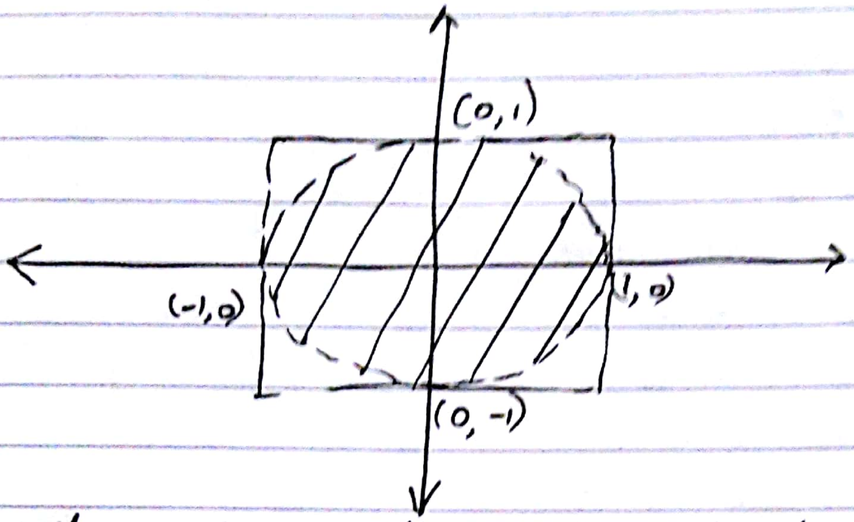
1.

(a)



All points in the ~~circle~~ rectangle with vertices (−1, −1), (−1, 1), (1, −1), (1, 1) are equally probable according to definition of random variable.

$$\text{so probability} = \frac{\text{Area of circle}}{\text{Area of rectangle}}$$

$$= \frac{\pi}{2 \times 2} = \pi/4$$

(b) Let's define a binomial random variable 'X' which has 2 components X_1 in the x-direction and X_2 in the y-direction. If in any trial $X_1^2 + X_2^2 \leq 1$ I'll set $X = 1$ otherwise as 0, π will be estimated as $= \frac{\text{No. of times}(X == 1)}{\text{No. of trials}} \times 4$.

~~(c) N=10 3.2000~~
~~N=100 3.0800~~
~~$N=10^3$ 3.1520~~

	π-value
(c) $N=10^1$	3.2000
$N=10^2$	3.1600
$N=10^3$	3.2080
$N=10^4$	3.1308
$N=10^5$	3.1440
$N=10^6$	3.1386
$N=10^7$	3.1421
$N=10^8$	3.1414

for $N=10^9$, no issue shall occur as I am not storing the values and just looping.

(d) $X =$ ~~non off~~ $\leq$ number of times the point lies in the circle.

$$Y = \frac{4X}{n} \qquad E(Y) = \pi.$$

$p = \pi/4.$

$$Var\left(\frac{4X}{n}\right) = \frac{16}{n^2} np(1-p)$$

$$p(|Y-\pi|) \leq 0.01$$

$$Z = \frac{Y - \pi}{\sqrt{\frac{16}{n} p(1-p)}} \longrightarrow \text{is a } (0,1) \text{ Gaussian distribution,}$$

$$P\left(\left|\sqrt{\frac{16}{n} p(1-p)}\, Z\right| \leq 0.01\right) = 0.95.$$

$$P\left(|Z| \leq \frac{0.01}{\sqrt{\frac{16}{n} p(1-p)}}\right) = 0.95.$$

$$\therefore \frac{0.01}{\sqrt{\frac{16}{n} p(1-p)}} = 2.$$

$$\frac{1}{200} = \sqrt{\frac{16}{n} p(1-p)}.$$

$$n = 16\,p(1-p) \times 200 \times 200.$$

$$n = 1.08 \times 10^5$$

Value of $\pi = 3.1423$ for $n = 108000$.

2,

(a) $X = Aw + u$. and $C = AA^T$ —(i)

By spectral theorem we can write $C = USU^T$.

and so $C = U\sqrt{S}\sqrt{S}\,U^T$ —(ii).

Using (i) and (ii) we can say that one of the solution is $\underline{\underline{A = U\sqrt{S}.}}$ U and S can be found by eig() function

Using 'A's value we can find $X = Aw + u$.

↳ given

↳ random gaussian distribution

3.

(a) In the x vs y graph, first I'll find the mean which is $\frac{\sum x_i}{N}, \frac{\sum y_i}{N}$. (Through which the line will pass). The second step is to find the slope of the line. For this we shall create a matrices b[i] which has value $b[i].x = x(i) - \mu_x$ and $b[i].y = y(i) - \mu_y$. And create a ~~ma~~ 2x2 matrix a which is equal to $\frac{\sum_{i=1}^{n} (b[i]).(b[i])^T}{n}$. The vector corresponding to the highest eigen value of matrix 'a' is the direction in which the line should point.

We have found the point and the slope, so we have found the line.

(c) The first dataset corresponds to a linear data set and so a good approximation ~~so~~ can be created via a straight line but the second dataset is parabolic ~~so~~ and we have forcably created a straight line approximation which is obviously a bad approximation.

4.

(a) Only 30-50 significant modes of variation are observed. These are far less than 784 that is 28^2. This is because data is highly correlated and eigen values are really small.

(b) We find that the three images are rotated a bit, this is because people tend to write the same digit differently.

(c)

4 d) For all the digits, $\mu - \sqrt{\lambda_i}v$, μ, $\mu + \sqrt{\lambda_i}v$, is in order of either tilted digit to straight or straight to tilted depending on the sign of v which is immaterial to us (eig gives just unit v). Adding or subtracting $\sqrt{\lambda_i}v$, too will mean that that influence of v. i.e. (v indicates the direction which describes relation b/w data points best). So, the correlation among attributes for let say 1 is that only a small portion of values in matrix (x-axis values) are 1 for all y-axis values. So, adding v means adding 1 values in the matrix of μ along y-axis for some interval in x.

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Q.5 b)

Let $a\vec{u} + b\vec{v}_1 + c\vec{v}_2 + d\vec{v}_3 + e\vec{v}_4$ be the closest representation of $\vec{v}$

$\Rightarrow \| a\vec{u} + b\vec{v}_1 + c\vec{v}_2 + d\vec{v}_3 + e\vec{v}_4 - \vec{v} \|_{Frob.}$ is min

$$\Rightarrow S = \sum_{i=1}^{19200} (au_i + bv_{1i} + cv_{2i} + dv_{3i} + ev_{4i} - v_i)^2 \text{ is min}$$

$$\Rightarrow \frac{\partial S}{\partial a} = 0, \quad \frac{\partial S}{\partial b} = 0 \quad -- \quad \text{\& so on}$$

$$\Rightarrow \sum_{i=1}^{19200} 2u_i (au_i + bv_{1i} + cv_{2i} + dv_{3i} + ev_{4i} - v_i) = 0 \quad -(1)$$

& (4) more equations

$$\Rightarrow a\langle u,u\rangle + b\langle v_1,u\rangle + c\langle v_2,u\rangle + d\langle v_3,u\rangle + e\langle v_4,u\rangle = \langle v_i, v\rangle \ (1)$$

$\vdots$

$$a\langle u,v_4\rangle + b\langle v_1,v_4\rangle + c\langle v_2,v_4\rangle + \ -- \ e\langle v_4,v_4\rangle = \langle v_i, v_4\rangle - (5)$$

$$\Rightarrow \begin{bmatrix} \langle u,u\rangle & -- & \langle v_4,u\rangle \\ \vdots & & \\ \langle u,v_4\rangle & -- & \langle v_4,v_4\rangle \end{bmatrix} \begin{bmatrix} a \\ b \\ \vdots \\ e \end{bmatrix} = \begin{bmatrix} \langle u,v\rangle \\ \vdots \\ \langle v_4,v\rangle \end{bmatrix}$$

$$AX = B$$

$$\Rightarrow X = A^{-1}B$$

Hence we got a, b, c, d, e & hence the closest representation of $\vec{v}$.

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c) $\underset{D\times 1}{X} = \underset{D\times N}{A}\,\underset{N\times 1}{W} + \underset{D\times 1}{\mu}$

In this case, since 4 eigenvectors are to be used, $N=4$, $D=19200$.

For a general case, where all eigenvectors can be used,

$C = USU^T$ (Spectral theorem)

& $C = AA^T$

Let $S = S'S'$ [S has positive entries on diag]

$\Rightarrow C = (US')(S'U)^T$ [S' is also diag].

$\Rightarrow A = US'$ is a solution to $C = AA^T$

where we have used all eigen vectors as columns of U.

For this case, since 4 eigenvectors are to be used, we can use U as 19200x4 matrix with columns as eig-vectors corresponding to 4 largest eig-values.

& S' a 4x4 diag. matrix.

Hence

$\underset{(19200\times 4)}{A} = \underset{(19200\times 4)}{U}\,\underset{(4\times 4)}{S'}$

And, then we get random samples as $X = AW + \mu$ where W is 4x1 random MVG.