

Use the last 5 digits of your roll number as the seed of the random number generator.

Q1. A) Choose a random 5D vector W , where the first dimension lies between (0,1), 2nd dimension between (1,2), 3rd between (2,3) etc. [1 mark]

B) Use it to generate $N=1000$ data-points $\{X_i, Y_i\}$, where $Y_i = W \cdot X_i + e_i$, where X_i is a 5D feature vector where each feature is uniformly between (-10, 10) and e_i is a random value between -1 and 1. [1 mark]

C) Fit a linear regression model on (X_i, Y_i) and estimate W_1 . [2 marks]

D) Calculate the estimation error $\|W - W_1\|_2$ and the MSE $(1/N) \sum (Y_i - W_1 \cdot X_i)^2$. [2 marks]

E) Fit a LASSO regression model on (X_i, Y_i) and estimate W_2 for different values of λ (1 to 20). [2 marks]

F) Sparsify each W_2 to obtain W_3 by setting all values below a certain threshold to 0 (you can choose the threshold). Plot the MSE $(1/N) \sum (Y_i - W_3 \cdot X_i)^2$ as a function of λ . [2 marks]

You can use library functions for Linear and LASSO regression.

Q2. A) Choose a random 10D vector W , where each value lies between -1 and 1. Generate $N=100$ datapoints (X_i, Y_i) such that X_i is a 10D vector whose elements lie between either (-10,-1) or (1,10), and $Y_i = \text{sign}(W \cdot X_i)$. Basically you create a linearly separable dataset with wide margin. [2 marks]

B) Implement perceptron algorithm to find a separating hyperplane. **You should not use library functions in this case.** [3 marks]

C) Plot the number of updates you make in each iteration. [1 mark]

D) Now, reduce the margin of the dataset by adding 10 points, whose features lie between (-1,0) and (0,1). Labels are still generated as $Y_i = \text{sign}(W \cdot X_i)$, i.e. the dataset is still linearly separable but now has narrower margin. [1 mark]

E) Run the perceptron algorithm again and plot the number of updates in each iteration. [1 mark]

F) Now add 5 "outliers" on both sides of W . [1 mark]

G) Run the perceptron and plot the number of updates in each iteration. [1 mark]