^2}{2R^{9/4}}\right) + \left(\frac{1}{R^{5/4}} - \frac{5y^2}{2R^{9/4}}\right) + \left(\frac{1}{R^{5/4}} - \frac{5z^2}{2R^{9/4}}\right)\right]$

substituting $\boxed{x^2+y^2+z^2 = R}$

$$\nabla^2\phi = -\frac{1}{2}\left[\frac{3}{R^{5/4}} - \frac{5R}{2R^{9/4}}\right] = \frac{-1}{4R^{5/4}} = \frac{-1}{4(x^2+y^2+z^2)^{5/4}}$$

So $\rho = -\varepsilon_0 \nabla^2\phi$

$$\rho = \frac{+\varepsilon_0}{4(x^2+y^2+z^2)^{5/4}}$$

full 3 Marks

Q.9) $\vec{A} = yz\,\hat{x} + 4xy\,\hat{y} + y\,\hat{z}$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & 4xy & y \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial}{\partial y} y - \frac{\partial}{\partial z} 4xy\right) - \hat{y}\left(\frac{\partial}{\partial x} y - \frac{\partial}{\partial z} yz\right)$$

$$+ \hat{z}\left(\frac{\partial}{\partial x} 4xy - \frac{\partial}{\partial y} yz\right)$$

No part marking.

$$= \hat{x}(1-0) - \hat{y}(0-y) + \hat{z}(4y-z)$$

$$= \hat{x} + y\hat{y} + \hat{z}(4y-z)$$

2