Monty Hall and Extensions

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1 Generalization

Though there are many variations, we will look at an optimal strategy where the participant is given many chances to change their mind. Given,

$$\begin{aligned} n &= \text{number of doors} \\ r &= \text{number of prizes} \\ k &= \text{number of times the participant can change their mind} \end{aligned}$$

We conjecture that the optimal strategy is to change their choice in the end i.e. $[(n-r-k), ...(n-r-1)]$. Lets prove it by induction.

Case i=1

Let x be the point at which the participant makes a new choice, then probability of getting the prize is

$$\left(\frac{r-1}{n-x}\right)\left(\frac{r}{n}\right) + \left(\frac{r}{n-x}\right)\left(\frac{n-r}{n}\right) \text{where } x \in [1, 2...(n-r-1)] \tag{1}$$

which realizes a maximum when $x = (n-r-1)$

Case i=k

Assuming the conjecture is true for i=(k-1), meaning the participant has already placed his optimal (k-1) choices in $[(n-r-k+1), ...(n-r-1)]$

Let x be the point at which the participant makes a new choice, then probability of getting the prize is

$$\left(\frac{r-1}{n-x}\right)\left(\frac{r}{n}\right) + \left(\frac{r}{n-x}\right)\left(\frac{n-r}{n}\right) \text{where } x \in [1, 2...(n-r-k)] \tag{2}$$

which realizes a maximum when $x = (n-r-k)$

The stacking works because this new decision was taken at a point when the other (k-1) decisions were not a part of the filtration.