Concrete Compressive Strength Prediction

Report submitted for the course

"Regression Analysis and Time Series Models": 2023-2024



Prof. Buddhananda Banerjee

Department of Mathematics

Indian Institute of Technology Kharagpur

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Aakansh Singh (21CE3FP20)
Aadit Shah (21MI31001)
Anand Kumar (21MI3FP25)

Samyak Singhvi (21MI3FP11)

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Abstract:

The most crucial component in civil engineering is concrete. Age and composition have a very nonlinear effect on the compressive strength of concrete. These components include cement, fly ash, blast furnace slag, Superplasticizer, water, coarse and fine aggregate, and water.

Sources:

The dataset is collected from Kaggle wherein actual author and owner of the dataset is

Prof. I-Cheng Yeh

Department of Information Management

Chung-Hua University,

Hsin Chu, Taiwan 30067, R.O.C.

E-mail: icyeh@chu.edu.tw

Dataset Characteristics:

The actual concrete compressive strength (MPa) for a given mixture under a specific age (days) was determined from laboratory. Data is in raw form (not scaled).

Summary Statistics:

Number of instances (observations): 1030

Number of Attributes: 9

Attribute breakdown: 8 quantitative input variables, and 1 quantitative output variable

Missing Attribute Values: None

Importing Modules:

To implement the linear regression and to handle the data we have to import essential libraries.

```
# Importing essential libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns
from sklearn.linear_model import LinearRegression
import warnings
warnings.filterwarnings("ignore")
```

So, we import pandas for data analysis, NumPy for calculating N-dimensional array, seaborn and matplotlib to visualize the data

Reading data:

The dataset is in CSV file so we used to pandas (pd.read csv(")) to read the data

	Cement	Blast Furnace Slag	Fly Ash	Water	Superplasticizer	Coarse Aggregate	Fine Aggregate	Age (day)	strength
0	540.0	0.0	0.0	162.0	2.5	1040.0	676.0	28	79.986111
1	540.0	0.0	0.0	162.0	2.5	1055.0	676.0	28	61.887366
2	332.5	142.5	0.0	228.0	0.0	932.0	594.0	270	40.269535
3	332.5	142.5	0.0	228.0	0.0	932.0	594.0	365	41.052780
4	198.6	132.4	0.0	192.0	0.0	978.4	825.5	360	44.296075
		•••				(and	***		
1025	276.4	116.0	90.3	179.6	8.9	870.1	768.3	28	44.284354
1026	322.2	0.0	115.6	196.0	10.4	817.9	813.4	28	31.178794
1027	148.5	139.4	108.6	192.7	6.1	892.4	780.0	28	23.696601
1028	159.1	186.7	0.0	175.6	11.3	989.6	788.9	28	32.768036
1029	260.9	100.5	78.3	200.6	8.6	864.5	761.5	28	32.401235

The dataset contains 1030 examples having 9 columns in which 8 is features and 1 column (strength) is target variable.

Prepare dataset:

Separated 8 features and target variable as shown in the code.

```
# Separating dependent and independent variable.
X_raw = concrete_df.drop(columns=['strength '], axis=1)
Y = concrete_df['strength ']
```

Written a function to prepare the data to make suitable for fitting linear regression and then printed the data which shows all the column names are replaced by X variable.

 X_0
 X_1
 X_2
 X_3
 X_4
 X_5
 X_6
 X_7
 X_8

 0
 1
 540.0
 0.0
 0.0
 162.0
 2.5
 1040.0
 676.0
 28

 1
 1
 540.0
 0.0
 0.0
 162.0
 2.5
 1055.0
 676.0
 28

 2
 1
 332.5
 142.5
 0.0
 228.0
 0.0
 932.0
 594.0
 270

 3
 1
 332.5
 142.5
 0.0
 228.0
 0.0
 932.0
 594.0
 365

 4
 1
 198.6
 132.4
 0.0
 192.0
 0.0
 978.4
 825.5
 360

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Parameter estimation:

A function is written to estimate the weight parameters.

The weight parameters are found as:

Fitting Linear Regression:

Coded a linear regression function from scratch and fitted the linear regression and then predicted the target variable using the fitted equation.

The **fitted equation** and predicted values are printed as:

Error calculation:

The function written to determine **Mean Square error** and **Root mean square error** is as

```
#Error calculation:
def Errors(y_true, y_pred):
    e = y_true - y_pred
    mse = ((e**2).sum())/X.shape[0]
    return mse
mse = Errors(Y,y_hat)

print("Mean squared error is: ", mse)
rmse = np.sqrt(mse)
print("Root mean squared error is: {}".format(rmse))
```

The Mean square error and Root mean square error is printed as

```
Mean squared error is: 107.21180273479736
Root mean squared error is: 10.354313243030527
```

One can see that the error is huge for the fitted equation.

Hypothesis Testing (ANOVA):

A function is coded to create ANOVA table using the equation and formulas taught in class.

```
# Function to create ANOVA Table
def Anova(y true,y pred):
    # Total variation
    SSt = ((y true-y true.mean())**2).sum()
    degree_t = X.shape[0] - 1
    # Residual variation
    SSres= ((y true - y pred)**2).sum()
    degree res = X.shape[0] - X.shape[1]
   MSres = SSres/degree_res
    # variation due to regression
    SSreg = SSt-SSres
    degree reg = X.shape[1]-1
   MSreg= SSreg/degree reg
    F = MSreg/MSres
    return degree_res,degree_reg, SSres,SSreg,MSres,MSreg, F
degree_res, degree_reg, SSres,SSreg,MSres,MSreg, F = Anova(Y,y_hat)
```

The ANOVA table generated from the code is printed as:

 DF
 SS
 MS
 F

 Regression
 8
 176744.871659
 22093.108957
 204.269137

 Residual
 1021
 110428.156817
 108.156863
 204.269137

 Total
 1029
 287173.028476
 22201.265820
 204.269137

The ANOVA null hypothesis testing is performed as shown and it is found that the hypothesis is rejected.

```
# Testing Null hypothesis
print("
print(f"H0: b0=b1=.....bk-1=0 against \nH1: bi!=0 for i=0 to k-1")
print("If F>F{alpha, k-1, n-k}, The H0 is rejested")
print("_
import scipy.stats
from scipy.stats import f
alpha = 0.05
q = 1 - alpha
f = f.ppf(q, degree_reg, degree_res)
print(f"The calcylated f value is: {f}" )
print(f"The observed f value is {F}")
if(abs(F)>f):
       print("The null hypothesis H0 is rejected")
else:
        print("The null hypothesis H0 is accepted")
```

```
H0: b0=b1=.....bk-1=0 against
H1: bi!=0 for i=0 to k-1
If F>F{alpha, k-1, n-k}, The H0 is rejested

The calcylated f value is: 1.9474558084667661
The observed f value is 204.2691365659187
The null hypothesis H0 is rejected
```

Since the hypothesis testing is rejected which suggest the all the features are significant, then proceeded to **Partial (Marginal) test.**

Predicting R² and R²_{adj}:

The R^2 and R^2_{adj} value is determined of the fitted equation and it is found that R^2 value is 61.55% which is not decent.

```
# Predict R^2 value and adjusted R^2 value:
R_sq = (SSreg/ (SSres+SSreg) ) * 100
AdjR_sq = (1- MSres/(MSres+MSreg)) * 100
print(f"The R square value is:{R_sq}")
print(f"Adjusted R square value is: {AdjR_sq}")
```

```
The R square value is:61.54647342657952
Adjusted R square value is: 99.51283470241574
```

Since the score of the fitted equation is not satisfying so tested the significant of each regressor individually by Partial (Marginal) test.

Partial Test or Marginal test:

```
# Test on individual regression coefficient (Partial test or Marginat Test)
Corr = np.linalg.inv(X.T@X)
from scipy.stats import t
def Marginal_test(beta, C, MSres,X):
    n = X.shape[1]
   for i in range(n):
        T = beta[i]/np.sqrt(MSres*C[i][i])
        if(abs(T)>t.ppf(1-0.05, degree_res)):
            print(f"H0:b{i}=0 is rejected")
        else:
            print(f"HØ:b{i}=0 is accepted")
Marginal_test(b_hat,Corr,MSres,X)
HØ:b0=0 is accepted
H0:b1=0 is rejected
H0:b2=0 is rejected
H0:b3=0 is rejected
H0:b4=0 is rejected
H0:b5=0 is rejected
```

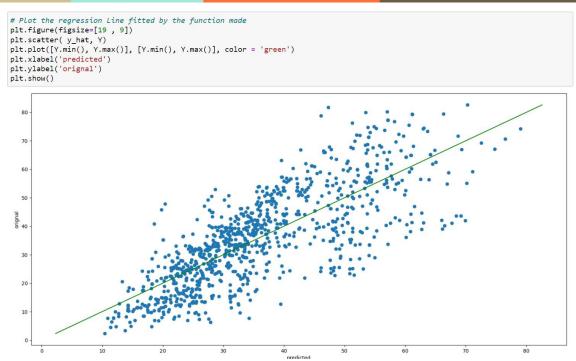
Notice that the null hypothesis for $\beta_0=0$ is accepted which means β_0 is not a significant parameter, therefore X_0 is not a significant regressor.

Further experimented fitting the linear regression discarding X_0 using similar functions as described above but the performance of the equation didn't change the errors much and score was almost the same as the previous one.

Plot of the equation:

H0:b6=0 is rejected H0:b7=0 is rejected H0:b8=0 is rejected

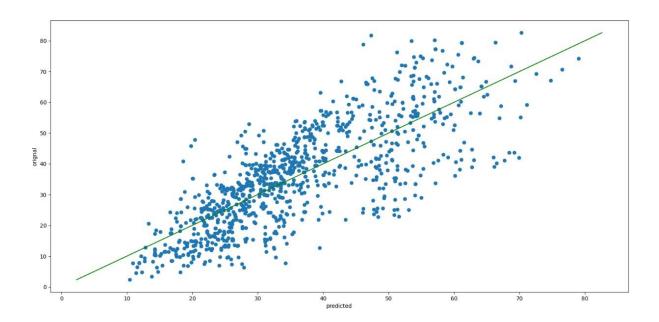
Plotted the fitted equation using matplotlib library from python and the plot is shown how the line is fitted to the scattered data.



The performance of the created function from scratch was not satisfactory so tried python inbuilt regression function to check the credibility of the created function.

Using python inbuilt regression:

```
# trying the inbuilt regression functionn
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error
lr = LinearRegression()
fit = lr.fit(X,Y)
print( '.....
y predict = lr.predict(X)
print( 'mean_ squred_error is ==',mean_squared_error(Y, y_predict))
rms = np.sqrt(mean_squared_error(Y,y_predict))
The predicted values are
y_predict
mean_ squred_error is == 107.21180273479739
root mean squared error is == 10.354313243030528
                                           __The predicted values are_
array([53.4728591 , 53.74331185, 56.81194746, ..., 26.47099254,
      29.11564722, 31.89398622])
```



It is found that the error and the predicted value obtained from the created function from scratch and from the python inbuilt function is exactly same. And the plot of the regression is also found the same as shown above. This proves that that the function created from scratch is accurate and less performance of linear regression gives hint to try other regression model like Polynomial Regression.

Before applying polynomial regression, we first checked for Multicollinearity and then cleaned the data to improve the performance of the model.

Multicollinearity Check:

Using seaborn, plotted the heatmap which shows the correlation of each variable with one another.

```
# Plot heatmap to check multicollinearity
plt.figure(figsize = (14,7))
sns.heatmap(concrete_df.corr(), annot=True, cmap='viridis')
<Axes: >
                                                                                                                                 0.8
     Blast Furnace Slag
                                    1
                                                                                                                                 0.6
              Fly Ash
                                                          1
                                                                    -0.66
                                                                                          -0.45
                                                                                                                -0.29
               Water
       Superplasticizer -
                                                                                1
                                                                                                                                 0.0
     Coarse Aggregate -
       Fine Aggregate
                                                                                           1
                                   -0.28
                                                                                                                                  -0.2
            Age (day)
                                                                                                      1
            strength
                                               Fly Ash
                                                          Water
                                                                                           Fine Aggregate
                                                                                                      Age (day)
```

It is noticed that the none of the variables have correlation greater than 0.5 which means the variables are not strongly correlated therefore the problem of Multicollinearity is not severe.

Data Cleaning:

First checked for the duplicates in the data, because duplicated data may bias the model.

```
# check for any duplicate values in the data
duplicates = concrete_df.duplicated()
concrete_df[duplicates]
duplicates.value_counts()

False    1005
True    25
Name: count, dtype: int64
```

There are 25 duplicates are found as shown in the code block.

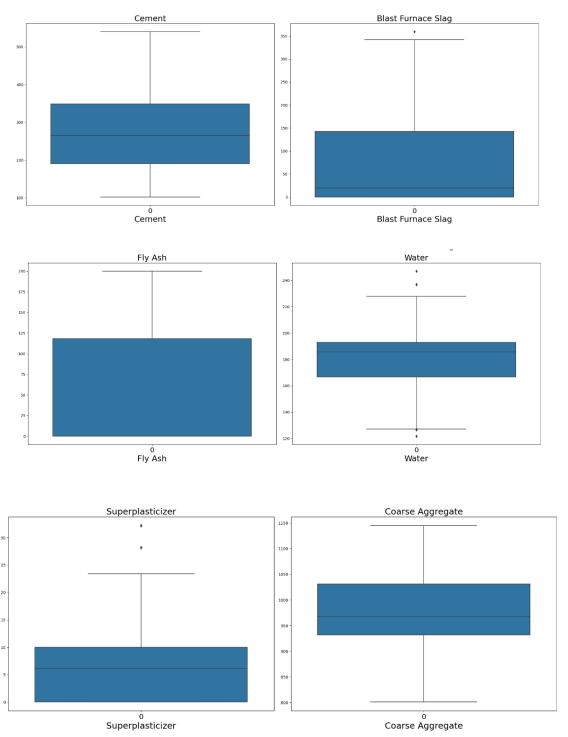
All the duplicates are dropped and then we printed the data frame.

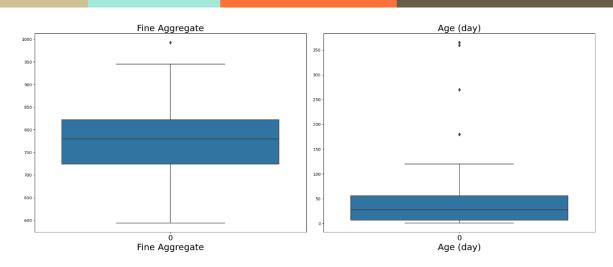
```
# Dropping duplicate values.
concrete_df=concrete_df.drop_duplicates()
concrete_df
```

	Cement	Blast Furnace Slag	Fly Ash	Water	Superplasticizer	Coarse Aggregate	Fine Aggregate	Age (day)	strength
0	540.0	0.0	0.0	162.0	2.5	1040.0	676.0	28	79.986111
1	540.0	0.0	0.0	162.0	2.5	1055.0	676.0	28	61.887366
2	332.5	142.5	0.0	228.0	0.0	932.0	594.0	270	40.269535
3	332.5	142.5	0.0	228.0	0.0	932.0	594.0	365	41.052780
4	198.6	132.4	0.0	192.0	0.0	978.4	825.5	360	44.296075
	200	1510	922	1311	1757	0.00	892	1220	500
1025	276.4	116.0	90.3	179.6	8.9	870.1	768.3	28	44.284354
1026	322.2	0.0	115.6	196.0	10.4	817.9	813.4	28	31.178794
1027	148.5	139.4	108.6	192.7	6.1	892.4	780.0	28	23.696601
1028	159.1	186.7	0.0	175.6	11.3	989.6	788.9	28	32.768036
1029	260.9	100.5	78.3	200.6	8.6	864.5	761.5	28	32.401235

1005 rows × 9 columns

Note that 25 duplicates are dropped and the number of example changed from 1030 to 1005. Any outliers present in data is checked using boxplot method the plots are shown below:





It is noted that some outliers are present in "slag", "water", "fine aggregate", "superplastic" and "age". Further created a function to remove outlier and removed the outlier from each variable whereit was present.

The function to remove outlier is shown below:

```
# Function to remove outlier
def remove_outlier(col):
    sorted(col)
    Q1,Q3=col.quantile([0.25,0.75])
    IQR=Q3-Q1
    lower_range=Q1-(1.5*IQR)
    upper_range=Q3+(1.5*IQR)
    return lower_range, upper_range
```

After preprocessing and cleaning the whole data once again, fitted the linear regression to the data using the function created from scratch.

The observation are as follows:

```
_The fitted equation is_
y_hat=0.115X_1+0.090X_2+0.067X_3+-0.177X_4+0.266X_5+0.005X_6+0.010X_7+0.308X_8}
                                         The Predicted Values are
1
        54.938918
5
         39.7736
7
        31.327257
8
        20.700388
9
        33.665311
1025
       39.747941
1026
       33.914072
1027
       25.553784
1028
       28.750184
1029
       31.866135
Length: 911, dtype: object
```

The errors are found as:

```
Mean squared error is: 60.91449792340837
Root mean squared error is: 7.804774046915668
```

It is notable that after cleaning and preprocessing the data. The mean square error has reduced from **107.21** to **60.91** and rmse has reduced from **10.354** to **7.805**. Though decrement in error is not very profound but it proves that the cleaning and preprocessing has improved the performance of the model.

Polynomial Regression:

Having tried various technique it is observed that the performance of the model has not increased very significantly, therefore using the polynomial regression to check if it helps in improving the performance of model.

Imported the polynomial features function from sklearn.preprocessing and then fitted the model on preprocessed and cleaned data as:

```
from sklearn.preprocessing import PolynomialFeatures
from sklearn import linear_model
poly = PolynomialFeatures ( degree=3, interaction_only=False , include_bias=True, order= 'C' )
x = poly.fit_transform(X)
poly_clf = linear_model.LinearRegression()
poly_clf.fit (x, Y)
print(poly_clf.score(x,Y))
0.9127464768959767
```

Printed the error value determined for the model as shown:

root mean squared error is == 4.6799769053473925

```
print( '...
y_predict = poly_clf.predict(x)
print( 'mean_squared_error is ==' , mean_squared_error(Y,y_predict) )
rms = np.sqrt(mean_squared_error(Y,y_predict))
print( 'root mean squared error is == {} '.format(rms))

mean_squared_error is == 21.90218383458496
```

Note that the mean square error has drastically decreased from **107** to **21.9** and root mean squared error decreased from **10.35** to **4.68** which is remarkable.

Created the ANOVA table for the polynomial regression using the created function, the table is:

		ANOVA Table								
	DF	SS	MS	F						
Regression	7	173184.070842	24740.581549	402.585944						
Residual	903	55493.107608	61.454161	402.585944						
Total	910	228677.178451	24802.035710	402.585944						

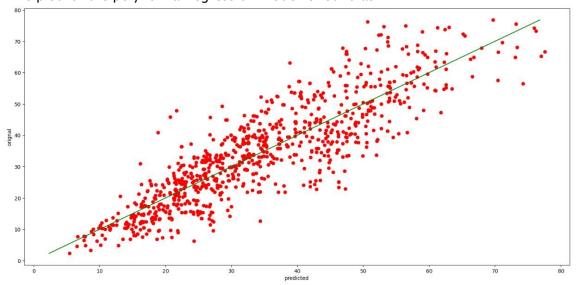
Further printed the R^2 and R^2_{adj} value for the polynomial regression as shown:

```
# Predict R^2 value and adjusted R^2 value:
R_sq = (SSreg /(SSreg + SSres)) * 100
AdjR_sq = (1-MSres/(MSres+MSreg)) * 100
print(f"The R square value is: {R_sq}")
print(f"Adjusted R square value is: {AdjR_sq}")
```

The R square value is: 75.73299269117922 Adjusted R square value is: 99.75222130164006

It is worth to note that the value of \mathbb{R}^2 has boosted from **61.5%** to **75.7%** which is remarkable, thus the polynomial regression is the best fit for the data.

The plot for the polynomial regression model is found as:



Note that the plot looks more fitting to the data than the linear regression model.

Results:

The weights parameters of the polynomial regression are printed as:

```
poly_clf.coef_
array([ 1.17606455e+06, -1.21797733e+02, -9.77272573e+01, -3.53238111e+02,
       -3.76676743e+02, 7.75298952e+02, -1.18724136e+02, -1.19179471e+02,
        6.76041298e+01, 4.58569683e-02, 9.58166304e-02, 2.45367884e-01,
        2.12243922e-01, -5.60072212e-01, 8.08168849e-02, 1.11840965e-01,
       -2.67501695e-02, 3.75291867e-02, 2.29013504e-01, 1.98304860e-01,
       -3.92699355e-01, 3.46025890e-02, 1.13145075e-01, -1.88148404e-02,
        2.29256932e-01, 6.41643543e-01, -4.57355725e-02, 2.31437232e-01,
        3.36810366e-01, -9.65166160e-02, 3.98257257e-01, -1.86652967e+00,
        3.21019872e-01, 2.71458223e-01, -1.92018557e-01, -4.76295214e-01,
       -2.95132720e-01, -9.73777622e-01, 5.63309521e-01, 4.51403166e-02,
       7.60088285e-02, -6.04224988e-02, 4.35843077e-02, -3.54123114e-02,
        8.47042601e-03, -6.06690449e-06, -2.20257439e-05, -4.76341834e-05,
       -2.63209791e-05, 1.38002560e-04, -1.53867488e-05, -2.34148459e-05,
        2.89178895e-06, -2.38578358e-05, -9.52082411e-05, -6.92818356e-05,
        2.02832857e-04, -2.33398681e-05, -5.41903069e-05, 7.61163967e-06,
       -7.55986215e-05, -1.91994405e-04, 1.47060437e-04, -7.83991785e-05,
       -1.23243164e-04, 3.86209393e-05, -7.56230069e-05, 6.22996491e-04,
       -7.63595393e-05, -1.11817284e-04, 3.69674159e-05, 1.42900063e-04,
        1.13123157e-04, 3.26173952e-04, -3.76857311e-04, -1.38788017e-05,
       -3.44716823e-05, 9.40923818e-06, -2.42730236e-05, 1.26710662e-05,
       -1.86757958e-05, -4.97889407e-06, -4.65634622e-05, -1.95807905e-05,
       1.57180910e-04, -4.83034385e-06, -2.69114087e-05, -3.52807168e-06,
       -7.93913694e-05, -1.84597322e-04, 6.19072050e-05, -5.91607543e-05,
       -1.27288675e-04, 2.64163429e-05, -1.14313037e-04, 2.62856617e-04,
       -3.53889653e-05, -1.23903297e-04, 3.98773406e-05, -2.99252062e-04,
       1.05529347e-04, 2.23772913e-04, -2.23501057e-04, -1.78748337e-07,
       -2.47185019e-05, 6.14690295e-06, -2.84803050e-05, 8.87638963e-06,
       -2.67067311e-05, -4.75932313e-05, -2.07327678e-04, -1.49503068e-04,
       -6.97120488e-05, -1.14466678e-04, 3.70893451e-05, -2.51396202e-04,
        4.58598045e-04, -2.05987040e-04, -3.45137377e-04, 9.02872960e-05,
       2.56145967e-04, -1.17654804e-04, 9.06443595e-05, -2.76394334e-04,
       -3.75746448e-05, -1.13619475e-04, 3.06418525e-05, -7.19746603e-05,
       4.61589425e-05, -2.17619235e-05, -9.22903603e-05, 1.12007320e-03,
       -2.14705714e-04, -1.38522589e-04, 1.02401524e-04, -2.17934993e-03,
       1.68743282e-04, 1.40964584e-03, -2.74292131e-04, -7.58364327e-05,
       -8.54057714e-05, 9.10085563e-05, -5.93615726e-05,
                                                          5.62802645e-05,
       5.38778679e-06, 2.62786175e-05, -5.74894183e-05,
                                                          8.53079265e-04.
       -1.88916884e-08, 3.27364966e-06, 3.03961969e-04, -2.07485180e-04,
        2.02738758e-04, -2.17015434e-04, -2.99990484e-05, -5.14818501e-06,
       -1.41635045e-05, 1.48010070e-05, -1.19784130e-05, 1.43269037e-05,
       -9.54195496e-06, -4.54048116e-06, 3.98642397e-06, -1.09146830e-05,
        7.88325444e-05])
```

The weight parameters of the linear regression are printed as:

Having experimented, the results of different fitted regression models are summarized below in the given table:

	Mean Squared Error	Root mean squared error	$R^2 \%$	$R^2_{adj} \%$
Linear Regression	60.914	7.805	61.55	99.51
Polynomial Regression	21.902	4.680	75.73	99.75

Conclusion:

In this report, we have analyzed the compressive strength prediction of concrete using linear regression and ANOVA hypothesis testing. We used a dataset consisting of 1030 examples having 8 features and 1 target variable. We prepared the data by separating the features and the target variable and then applied linear regression to fit the model. We calculated the mean square error and root mean square error for the model, and we performed ANOVA hypothesis testing to determine if all the features were significant or not. The null hypothesis was rejected, which means that all the features are significant. However, we also performed a partial (marginal) test and it was found that one of the regressors is not significant. We further experimented with fitting the linear regression model by discarding the non-significant regressor and found that the performance of the model did not change significantly. We then checked for multicollinearity and cleaned the data by removing both duplicates and outliers. We also checked the performance of the model using the Python inbuilt regression function and found that the performance was similar to the model we created from scratch. Overall, we concluded that the performance of the linear regression model was not satisfactory and this prompted us to try other regression models like polynomial regression.

Trying polynomial regression has resulted to be the best fit for the data. The model has boosted the performance and reduced the errors drastically.