# Practice_Set_2_Solution

**Question 1. Solution:**

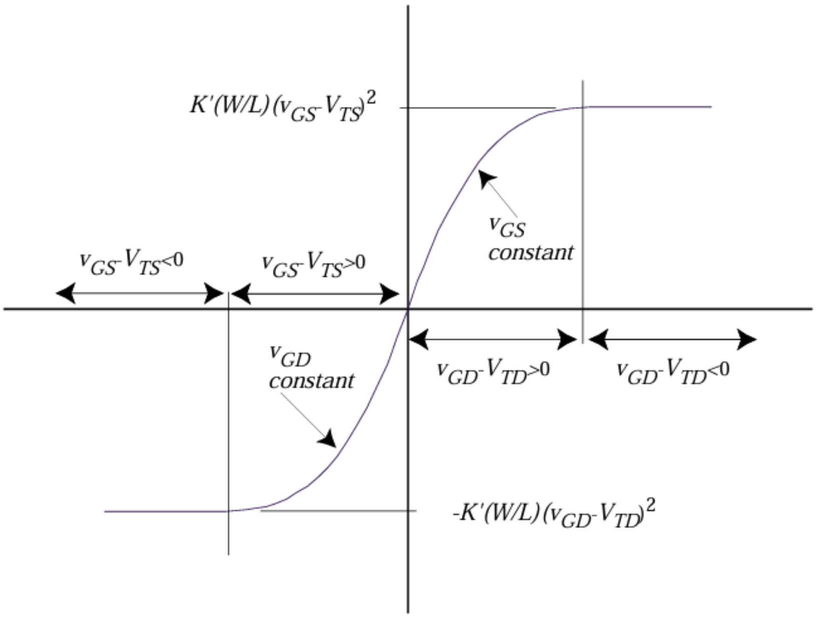


Figure 1: Id versus Vds

**Question 2. Solution:**

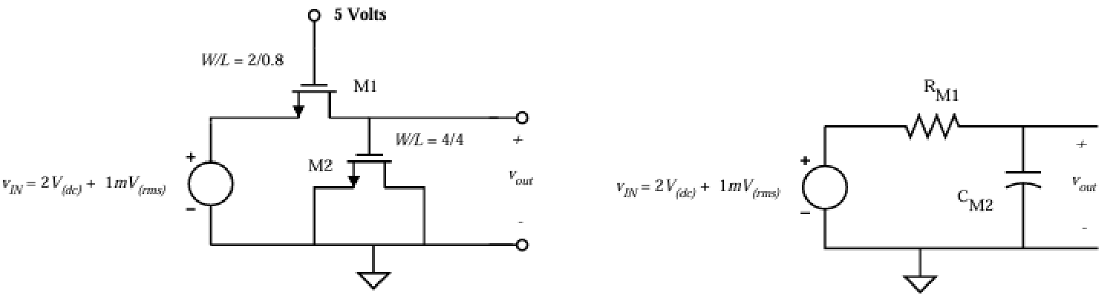


Figure 2

$$\frac{v_{\text{out}}(s)}{v_{\text{IN}}(s)} = \frac{\dfrac{1}{sC_{M2}}}{R_{M1} + \dfrac{1}{sC_{M2}}} = \frac{1}{sC_{M2}R_{M1} + 1}$$

$$V_{T1} = V_{T0} + \gamma\Big(\sqrt{2|\phi_F| + v_{SB}} \;-\; \sqrt{2|\phi_F|}\Big)$$

$$V_{T1} = 0.7 + 0.4\big(\sqrt{0.7 + 2.0} \;-\; \sqrt{0.7}\big) = 1.02$$

$$R_{M1} = \frac{1}{K'(W/L)_{M1}\,(v_{GS1} - V_{T1})} \;=\; 1.837 \text{ k}\Omega$$

$$C_{M2} = W_{M2}\,L_{M2}\,C_{\text{ox}} = (4\times10^{-6})(4\times10^{-6})(24.7\times10^{-4}) = 39.52\times10^{-15} \text{ F}$$

$$R_{M1}C_{M2} = (1.837 \times 10^3) \times (39.52 \times 10^{-15}) = 72.6 \times 10^{-12} \text{ s}$$

$$\frac{v_{\text{out}}(s)}{v_{\text{IN}}(s)} = \frac{1}{s(R_{M1}C_{M2}) + 1} = \frac{1}{s(72.6 \times 10^{-12}) + 1} = \frac{1}{\dfrac{s}{13.77 \times 10^9} + 1}$$

**Question 3. Solution:**

*The expression for drain current in saturation is:*

$$I_D \;=\; \frac{K'W}{2L}\,(v_{GS} - v_T)^2\,(1 + \lambda v_{DS})$$

*For multiple transistors with the same drain, gate, and source voltage, the drain current can be expressed simply as*

$$I_{D(i)} \;=\; \left(\frac{W}{L}\right)_i (v_{GS} - v_T)^2\,(1 + \lambda v_{DS})$$

*The drain current in each transistor is additive to the total current, thus*

$$I_{D(\text{TOTAL})} \;=\; (v_{GS} - v_T)^2\,(1 + \lambda v_{DS})\left[\sum_i \left(\frac{W}{L}\right)_i\right]$$

*Since the lengths are the same, we have*

$$I_{D(\text{TOTAL})} \;=\; \frac{1}{L}\,(v_{GS} - v_T)^2\,(1 + \lambda v_{DS})\left[\sum_i W_i\right]$$

**Question 4. Solution:**

*Error in problem statement : replace "parallel" with "series"*

*Assume that all devices are in the non-saturation region. Consider the case for two transistors in series as illustrated below.*

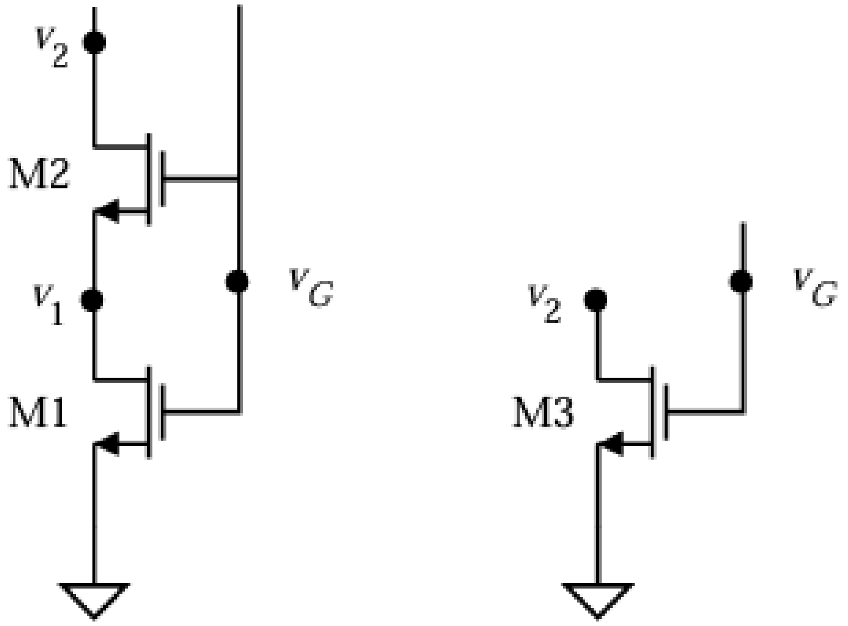


Figure 3

*The drain current in M1 is*

$$i_1 \;=\; \frac{K'W}{L}\Big[(v_{GS}-v_T)v_{DS}-\frac{v_{DS}^2}{2}\Big]$$

*Thus, for the node voltage* $v_1$ *(with notation* $v_{DS}=v_1$ *etc.)*

$$i_1 \;=\; \beta_1\big[(v_{GS}-v_T)v_1-\tfrac{v_1^2}{2}\big] \;=\; \beta_1\big[(v_G-v_T)v_1-\tfrac{v_1^2}{2}\big] \;=\; \beta_1\big[V_{\text{on}}\,v_1-\tfrac{v_1^2}{2}\big],$$

*where*

$$V_{\text{on}}=v_G-v_T.$$

*Solving for* $v_1$ *in terms of* $i_1$ *gives*

$$v_1 \;=\; V_{\text{on}}-\sqrt{V_{\text{on}}^2-\frac{2i_1}{\beta_1}}.$$

*Squaring (and expanding) yields*

$$v_1^2 \;=\; 2V_{\text{on}}^2-2V_{\text{on}}\sqrt{V_{\text{on}}^2-\frac{2i_1}{\beta_1}}-\frac{2i_1}{\beta_1}.$$

*The drain current in M2 is*

$$i_2 \;=\; \beta_2\Big[(v_G-v_1-v_T)(v_2-v_1)-\frac{(v_2-v_1)^2}{2}\Big].$$

*Using $V_{\text{on}} = v_G - v_T$ we write*

$$i_2 \;=\; \beta_2\Big[(V_{\text{on}} - v_1)(v_2 - v_1) - \frac{(v_2 - v_1)^2}{2}\Big].$$

*Expanding gives the form used in the notes:*

$$i_2 \;=\; \beta_2\Big[V_{\text{on}}v_2 - V_{\text{on}}v_1 + \tfrac{v_1^2}{2} - \tfrac{v_2^2}{2}\Big].$$

*Substitute the earlier expression for $v_1$ and, after algebra and equating the drain currents (i.e. $i_1 = i_2$ for series devices), one obtains the compact result*

$$i_2 \;=\; \frac{\beta_1\beta_2}{\beta_1 + \beta_2}\Big[V_{\text{on}}v_2 - \frac{v_2^2}{2}\Big].$$

*The expression for the current in M3 is*

$$i_3 \;=\; \beta_3\Big[(v_{GS} - v_T)\,v_2 - \frac{v_2^2}{2}\Big] \;=\; \beta_3\Big[V_{\text{on}}v_2 - \frac{v_2^2}{2}\Big].$$

*The drain current in M3 must be equivalent to the drain current in M1 and M2, thus*

$$\beta_3 \;=\; \frac{\beta_1\beta_2}{\beta_1 + \beta_2} \;=\; \Big(\frac{1}{\beta_1} + \frac{1}{\beta_2}\Big)^{-1} \;=\; \Big(\frac{L_1}{K'W_1} + \frac{L_2}{K'W_2}\Big)^{-1}.$$

*Since the* ***widths are equal*** *and the transconductances are equal, the notes state*

$$\beta_3 \;=\; \frac{1}{K'W}\,(L_1 + L_2).$$

*(Analysis may be extended to any number of transistors in series; repeat the two-at-a-time analysis with M3 and another device in series.)*

*Thus the equivalent length is*

$$L_{\text{EQUIVALENT}} \;=\; \sum_i L_i.$$

**Question 5. Solution:**

$$g_m \;=\; \frac{W}{L}\left(\frac{1}{n(kT/q)}\right) I_{DO}\exp\!\left(\frac{v_{GS}}{n(kT/q)}\right)$$

*Square both sides and equate to the expression involving $I_D$:*

$$\left(\frac{W}{L}\right)^2\left(\frac{1}{n(kT/q)}\right)^2 I_{DO}^2\exp\!\left(\frac{2v_{GS}}{n(kT/q)}\right) \;=\; (2K')\frac{W}{L}\,I_D$$

*Divide by* $\frac{W}{L}$ *to simplify:*

$$\frac{W}{L}\left(\frac{1}{n(kT/q)}\right)^2 I_{DO}^2 \exp\left(\frac{2v_{GS}}{n(kT/q)}\right) \;=\; 2K' I_D$$

*Noting that* $I_D = \frac{W}{L} I_{DO} \exp(\frac{v_{GS}}{n(kT/q)})$, *we have*

$$I_{DO} \exp\left(\frac{v_{GS}}{n(kT/q)}\right) \;=\; \frac{I_D}{W/L}.$$

*Substitute this into the previous relation to obtain*

$$2K' \left[n(kT/q)\right]^2 \;=\; I_{DO} \exp\left(\frac{v_{GS}}{n(kT/q)}\right) \;=\; \frac{I_D}{W/L}.$$

*Therefore the final compact form is*

$$\boxed{I_D \;=\; 2K' \frac{W}{L} \left[n(kT/q)\right]^2}$$

**Question 6. Solution:**

*The equation for threshold voltage with absolute values so that it can be applied to n-channel or p-channel transistors without confusion.*

$$|V_T| = |V_{T0}| + \gamma\left[\sqrt{2|\phi_F| + |V_{SB}|} \;-\; \sqrt{2|\phi_F|}\right]$$

$$r_{ON} \;=\; \frac{1}{\partial I_D/\partial V_{DS}} = \frac{L}{K'W(|V_{GS}| - |V_T| - |V_{DS}|)} = \frac{L}{K'W(|V_{GS}| - |V_T|)} \qquad (when\ V_{DS}$$

*For n-channel device,*

$$V_{T0} = 0.7$$

$$\gamma = 0.4$$

$$2|\phi_F| = 0.7$$

*The table below shows the value of* $V_{GS}$ *and* $V_{SB}$ *for each value of* $V_S$

| $V_S$ (volts) | $V_{GS}$ (volts) | $V_{SB}$ (volts) |
|---|---|---|
| 0.0 | 5 | 0 |
| 1.0 | 4 | 1 |
| 2.0 | 3 | 2 |
| 3.0 | 2 | 3 |
| 4.0 | 1 | 4 |
| 5.0 | 0 | 5 |

*Using* $V_S = 0$, *calculate* $V_T$

$$|V_T| = |V_{T0}| + \gamma\left[\sqrt{2|\phi_F| + |V_{SB}|} - \sqrt{2|\phi_F|}\right] = 0.7 + 0.4\left[\sqrt{0.7 + 0.0} - \sqrt{0.7}\right] = 0.7$$

*Calculate* $r_{on}$

$$r_{ON} = \frac{1}{\partial I_D / \partial V_{DS}} = \frac{L}{K'W(|V_{GS}| - |V_T| - |V_{DS}|)} = \frac{1}{110\mu \times 2(5 - 0.7 - 0)} = 1057\,\Omega$$

*Repeat for* $V_S = 1$

$$|V_T| = 0.7 + 0.4\left[\sqrt{0.7 + 1.0} - \sqrt{0.7}\right] = 0.887$$

$$r_{ON} = \frac{1}{110\mu \times 2(4 - 0.887 - 0)} = 1460\ \Omega$$

*Repeat for* $V_S = 2$

$$|V_T| = 0.7 + 0.4\left[\sqrt{0.7 + 2.0} - \sqrt{0.7}\right] = 1.023$$

$$r_{ON} = \frac{1}{110\mu \times 2(3 - 1.023 - 0)} = 2299\ \Omega$$

*Repeat for* $V_S = 3$

$$|V_T| = 0.7 + 0.4\left[\sqrt{0.7 + 3.0} - \sqrt{0.7}\right] = 1.135$$

$$r_{ON} = \frac{1}{110\mu \times 2(2 - 1.135 - 0)} = 5253\ \Omega$$

*Repeat for* $V_S = 4$

$$|V_T| = 0.7 + 0.4\left[\sqrt{0.7 + 4.0} - \sqrt{0.7}\right] = 1.233$$

$$r_{ON} = \frac{1}{110\mu \times 2(1 - 1.233 - 0)} = -19549\ \Omega$$

*The negative sign means that the device is off due to the fact that* $V_{GS} < V_T$*. Thus*

$$r_{ON} = \infty$$

*Repeat for* $V_S = 5$

$$|V_T| = 0.7 + 0.4\left[\sqrt{0.7 + 5.0} - \sqrt{0.7}\right] = 1.320$$

$$r_{ON} = \frac{1}{110\mu \times 2(0 - 1.320 - 0)} = -3442\ \Omega$$

*The negative sign means that the device is off due to the fact that* $V_{GS} < V_T$*. Thus*

$$r_{ON} = \infty$$

*Summary:*

| $V_S$ *(volts)* | *R (ohms)* |
|---|---|
| *0.0* | *1057* |
| *1.0* | *1460* |
| *2.0* | *2299* |
| *3.0* | *5253* |
| *4.0* | *infinity* |
| *5.0* | *infinity* |

**Question 7. Solution:**

*Dividing this problem into two part. First part already discussed in above question.*

Table 1: Summary for n-channel device from Problem 6

| $V_S$ **(volts)** | **R (ohms)** |
|---|---|
| 0.0 | 1057 |
| 1.0 | 1460 |
| 2.0 | 2299 |
| 3.0 | 5253 |
| 4.0 | infinity |
| 5.0 | infinity |

*Now in the 2nd step:*

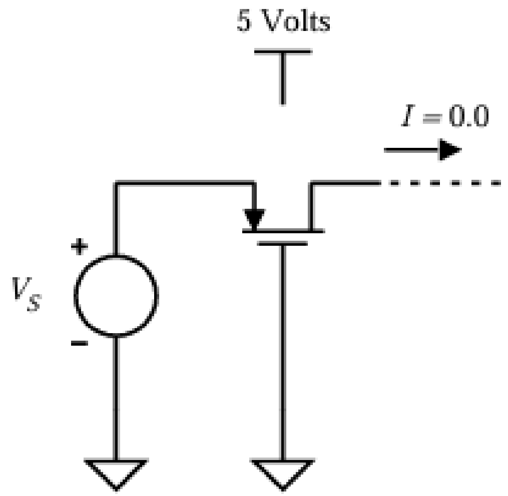


Figure 4

*The equation for threshold voltage with absolute values so that it can be applied to n-channel or p-channel transistors without confusion.*

$$|V_T| = |V_{T0}| + \gamma\left[\sqrt{2|\phi_F| + |V_{SB}|} \; - \; \sqrt{2|\phi_F|}\right]$$

$$r_{ON} \; = \; \frac{1}{\partial I_D / \partial V_{DS}} = \frac{L}{K'W(|V_{GS}| - |V_T| - |V_{DS}|)} = \frac{L}{K'W(|V_{GS}| - |V_T|)} \qquad (when\ V_{DS}$$

*For p-channel device,*

$$|V_{T0}| = 0.7$$

$$K' = 50\mu$$

$$\gamma = 0.57$$

$$2|\phi_F| = 0.8$$

*The table below shows the value of $V_{GS}$ and $V_{SB}$ for each value of $V_S$*

| $V_S$ *(volts)* | $V_{GS}$ *(volts)* | $V_{BS}$ *(volts)* |
|---|---|---|
| *0.0* | *0* | *5* |
| *1.0* | *1* | *4* |
| *2.0* | *2* | *3* |
| *3.0* | *3* | *2* |
| *4.0* | *4* | *1* |
| *5.0* | *5* | *0* |

*Using* $V_S = 5,$ *calculate* $V_T$

$$|V_T| = |V_{T0}| + \gamma\left[\sqrt{2|\phi_F| + |V_{SB}|} - \sqrt{2|\phi_F|}\right] = 0.7 + 0.57\left[\sqrt{0.8+0.0} - \sqrt{0.8}\right] = 0.7$$

*Calculate* $r_{on}$

$$r_{ON} = \frac{1}{\partial I_D/\partial V_{DS}} = \frac{L}{K'W(|V_{GS}| - |V_T| - |V_{DS}|)} = \frac{1}{50\mu \times 4(5-0.7-0)} = 1163\ \Omega$$

*Repeat for* $V_S = 4$

$$|V_T| = 0.7 + 0.57\left[\sqrt{0.8+1.0} - \sqrt{0.8}\right] = 0.955$$

$$r_{ON} = \frac{1}{50\mu \times 4(4-0.955-0)} = 1642\ \Omega$$

*Repeat for* $V_S = 3$

$$|V_T| = 0.7 + 0.57\left[\sqrt{0.8+2.0} - \sqrt{0.8}\right] = 1.144$$

$$r_{ON} = \frac{1}{50\mu \times 4(3-1.144-0)} = 2694\ \Omega$$

*Repeat for* $V_S = 2$

$$|V_T| = 0.7 + 0.57\left[\sqrt{0.8+3.0} - \sqrt{0.8}\right] = 1.301$$

$$r_{ON} = \frac{1}{50\mu \times 4(2-1.301-0)} = 7145\ \Omega$$

*Repeat for* $V_S = 1$

$$|V_T| = 0.7 + 0.57\left[\sqrt{0.8+4.0} - \sqrt{0.8}\right] = 1.439$$

$$r_{ON} = \frac{1}{50\mu \times 4(1-1.439-0)} = -11390\ \Omega$$

*The negative sign means that the device is off due to the fact that* $V_{GS} < V_T$. *Thus*

$$r_{ON} = \infty$$

*Repeat for* $V_S = 0$

$$|V_T| = 0.7 + 0.57\left[\sqrt{0.8 + 5.0} - \sqrt{0.8}\right] = 1.563$$

$$r_{ON} = \frac{1}{50\mu \times 4(0 - 1.563 - 0)} = 3199\ \Omega$$

*The negative sign means that the device is off due to the fact that $V_{GS} < V_T$. Thus*

$$r_{ON} = \infty$$

*Summary:*

| $V_S$ *(volts)* | *R (ohms)* |
|---|---|
| *0.0* | *infinity* |
| *1.0* | *infinity* |
| *2.0* | *7145* |
| *3.0* | *2694* |
| *4.0* | *1642* |
| *5.0* | *1163* |

*Combining both steps, we get-*

Table 2: Table showing both and their parallel combination:

| $V_S$ (volts) | R (ohms), n-channel | R (ohms), p-channel | R (ohms), parallel |
|---|---|---|---|
| 0.0 | 1057 | infinity | 1057 |
| 1.0 | 1460 | infinity | 1460 |
| 2.0 | 2299 | 7145 | 1739 |
| 3.0 | 5253 | 2694 | 1781 |
| 4.0 | infinity | 1642 | 1642 |
| 5.0 | infinity | 1163 | 1163 |

**Question 8. Solution:**

*Using Eq. ,*

$$V_{REF} = V_T - \frac{1}{\beta_R} + \sqrt{\frac{2(V_{DD} - V_T)}{\beta_R} + \frac{1}{\beta_R^2 R^2}}$$

$$\beta R = 220 \times 10^{-6} \times 10^5 = 22$$

$$V_{REF} = 0.7 - \frac{1}{22} + \sqrt{\frac{2(5 - 0.7)}{22} + \left(\frac{1}{22}\right)^2}$$

$$V_{REF} = 1.281$$

$$\frac{1}{R}\frac{dR}{dT} = 1500 \ ppm/^{\circ}C$$

$$\frac{dV_{REF}}{dT} = -\alpha + \frac{\sqrt{V_{DD} - V_{REF}}}{2\beta R}\left(\frac{1.5}{T} - \frac{1}{R}\frac{dR}{dT}\right) \Bigg/ \left(1 + \frac{1}{\sqrt{2\beta R\left(V_{DD} - V_{REF}\right)}}\right)$$

$$\frac{dV_{REF}}{dT} = \frac{-2.3 \times 10^{-3} + \sqrt{\frac{5-1.281}{2(22)}}\left(\frac{1.5}{300} - 1500 \times 10^{-6}\right)}{1 + \frac{1}{\sqrt{2(22)\left(5-1.281\right)}}}$$

$$\frac{dV_{REF}}{dT} = -1.189 \times 10^{-3} \ \ V/^{\circ}C$$

*The fractional temperature coefficient is given by*

$$TC_f = \frac{1}{V_{REF}}\frac{dV_{REF}}{dT}$$

*giving, for this example,*

$$TC_f = -1.189 \times 10^{-3}\left(\frac{1}{1.281}\right)^{\circ} C^{-1} = -928 \ ppm/^{\circ}C$$

**Question 9. Solution:**

***PART-A:*** *The small-signal model of this circuit is shown below:*

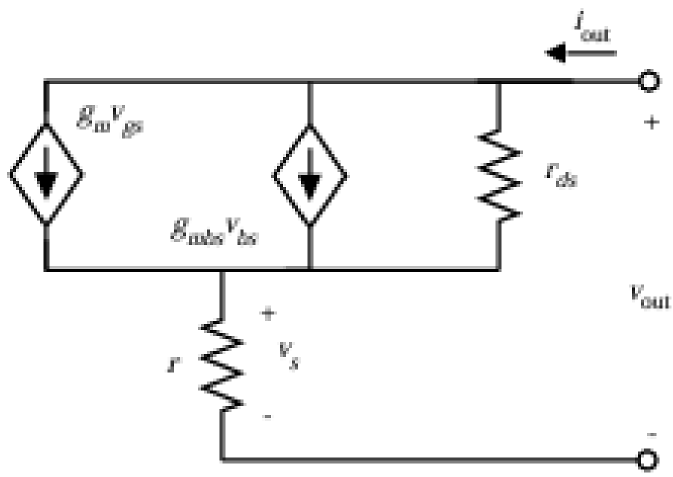


Figure 5

*First calculate dc terminal conditions.*

$$I_D = 10\,\mu A$$

$$V_S = I_D \times R = 10 \times 10^{-6} \times 100 \times 10^{3} = 1\ volt$$

$$V_S = V_{SB}$$

$$r_{out} = \frac{v_{out}}{i_{out}} = r + r_{ds} + \big[(g_m + g_{mb})r_{ds}\big]r \equiv (g_m r_{ds})r$$

$$g_m \equiv \sqrt{(2K'W/L)I_D} = \sqrt{2 \times 110 \times 10^{-6} \times 2/1 \times 10 \times 10^{-6}} = 66.3 \times 10^{-6}$$

$$g_{mbs} = g_m \frac{\gamma}{2(2|\phi_F| + V_{SB})^{1/2}} = 66.3 \times 10^{-6} \times \frac{0.4}{2(0.7+1)^{1/2}} = 10.17 \times 10^{-6}$$

$$g_{ds} \equiv I_D \lambda = 10 \times 10^{-6} \times 0.04 = 400 \times 10^{-9}$$

$$r_{ds} = \frac{1}{g_{ds}} = 2.5 \times 10^{6}$$

*thus*

$$r_{out} = 100\times10^{3} + 2.5\times10^{6} + \big[(66.3\times10^{-6} + 10.17\times10^{-6})\,2.5\times10^{6}\big]\,100\times10^{3} = 21.7\times10^{6}$$

$$r_{out} = 21.7 \times 10^{6}$$

***PART-B:*** *The minimum voltage across drain and source while remaining in saturation is $V_{ON}$*

$$V_{ON} = \sqrt{\frac{2I_D}{\beta}} = \sqrt{\frac{2 \times 10 \times 10^{-6}}{2 \times 110 \times 10^{-6}}} = \sqrt{\frac{10}{110}} = 0.302$$

*The minimum drain voltage is*

$$V_{D(\min)} = V_S(\min) + V_{ON} = 1 + 0.302 = 1.302$$

***PART-C:***

$$r_{DS1} = \frac{1}{g_{m1}}$$

$$\frac{1}{g_m} = 100\ k\Omega$$

$$g_{m1} = \frac{1}{100\ k\Omega} \equiv \sqrt{2K'(W/L)_1 I_D} = \sqrt{2 \times 110 \times 10^{-6} \times 10 \times 10^{-6}}\,\sqrt{(W/L)_1}$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{10^{-5}}{\sqrt{2 \times 110 \times 10^{-6} \times 10 \times 10^{-6}}}\right)^2 = \frac{1}{22}$$

*From the previous problem,*

$$g_{m2} = 66.3 \times 10^{-6}$$

$$r_{ds2} = 2.5 \times 10^6$$

***Note that the terminal conditions of M2 must change in order to support the larger gate voltage required on M1. This will be addressed in the next problem.***

**Question 10. Solution:**

*First calculate node voltages and currents.*

*Assume a near perfect current mirror so that the current in all devices is 10 microamps.*

*Calculate node voltages.*

$$V_{GS4} = V_{G4} = \sqrt{\frac{2i_D}{\beta}} + V_T = \sqrt{\frac{2 \times 10 \times 10^{-6}}{1 \times 110 \times 10^{-6}}} + 0.7 = \sqrt{\frac{20}{110}} + 0.7 = 1.126$$

$$V_{GS3} = V_{G3} = \sqrt{\frac{2i_D}{\beta}} + V_T = \sqrt{\frac{2 \times 10 \times 10^{-6}}{4 \times 110 \times 10^{-6}}} + 0.7 = \sqrt{\frac{20}{440}} + 0.7 = 0.913$$

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*VGS of M2 must be solved taking into account the back-bias voltage and its effect on threshold voltage. The following equations relate to M2 terminals (subscripts dropped for simplicity)*

$$V_{GS} = V_G - V_S = \sqrt{\frac{2i_D}{\beta}} + V_{T0} + \gamma\left(\sqrt{2|\phi_f| + V_{SB}} - \sqrt{2|\phi_f|}\right)$$

*Noting that the bulk terminal is ground we get*

$$V_G - V_S = \sqrt{\frac{2i_D}{\beta}} + V_{T0} + \gamma\left(\sqrt{2|\phi_f| + V_S} - \sqrt{2|\phi_f|}\right)$$

$$V_G - V_S - \sqrt{\frac{2i_D}{\beta}} - V_{T0} + \gamma\sqrt{2|\phi_f|} = \gamma\left(\sqrt{2|\phi_f| + V_S}\right)$$

$$V_G - \sqrt{\frac{2i_D}{\beta}} - V_{T0} + \gamma\sqrt{2|\phi_f|} - V_S = \gamma\left(\sqrt{2|\phi_f| + V_S}\right)$$

$$A - V_S = \gamma\left(\sqrt{2|\phi_f| + V_S}\right)$$

*where*

$$A = V_G - \sqrt{\frac{2i_D}{\beta}} - V_{T0} + \gamma\sqrt{2|\phi_f|}$$

$$(A - V_S)^2 = \gamma^2(2|\phi_f| + V_S)$$

$$A^2 - 2AV_S + V_S^2 = \gamma^2(2|\phi_f| + V_S)$$

$$V_S^2 - V_S(2A + \gamma^2) + A^2 - \gamma^2(2|\phi_f|) = 0$$

*Now solving numerically:*

$$A = V_G - \sqrt{\frac{2i_D}{\beta}} - V_{T0} + \gamma\sqrt{2|\phi_f|} = 1.126 - \sqrt{\frac{20}{440}} - 0.7 + 0.4\sqrt{0.7} = 0.5475$$

$$V_S^2 - V_S[2(0.5475) + 0.4^2] + 0.5475^2 - 0.4^2(0.7) = 0$$

$$V_S^2 - V_S(1.255) + 0.1877 = 0$$

$$V_S = 0.1736$$

$$V_{ON} = \sqrt{\frac{2i_D}{\beta}} = \sqrt{\frac{20}{440}} = 0.2132$$

$$V_{OUT(min)} = V_{ON} + V_S = 0.2132 + 0.1736 = 0.3868$$

*Small signal calculation of output resistance:*

$$g_{m1} = g_{m2} \equiv \sqrt{(2K'W/L)I_D} = \sqrt{2 \times 110 \times 10^{-6} \times 4/1 \times 10 \times 10^{-6}} = 93.81{\times}10^{-6}$$

$$g_{mbs2} = g_{m2}\frac{\gamma}{2(2|\phi_f| + V_{SB})^{1/2}} = 93.81{\times}10^{-6}{\times}\frac{0.4}{2(0.7 + 0.1736)^{1/2}} = 20.07{\times}10^{-6}$$

$$r_{out} = \frac{v_{out}}{i_{out}} = r_{ds1} + r_{ds2} + [(g_{m2} + g_{mbs2})r_{ds2}]\, r_{ds1}$$

$$g_{ds1} = g_{ds2} \equiv I_D\lambda = 10 \times 10^{-6} \times 0.04 = 400 \times 10^{-9}$$

$$r_{ds1} = r_{ds2} = \frac{1}{g_{ds}} = 2.5 \times 10^6$$

$$r_{out} = r_{ds1} + r_{ds2} + [(g_{m2} + g_{mbs2})r_{ds2}]\, r_{ds1} = 2.5 \times 10^6 + 2.5 \times 10^6$$

$$r_{out} = 2.5{\times}10^6{+}2.5{\times}10^6{+}[(93.81{\times}10^{-6}{+}20.07{\times}10^{-6})\, 2.5{\times}10^6]\, 2.5{\times}10^6$$

$$r_{out} = 717 \times 10^6$$

**Question 11. Solution:**

*By comparison with the circuit in P4.3-6, the output transistors are identical but the bias currents are halved. In order to achieve the same gate voltages on M1 and M2, the W/L of M3 and M4 must be half of those in Fig P4.3-6. This is illustrated in the following equations.*

$$V_{GS} = \sqrt{\frac{2i_D}{K'(W/L)}} + V_T$$

$$V_{GS}(5\mu A) = \sqrt{\frac{2(5\mu A)}{K'(W/L)_{5\mu A}}} + V_T = V_{GS}(10\mu A) = \sqrt{\frac{2(10\mu A)}{K'(W/L)_{10\mu A}}} + V_T$$

$$\sqrt{\frac{2(5\mu A)}{K'(W/L)_{5\mu A}}} = \sqrt{\frac{2(10\mu A)}{K'(W/L)_{10\mu A}}}$$

$$\frac{5\mu A}{(W/L)_{5\mu A}} = \frac{10\mu A}{(W/L)_{10\mu A}}$$

$$\frac{(W/L)_{10\mu A}}{(W/L)_{5\mu A}} = \frac{10\mu A}{5\mu A} = 2$$

$$(W/L)_{10\mu A} = 2\,(W/L)_{5\mu A}$$

*Thus for Fig. 4.3-7*

$$(W/L)_4 = \frac{1}{2}$$

$$(W/L)_3 = 2/1$$

**Question 12. Solution:**

$$i_D = K'\frac{W}{L}(v_{GS} - V_T)^2$$

*and*

$$v_{GS} = \sqrt{\frac{2i_D}{K'(W/L)}} + V_T$$

*Thus, combining these expressions for the circuit in Fig. P4.4-1,*

$$i_O = K_2'\left(\frac{W}{L}\right)_2 (v_{GS2} - V_{T2})^2$$

$$i_O = K_2'\left(\frac{W}{L}\right)_2 \left(\sqrt{\frac{2 \times 20 \times 10^{-6}}{K_1'(W/L)_1}} + V_{T1} - V_{T2}\right)^2$$

*Minimum and Maximum occurs under the following conditions*

| | $K_1'$ | $K_2'$ | $(W/L)_1$ | $(W/L)_2$ | $V_{T1}$ | $V_{T2}$ |
|---|---|---|---|---|---|---|
| $i_{O(min)}$ | *Max* | *Min* | *Max* | *Min* | *Min* | *Max* |
| $i_{O(max)}$ | *Min* | *Max* | *Min* | *Max* | *Max* | *Min* |

*Substituting in the expression for drain current yields:*

| | $K_1'$ | $K_2'$ | $(W/L)_1$ | $(W/L)_2$ | $V_{T1}$ | $V_{T2}$ |
|---|---|---|---|---|---|---|
| *27.82μ* | *115.5μ* | *104.5μ* | *3.316* | *2.714* | *0.695* | *0.705* |
| *56.93μ* | *104.5μ* | *115.5μ* | *2.714* | *3.316* | *0.705* | *0.695* |

**Question 13. Solution:**

*The small-signal model for Fig. 4.4-2 is:*

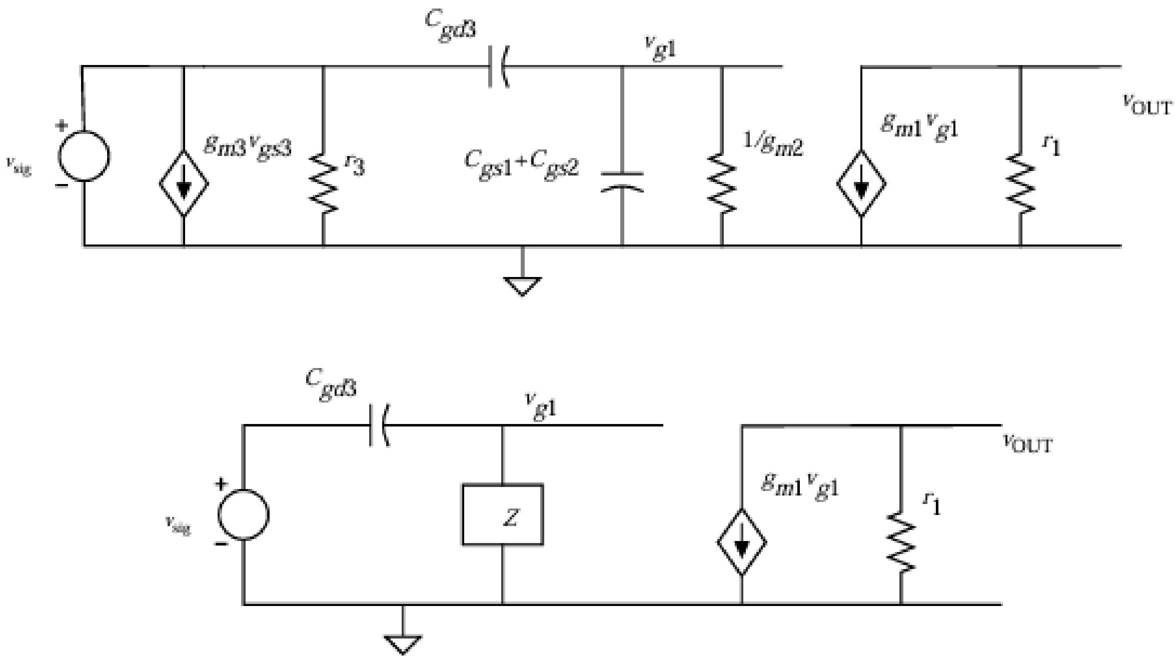


Figure 6

$$v_{g1} = v_{sig}\left(\frac{Z}{Z + 1/sC_{gd3}}\right)$$

$$v_{OUT} = -g_{m1}r_1\, v_{g1} = -g_{m1}r_1\left(\frac{Z\, v_{sig}}{Z + 1/sC_{gd3}}\right)$$

$$\frac{v_{OUT}}{v_{sig}} = -g_{m1}r_1\left(\frac{s\, C_{gd3}}{s(C_{gd3} + C_{gs1} + C_{gs2}) + g_{m1}}\right)$$

$$\left|\frac{v_{OUT}(\omega)}{v_{sig}(\omega)}\right| = \left|-g_{m1}r_1\left(\frac{\omega\, C_{gd3}}{\sqrt{[\omega(C_{gd3} + C_{gs1} + C_{gs2})]^2 + g_{m1}^2}}\right)\right|$$

*The transfer function has the following poles and zeros.*

$$\omega_p = \left(\frac{g_{m1}}{C_{gd3} + C_{gs1} + C_{gs2}}\right)$$

$$\omega_z = \frac{g_{m1}}{C_{gd3}}$$

$$r_1 = \frac{1}{\lambda i_d} = \frac{1}{0.04 \times 10 \times 10^{-6}} = 2.5 \times 10^6$$

$$g_{m1} = \sqrt{2K'(W/L)I_D} = \sqrt{2 \times 110 \times 10^{-6} \times 2 \times 10 \times 10^{-6}} = 66.33 \times 10^{-6}$$

$$C_{gs1} = \frac{2}{3}C_{ox} \times W \times L + C_{GSO} \times W = 3.29\ fF + 0.44\ fF = 3.73\ fF$$

$$C_{gs1} = C_{gs2}$$

$$C_{gd3} = C_{GSO} \times W = 0.44\ fF$$

*Substituting numerical values yields:*

$$\left|\frac{v_{OUT}(\omega)}{v_{sig}(\omega)}\right| = 66.33 \times 10^{-6} \times 2.5 \times 10^{6} \times \left( \frac{6.28 \times 10^{6} \times 0.44 \times 10^{-}}{\sqrt{[6.28 \times 10^{6}(0.44 \times 10^{-15} + 3.73 \times 10^{-15} + 3.73 \times}} \right.$$

$$\left|\frac{v_{OUT}(\omega)}{v_{sig}(\omega)}\right| = 6.91 \times 10^{-3} \quad at\ \omega = 6.28\ Mrps$$

*For* $v_{sig} = 100\ mV$

$$v_{OUT} = v_{sig} \times 6.91 \times 10^{-3} = 100 \times 10^{-3} \times 6.91 \times 10^{-3} = 691\ \mu V$$

**Question 14. Solution:**

$$V_{REF}\Big|_{T=T_0} = V_{C0} + V_0\left(\gamma - \alpha\right) = 1.262\ @\ 300\ K$$

$$KV_0 = V_{C0} - V_{BE0} + V_0\left(\gamma - \alpha\right)$$

$$K = \left(\frac{R_2}{R_1}\right)\ln(10) = \frac{V_{C0} - V_{BE0} + V_0\left(\gamma - \alpha\right)}{V_0}$$

$$V_{BE0} = \frac{kT}{q}\ln\left(\frac{I}{I_S}\right)$$

$$I = \frac{\Delta V_{BE}}{R_1} = 1\ \mu A$$

$$R_1 = \frac{0.0259\ \ln(10)}{1\ \mu A} = 59.64\ k\Omega$$

$$K = \frac{1.205 - 0.53 + 0.0259(2.2)}{0.0259} = 28.26\ k\Omega = \left(\frac{R_2}{R_1}\right)\ln(10)$$

$$R_2 = 732\ k\Omega$$

*Stacking bipolar transistors reduces sensitivity to amplifier offset.*

**Question 15. Solution:**

***PART-A:*** *Given* $V_T = 0.2V_{DD}$ *and* $\left(V_{out(\max)} - V_{out(\min)}\right) = 0.7V_{DD}$

*From Eq. (5.1-1) and (5.1-5)*

$$V_{out(\max)} - V_{out(\min)} = \frac{(V_{DD} - V_T)}{\sqrt{1 + \dfrac{\beta_2}{\beta_1}}}$$

*or,*

$$0.7V_{DD} = \frac{(V_{DD} - 0.2V_{DD})}{\sqrt{1 + \dfrac{\beta_2}{\beta_1}}} \quad \Rightarrow \quad \left(1 + \frac{\beta_2}{\beta_1}\right) = \left(\frac{8}{7}\right)^2 \quad \Rightarrow \quad \frac{\beta_2}{\beta_1} = 0.306$$

*The small-signal voltage gain can be given by*

$$A_v = \frac{g_{m1}}{g_{m2}} = -\sqrt{\frac{\beta_1}{\beta_2}} = -1.8\ V/V$$

***PART-B:*** *Assuming* $M_1$ *is operated in saturation*

$$I_{D1} = K_n'\left(\frac{W}{L}\right)_1\left(\frac{(V_{in} - V_T)^2}{2}\right)$$

*or,*

$$100\ \mu = (110\ \mu)(5)\left(\frac{(V_{in} - 0.7)^2}{2}\right) \quad \Rightarrow \quad V_{in} = 1.303\ V$$

*The small-signal gain can be given by*

$$A_v \cong -\frac{g_{m1}}{g_{m2}} = -\sqrt{\frac{K_N'}{K_P'}\left(\frac{W}{L}\right)_1\left(\frac{L}{W}\right)_2} = -2.345\ V/V$$

*The output resistance can be given by*

$$R_{out} \cong \frac{1}{g_{m2}} = 7.07\ k\Omega$$

**Question 16. Solution:**

*Assuming all transistors are in saturation and ideal current mirroring*

$$I_{D1} = K'\left(\frac{W}{L}\right)\left(\frac{(V_{in}-V_T)^2}{2}\right)$$

*or,*

$$110\mu = (110\mu)(2)\left(\frac{(V_{in}-0.7)^2}{2}\right) \quad \Rightarrow \quad V_{in} = 1.7\,V$$

*The small-signal voltage gain can be given by*

$$A_v \equiv -\frac{g_{m1}}{g_{m2}} = -\sqrt{\frac{K_1'}{K_2'}}\left(\frac{W}{L}\right)\sqrt{\frac{I_{D1}}{I_{D2}}} = -6.95\ V/V$$

*where,* $I_{D3} = I_{D4} = 100\ \mu A$, *and* $I_{D2} = 10\ \mu A$.

*The output resistance can be given by*

$$R_{out} \equiv \frac{1}{g_{m2}} = 31.6\ k\Omega$$

**Question 17. Solution:**

*a)* $V_{GG} = V_{T2} + V_{dsat2}$

$V_{GG} = V_{T2} + \sqrt{\frac{2I_{D2}}{K_n'(W/L)_2}} = 2.05\ V$

*b)* $V_{in} = V_{DD} - V_{T1} - \sqrt{\frac{2I_{D1}}{K_p'(W/L)_1}} = 3.406\ V$

*c)* $A_v = \frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{(g_{ds1}+g_{ds2})} = -24.85\ V/V$

*d)* $f_{-3dB} = \frac{(g_{ds1}+g_{ds2})}{2\pi\,(C_{gd1}+C_{gd2}+C_{bd1}+C_{bd2}+C_L)} = 2.51\ MHz$

**Question 18. Solution:**

| | Circuit 1 | Circuit 2 | Circuit 3 | Circuit 4 | Circuit 5 | Circuit 6 |
|---|---|---|---|---|---|---|
| $g_m$ | $g_{mN}=\sqrt{2}\,g_{mP}$ | $g_{mP}$ | $g_{mN}=\sqrt{2}\,g_{mP}$ | $g_{mP}$ | $g_{mN}=\sqrt{2}\,g_{mP}$ | $g_{mP}$ |
| $R_{out}$ | $\approx\frac{1}{g_{mN}+g_{mbN}} = \frac{0.707}{g_{mP}+g_{mbP}}$ | $\approx\frac{1}{g_{mP}+g_{mbP}}$ | $\approx\frac{1}{g_{mP}}$ | $\approx\frac{1}{g_{mN}} \approx \frac{0.707}{g_{mP}}$ | $\frac{1}{g_{dsN}+g_{dsP}} = \frac{1}{g_{dsP}(1+\sqrt{2})}$ | $\frac{1}{g_{dsN}+g_{dsP}} = \frac{1}{g_{dsP}(1+\sqrt{2})}$ |
| \|Gain\| | $\frac{g_{mP}}{g_{mP}+g_{mbP}}$ | $\frac{g_{mP}}{g_{mP}+g_{mbP}}$ | $\sqrt{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{2}\,g_{mP}}{g_{dsP}(1+\sqrt{2})}$ | $\frac{g_{mP}}{g_{dsP}(1+\sqrt{2})}$ |

Figure 7

*(a.) Circuit 5 has the highest gain.*
*(b.) Circuit 4 has the lowest gain (assuming normal values of $g_m/g_{mb}$).*
*(c.) Circuits 5 and 6 have the highest output resistance.*
*(d.) Circuit 1 has the lowest output resistance.*

**Question 19. Solution:**

*Referring to the figure*

$$V_{GS1} = V_{T1} + V_{dsat1}$$

*or,*

$$V_{GS1} = V_{T1} + \sqrt{\frac{2I_{D1}}{\beta_1}}$$

$$V_{GS2} = V_{T2} + V_{dsat2}$$

*or,*

$$V_{GS2} = V_{T2} + \sqrt{\frac{2I_{D2}}{\beta_2}}$$

*The input-offset voltage can de defined as*

$$|V_{OS}| = |V_{GS1} - V_{GS2}|$$

*or,*

$$|V_{OS}| = |V_{T1} - V_{T2}| + \left|\sqrt{\frac{2I_{D1}}{\beta_1}} - \sqrt{\frac{2I_{D1}}{\beta_2}}\right|$$

*Considering the transistors $M_3$ and $M_4$, mismatches in these two transistors would cause an offset voltage between the output nodes. But, if it is assumed that this offset voltage between the output nodes is small as compared to the drain-to-source voltages of the transistors $M_1$ and $M_2$, then*

$$V_{DS1} \cong V_{DS2}$$

*Thus, it is assumed here that*

$$I_{D1} = I_{D2} = I$$

*So, the input-offset voltage becomes*

$$|V_{OS}| = |V_{T1} - V_{T2}| + \left|\sqrt{\frac{2I}{\beta_1}} - \sqrt{\frac{2I}{\beta_2}}\right|$$

*Assuming $I = 50\ \mu A$, the worst-case input offset voltage can be given by*

$$|V_{OS}| = (1.01 - 0.99) + \left[\sqrt{\frac{2(50\mu)}{0.95(10\mu)}} - \sqrt{\frac{2(50\mu)}{1.05(10\mu)}}\right]$$

*or,* $\quad VOS(max) = 0.18\ V$

**Question 20. Solution:**

*Assume $g_{m1} = g_{m2}$ otherwise multiply the gain of circuits 1 and 2 by* $\dfrac{g_{m2}}{g_{m1}+g_{m2}}$.

| Circuit | $R_{out}$ | $\frac{v_{out}}{v_{in}}$ |
|---|---|---|
| 1 | $\frac{1}{g_{ds2}+g_{m8}+g_{ds8}}$ | $\frac{g_{m1}g_{m2}}{(g_{m1}+g_{m2})(g_{ds2}+g_{m8}+g_{ds8})} = \frac{0.5\,g_{m2}}{g_{ds2}+g_{m8}+g_{ds8}}$ |
| 2 | $\frac{1}{g_{ds2}+g_{ds8}}$ | $\frac{g_{m1}g_{m2}}{(g_{m1}+g_{m2})(g_{ds2}+g_{ds8})} = \frac{0.5\,g_{m2}}{g_{ds2}+g_{ds8}}$ |
| 3 | $\frac{1}{g_{ds2}+g_{ds8}}$ | $\frac{g_{m1}+g_{m2}}{2\,(g_{ds2}+g_{ds8})}$ |
| 4 | $\frac{1}{g_{ds2}+\frac{g_{ds6}g_{ds8}}{g_{m6}}} = \frac{g_{m6}}{g_{ds6}g_{ds8}+g_{m6}g_{ds2}}$ | $\frac{(g_{m1}+g_{m2})\,g_{m6}}{2\,(g_{m6}g_{ds2}+g_{ds6}g_{ds8})}$ |
| 5 | $\frac{g_{m4}g_{m6}}{g_{ds2}g_{m6}g_{ds4}+g_{m6}g_{ds4}g_{ds8}}$ | $\frac{(g_{m1}+g_{m2})\,g_{m4}g_{m6}}{2\,(g_{ds2}g_{m6}g_{ds4}+g_{m6}g_{ds4}g_{ds8})}$ |