

# Elections

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## 1 Betting odds

If the betting odds are  $q/1$ , then for every dollar I spend, I will get  $q$  dollars back if I win.

$$\begin{aligned}\text{win} = E1 &\Rightarrow \text{Payoff}(E1) = q \\ \text{lose} = E2 &\Rightarrow \text{Payoff}(E2) = -1\end{aligned}$$

Due to market equilibrium, the expectation of the bet is 0, meaning

$$0 = qp - (1 - p) \Rightarrow p = \frac{1}{1 + q}$$

Heard that there is 50/1 odds against Dwayne Johnson winning the 2020 elections. The implied winning probability of 2% is too high. My thought is that betting gives fatter tailed distributions as the gamblers are not rational and take more risks giving more weights to the outlier events.

## 2 Winner's bump

Suppose that the election winner  $X$  has a probability  $p$  of being voted while the losers  $Y$  have an aggregate probability  $(1 - p)$  of getting voted by the eligible citizens.

Let  $Z_t$  be the random variable representing the vote lead taken by the winner; while  $X_t$  and  $Y_t$  are the random variables representing the votes received by the winner and loser. Time is defined in such a way that at  $t_n$ ,  $n$  chits have been read. Consequently,

$$X_{t_n} \sim \text{Binomial}(n, p) \tag{1}$$

$$Y_{t_n} \sim \text{Binomial}(n, 1 - p) \tag{2}$$

Now, we compute the expectation of  $Z_t$ . Using linearity of expectation and (1),

$$\begin{aligned}E(Z_{t_n}) &= E(X_{t_n}) - E(Y_{t_n}) \\ &= np - n(1 - p) \\ &= n(2p - 1)\end{aligned}$$

Clearly, in an expected sense, we can see that the lead taken by the winner increases with time as more chits get tallied and they consolidate their wins.