# PRACTICAL NO :- 1 :

* Topic :- limits & continuity :-

① $\lim_{x \to a} \left[ \dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

② $\lim_{x \to 0} \left[ \dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

③ $\lim_{x \to \pi/6} \left[ \dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

④ $\lim_{x \to \infty} \left[ \dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

⑤ Examine the continuity of the following function at give points.

(i) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $0 < x \leq \pi/2$
for $\pi/2 < x \pi$, $\Big\}$ at $x = \pi/2$

(ii) $f(x) = \dfrac{x^2-9}{x-3}$ $\quad 0 < x < 3$
$= x+3$ $\quad 3 \leq x \leq 6$
$= \dfrac{x^2-9}{x+3}$ $\quad 6 \leq x \leq 9$ $\Big\}$ at $x = 3$ & $x = 6$ ;

ISO

(6) Find values of K, so that the function $f(x)$ is at at the indicated point.

(i) $f(x) = \frac{1-\cos 4x}{x^2}$   $x<0$ ; $= K$   $x=0$   } at $x=0$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x}$   $x \neq 0$ ; $= K$   $x = 0$   } at $x=0$

(iii) $f(x) = \frac{\sqrt{3}-\tan x}{\pi - 3x}$   $x \neq \frac{\pi}{3}$ ; $= K$   $x = \pi/3$   } at $x = \frac{\pi}{3}$

7) Discuss the continuity of the foll fun which of the fun have a removable discontinuity. redefine the fun so as to remove the discontinuity.

(i) $f(x) = \frac{1-\cos 3x}{x \tan x}$   $x \neq 0$ ; $= 9$   $x = 0$   } at $x = 0$

(ii) $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2}$   $x \neq 0$ ; $= \frac{\pi}{60}$   $x = 0$   } at $x = 0$

8) If $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is cts at $x=0$ find $f(0)$

9) If $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \frac{\pi}{2}$ is cts at $x = \frac{\pi}{2}$ find $f(\pi/2)$.

## Solutions

① $$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$\therefore \lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$$

$$\therefore \lim_{x\to a} \frac{(a+2x-3x)}{(3a+x-4x)} \frac{(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{a+2x}+\sqrt{3x})}$$

$$\therefore \lim_{x\to a} \frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\therefore \frac{1}{3}\lim_{x\to a} \frac{\cancel{(a-x)}(\sqrt{3a+x}+2\sqrt{x})}{\cancel{(a-x)}(\sqrt{a+2x}+\sqrt{3x})}$$

$$\therefore \lim_{x\to a} \frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$\therefore \lim_{x\to a} \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\therefore \frac{1}{3} \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$ $\quad \therefore \frac{1}{3} \frac{4\sqrt{a}}{2\sqrt{3a}}$

$\therefore \frac{1}{3} \cdot \frac{\cancel{4}\sqrt{a}}{\cancel{2}\sqrt{3a}}$ $\quad \therefore \frac{1}{3}$

$\therefore \frac{2}{3\sqrt{3}}$

2) $\lim_{y\to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}}\right]$

$\lim_{y\to 0} \left(\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}}\right)$

$\therefore \lim_{y\to 0} \frac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$

$\therefore \lim_{y\to 0} \frac{1}{\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}$

$\therefore \frac{1}{\sqrt{a}\,(\sqrt{a}+\sqrt{a})}$ $\quad \therefore \frac{1}{2a}$ //

3) $\frac{\left(\cos\frac{\sqrt{3}}{2}h - \sin\frac{h}{2}\right) - \left(\sin\frac{3h}{2} + \cos\frac{\sqrt{3}}{2}h\right)}{6h}$ $\quad \therefore \left(\cos\frac{\sqrt{3}h}{2} - \cos\frac{\sqrt{3}}{2}h\right) = 0$

$\therefore \frac{\cancel{-}\sin 4h/2}{\cancel{-}\, 6h}$

$\therefore \frac{\sin 4h/2}{6h}$ $\quad \therefore \frac{\sin 4h}{12h}$ $\quad \therefore \frac{1}{3} \times \frac{\sin h}{h}$

$\therefore \frac{1}{3}$ //

4) $\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$

$\therefore \lim_{x\to a} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}} \times \frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\right]$

$\therefore \lim_{x\to a} \frac{(\cancel{x^2}+5-\cancel{x^2}+3)\,(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\cancel{x^2}+3-\cancel{x^2}-1)\,(\sqrt{x^2+5}+\sqrt{x^2-3})}$

$\frac{\cancel{8}\,4}{\cancel{2}} \left(\frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+5} + \sqrt{x^2-3}}\right)$

$\therefore \frac{4\left(\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{y^2\left(1+\frac{1}{x^2}\right)}\right)}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$

$$\therefore 4 \lim_{x\to 0} \frac{\sqrt{x^2(1+3/x^2)}}{\sqrt{x^2(1+5/x^2)}} + \frac{\sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1-3/x^2)}}$$

$\therefore 4$

5) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $0 < x \leq \pi/2$

$\therefore \dfrac{\cos x}{\pi - 2x}$, for $\dfrac{\pi}{2} < x < \pi$ } at $x = \pi/2$

$\rightarrow f(\frac{\pi}{2}) = \dfrac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\quad \therefore f(\pi/2) = 0$

$\therefore f$ at $x = \dfrac{\pi}{2}$ define

Q2)

a) $x - \dfrac{\pi}{2} = h$ $\quad \therefore x = h + \dfrac{\pi}{2}$ where $h \to 0$.

$$\therefore \lim_{x\to 0} \frac{\cos(h+\pi/2)}{\pi - 2\cdot(h+\pi/2)}$$

$$\therefore \lim_{x\to 0} \frac{\cos(h+\pi/2)}{\pi - \cancel{2}\left(\frac{2h+\pi}{\cancel{2}}\right)} \quad \therefore (\cancel{\pi} - 2h - \cancel{\pi})$$

$$\therefore \lim_{x\to 0} \frac{\cos(h+\pi/2)}{-2h}$$



$$\lim_{x\to 0} \frac{\cos h \cdot \cos\pi/2 - \sin h \cdot \sin\pi/2}{-2h}$$

$$\therefore \lim_{x\to 0^+} \cos h \cdot 0 \frac{\cancel{-}\sin h}{\cancel{-}\cdot 2h}$$

$\therefore 1/2$

b) $\displaystyle\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$$\therefore \lim_{x\to\pi/2} \frac{2\sin x\cos x}{\sqrt{2\sin^2 x}}$$

$$\therefore \lim_{x\to\pi/2} \frac{\overset{\sqrt{2}}{\cancel{2}}\cancel{\sin x}\cos x}{\sqrt{2}\cancel{\sin x}} \quad \therefore \frac{2}{\sqrt{2}} \lim_{x\to\pi/2^+} \cos x$$

$\therefore$ LHS $\neq$ RHS.

3) (i) at $x = 3$

$f(3) = \dfrac{x^2-9}{x-3} = 0$.

$\therefore f$ at $x = 3$ define.

(2) $\displaystyle\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}$.

(1) $f(3) = x+3 = 3+3 = 6$

is define at $x = 3$

(2) $\displaystyle\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$$

$\therefore$ LHS = RHS

$f$ is continuous at $x = 3$.

For $x=6$

(i) $f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9}$

2) $\lim_{x\to 6^+} = \frac{x^2-9}{x+3}$

$\therefore \lim_{x\to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$ $\therefore \lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\therefore \lim_{x\to 6^-} x+3 = 3+6 = 9$

$\therefore$ LHL $\neq$ RHL

function is not continous.

6) (i) $f$ is cts at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$\therefore \lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$

$2 \lim_{x\to 0} \frac{2\sin^2 x}{x^2} = k$

$\therefore 2 \lim_{x\to 0} \left(\frac{\sin^2 2x}{x}\right)^2 = k$

$2(2)^2 = k$

$k = 8$



(ii) $\frac{\frac{\cancel{\sqrt{3}} - 3\tan h - \cancel{\sqrt{3}} - \tan h}{1-\sqrt{3}\tan h}}{-3h}$

$\therefore \frac{\frac{-4\tan h}{(-3h)(1-\sqrt{3}\tan h)}}{-3h}$ $\therefore \left(\frac{4}{3}\right)\left(\frac{1}{1-0}\right)$

(iii) $x - \frac{\pi}{3} = h$

$x = h + \frac{\pi}{3}$ $\therefore h \to 0$

$\therefore f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$\therefore \frac{\sqrt{3} - \frac{\tan \pi/3 + \tan h}{1 - \tan\pi/3 \cdot \tan h}}{\pi - \pi - 3h}$

$\therefore \frac{\frac{\cancel{\sqrt{3}} - \sqrt{3}\times\sqrt{3}\tan h - \cancel{\sqrt{3}} - \tan h}{1-\sqrt{3}\cdot\tan h}}{-3h}$

$\therefore \frac{\frac{\cancel{\sqrt{3}} - 3\cdot\tan h - \cancel{\sqrt{3}}\cdot\tan h}{1-\sqrt{3}\cdot\tan h}}{-3h}$

$\therefore \dfrac{\dfrac{-4\tan h}{1-\sqrt{3}\tan h}}{-3h}$ $\quad \therefore \dfrac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$

$\therefore \frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \cdot \lim_{h\to 0} \frac{1}{1-\sqrt{3}\tan h}$

$\therefore \frac{4}{3} \frac{1}{1-3(0)}$

$\therefore \frac{4}{3}$ //

vii)

① $f(x) = \dfrac{1-\cos 3x}{x\tan x}$ $\quad \therefore \dfrac{2\sin^2 3/2\, x}{x\tan x}$

$\therefore \dfrac{\dfrac{2\sin^2 3x/2}{x^2}}{x \cdot \dfrac{\tan x}{x^2}} \times x^2$

$\therefore 2 \lim_{x\to 0} (3/2)^2$

$\therefore 2 \times \frac{9}{4} = 9/2$ //

$\therefore \lim_{x\to 0} f(x) = \frac{9}{2} \quad 9 = f(0)$



$\therefore f$ is not cts at $x=0$

Redefine funct<sup></sup>

$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ 9/2 & x=0 \end{cases}$

Now $\lim_{x\to 0} f(x) = f(0)$

$f$ has removable dis cts at $x=0$

(ii) at $x=0$

$\lim_{x\to 0} \dfrac{(e^{3x}-1)\cdot(\sin(\pi x/180))}{x^2}$

$\therefore 3 \lim_{x\to 0} \dfrac{e^{3x}-1}{3x} \cdot \lim_{x\to 0} \dfrac{\sin(\pi x/180)}{x}$

$\therefore 3 \log_e e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$

$\therefore f$ is cts at $x=0$

8) $f(x) = \dfrac{e^{x^2}-\cos x}{x^2} \quad x=0$

$\therefore$ is continous at $x=0$

$\therefore f$ is continous at $x=0$

$\therefore \lim_{x\to 0} f(x) = f(0)$

$\therefore \dfrac{e^{x^2}-\cos x}{x^2} = f(0)$

$\therefore \dfrac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$\therefore \dfrac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$

$\therefore \dfrac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \dfrac{2 \sin^2 x/2}{x^2}$

$\therefore \log e + 2 \left(\dfrac{\sin x/2}{x}\right)^2$

$\therefore$ Multiply with 2 on Num & denominator

$\therefore 1 + 2 \times \frac{1}{4} = 3/2 = f(0).$

9) $f(x) = \dfrac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x} \quad x \neq \pi/2$

$\therefore f(0)$ is cts at $x = \pi/2$

$\dfrac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x} \times \dfrac{\sqrt{2} + \sqrt{1 + \sin x}}{\sqrt{2} + \sqrt{1 + \sin x}}$

$\therefore \dfrac{\cancel{2 - 1} + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1 + \sin x})}$

$\therefore \dfrac{1 + \sin x}{1 - \sin^2 x (\sqrt{2} + \sqrt{1 + \sin x})}$

$\therefore \dfrac{1}{(1 - \sin x)(\sqrt{2} + \sqrt{1 + \sin x})}$

$\therefore \dfrac{1}{2(\sqrt{2} + \sqrt{2})} \quad \therefore \dfrac{1}{4\sqrt{2}} \quad \therefore f(\pi/2) = \dfrac{1}{4\sqrt{2}}$



6)(ii) $f(x) = (\sec^2 x)^{\cot^2 x} \quad x \neq 0$
$= K \quad x = 0$ } at $x = 0$

$\therefore (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$

$\therefore e = K$

4/02/2020

2) PRACTICAL No :- 2

- Topic :- Derivation

Q1) Show that the following function define form $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

(i) $\cot x$ (ii) $\operatorname{cosec} x$ (iii) $\sec x$

Q2) If $f(x) = 4x+1$, $x \le 2$
$= x^2+5$, $x>0$ of $x=2$, then find $f$ is
6) differentiable or not?

Q3) If $f(x) = 4x+7$, $x<3$
$= x^2+3x+1$, $x \ge 3$ at $x=3$, then find $f$ is differentiable or not?

Q4) If $f(x) = 8x-5$, $x \le 2$
$= 3x^2-4x+7$, $x>2$ at $x=2$ then find $f$ is differentiable or not?

- Answer :-



Q1. i) $\cot x$.

$\therefore$ To prove that the ~~following~~ $\cot x$ function is $\mathbb{R}$ to $\mathbb{R}$ are differentiable

$\therefore \cot x = f(x)$

$\therefore \frac{d}{dx} \cot x$

$\therefore Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$

$= \lim_{x \to a} \frac{\cot x - \cot a}{x-a}$ $\therefore \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x-a}$

$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a)\tan x \cdot \tan a}$ $\therefore$ put $x-a=h$, $x=a+h$

$\therefore$ as $x \to a$, $h \to a$

$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$

$\therefore \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$

$\therefore$ Formula $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\therefore \tan A \cdot \tan B = \tan(A-B)(1+\tan A \cdot \tan B)$

$= \lim_{h \to 0} \tan(a-a-h) - (1+\tan a \tan(a+h))$

$= \lim_{h \to 0} \frac{-\tan h}{h} \cdot \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$

$\therefore -1 \cdot \frac{1+\tan^2 a}{\tan^2 a}$ $\therefore \frac{-\sec^2 a}{\tan^2 a}$

$\therefore \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$

$\therefore -\operatorname{cosec}^2 a$

$\therefore Df(a) = -\cos^2 a$

$\therefore f$ is differentiable $\forall a \in \mathbb{R}$.

(ii) $\operatorname{cosec} x$

$\therefore f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\therefore = \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x - a} \qquad \therefore \lim_{x \to a} \frac{\sin a - \sin x}{(x-a) \sin a \sin x}$$

$\therefore$ put $x - a = h$

$\therefore x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a) \sin a \sin(a+h)}$$

Formula

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$$

$$\therefore \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \cdot \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$\therefore -\frac{1}{\cancel{2}} \times \frac{\cancel{2}\cos\left(\frac{2a+h}{2}\right)}{\sin(a+0)}$$

$$\therefore \frac{-\cos a}{\sin a} = -\cot a \cdot \operatorname{cosec} a$$

(iii) $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x - a}$$



$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a) \cos a \cdot \cos x}$$

$\therefore$ Put $x - a = h$ $\qquad \therefore x = a + h$

$\therefore$ as $x \to a$, $h \to 0$

$$D(f)(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \cdot \cos a \cos(a+h)}$$

$\therefore$ Formula :- $-2\sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

$$\therefore \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{\cancel{a}-\cancel{a}-h}{2}\right)}{h \cdot \cos a \cdot \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right) \cdot \sin -h/2}{\cos a \cos(a+h) \times -h/2} \times \frac{-1}{2}$$

$$= -\frac{1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+0)}$$

$$= -\frac{1}{\cancel{2}} \cdot -\cancel{2} \frac{\sin a}{\cos a \cdot \cos a}$$

$\therefore \tan a \cdot \sec a$ //

Q2) LHD :-

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x + 1 - (4 \times 2 + 1)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x + 1 - 9}{x - 2} \qquad \therefore \lim_{x \to 2^-} \frac{4x - 8}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4\cancel{(x-2)}}{\cancel{(x-2)}} = 4$$

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$Df(2^-) = 4$

RHD:-

$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$

$= \lim_{x \to 2^+} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$

$= 2+2 = 4$

$\therefore Df(2^+) = 4$

RHD = LHD //

$\therefore$ f is differentiable at $x=2$

Q3) RHD:- $Df(3^+) = \lim_{x \to 3^+} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3+3+1)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3} = \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$

$= \lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$

$= \lim_{x \to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}} = 3+6 = 9$ //

$Df(3^+) = 9$

LHD:- $Df(3^-)$

$= \lim_{x \to 3^-} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3} = \lim_{x \to 3^-} \frac{4x-12}{x-3}$

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$= \lim_{x \to 3} \frac{4(x-3)}{(x-3)}$

$\therefore D(f) = 4$

$\therefore$ RHD $\neq$ LHD

f is not differentiable at $x=3$

Q4) $f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD:-

$Df(2^+) = \lim_{x \to 2^+} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2-4x+7-11}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2-4x-4}{x-2}$ $\therefore \lim_{x \to 2^+} \frac{3x(x-2)+2(x}{x-2}$

$= \lim_{x \to 2} + \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$ $= 3 \times 2 + 2 = 8$

$Df(2^+) = 8$

LHD:-

$Df(2^-) = \lim_{x \to 2^-} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{8x-5-11}{x-2} = \lim_{x \to 2^-} \frac{8x-16}{x-2}$

$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)} = 8$

$Df(2^-) = 8$

LHD = RHD

$\therefore$ f is differentiable at $x=3$ //

(ii)

Topic:- Application derivatives

Q1. Find the interval in which function is increasing & decreasing

(i) $f(x) = x^3 - 5x - 11$  (ii) $f(x) = x^2 - 4x$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$  (iv) $f(x) = x^3 - 27x + 5$

(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

Q2. Find the intervals in which fun is concave upwards and concave downwards.

(i) $y = 3x^2 - 2x^3$  (ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

(iii) $y = x^3 - 27x + 5$  (iv) $y = 69 - 24x - 9x^2 + 2x^3$

Q (v) $y = 2x^3 + x^2 - 20x + 4$.

Answers

Q1)

(i) $f(x) = x^3 - 5x - 11$

→ $\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 3x^2 - 5 > 0$

$\therefore 3x^2 > 5$

$\therefore x^2 > 5/3$  $\therefore x > \pm\sqrt{\frac{5}{3}}$

$+ \quad -\sqrt{\frac{5}{3}} \quad - \quad \sqrt{\frac{5}{3}} \quad +$

(iii) $\therefore x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

$\therefore f$ is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3x^2 < 5$

$\therefore x^2 < \frac{5}{3}$

$\therefore x < \pm\sqrt{\frac{5}{3}}$

$+ \quad -\sqrt{\frac{5}{3}} \quad - \quad \sqrt{\frac{5}{3}} \quad +$



$\therefore x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

(ii) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) > 0$  $\therefore x - 2 > 0$  $\therefore x > 2$

$\therefore x \in (2, \infty)$

$\therefore f$ is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$  $\therefore x - 2 < 0$  $\therefore x < 2$

$\therefore x \in (-\infty, 2)$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 6x^2 + 12x - 10x - 20 > 0$

$\therefore 6x(x+2) - 10(x+2) > 0$

$\therefore (6x - 10)(x+2) > 0$

$+ \quad -2 \quad 10/6 \quad +$  $\therefore x \in (-\infty, -2) \cup \left(\frac{10}{6}\right.$

$\therefore f$ is decreasing iff $f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$6x^2 + 12x - 10x - 20 < 0$

$6x(x+2) - 10(x-2) < 0$

$\therefore (6x - 10)(x+2) < 0$

(ii)

\+ $-2$ \+ $\frac{10}{6}$ $\therefore x \in (-\infty,-2) \cup \left(\frac{10}{6},\infty\right)$

$\therefore f$ is decreasing iff $f'(x)<0$

$\therefore 6x^2+2x-20<0$

$\therefore 6x^2+12x-10x-20<0$

$\therefore 6x(x+2)-10(x+2)<0$

$(6x-10)(x+2)<0$

\+ $-2$ − $10/6$ \+

$\therefore x \in \left(-2, \frac{10}{6}\right)$

(iv) $f(x) = x^3-27x+5$

$f'(x) = 3x^2-27$

$= 3(x^2-9)$

$\therefore f$ is increasing iff $f'(x)>0$

$\therefore 3(x^2-9)>0$

$x^2-9>0$

$\therefore (x-3)(x+3)>0$

(iii)

\+ $3$ − $3$ \+ $\therefore x \in (-\infty,-3) \cup (3,\infty)$

$\therefore$ is decreasing iff $f'(x)<0$

$\therefore 3(x^2-9)<0$

$x^2-9<0$

$(x-3)(x+3)<0$



\+ $-3$ − $3$ \+

$\therefore x \in (-3,3)$

$\therefore f(x) = 69-24x-9x^2+2x^3$

$\therefore f'(x) = -24-18x+6x^2$

i.e $6x^2-18x-24$

$6(x^2-3x-4)$

$\therefore f$ is increasing iff $f'(x)>0$

$\therefore 6(x^2-3x-4)>0$

$\therefore x^2-3x-4>0$

$\therefore x^2-4x+x-4>0$

$\therefore x(x-4)+1(x-4)>0$

$\therefore (x+1)(x-4)>0$

\+ $-1$ − $4$ \+ $\therefore x \in (-\infty,-1) \cup ($

$\therefore f$ is decreasing iff $f'(x)<0$

$\therefore 6(x^2-3x-4)<0$

$\therefore x^2-3x-4<0$

$\therefore x^2-4x+x-4<0$

$\therefore x(x-4)+1(x-4)<0$

$\therefore (x+1)(x-4)<0$

\+ $-1$ − $4$ \+ $\therefore x \in (1,4)$

(ii) Q2) (i) $y = 3x^2 - 2x^3$

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Let,

$f(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$
$= 6(1-2x)$

$\therefore f''(x)$ is concave upwards iff,

$\therefore f''(x) > 0$ $\quad \therefore 1 - 2x > 0$

$\therefore 6(1-2x) > 0$ $\quad \therefore -2x > -1$

$\therefore 2x < 1$ $\quad \therefore x < \frac{1}{2}$

$\therefore x \in (-\infty, 1/2)$

$\therefore f''(x)$ is concave downwards iff

$\therefore f''(x) < 0$

$\therefore 6(1-2x) < 0$ $\quad \therefore 1 - 2x < 0$ $\quad \therefore -2x < -1$

$\therefore 2x > 1$ $\quad x > \frac{1}{2}$ $\quad \therefore x \in \left(\frac{1}{2}, \infty\right)$

(ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

Let,

$f(x) = y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$

$\therefore f''(x) = 12x^2 - 36x + 24$
$= 12(x^2 - 3x + 2)$

(iii) $\therefore f''(x)$ is concave upwards iff

$f''(x) > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\cancel{x^2 - 3x + 2 > 0}$

$x^2 - x - 2x + 2 > 0$

$x(x-1) - 2(x-1) > 0$

$(x-2)(x-1) > 0$

+ − +

1 2

$\therefore x \in (-\infty, 1) \cup (2, \infty)$

$\therefore f''(x)$ is concave downwards iff

$\therefore f''(x) < 0$

$\therefore 12(x^2 - 3x + 2) < 0$

$\therefore x^2 - 3x + 2 < 0$

$\therefore x^2 - x - 2x + 2 < 0$ $\quad \therefore x(x-1) - 2(x-2) < 0$

$\therefore (x-2)(x-1) < 0$

+ − +

1 2

$\therefore x \in (1, 2)$

(iii) $y = x^3 - 27x + 5$

Let

$f(x) = y = x^3 - 27x + 5$

$\therefore f(x) = 3x^2 - 27$

$\therefore f''(x) = 6x$

$\therefore f''(x)$ is concave ~~iff~~ upwards iff,

$\therefore f''(x) > 0$

$\therefore 6x > 0$ $\quad \therefore x > 0$ $\quad \therefore x \in (0, \infty)$

$\therefore f''(x)$ is concave downwards iff,

$\therefore f''(x) < 0$

$\therefore 6x < 0$

$x < 0$ $\quad \therefore x \in (-\infty, 0]$

(ii)

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iv) $f''(x)$ is concave downwards iff,
$f''(x) < 0$ $\quad \therefore 6x < 0 \quad x < 0 \quad \therefore x \in (-\infty, 0)$

(iv) $y = 69 - 24x - 9x^2 + 2x^3$

let,
$f(x) = y = 69 - 24x - 9x^2 + 2x^3$
$\therefore f'(x) = -24 - 18x + 6x^2$
$\therefore f''(x) = -18 + 12x$

$\therefore f'(x)$ is concave upwards iff,
$f''(x) > 0$
$\therefore -18 + 12x > 0$
$\therefore 12x > 18 \quad \therefore x > 18/12$
$\therefore x \in (3/2, \infty)$
$\therefore f(x)$ is concave downwards iff.
$f''(x) < 0$
$\therefore -18x + 12x < 0$
$\therefore 12x < 18 \quad x < \frac{18}{12}$

(iii) (v) $y = 2x^3 + x^2 - 20x + 4$

let,
$f(x) = y = 2x^3 + x^2 - 20x + 4$
$\therefore f'(x) = 6x^2 + 2x - 20$
$\therefore f''(x) = 12x + 2$
$= 2(6x + 1)$

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$\therefore f''(x)$ is concave upwards iff,
$f''(x) > 0$
$\therefore 2(6x - 1) > 0$
$\therefore 6x - 1 > 0$
$\therefore 6x > 1 \quad \therefore x > \frac{1}{6} \quad \therefore x \in (\frac{1}{6}, \infty)$

$\therefore f''(x)$ is concave downwards iff,
$\therefore f''(x) < 0$
$\therefore 2(6x + 1) < 0$
$\therefore 6x + 1 < 0$
$\therefore 6x < -1$
$\therefore x < -\frac{1}{6}$
$\therefore x \in (-\infty, -\frac{1}{6})$

20/12/19

— x —

(ii)

(1-10)

Topic:- Application of derivatives & Newton method:

Q.1) Find maximum & minimum values of foll function

(i) $f(x) = x^2 + \frac{16}{x^2}$

(ii) $f(x) = 3 - 5x^3 + 3x^5$

(iii) $f(x) = x^3 - 3x^2 + 1$ in $[-1/2, 4]$

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Q.2) Find the root of foll eq$^n$ by newton's method (Take 4 iteration only) correct upto 4 decimal.

(i) $f(x) = x^3 - 3x^2 - 55x + 95$ (take $x_0 = 0$)

(ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

(iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Answer

Q.1

(i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

Now consider,

$f'(x) = 0$

$\therefore 2x - \frac{32}{x^3} = 0$

$\therefore 2x = \frac{32}{x^3}$

$\therefore x^4 = \frac{32}{2}$

$\therefore x^4 = 16$

$x = \pm 2$

$\therefore f''(x) = 2 + \frac{96}{x^4}$

$\therefore f''(2) = 2 + \frac{96}{2^4}$

$\therefore f''(2) = 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$

(iii)

$\therefore f(2) = 2^2 + \frac{16}{2^2}$ $\therefore 4 + \frac{16}{4}$ $\therefore 4+4$ $\therefore 8$.

$\therefore f(-2) = 2 + \frac{96}{-2^4}$ $\therefore 2 + \frac{96}{16}$ $= 2+6$ $\therefore 8 > 0$

$\therefore f$ has minimum value at $x = -2$

$\therefore$ function reaches minimum value at $x = 2$ and $x = -2$ //

(ii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

$\therefore$ Consider,

$\therefore f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$\therefore 15x^4 = 15x^2$

$\therefore x^2 = 1$

$\therefore x = \pm 1$.

$\therefore f''(x) = -30x + 60x^3$

$\therefore f(1) = -30 + 60$

$= 30 > 0$

$\therefore f$ has minimum value at $x = 1$

$\therefore f$ has

$\therefore \cancel{f(1)} = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore f$ has maximum value at $x = -1$

(ii)

$\therefore f(-1) = 3-5(-1)^3+3(-1)^5$
$= 3+5-3$
$= 5$

$\therefore$ f has the minimum value 5 at $x=-1$ and the minimum value 1 at $x=1$.

(iii) $f(x) = x^3-3x^2+1$
$\therefore f'(x) = 3x^2-6x$
Consider,
$f'(x)=0$
$\therefore 3x^2-6x=0$
$\therefore 3x(x-2)=0$
$\therefore 3x=0$ or $x=2$
$\therefore x=0$
$\therefore f''(x) = 6x-6$
$\therefore f(0) = -6$
$\therefore$ f has maximum value at $x=0$

$\therefore f(0) = (0)^3-3(0)^2+1 = 1$

(iii) $\therefore f''(2) = 6(2)-6$
$= 12-6$
$= 6>0$
$\therefore$ f has minimum value at $x=2$
$\therefore f(2) = (2)^3-3(2)^2+1$
$= 8-3(4)+1$
$= 9-12$
$= -3$

$\therefore$ f has maximum value 1 on $x=0$ and f has minimum value $-3$ on $x=2$.

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(iv) $f(x) = 2x^3-3x^2-12x+1$
$\therefore f'(x) = 6x^2-6x-12$
$\therefore 6(x^2-x-2)=0$
$\therefore x^2-x-2=0$
$\therefore x^2+x-2x-2=0$
$\therefore x(x+1)-2(x+1)=0$
$\therefore (x-2)(x+1)=0$
$\therefore x=2$ or $x=-1$
$\therefore f''(x) = 12x-6$
$\therefore f''(2) = 12(2)-6$
$= 24-6$
$= 18>0$
$\therefore$ f has minimum value at $x=2$

$\therefore f(2) = 2(2)^3-3(2)^2-12(2)+1$
$= 2(8)-3(4)-24+1$
$= 16-12-24+1$
$= -19$

$\therefore f''(-1) = 12(-1)-6$
$= -12-6$
$= -18<0$
$\therefore$ f has maximum value at $x=-1$.

$\therefore f(-1) = 2(-1)^3-3(-1)^2-12(-1)+1$
$= -2-3+12+1$
$= 8$
$\therefore$ f has maximum value 8 at $x=-1$ and f has minimum value $-19$ at $x=2$

Q.2)

(i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ . $x_0 = 0 \rightarrow$ Given.

$\therefore f'(x) = 3x^2 - 6x - 55$

$\therefore$ By Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ $\therefore x_1 = 0 + \frac{9.5}{55}$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ $\therefore 0.1727 - \frac{0.0829}{55.9467}$

$\therefore 0.1712$

$\therefore f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$\therefore f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$



$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 0.1712 + \frac{0.0011}{55.9393}$

$= 0.1712$

$\therefore$ The root of the equation is 0.1712

(ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$.

$\therefore f'(x) = 3x^2 - 4$

$\therefore$ By newton's method

$\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$\therefore f(2) = 2^3 - 4(2) - 9$ $\therefore f(3) = 3^3 - 4(3)$

$= 8 - 8 - 9$ $= 27 - 12$

$= -9$ $= 6$

$\therefore$ let $x_0 = 3$ be the initial approximation

$\therefore$ By newton's method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$\therefore x_1 = x_0 - f(x_0) / f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$\therefore f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$\therefore f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

(ii)

EAO

$x_2 = x_1 - f(x_1) / f'(x_1)$
$= 2.7392 - 6.596 / 18.5096$
$= 2.7071$

$\therefore f(x_2) = (2.7071)^3 - 4(2.7071)$
$= 19.8386 - 10.8284$
$= 0.0102$

$\therefore f'(x_2) = 3(2.7071)^2 - 4$
$= 21.9851 - 4 \quad = 17.9851$

$\therefore 2.7071 - \dfrac{0.0102}{17.9851}$

$\therefore 2.7071 - 0.0056 = 2.7015$

$\therefore f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= 19.7158 - 10.806 - 9 = -0.0901$

$\therefore f'(3) = 3(2.7015)^2 - 4 = 21.8943 - 4 = 17.8943$

$\therefore x_4 = 2.7015 + 0.0901/17.8943 = 2.7015 + 0.0050$
$= 2.7065$.

(iii) 3) $f(x) = x^3 - 1.8x^2 - 10x + 17 \quad [1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$\therefore f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$
$= -1.8 - 10 + 17$
$= 6.2$

$\therefore f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 - 20 + 17 = -2.2$

Let $x_0 = 2$ be initial approximation. By newton's method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$ 041

$x_1 = x_0 - f(x_0) / f'(x_0)$
$= 2 - 2.2/5.2$
$= 2 - 0.4230 = 1.577$

$\therefore f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$
$= 3.9219 - 4.4764 - 15.77 + 17$
$= 6.6755$

$\therefore f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4608 - 5.6772 - 10$
$= -8.2164$

$\therefore x_2 = x_1 - f(x_1) / f'(x_1)$
$= 1.577 + 0.6755/8.2164$
$= 1.577 + 0.0822$
$= 1.6592$

$\therefore f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 4.5677 - 4.9553 - 16.592 + 17 = 0.0204$.

$\therefore f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= 8.2588 - 5.97312 - 10$
$= -7.7143$

$\therefore x_3 = x_2 - f(x_2) / f'(x_2)$ ~~$\therefore f(x_3) = (1.6618)^2$~~
$= 1.6592 + 0.0204/7.7143$
$= 1.6592 + 0.0026$
$= 1.6618$

$\therefore f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$
$= 4.5892 - 4.9708 - 16.618 + 17$
$= 0.0004$

$\therefore f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= 8.2847 - 5.9824 - 10 = -7.6977$

$\therefore x_4 = x_3 - f(x_3) / f'(x_3)$
$= 1.618 + \dfrac{0.0004}{7.6977} = 1.6618$.

PRACTICAL NO:- 5

IAO

Q1. Solve the foll integration :-

(i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$ (ii) $\int (4e^{3x}+1)\,dx$ (iii) $\int (2x^2-3\sin x+5\sqrt{x})\,dx$

(iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$ (v) $\int t^7 \sin(2t+4)\,dt$

(vi) $\int \sqrt{x}\,(x^2-1)\,dx$ (vii) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

(viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$ (ix) $\int e^{\cos^2 x}\sin 2x\,dx$

(x) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right)dx$

Answer

(i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$ $\therefore \int \frac{1}{\sqrt{x^2+2x+1-4}}$ # $a^2+2ab+b^2=(a+b)^2$

$\therefore \int \frac{1}{\sqrt{(x+1)^2-4}}\,dx$ $\therefore$ Substitute $x+1=t$

$\therefore dx = \frac{1}{t} \times dt$ where $t = 1$ …

$\therefore \int \frac{1}{\sqrt{t^2-4}}$

Using # $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(|x+\sqrt{x^2-a^2}|\right)$

$\therefore \ln\left(|t+\sqrt{t^2-4}|\right)$ $\therefore t = x+1$

$\therefore \ln\left(|x+1+\sqrt{(x+1)^2-4}|\right)$

$\therefore \ln\left(|x+1+\sqrt{x^2+2x-3}|\right)$

$\therefore \ln\left(|x+1+\sqrt{x^2+2x-3}|\right)+C$



2) $\int (4e^{3x}+1)\,dx$

$\therefore \int 4e^{3x}\,dx + \int 1\,dx$

$\therefore 4\int e^{3x}\,dx + \int 1\,dx$ # $\int e^{ax}dx = \frac{1}{a}e^{ax}$

$\therefore \frac{4e^{3x}}{3} + x$ $\therefore \frac{4e^{3x}}{3} + x + C$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$ # $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2\,dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10 \times \sqrt{x}}{3} + C$

$= \frac{2x^3+10x\sqrt{x}}{3} + 3\cos x + C$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$\therefore \int \frac{x^3+3x+4}{x^{1/2}}\,dx$

# split the denomenator

$\therefore \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx$ $\therefore \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\,dx$

$\therefore \int x^{5/2}\,dx + \int 3x^{1/2}\,dx + \int \frac{4}{x^{1/2}}\,dx$

$\therefore \frac{x^{5/2+1}}{5/2+1}$

$\therefore \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

5) $\int t^7 \times \sin(2t^4)\, dt$

$\therefore$ put $u = 2t^4$

$du = 2 \times 4t^3$

$\therefore \int t^7 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3}\, du$

$\therefore \int t^4 \sin(2t^4) \times \frac{1}{2 \times 4}\, du$

$\therefore \int \frac{t^4 \times \sin(2t^4)}{8}\, du$

$\therefore$ Substitute $t^4$ with $u/2$

$\therefore \int \frac{u/2 \times \sin(u)\, du}{8}$

$\therefore \int \frac{u \times \sin(u)}{2} / 8\, du$

$\therefore \int \frac{u \times \sin(u)}{16}\, du$

$\therefore \frac{1}{16} \int u \times \sin(u)\, du$

# $\int u\, dv = uv - \int v\, du$

where $u = u$

$dv = \sin(v) \times du$

$du = \cancel{1 du}$ $\quad v - \cos(v)$

$\frac{1}{16} (u \times (-\cos(u)) - \int -\cos(u)\, du$

$\therefore \frac{1}{16} \times (u \times (-\cos u)) + \int \cos(v)\, du$



# $\int \cos x\, dx = \sin(x)$

$\therefore \frac{1}{16} (u \times (-\cos u)) + \sin(u))$

return the substitution $u = 2t^4$

$\therefore \frac{1}{16} (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$\therefore \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$

vi) $\int \sqrt{x}\, (x^2 - 1)\, dx$

$\therefore \int \sqrt{x}\, x^2 - \sqrt{x}\, dx \quad \therefore \int x^{1/2} \cdot x^2 - \cancel{x} x^{1/2}\, dx$

$\therefore \int x^{5/2} - x^{1/2}\, dx \quad \therefore \int x^{5/2}\, dx - \int x^{1/2}\, dx$

$\therefore I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$\therefore I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$\therefore \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + C$

viii) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\, dx \quad \therefore \int \frac{\cos x}{\sin x^{2/3}}\, dx$

$\therefore$ put $t = \sin(x)$

$t = \cos x$

$\therefore \int \frac{\cos(x)}{\sin(x)^{3/2}} \cdot \frac{1}{\cos(x)}\, dt$

$\therefore \frac{1}{\sin x^{2/3}} dt \quad \therefore \frac{1}{t^{2/3}} dt$

$\therefore I = \int \frac{1}{t^{2/3}} dt = \frac{-1}{(\frac{2}{3}-1) t^{2/3-1}} = \frac{-1}{-1/3\, t^{2/3-1}} = \frac{1}{1/3\, t^{-1/3}}$

$\therefore \frac{t^{1/3}}{1/3} = 3t^{1/3} \quad \therefore 3\sqrt[3]{t}$

$\therefore$ Resubsituting $t = \sin(x)$

$\therefore 3\sqrt[3]{\sin(x)} + C$ //

(x) $\int \frac{x^2-2x}{x^3-3x^2+1} dx$

put $x^3-3x^2+1 = dt$

$\therefore I = \int \frac{x^2-2x}{x^3-3x^2+1} \cdot \frac{1}{3x^2-3\times 2x} dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \cdot \frac{1}{3x^2-6x} dt$

$\therefore \int \frac{x^2-2x}{x^3-3x^2+1} \cdot \frac{1}{3(x^2-2x)} dt$

$\therefore \int \frac{1}{x^3-3x^2+1} \cdot \frac{1}{3} dt$

$\therefore \int \frac{1}{3(x^3-3x^2+1)} dt = \int \frac{1}{3t} dt$

$\therefore \frac{1}{3} \int \frac{1}{t} dt \quad \int \frac{1}{x} dx = \ln|x|$

$\therefore \frac{1}{3} \times \ln|t| + C$

$\therefore 1/3 \times \ln(|x^3-3x^2+1|) + C$



$\int e^{\cos^2 x} \sin(2x) dx$.

# $\sin(2x) = 2\cos(x) \cdot \sin(x)$

$\therefore \int -(-e^{\cos^2 x}) \cdot 2\cos(x) \sin(x) dx$

$\therefore$ Subsitutie $u = \cos^2(x) \rightarrow \frac{du}{dx} = -2\cos(x)\sin(x)$.

$\therefore dx = -\frac{1}{2\cos(x)\sin(x)} du$

$\therefore -\int e^u du$

$\therefore -e^u$

$\therefore$ resubsituting $u = \cos^2(x)$

$\therefore -e^{\cos^2 x}$

$\therefore -e^{\cos^2 x} + C$ //

03/01/2020

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PRACTICAL:- 2+4

* Topic:- Application of integration & numerical integration

Q.1(i) $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} dt$

$\therefore x = 1 - \sin t$ $\quad \therefore \frac{dx}{dt} = 1 - \cos t$ $\quad \therefore \frac{dy}{dt} = 1 - (-\sin t) = \cos$

$y = 1 - \cos t$

$\therefore L = \int_0^{2\pi} \sqrt{(1 - \cos t + \sin t)^2 + (\sin t)^2} dt$

$\therefore \int_0^{2\pi} \sqrt{1 - 2\cos t + 1} \; dt$

$\therefore \int_0^{2\pi} \sqrt{2 - 2\cos t} \; dt = \int_0^{2\pi} \sqrt{2(1-\cos t)} = \int_0^{2\pi} \sqrt{2 \times 2 \sin t/2} = \sqrt{4 \sin}$

$\therefore \int_0^{2\pi} 2 \left|\sin \frac{t}{2}\right| dt$ $\quad \therefore \sin \frac{t}{2} = \frac{1 - \cos t}{2}$

$\therefore \int_0^{2\pi} 2 \sin \frac{t}{2} dt$

$\therefore (-4\cos(t/2\pi))^{2\pi} = (-4\cos\pi - (-4\cos 0))$

$\therefore 4 + 4 = 8$.

(ii) $y = \sqrt{4 - x^2}$ $\quad x \in [-2, 2]$

$\therefore I = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

$\therefore y = \sqrt{4 - x^2}$ $\quad \therefore \frac{dy}{dx} = 2\int_0^2 \left(\frac{-x}{\sqrt{4-x^2}}\right)^2 dx$

$\therefore 2\int_0^2 \sqrt{1 + \frac{x^2}{4 - x^2}} dx$ $\quad \therefore 4\int_0^2 \frac{1}{\sqrt{4 - x^2}} dx$ $\quad \therefore 4(\sin^{-1}(x/2))_0^2$

$\therefore 2\pi$

(iii) $y = x^{3/2}$ in $[0, 4]$

$f(x) = \frac{3}{2} x^{1/2}$ $\quad \therefore L = \int_a^b \sqrt{1 + (f'(x))^2} dx$

$\therefore \int_a^b \sqrt{1 + \frac{9}{4} x} \, dx$ $\quad \therefore \int_0^4 \sqrt{\frac{4 + 9x}{4}} \; dx$

$\therefore \int_0^4 \sqrt{4 + 9}$ $\quad \therefore \frac{1}{2}\left[\frac{(4 + 9x)^{1/2+1}}{3/2 + 1}\right]_0^4 dx$

$\therefore \frac{1}{27}\left[(4 + 9x)^{3/2}\right]_0^4$ $\quad \therefore \frac{1}{27}\left[(4+0)^{3/2} - (4 + 36)^{3/2}\right]$

$\therefore \frac{1}{27}\left((4)^{3/2} - (40)^{3/2}\right]$

(iv) $x = 3\sin t \quad y = 3\cos t$
RAD

$\therefore \frac{dx}{dt} = 3\cos t \quad \therefore \frac{dy}{dt} = -3\sin t$

$\therefore I = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$

$\therefore \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt \quad \therefore \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$

$\therefore \int_0^{2\pi} \sqrt{9(1)}\, dt \quad \therefore \int_0^{2\pi} 3\, dt = 3\int_0^{2\pi} dt = 3[x]_0^{2\pi}$
$= 3(2\pi - 0)$
$= 6\pi$

(v) $x = \frac{1}{4}y^3 + \frac{1}{2y} \quad y : [1, 2]$

$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4 - 1}{2y^2} \quad \therefore \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$\therefore \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy \quad \therefore \int_1^2 \frac{y^4+1}{2y^2}\, dy$

$\therefore \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$

$\therefore \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1}}{1}\right]_1^2 \quad \therefore \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$

$\therefore \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right] = \frac{17}{12}$



Q.2) (i) $\int_0^2 e^{x^2}\, dx$ with $n = 4$

$a = 0, \; b = 2, \; n = 4$

$h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
| $y$ | 1 | 1.2840 | 2.7182 | 9.4877 | 54.5981 |

$\therefore$ By simpson's Rule

$\therefore \int_0^2 e^{x^2}\, dx = \frac{0.5}{3}[(1 + 54.5981) + 4(1.2840 + 9.4877) + 2(2.7182$

$\therefore 17.3535$

(ii) $\int_0^4 x^2\, dx$ with $n = 4$

$a = 0, \; b = 4, \; n = 4 \quad \therefore h = \frac{b-a}{n} = \frac{4-0}{4} = 1$

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 |
| $y$ | 0 | 1 | 4 | 9 | 16 |

By simpson's rule.

$\int_0^{\pi/3} \sqrt{\sin x} = \frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)$

$\therefore \frac{\pi}{54}[(0+0.9306)+4(0.4167+0.7071+0.8512)+2(0.5[illegible]$

$\therefore \frac{\pi}{54} \times 011.6996 \quad \therefore 0.6806$

$\therefore \int_0^{\pi/3} \sqrt{\sin x} = 0.6806.$

# PRACTICAL No:- 7



Topic:- Differential equations

(i) $x\frac{dy}{dx} + y = e^x$

$\therefore \frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$\therefore P(x) = \frac{1}{x} \quad ; Q(x) = \frac{e^x}{x}$

$\therefore I.F = e^{\int 1/x\, dx} = e^{\log x}$

$\therefore I.F = x$

$\therefore y(I.F) = \int Q(x)(I.F)\,dx$

$\therefore y(x) = \int \frac{e^x}{x} \cdot x\,dx$

$\therefore xy = e^x + c$

(ii) $e^x \frac{dy}{dx} + 2e^x y = 1$

$\therefore e^x\left(\frac{dy}{dx} + 2y\right) = 1 \quad \therefore \left(\frac{dy}{dx} + 2y\right) = \frac{1}{e^x}$

$\therefore$ comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$\therefore P(x) = 2 \quad ; \quad Q(x) = \frac{1}{e^x}$

$\therefore I.F = e^{\int 2dx} = e^{2x}.$

$\therefore y(I.F) = \int Q(x)(I.F)\,dx$

$ye^{2x} = \int \frac{1}{e^x} e^{2x}\,dx$

$\therefore ye^{2x} = \int e^{-x} \cdot e^{2x}\,dx \quad \therefore ye^{2x} = \int e^{x}\,dx$

$\therefore ye^{2x} = e^{x} + c$ //

Q (iii) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x\frac{dy}{dx} + 2y = \frac{\cos x}{x}$

$\therefore \frac{dy}{dx} + \left(\frac{2}{x}\right)y = \frac{\cos x}{x^2}$

$\therefore$ Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$.

$\therefore P(x) = \frac{2}{x} \quad ; \quad Q(x) = \frac{\cos x}{x^2}$

$\therefore I.F = e^{\int \frac{2}{x} dx}$

$= e^{2\log x}$

$= e^{\log x^2}$

$\therefore I.F = x^2$

$\therefore y(I.F) = \int Q(x)(I.F)\,dx$

$y(x^2) = \int \frac{\cos x}{\cancel{x^2}} \cancel{(x^2)}\,dx$

$\therefore x^2 y = \sin x + c$.



iv) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\therefore \frac{dy}{dx} + \frac{3}{x}y = \frac{\sin x}{x^3}$

$\therefore$ Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$\therefore P(x) = 3x^{-1} \quad . \quad Q(x) = \frac{\sin x}{x^3}$

$\therefore I.F = e^{\int \frac{3}{x} dx}$

$= e^{3\log x} = e^{\log x^3} = x^3$

$\therefore y(I.F) = \int Q(x)(I.F)\,dx$.

$y(x^3) = \int \frac{\sin x}{\cancel{x^3}} \cancel{x^3}\,dx$

$\therefore x^3 y = -\cos x + C$

v) $\therefore e^{3x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$\therefore e^{2x}\left(\frac{dy}{dx} + 2y\right) = 2x$

$\therefore \frac{dy}{dx} + 2y = \frac{2x}{e^{3x}}$.

$\therefore$ Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$\therefore P(x) = 2 \quad \therefore Q(x) = \frac{2x}{e^{2x}}$

$\therefore I.F = e^{\int 2 dx}$

$= e^{2x}$.

$\therefore y(I.F) = \int Q(x)(I.F)\,dx$

$\therefore y(e^{2x}) = \int \frac{2x}{e^{2x}}(e^{2x})\,dx$

$\therefore ye^{2x} = \frac{2x^2}{2} + c$

$\therefore ye^{2x} = x^2 + c$ //.

vi) $\sec^2 x \tan y\,dx + \sec^2 y \tan x \frac{dy}{dx} = 0$

$\therefore \sec^2 x \tan y\,dx = -\sec^2 y \tan x\,dy$

$\therefore \frac{\sec^2 x}{\tan x}\,dx = \frac{-\sec^2 y}{\tan y}$

$\therefore \int \frac{\sec^2 x}{\tan x}\,dx = -\int \frac{\sec^2 y}{\tan y}$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\therefore \log|\tan x| + \log|\tan y| = c$

$\therefore \log|\tan x.\tan y| = c \Rightarrow \tan x \cdot \tan y = e^c$

vii) $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x-y+1 = v$.

$\therefore 1 - \frac{dy}{dx} = \frac{dv}{dx}$ $\quad \therefore \frac{dy}{dx} = 1 - \frac{dv}{dx}$

$\therefore 1 - \frac{dv}{dx} = \sin^2 v$



$\therefore 1 - \sin^2 v = \frac{dv}{dx}$ $\quad \therefore dx = \frac{dv}{1-\sin^2 v}$

$\therefore \int dx = \int \sec^2 v$ $\quad \therefore x = \tan v + c$

$\therefore$ Put $v = x+y-1$ $\quad \therefore x = \tan(x+y-1) + c$.

viii) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$ $\quad \therefore \frac{dy}{dx} = \frac{2x+3y-1}{3(2x+3y+2)}$

$\therefore$ Put $2x+3y = v$

$\therefore 2+3\frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx}-2\right)$ $\quad \therefore \frac{1}{\cancel{3}}\left(\frac{dv}{dx}-2\right) = \frac{v-1}{\cancel{3}(v+2)}$

$\therefore \frac{dv}{dx} = \frac{v-1}{v+2} + 2$ $\quad \therefore \frac{dv}{dx} = \frac{v-1+2v+4}{2+2}$

$\therefore \frac{dx}{dv} = \frac{3v+3}{v+2}$ $\quad \therefore \frac{v+2dv}{3(v+1)} = dx$

$\therefore \frac{1}{3}\int \frac{(v+1+1)}{v+1}\,dv = \int dx$ $\quad \therefore \frac{1}{3}\int 1 + \frac{1}{v+1} = \int dx$

$\therefore \frac{1}{3}(v + \log(v+1)) = x + c$ $\quad \therefore$ But $v = 2x+3y$

$\therefore 2x+3y + \log|2x+3y+1| = 3x + c$

$\therefore 3y = x - \log|2x+3y+1| + c$ //.

10/01/2020

PRACTICAL:



Euler's method

Q1) $\frac{dy}{dx} = y + e^x - 2$

$\therefore f(x,y) = y + e^x - x$ , $y_0 = 2$ , $x_0 = 0$ , $h = 0.5$

vi)

| n | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 2 | 1 | 2.5 |
| 1 | 0.5 | 2.5 | 2.487 | 3.57435 |
| 2 | 1 | 3.5743 | 4.2925 | 5.3615 |

$\therefore y_{n+1} = y_n + h f(x_n, y_n)$

| n | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 3 | 1.5 | 5.3615 | 7.8431 | 9.28305 |
| 4 | 2 | 9.2831 | | |

$\therefore$ By Euler's formula :-

$y(2) = 9.2831$ //.

vii) Q2) $\frac{dy}{dx} = 1 + y^2$

$f(x,y) = 1 + y^2$ , $y_0 = 0$ , $x_0 = 0$ , $h = 0.2$

Using Euler's interation formula

$y_{n+1} = y_n + h f(x_n, y_n)$



| n | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0.2 |
| 1 | 0.2 | 0.2 | 0.104 | 0.408 |
| 2 | 0.4 | 0.408 | 1.1665 | 0.6413 |
| 3 | 0.6 | 0.6413 | 1.4113 | 0.9236 |
| 4 | 0.8 | 0.9236 | 1.8630 | 1.2942 |
| 5 | 1 | 1.2942 | | |

$\therefore$ By Euler's formula

$y(1) = 1.2942$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $x_0 = 0$ , $h = 0.2$

$\therefore$ Using Euler's iteration formula,

$y_{n+1} = y_n + h f(x_n, y_n)$

| n | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 |
| 1 | 0.2 | 0 | | |
| 2 | 0.4 | | | |
| 3 | 0.6 | | | |
| 4 | 0.8 | | | |
| 5 | 1 | | | |

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4) $\frac{dy}{dx} = 3x^2+1 \quad y_0 = 2, \; x_0 = 1, \; h = 0$

For $h = 0.5$

Using Euler's iteration formula

$$y_{n+1} = y_n + h\,f(x_n, y_n)$$

| n | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 1 | 2 | 4 | 4 |
| 1 | 1.5 | 4 | 49 | 28.5 |
| 2 | 2 | 28.5 | | |

$\therefore$ By Euler's formula

$y(2) = 28.5$

For $h = 0.25$

| n | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 1 | 2 | 4 | 3 |
| 1 | 1.25 | 3 | 5.6875 | 4.4219 |
| 2 | 1.5 | 4.4219 | 7.75 | 6.3594 |
| 3 | 1.75 | 6.2594 | 10.1815 | 8.9048 |
| 4 | 2 | 8.9048 | | |

$\therefore$ By Euler's formula

$y(2) = 8.9048$



5) $\frac{dy}{dx} = \sqrt{xy} + 2 \quad y_0 = 1, \; x_0 = 1, \; h = 0.2$

Using Euler's iteration formula,

$$y_{n+1} = y_n + h f(x_n, y_n)$$

| n | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 1 | 1 | 3 | 1.6 |
| 1 | 1.2 | 1.6 | | |

$\therefore$ By Euler's formula,

$y(1.2) = 1.6$

17/01/2020

— x —

# PRACTICAL :- 9

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Topic :- Limits & Partial Order derivatives

Q1) (i) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3 - 3y + y^2 - 1}{xy + 5}$

$\therefore$ At $(-4,-1)$, denominator $\neq 0$

$\therefore$ By applying limit

$\therefore \dfrac{(-4)^3 - 3(-1) + (-1)^2 - 1}{-4(-1) + 5}$

$\therefore \dfrac{-64 + 3 + 1 - 1}{4 + 5} \quad \therefore \dfrac{-61}{9}$ //

(ii) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

$\therefore$ At $(2,0)$ denominator $\neq 0$

$\therefore$ By applying limit,

$\therefore \dfrac{(0+1)((2)^2 + 0 - 4(2))}{2 + 0} \quad \therefore \dfrac{1(4 + 0 - 8)}{2}$

$\therefore \dfrac{-4}{2} \quad \therefore -2$ //

(iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2 - y^2 z^2}{x^3 - x^2 yz}$

$\therefore$ At $(1,1,1)$, denominator $= 0$

$\therefore \lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2 - y^2 z^2}{x^3 - x^2 yz}$

$\therefore \lim_{(x,y,z)\to(1,1,1)} \dfrac{\cancel{(x - yz)}(x + yz)}{x^2 \cancel{(x - yz)}}$

$\therefore \lim_{(x,y,z)\to(1,1,1)} \dfrac{x + yz}{x^2}$

$\therefore$ On applying limit

$\therefore \dfrac{1 + 1(1)}{(1)^2} = 2$ //.

Q2) (i) $f(x,y) = xye^{x^2+y^2}$

$\therefore f_x = \dfrac{\partial}{\partial x}(f(x,y)) \quad \therefore \dfrac{\partial}{\partial x}(xye^{x^2+y^2})$,

$\therefore ye^{x^2+y^2}(2x) \quad \therefore f_x = 2xye^{x^2+y^2}$

$\therefore f_y = \dfrac{\partial}{\partial y} f(x,y) = \dfrac{\partial}{\partial y}(xye^{x^2+y^2}) = xe^{x^2+y^2}(2y$

$\therefore f_y = 2yxe^{x^2+y^2}$

(ii) $f(x,y) = e^x \cos y$

$\therefore f_x = \dfrac{\partial}{\partial x}(f(x,y)) \quad \therefore \dfrac{\partial}{\partial x}(e^x \cos y)$

$\therefore f_x = e^x \cos y \quad \therefore f_y = \dfrac{\partial}{\partial y} f(x,y)$

$\therefore \dfrac{\partial}{\partial y}(e^x \cos y) \quad \therefore f_y = -e^x \sin y$.

055

(iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$\therefore \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f_x = 3x^2y^2 - 6xy$ $\quad \therefore f_y = \frac{\partial}{\partial y}(f(x,y))$

$\therefore \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f_y = 2x^3y - 3x^2 + 3y^2$

(3)(ii) $f(x,y) = \frac{2x}{1+y^2}$ $\quad \therefore f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$

$\therefore \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{d}{\partial x}(1+y^2)}{(1+y^2)^2}$

$\therefore \frac{2 + 2y^2 - 0}{(1+y^2)^2}$ $\quad \therefore \frac{2\cancel{(1+y^2)}}{\cancel{(1+y^2)}(1+y)^2}$

$\therefore \frac{2}{1+y^2}$

$\therefore$ At $(0,0)$

$= \cancel{\frac{2}{1+0}} = 2$ //.

$\therefore f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$



$\therefore \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$

$\therefore \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$

$\therefore \frac{-4xy}{(1+y^2)^2}$

$\therefore$ At $(0,0)$

$\therefore \frac{-4(0)(0)}{(1+0)^2} = 0$ //.

4)(i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$\therefore f_x = \frac{x^2 \frac{d}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$

$\therefore \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$ $\quad \therefore \frac{-x^2y - 2x(y^2 - xy)}{x^4}$

$\therefore f_y = \frac{2y - x}{x^2}$

$\therefore f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2y - 2x(y^2 - xy)}{x^4}\right)$

$\therefore \frac{x^4\left(\frac{\partial}{\partial x}(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy + 2x^2y)\frac{d}{\partial x}(x^4)}{(x^4)^2}$

$\therefore \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{(x^4)^2}$ —(1)

$\therefore f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y-x}{x^2}\right) = \frac{2-0}{x^2} = \frac{2}{x^2}$

$\therefore f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$

$\therefore \frac{-x^2 - 4xy + 2x^2}{x^4}$

$\therefore f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y-x}{x^2}\right)$

$= \frac{x^2\frac{\partial}{\partial x}(2y-x) - (2y-x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$

$\therefore \frac{-x^2 - 4xy + 2x^2}{x^4}$

$\therefore$ From 3 & 4

$\therefore f_{xy} = f_{yx}$

(ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$\therefore f_x = \frac{\partial}{\partial x}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$

$\therefore f_y = \frac{\partial}{\partial y}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 0 + 6x^2y - 0$

$= 6x^2y$

$\therefore f_{xx} = 6x + 6y^2 - \left(\frac{x^2+1\frac{\partial(2x)}{\partial x} - 2x\frac{\partial(x^2+1)}{\partial x}}{(x^2+1)^2}\right)$

$\therefore 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right)$ —— (1)



$f_{yy} = \frac{\partial}{\partial y}(6x^2y)$

$= 6x^2$

$\therefore f_{xy} = \frac{\partial}{\partial y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$= 0 + 12xy - 0$

$= 12xy$

$\therefore f_{yx} = \frac{\partial}{\partial x}(6x^2y) = 12xy$.

$\therefore$ From (3) & (4)

$\therefore f_{xy} = f_{yx}$.

(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$\rightarrow f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$\therefore f_{xx} = \frac{\partial}{\partial x}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)\cdot(y) + e^{x+y}(1)$

$= -y^2\sin(xy) + e^{x+y}$

$\therefore f_{yy} = \frac{\partial}{\partial y}(x\cos(xy) + e^{x+y})$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$-x^2 \sin(xy) + e^{x+y}$

$\therefore f_{xy} = -y^2 \sin(xy) + \cos(xy) + e^{x+y}$ ——— (3)

$\therefore f_{yx} = \frac{\partial}{\partial x}\left(x\cos(xy) + e^{x+y}\right)$

$= -x^2 \sin(xy) + \cos(xy) + e^{x+y}$ ——— (4)

$\therefore$ from (3) & (4)

$\therefore f_{xy} \neq f_{yx}$

Q5)

(i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

→ $f(1,1) = \sqrt{1^2+1^2} = \sqrt{2}$

$\therefore f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$ $\quad$ $f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\quad$ $= \frac{y}{\sqrt{x^2+y^2}}$

$\therefore f_x$ at $(1,1) = \frac{1}{\sqrt{2}}$ $\quad$ $\therefore f_y$ at $(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$\therefore \cancel{\sqrt{2}} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \cancel{\frac{2}{\sqrt{2}}}$ $\quad$ $\therefore \frac{x+y}{\sqrt{2}}$ ||



(ii) $f(x,y) = 1 - x + y\sin x$ at $\left(\frac{\pi}{2}, 0\right)$

$\therefore f\left(\frac{\pi}{2}, 0\right) = 1 - \frac{\pi}{2} + 0 = 1 - \frac{\pi}{2}$

$\therefore f_x = 0 - 1 + y\cos x$ $\quad$ $\therefore f_y = 0 - 0 + \sin x$

$\therefore f_x$ at $\left(\frac{\pi}{2}, 0\right) = -1 + 0 = -1$ $\quad$ $\therefore f_y$ at $\left(\frac{\pi}{2}, 0\right) = \sin\frac{\pi}{2}$

$\therefore f(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \frac{\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y-0)$

$= 1 - \cancel{\frac{\pi}{2}} - x + \cancel{\frac{\pi}{2}} + y$

$= 1 - x + y$

(iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$\therefore f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\quad$ $f_y = 0 + \frac{1}{y}$

$\therefore f_x$ at $(1,1) = 1$ $\quad$ $\therefore f_y$ at $(1,1) = 1$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$ ||

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# PRACTICAL :- 10

Q1) Find ~~the~~ directional derivative of the following function at given points & in the direction of given vector.

(i) $f(x,y) = x + 2y - 3$ $\quad a = (1,-1) \quad u = 3i - j$

Here, $u = 3i - j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

Unit vector along $u$ is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$\therefore f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1 - 2 - 3 = -4$

$\therefore f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)$

$\therefore f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$\therefore f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$\therefore D_uf(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{\cancel{-4} + \cancel{h}/\sqrt{10} + \cancel{4}}{\cancel{h}}$

$D_uf(a) = \frac{1}{\sqrt{10}}$ //

(ii) $f(x) = y^2 - 4x + 1$ $\quad a = (3,4) \quad u = i + 5j$

Here, $u = i + 5j$ is not a unit vector

$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

$\therefore$ Unit Vector along $u$ is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$\therefore \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$\therefore f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$\therefore f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$\therefore f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

$\therefore f_{x,y}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$\therefore \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$\therefore \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$

$\therefore \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$\therefore D_uf(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + \cancel{5} - \cancel{5}}{h}$

$$\therefore \frac{\cancel{h}\left(\frac{25h}{76} + \frac{36}{\sqrt{76}}\right)}{\cancel{h}} \qquad \therefore D_u f(a) = \frac{25h}{76} + \frac{36}{\sqrt{76}}$$

(iii) $2x+3y$ $\quad a=(1,2)$, $u=(3i+4j)$

Here $u = 3i+4j$ is not a unit vector

$|\bar{U}| = \sqrt{(3)^2+(4)^2} = \sqrt{25} = 5$

$\therefore$ Unit Vector along $u$ is $\frac{U}{|U|} = \frac{1}{5}(3,4)$

$\therefore \left(\frac{3}{5}, \frac{4}{5}\right)$

$\therefore f(a) = f(1,2) = 2(1) + 3(2) = 8$

$\therefore f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1+\frac{3h}{5}, 2+\frac{4}{5}\right)$

$\therefore f(a+hu) = 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{18\cancel{h}}{5} + \cancel{8} - \cancel{8}}{\cancel{h}}$$

$= \frac{18}{5}$

Q2) Find gradient vector for the foll fun at given point

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(i) $f(x,y) = x^y + y^x$ $\quad a=(1,1)$

$\therefore f(x) = y \cdot x^{y-1} + y^x \log y$

$\therefore f(y) = x^y \log x + xy^{x-1}$

$\therefore f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^x \log y, x^y \log x + xy^{x-1})$

$\therefore f(1,1) = (1+0, 1+0)$

$= (1,1)$

(ii) $f(x,y) = (\tan^{-1}x) \cdot y^2$ $\quad a=(1,-1)$

$\therefore f(x) = \frac{1}{1+x^2} \cdot y^2 \qquad \therefore f(y) = 2y\tan^{-1}x$

$\therefore f(x,y) = (f(x), f(y))$

$= \left(\frac{y^2}{1+x^2}, 2y\tan^{-1}x\right)$

$\therefore f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right) = \left(\frac{1}{2}, \frac{\pi}{4}(-2)\right)$

$\therefore f(1,-1) = \left(\frac{1}{2}, \frac{-\pi}{2}\right)$

(iii) $f(x,y,z) = xyz - e^{x+y+z}$ $\quad a=(1,-1,0)$

$\therefore f(x) = yz - e^{x+y+z}$

$\therefore f(y) = xz - e^{x+y+z}$

$\therefore f(z) = xy - e^{x+y+z}$

$\therefore f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$\therefore f(1,-1,0) = ((-1)(0) - e^{-1+(-1)+0}, (1)(0) - e^{1+0+(-1)}, (1)(-1) - e^{1+(-1)+0})$

$\therefore (0 - e^0, 0 - e^0, -1 - e^0)$

$\therefore (-1, -1, -2)$ //

Q.3) Find the eq.$^n$ of tangent & normal to each of the following curves at given point.

(i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$\therefore f(x) = \cos y \, 2x + e^{xy} y$

$\therefore f(y) = x^2(-\sin y) + e^{xy} . x$

$\therefore (x_0, y_0) = (1,0)$ $\quad x_0 = 1, y_0 = 0$

$\therefore$ eq$^n$ of tangent

$\therefore f(x)(x - x_0) + f_y(y - y_0) = 0$

$\therefore f_x(x_0, y_0) = \cos 0 \, 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$ //

$\therefore f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 . 1$

$= 0 + 1 . 1$

$= 1$

$\therefore 2(x-1) + (y-0) = 0$

$\therefore 2x - 2 + y = 0$

$\therefore 2x + y - 2 = 0 \rightarrow$ It is the required eq$^n$ of tangent.

$\therefore$ Eq$^n$ of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$\therefore 1(x) + 2(y) + d = 0$

$\therefore 1 + 2y + d = 0$ at $(1,0)$

$\therefore 1 + 2(0) + d = 0$

$d + 1 = 0$ $\quad \therefore d = -1$ //



ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$\therefore f(x) = 2x + 0 - 2 + 0 + 0 = 2x - 2$

$\therefore f(y) = 0 + 2y - 0 + 3 + 0 = 2y + 3$

$\therefore (x_0, y_0) = (2,-2)$ $\quad \therefore x_0 = 2, y_0 = -2$

$\therefore f(x)(x_0, y_0) = 2(2) - 2 = 2$

$\therefore f_y(x_0, y_0) = 2(-2) + 3 = -1$

$\therefore$ Eq$^n$ of tangent

$\therefore f_x(x - x_0) + f_y(y - y_0) = 0$

$\therefore$ eq$^n$ of tangent

$\therefore f_x(x - x_0) + f_y(y - y_0) = 0$

$\therefore 2(x-2) + (-1)(y+2) = 0$

$\therefore 2x - 2 - y - 2 = 0$

$\therefore 2x - y - 4 = 0 \longrightarrow$ It is required eq$^n$ of tangent

$\therefore$ Eq$^n$ of Normal

$= ax + by + c = 0$

$\therefore bx + ay + d = 0$

$\therefore -1(x) + 2(y) + d = 0$

$\therefore -x + 2y + d = 0$ at $(2,-2)$

$\therefore -2 + 2(-2) + d = 0$ $\quad \therefore -2 - 4 + d = 0$

$\therefore -6 + d = 0$

$\therefore d = 6$

Q4) Find the eqⁿ of tangent & normal line to each of the foll surface

i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$\therefore f_x = 2x - 0 + 0 + z$

$\therefore f(x) = 2x + z$

$\therefore f(y) = 0 - 2z + 3 + 0 = 2z + 3$

$\therefore f(z) = 0 - 2y + 0 + x = -2y + x$

$\therefore (x_0, y_0, z_0) = (2,1,0) \quad \therefore x_0 = 2,\ y_0 = 1,\ z_0 = 0$

$\therefore f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$\therefore f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$\therefore f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

$\therefore$ Eqⁿ of tangent

$\therefore \cancel{f(x)}\ f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$\therefore 4(x-2) + 3(y-1) + 0(z-0) = 0$

$\therefore 4x - 8 + 3y - 3 = 0$

$\therefore 4x + 3y - 11 = 0 \rightarrow$ This is required eqⁿ of tangent.

$\therefore$ Eqⁿ of normal at $(4,3,-11)$

$$\therefore \frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_x}$$

$$\therefore \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$



(ii) $3xyz - x - y + z = -4$ at $(1,-1,2)$

$\therefore 3xyz - x - y + z + 4 = 0$ at $(1,-1,2)$

$\therefore f_x = 3yz - 1 - 0 + 0 + 0$
$= 3yz - 1$

$\therefore f_y = 3xz - 0 - 1 + 0 + 0 = 3xz - 1$

$\therefore f_z = 3xy - 0 - 0 + 1 + 0 = 3xy + 1$

$\therefore (x_0, y_0, z_0) = (1,-1,2) \quad \therefore x_0 = 1,\ y_0 = -1,\ z_0 = 2$

$\therefore f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$\therefore f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$\therefore f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

$\therefore$ Eqⁿ of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x - 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0 \rightarrow$ This is required eqⁿ of tangent

$\therefore$ Eqⁿ of normal at $(-7,5,-2)$

$$\therefore \frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$\therefore \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$ //

Q5) Find the local maxima & minima for the following.

(i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$\therefore f_x = 6x - 3y + 6 - 0$

$\therefore f_y = 0 + 2y - 3x + 0 - 4 = 2y - 3x - 4$

$\therefore f_x = 0$

$6x - 3y + 6 = 0$

$\therefore 3(2x - y + 2) = 0$

$\therefore 2x - y + 2 = 0$

$\therefore 2x - y = -2$ —— (1)

$\therefore f_y = 0$

$2y - 3x - 4 = 0$

$\therefore 2y - 3x = 4$ —— (2)

$\therefore$ Multiply eqn 1 with 2

$\therefore 4x - \cancel{2y} = -4$

$\therefore \cancel{2y} - 3x = 4 \quad \therefore x = 0$

$\therefore$ Subsituting value of x in eqn (1)

$\therefore 2(0) - y = -2$

$\therefore +y = +2 \quad \therefore y = 2$ //

$\therefore$ Critical point are $(0,2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

$\therefore$ Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0 \quad \therefore f$ has maximum at $(0,2)$



$\therefore 3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$\therefore 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$\therefore 0 + 4 - 0 + 0 - 8 = -4$ //

(ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$\therefore f_x = 8x^3 + 6xy$

$\therefore f_y = 3x^2 - 2y$.

$\therefore f_x = 0$

$8x^3 + 6xy = 0 \quad \therefore 2x(4x^2 + 3y) = 0$

$\therefore 4x^2 + 3y = 0$ — (1)

$\therefore f_y = 0$

$\therefore 3x^2 - 2y = 0$ — (2)

$\therefore$ Multiply eqn (1) with 3
(2) with 4

$\therefore \cancel{12x^2} + 9y = 0$

$-\cancel{12x^2} - 8y = 0$

$-17y = 0 \quad y = 0$ //.

Subsituting value of y in eqn (1)

$\therefore 4x^2 + 3(0) = 0$

$\therefore 4x^2 = 0 \quad x = 0$

$\therefore$ Critical point is $(0,0)$

$r = f_{xx} = 24x^2 + 6y$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

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$r$ at $(0,0)$

$= 24(0+6(0)) = 0$

$\therefore r = 0$

$f(x,y)$ at $(0,0)$

$2(0)^4 + 3(0)^2(0) = 0$

$= 0$

$\therefore rt - s^2 = 0(-2) - (5)^2 . 0$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

(iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$\therefore f_x = 2x + 2$

$\therefore f_y = -2y + 8$

$\therefore f_x = 0 \quad \therefore 2x + 2 = 0$

$\therefore x = -1$

$\therefore f_y = 0 \quad -2y + 8 = 0$

$\therefore y = 4$

$\therefore$ Critical points is $(-1,4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$\therefore rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

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$\therefore f(x,y)$ at $(-1,4)$

$(-1)^2 - 4^2 + 2(-1) + 8(4) - 70$

$\therefore 1 + 16 - 2 + 32 - 70$

$\therefore 17 + 30 - 70$

$\therefore 37 - 70 = -33$