

Coin Toss

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1 Expectation

We want to compute the expected number of trials required to get k consecutive heads. Trivially, expected number of tosses required for 0 consecutive heads is 0 i.e.

$$y_0 = 0 \tag{1}$$

Now, let's work on expected number of coin tosses to get n consecutive heads.

$$y_n = (y_{n-1} + 1)P(H) + (y_{n-1} + y_n + 1)P(T) \tag{2}$$

$$y_n = 2y_{n-1} + 2 \tag{3}$$

Using (1) and (3), we get

$$y_n = 2(2^n - 1) \tag{4}$$

2 Distribution

We want to compute the probability distribution of k consecutive heads over trials. We define a random variable X which denotes k heads ending in n th trial.

$$P(X = n) = P(\text{no } k \text{ heads in } [1, n - k - 1])P(\text{tails in } (n - k))P(k \text{ heads in } [n - k + 1, n]) \tag{5}$$

$$= \left(1 - (n - 3k - 1) \left(\frac{1}{2}\right)^{(n-k-1)}\right) \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^k \tag{6}$$