RM 294 – Optimization I
Project 1 – Linear Programming

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Problem Description:

Assume that your company is deciding how to spend a marketing budget of $10M. You work in the marketing department as a data scientist and the chief marketing officer has asked you write a report recommending how to spread this budget among several marketing mediums. Your department has employed an outside consulting firm to estimate the return on investment (ROI) of each marketing medium under consideration. The results are in the table below, and also in a CSV attached to this assignment:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	3.1%	4.9%	2.4%	3.9%	1.6%	2.4%	4.6%	2.6%	3.3%	4.4%

On top of these ROIs, your boss has decided to constrain your budget as follows:

a. The amount invested in print and TV should be no more than the amount spent on Facebook and Email.
b. The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) should be at least twice of SEO and AdWords.
c. For each platform, the amount invested should be no more than $3M.

Data Read Snippet :

CSV DATA IS READ BELOW

Please Note ROI_MAT is divided by 100 to maintain the uniformity with the previous file after reading it.

In [3]:
```
#Read ROI data
roi_data=pd.read_csv('ROI_data.csv')
print('ROI DATA')
display(roi_data)

print('ROI Monthly Data')
roi_mat=pd.read_csv('roi_mat.csv', index_col=0)

#DIVIDING ROI_MAT by 100 to maintain uniformity with ROI_DATA
roi_mat=roi_mat/100
display(roi_mat)
```

Question 1:

Formulate the marketing budget allocation problem as a linear program. Use gurobi to find the optimal budget allocation.

Marketing Budget allocation as a linear problem:

Let x_1 be the amount invested in print, x_2 be the amount invested in TV,
x_3 be the amount invested in SEO, x_4 be the amount invested in AdWords,

x_5 be the amount invested in Facebook, x_6 be the amount invested in LinkedIn,
x_7 be the amount invested in Instagram, x_8 be the amount invested in Snapchat,
x_9 be the amount invested in Twitter and x_{10} be the amount invested in Email

Objective Function: To maximize ROI

$$0.031x_1 + 0.049x_2 + 0.024x_3 + 0.039x_4 + 0.016x_5 + 0.024x_6 + 0.046x_7 + 0.026x_8 + 0.033x_9 + 0.044x_{10}$$

Constraints:

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} \leq 10$ (Total Budget Constraints)
$x_5 + x_6 + x_7 + x_8 + x_9 \leq 2(x_3 + x_4)$ (Inter-Platform Budget Constraint)
$x_1 + x_2 \leq x_5 + x_{10}$ (Inter-Platform Budget Constraint)
$x_i \leq 3 \; \forall \; i \in [1, 2, 3, 4, 5, 6, 7, 8 , 9, 10]$ (Platform Budget Constraint)

Using gurobi to solve the problem,

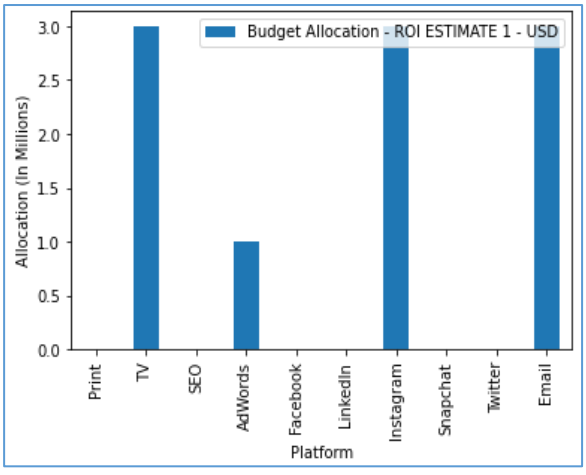
```
obj = roi_data.iloc[0][1:].values #Maximize ROI
A = np.zeros((13,10))
A[0,:] = [1,1,1,1,1,1,1,1,1,1]      #Total sum of all money spent be less that or equal to 10 million USD (budget)
A[1,:] = [1,1,0,0,-1,0,0,0,0,-1]    #Amount of money spent on print and TV be less than money spent on facebook and email
A[2,:] = [0,0,-2,-2,1,1,1,1,1,0]    #Total amount used in Social media atleast twice of SEO and adword
A[3:,:] = np.diag(np.ones(10))      #Money spent on any medium be less than or equal to 3 million USD

b = np.array([10000000,0,0,3000000,3000000,3000000,3000000,3000000,3000000,3000000,3000000,3000000,3000000])
sense = np.array(['<','<','>','<','<','<','<','<','<','<','<','<','<'])
```

```
ojModel = gp.Model()

ojModX = ojModel.addMVar(10)
ojModCon = ojModel.addMConstrs(A, ojModX, sense, b)
ojModel.setMObjective(None,obj,0,sense=gp.GRB.MAXIMIZE)

ojModel.Params.OutputFlag = 0
ojModel.optimize()

maximum_profit_roi_1=ojModel.objVal
```

We find that the optimal allocation is:

Budget Allocation - ROI ESTIMATE 1 - USD (Millions)

Platform	
Print	0.0
TV	3.0
SEO	0.0
AdWords	1.0
Facebook	0.0
LinkedIn	0.0
Instagram	3.0
Snapchat	0.0
Twitter	0.0
Email	3.0



Thus, the optimal allocation is to assign 3 million to TV, Instagram, Email each while assign 1 million to AdWords.

The total expected ROI from the optimal allocation is:

Summary	Amount (USD) - ROI estimate 1 (In Millions)
Total Budget or Investment	10.000
Optimized ROI	0.456

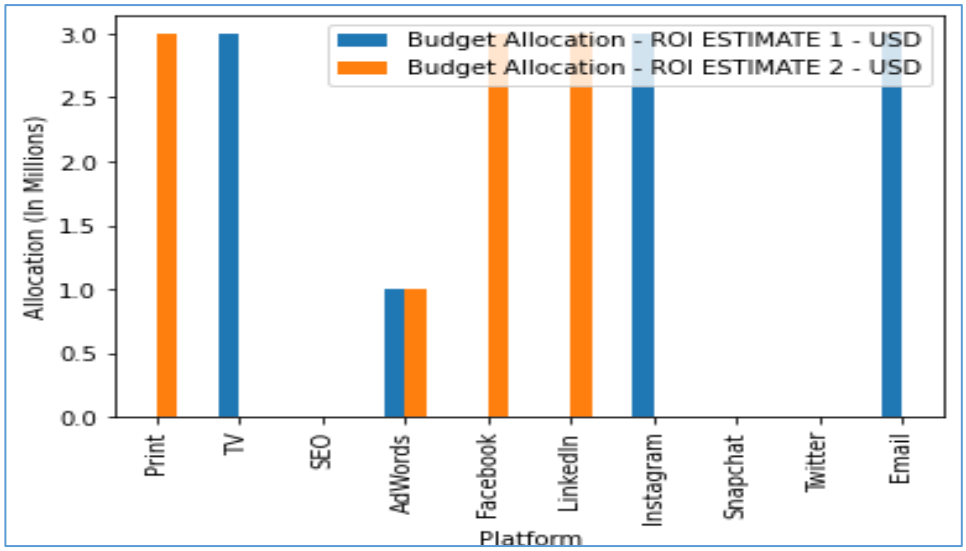
Question 2:

To be cautious about the decision, your team has decided to get another opinion about the ROI data and rerun the analysis. The second consulting firm returns the estimates of the ROI data in the table below (also in the CSV file mentioned above). You are asked to compare the two optimal allocations from these two ROI estimates.

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	4.9%	2.3%	2.4%	3.9%	4.4%	4.6%	2.6%	1.9%	3.7%	2.6%

Comparison of the two Optimal Allocations:

Platform	Budget Allocation - ROI ESTIMATE 1 - USD (Millions)	Budget Allocation - ROI ESTIMATE 2 - USD (Millions)
Print	0.0	3.0
TV	3.0	0.0
SEO	0.0	0.0
AdWords	1.0	1.0
Facebook	0.0	3.0
LinkedIn	0.0	3.0
Instagram	3.0	0.0
Snapchat	0.0	0.0
Twitter	0.0	0.0
Email	3.0	0.0



As we can see from the above table, the optimal allocation for AdWords remains the same i.e 1 million. The remaining 9 million can be equally divided amongst Print, Facebook and LinkedIn (3 million each)

Change in ROI with estimate 2:

Summary	Amount (USD) - ROI estimate 1 (In Millions)	Amount (USD) - ROI estimate 2
Total Budget or Investment	10.000	10.000
Optimized ROI	0.456	0.456

Question 3:

Are the allocations the same? Assuming the first ROI data is correct, if you were to use the second allocation how much lower would the objective be relative to the optimal objective? Assuming the second ROI data is correct, if you used the first allocation how much lower would the objective be relative to the optimal objective? Do you think the third constraint above, based on your boss' experience, is useful?

Assuming the first ROI is correct , with the Allocation of second ROI data:

If the first ROI is correct, we stand to lose 204,000 USD compared to the optimal objective

```
: ojModel1.objVal - sum(obj1*y2)
: 0.20400000000000007
```

Assuming the second ROI is correct , with the Allocation of second ROI data:

If the first ROI is correct, we stand to lose 192,000 USD compared to the optimal objective

```
ojModel2.objVal - sum(obj2*y1)
```

```
0.19200000000000006
```

```
Case 1 : Change in Objective(Maximum ROI) is :  204000.0
Case 2 : Change in Objective(Maximum ROI) is :  192000.0
```

Removing the third constraint:

The optimal allocation across platforms for the two ROI estimates after removing the third constraint is:

Platform	Budget Allocation (1)	Budget Allocation (2)
Print	0.0	5.0
TV	5.0	0.0
SEO	0.0	0.0
AdWords	0.0	0.0
Facebook	0.0	5.0
LinkedIn	0.0	0.0
Instagram	0.0	0.0
Snapchat	0.0	0.0
Twitter	0.0	0.0
Email	5.0	0.0

Optimal ROI without the constraint is $0.465 million. While the optimal ROI with the constraint is $0.456 million. Thus, we stand to gain $0.009 million or $9000 USD without the constraints.

However, as we can see the budget allocation for media spends is split only in two platforms without the constraint. Adding a constraint helps to avoid putting all the eggs in two baskets only. Especially given we are not completely sure about the projected ROI estimates for the platforms.

Question 6:

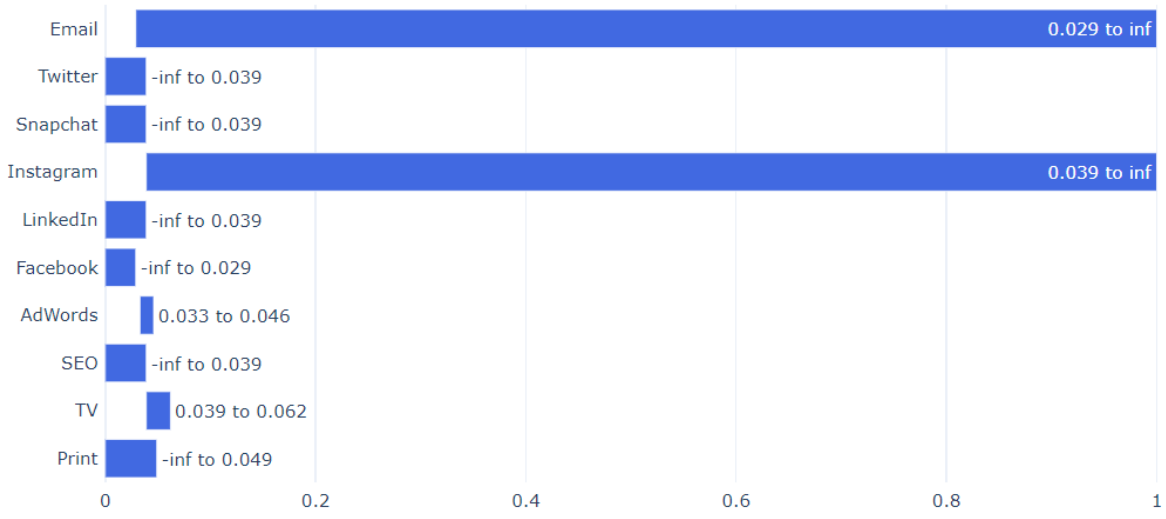
To explore this further perform some analysis of how your optimal allocation would change based on changes in the ROI data. Use the first ROI data as your starting point. By how much could each advertising medium's ROI increase or decrease and still result in the same optimal allocation you found in step (3)?

To do this we utilize the Sensitive Analysis properties in the Gurobi optimization model. -Inf and Inf values indicate non-binding constraints. This is because of the fact that each medium may hae a maximum of 3M in investment.

```
optimal_allocation_range['Minimum Val']=ojModX.SAObjLow
optimal_allocation_range['Maximum Val']=ojModX.SAObjUp
optimal_allocation_range['Current Value']= roi_data.iloc[0][1:].values
```

Below are the tabulated values. If we keep all other values constant, we can change the ROI for the medium of concern in the ranges below, to maintain the same Optimal Allocation.

Platform	Minimum Val	Maximum Val	Current Value
Print	-inf	0.049	0.031
TV	0.039	0.062	0.049
SEO	-inf	0.039	0.024
AdWords	0.033	0.046	0.039
Facebook	-inf	0.029	0.016
LinkedIn	-inf	0.039	0.024
Instagram	0.039	inf	0.046
Snapchat	-inf	0.039	0.026
Twitter	-inf	0.039	0.033
Email	0.029	inf	0.044



Question 7:

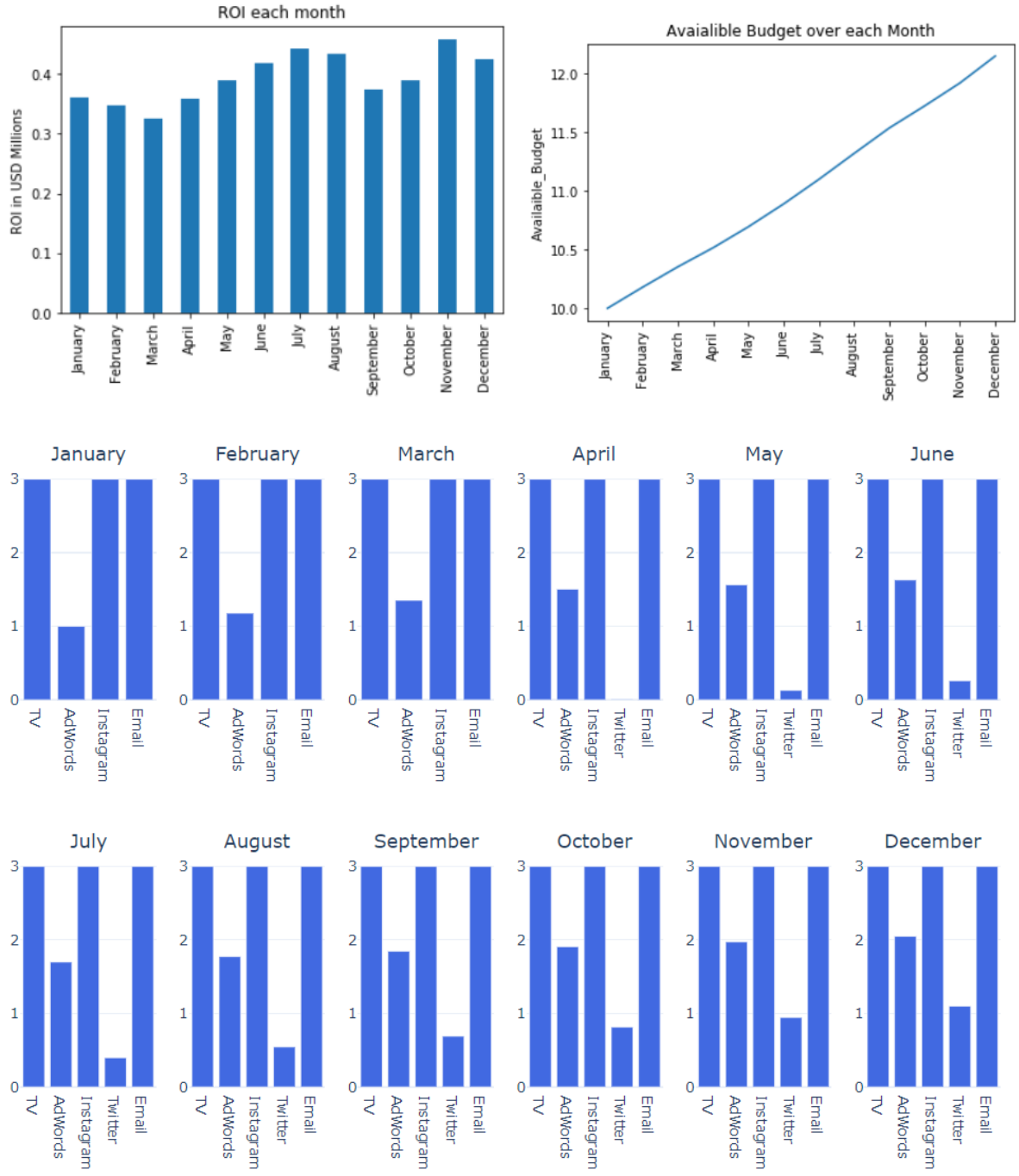
Your boss has gained permission to reinvest half of the return. For example, if the marketing obtains a 4% return in January, the budget of February will be $10M + $10M × 4% × 50% = $10.2M. The monthly ROI for next year is given in Project1.Rdata. The three constraints given by your boss are still in place for each month. What is the optimal allocation for each month?

We utilize the values forecasted as ROI by team in question 1. While we calculate the monthly return using the ROI for each respective month. We re-invest half the return of each month and add to the principal amount. This results in increased returns, as one of the constraints is now variable and gaining more margin each month.

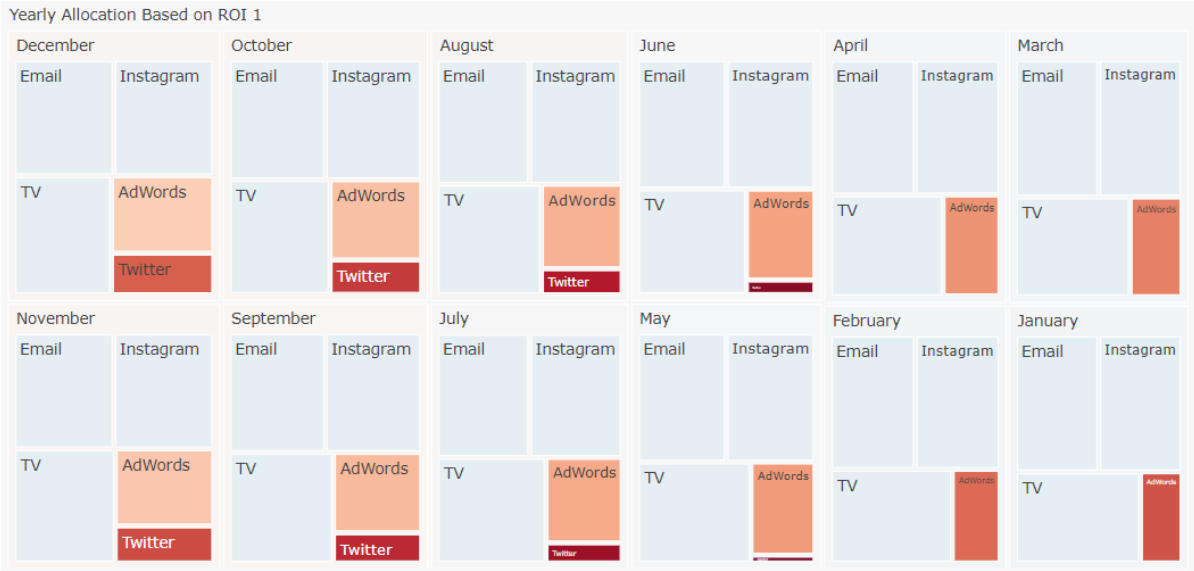
As per this please find below the Investment allocation and accumulated return on investment each month.

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	ROI	Availaible_Budget
January	0.0	3.0	0.0	1.000000	0.0	0.0	3.0	0.0	0.000000	3.0	0.360000	10.000000
February	0.0	3.0	0.0	1.180000	0.0	0.0	3.0	0.0	0.000000	3.0	0.347840	10.180000
March	0.0	3.0	0.0	1.353920	0.0	0.0	3.0	0.0	0.000000	3.0	0.324449	10.353920
April	0.0	3.0	0.0	1.505381	0.0	0.0	3.0	0.0	0.010763	3.0	0.357624	10.516144
May	0.0	3.0	0.0	1.564986	0.0	0.0	3.0	0.0	0.129971	3.0	0.389058	10.694957
June	0.0	3.0	0.0	1.629829	0.0	0.0	3.0	0.0	0.259657	3.0	0.419281	10.889486
July	0.0	3.0	0.0	1.699709	0.0	0.0	3.0	0.0	0.399418	3.0	0.441764	11.099126
August	0.0	3.0	0.0	1.773336	0.0	0.0	3.0	0.0	0.546672	3.0	0.434534	11.320008
September	0.0	3.0	0.0	1.845758	0.0	0.0	3.0	0.0	0.691517	3.0	0.375183	11.537275
October	0.0	3.0	0.0	1.908289	0.0	0.0	3.0	0.0	0.816578	3.0	0.390196	11.724866
November	0.0	3.0	0.0	1.973321	0.0	0.0	3.0	0.0	0.946643	3.0	0.458252	11.919964
December	0.0	3.0	0.0	2.049697	0.0	0.0	3.0	0.0	1.099393	3.0	0.424417	12.149090

Below, On the left we see the return earned each month. On the right we see the increase in budget across the months.



As we can see, there is only allocation to the 4 Mediums, TV, Instagram, Email and AdWords up until March. From April, however, due to the increased budget, there is a slight investment in Twitter. This becomes the 5th Medium of Investment and is allocated higher budget as we move across the months. Please find an alternate visualization as a treemap below.



1 Size of each Medium indicates the amount spent. As we can see the aloocated budget in Twitter steadily increases over the months.

Question 8:

A stable budget is defined as a monthly allocation such that for each platform the monthly change in spend is no more than $1M. Is the allocation you found stable? If it isn't, you do not need to solve a new optimization model. Describe how my might model this?

```
df=sumary_df.T
df_shift = sumary_df.T.apply(lambda x: x.shift(1))
df_shift.loc['January'] = 0

tempdf=df_shift-df
tempdf=tempdf[1:]
tempdf.drop(columns=['ROI','Availaible_Budget'] , inplace=True)
stable_check=tempdf.applymap(lambda x : abs(x)>=1)
```

From the code above, we can see that the delta in consecutive monthly allocation for each platform is not more than $1M. This indicates a stable budget.