LAB 4 ANALYSIS

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(Submitted with 1 of 4 grace days used)

Part A) Data Collection

The IMU is mounted at the car armrest near the center of the vehicle on a flat portion, while the GNSS puck is mounted on the roof of the car.

The collection is divided into two parts. The first segment (t = 56 to t = 190s) involved driving on a circular route of roughly 5m radius to calibrate the IMU's internal compass in the XY plane. The second segment (t = 215 to t = 1087s) involves driving the GPS IMU setup in a closed-loop forward-only path. The data collection zone is depicted in Fig. 1. Two total loops around the given trajectory were made as redundancy measures.

Lowpass IIR filtration with a cutoff frequency of 0.5 Hz is performed on the linear acceleration (X-Axis), yaw rate (Ω_Z) and the IMU Yaw angle (Θ_Z) to smoothen the time-series data and attenuate high frequency noise.

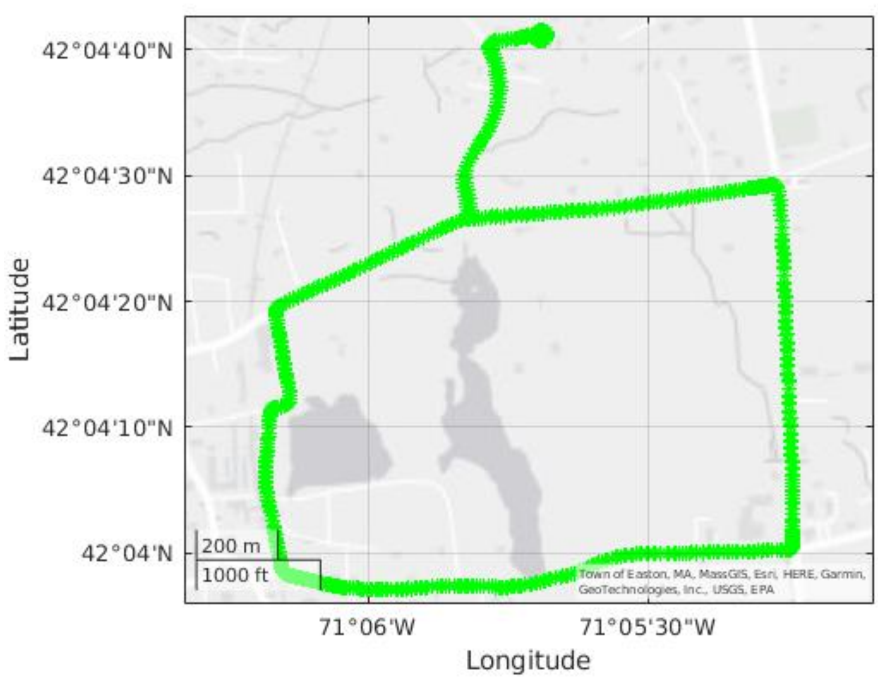

Figure 1. GPS trajectory followed

Part B1) Magnetometer Calibration

Figure 2 depicts the magnetometer calibration procedure with an optimization process being used to fit an ellipsoid (in this case, an ellipse in the XY plane, since the Z axis magnetic field is assumed to be held near constant for the calibration period), and then the ellipse's major and minor axes are scaled and shifted to the origin.

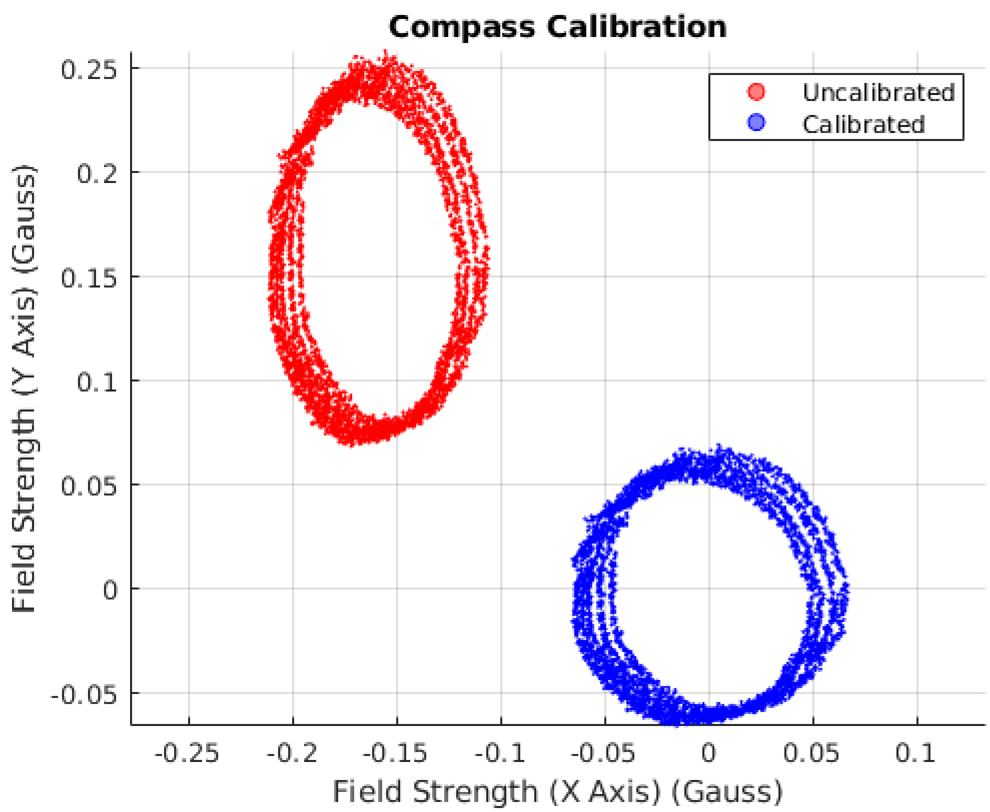

Figure 2. Compass calibration before and after

The transformation matrix for calibration

$$A = \begin{bmatrix} 1.2535 & 0 & 0 \\ 0 & 0.7116 & 0 \\ 0 & 0 & 1.1209 \end{bmatrix}$$

The translation vector for the ellipse center point $b = [-0.159 \;\; 0.161 \; -0.337]$

The expected magnetic field strength is estimated as 0.0627 Gauss.

And the net correction can be formulated as: $MAG_{Cal} = (MAG_{Uncal} - b) \cdot A$

From Fig. 2, it is seen that the magnetometer initially suffers from significant hard iron effects more than that of the soft iron influences. The magnetometer calibration procedure can recover a corrected measurement of these values which are by default tilt compensated with respect to the gravity vector.

Since the obtained magnetometer values are already tilt compensated, they can directly be used to recover the yaw angle using *atan2* function. The yaw angle here is given by $atan2(-MAG_{Cal,Y}, MAG_{Cal,X})$.

The yaw rate integration begins from 0, and as such a bias is applied to the integrated yaw rate after radian to degree conversion and wrapping around 180°. The initial bearing of the car is roughly -270° from the magnetic north which is Eastward. This includes the magnetometer calibration phase of the data collection as well.

The complementary filter includes parallel Lowpass and High-pass filters or a complementary weight α, filtering the high-frequency magnetometer noise, and the low frequency drift of the gyroscope. The complementary filter structure used for fusing the integrated yaw rate and magnetometer heading values is of the form $YAW_{Comp} = \alpha.MAG_{Yaw} + (1-\alpha)YAW_{YawRate}$, where $\alpha = \frac{T_c}{Tc+T_{Sampling}}$. The time constant of these filters is the same, and is determined partly by trial and error, and partly by design [1,2].

One way of determining the time constant T_C is through the use of manufactured specs regarding the worst-case in-run bias instability value (°/s) which for the VN-100S is 0.1167°/s and using at least half of this value as the time constant. Thus T_c is taken as 0.059. Another value of T_c which was tested is half of the in-run bias instability analysis ensemble size taken from LAB3, which is fairly close to what the mentioned reference points towards. This choice(s) of time constant lays more emphasis on the magnetometer data, with α coming out to be approximately 0.71. Figure 3 below shows the various outputs in order. It is observed that the yaw angle recovered from the magnetometer data resembles the filtered IMU Yaw angle more than that of the integrated yaw rate. As such, the choice of the weighting factor α is justified to keep the emphasis towards the magnetometer readings. The blue line plot shows the complementary filter output after weighting the magnetometer and biased integrated yaw rate values, while the green line plot shows the yaw angle read from the IMU data stream. The angles are wrapped around 180° after being converted to degrees and biased based on the starting yaw angle of the gyroscope output. There is an observable phase lag between the complementary filter's output and that of the IMU. Overall, weighting the complementary filter more towards the magnetometer yaw is more justified due to the magnetometer being an absolute source of heading information, unlike that of the yaw rate integration, which (previously established) suffer from angle random walk. Any phase lags in the complementary filter output could thus be attributed to the individual contributions of gyroscopic drifts and uncorrected calibration errors in the magnetometer.

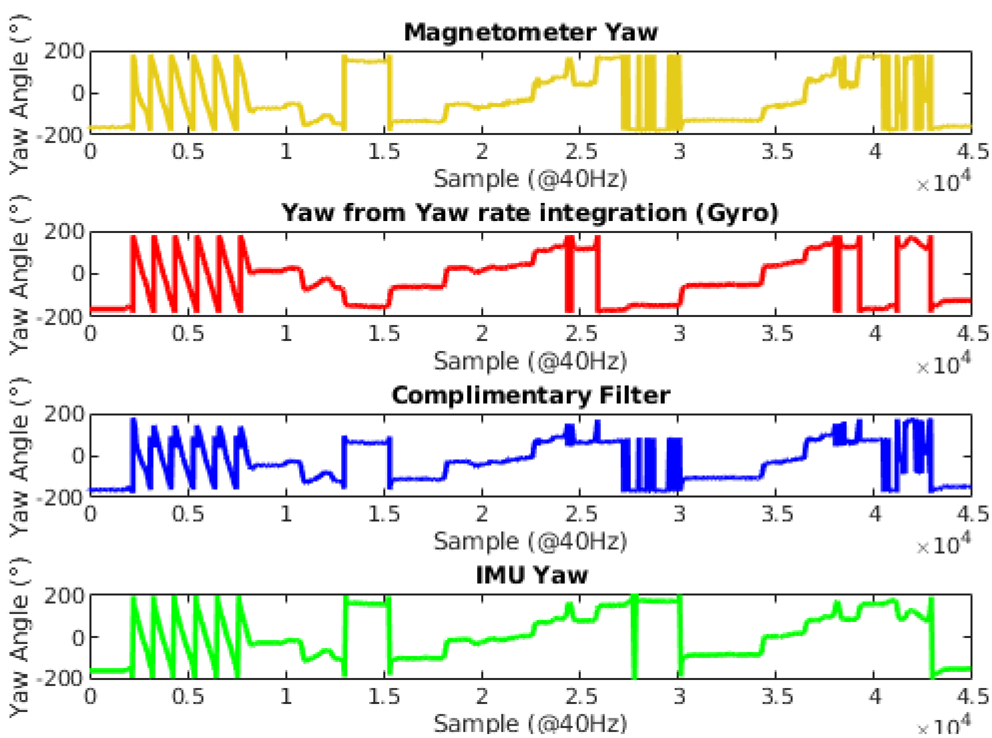

Figure 3. Comparison of complementary filter output with the IMU Yaw

Part B2) Forward Acceleration Integration

The forward velocity profile is found through double integration of the forward acceleration component. The linear acceleration profile for the second part of data collection (forward motion in a closed loop) is seen in Fig. 4. A lowpass filter with a spectrum cutoff frequency of 0.5Hz is utilized after observations made in the power spectrum that most of the signal content is concentrated near 0Hz.

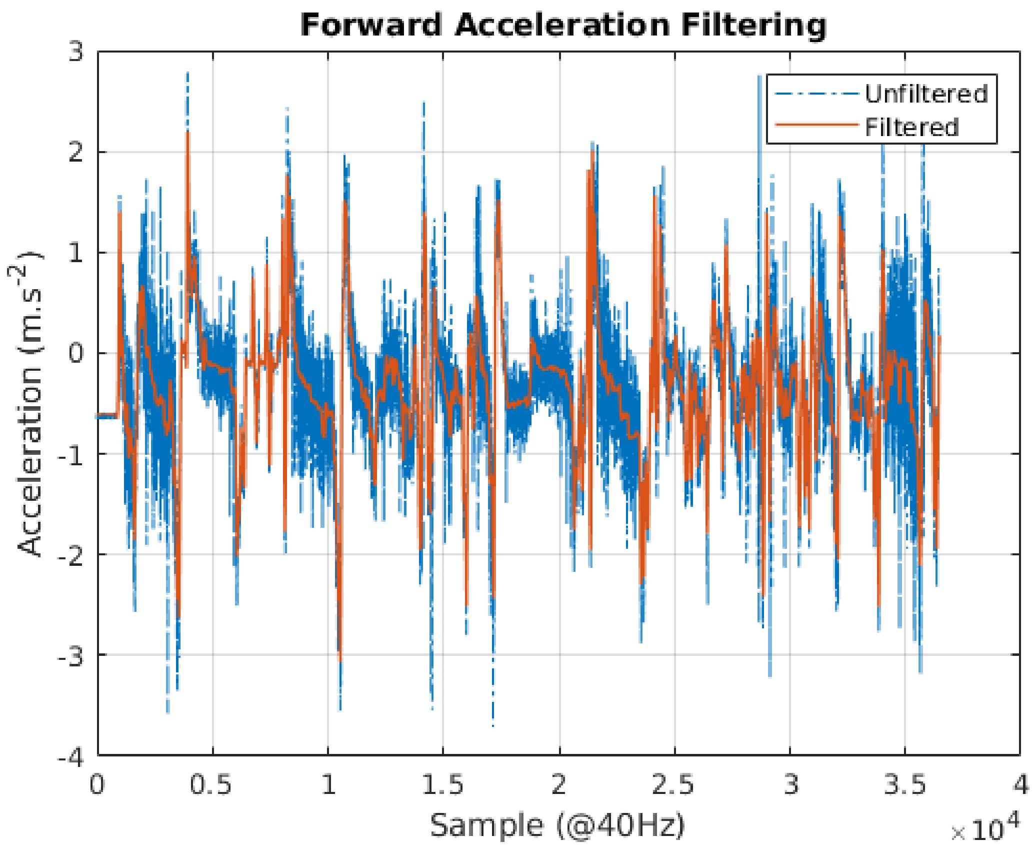

Figure 4. Forward acceleration profile

The linear acceleration profile suffers from in-run biases of varying magnitude. These biases are identified by observing regions of zero motion while recording the data. The mean of these stationary biases is then utilized to offset subsequent acceleration measurements until a new bias is found, and the process is repeated. In the obtained filtered acceleration data, an initial bias of $-0.6 ms^{-2}$ is observed followed by 6 additional spots where the acceleration is locally constrained to a very small value with an overall offset.

The algorithm can be stated as follows:

- *Generate window of size SZ (700 samples), and set standard deviation tolerance bounds as TOL ($0.1 m.s^{-2}$)*
- *Start window from index i=SZ, and slide window till end of timeseries.*
- *Record global index of first element of each window which satisfies tolerance constraint.*
- *Set flag to 1 whenever constraint is satisfied*
- *Calculate mean of window and assign bias value to flag index whenever flag = 1*

- *Offset subsequent samples when flag = 0 with prior bias till flag = 1 (a new bias is found)*

The GPS Velocity is obtained by considering the motion of the car on the geoid with a mean radius (assumed constant for small variations in latitude and longitude). This mean-radius results in a degree-to-meter conversion for two coordinates in decimal degrees. In this case, since most of the variation occurs along longitudinal values, a degree to meter conversion coefficient is taken as $\Delta = 29795778$ m/°

Differentiating this distance with respect to a small change in time provides the point-to-point velocity between two coordinate points in the Latitude Longitude axes. The peak velocities are in the vicinity of 16-18 ms^{-1}, with some error likely due to an incorrect estimation of the distance covered in meters per degree of latitude and longitude, along with some locally unaccounted variations in altitude between successive measurements.

Figs. 5 and 6 shows a comparison of the estimated GPS velocity compared with the forward IMU velocity obtained through the integrated forward acceleration. For data collection, due to unaccounted serial port malfunction, some data points were lost between time samples t = 700s to t =730s. It is thus worth noting that Fig. 6 shows a subset of the total samples recorded due to loss of signal during the redundancy loop traversed, and all subsequent calculations are done on this subset for this reason.

The dotted yellow line in subplot 1 of Fig. 5 shows the regions where bias offsets where calculated and applied. There are some scaling issues between the estimated velocities because of two major reasons:

1) Uncorrected Bias: The windowing approach is able to remove biases dynamically and in a causal fashion. There are two parameters which need consideration while selecting the window, and that has a degree of data dependence. In this case, the window tolerances are taken to be $0.1ms^{-2}$ and the window size is taken as 700 samples. But these two parameters are selected based on a bird's eye view of the acceleration data where it was observed that two major biases spanning over 1000 samples while there are some other smaller zones which are locally stagnant. A more meticulous approach to tuning these two parameters should ideally lead to better bias removal.

2) Incorrect estimation of GPS velocities: The decimal degrees to meter conversion relies on the distance between two points 1° Latitude or Longitude apart. For this case, only the degree to meter value along the latitude is taken, whereas some motion spills off into the longitude as well. *Why UTM coordinates were not used is because UTM coordinates do not have smooth transition position values between two gridlines. The data collected involved two different UTM Easting zones, and as such calculation of velocities would have resulted in a sharp unrealistic velocity spike wherever the coordinates crossed into another zone.*

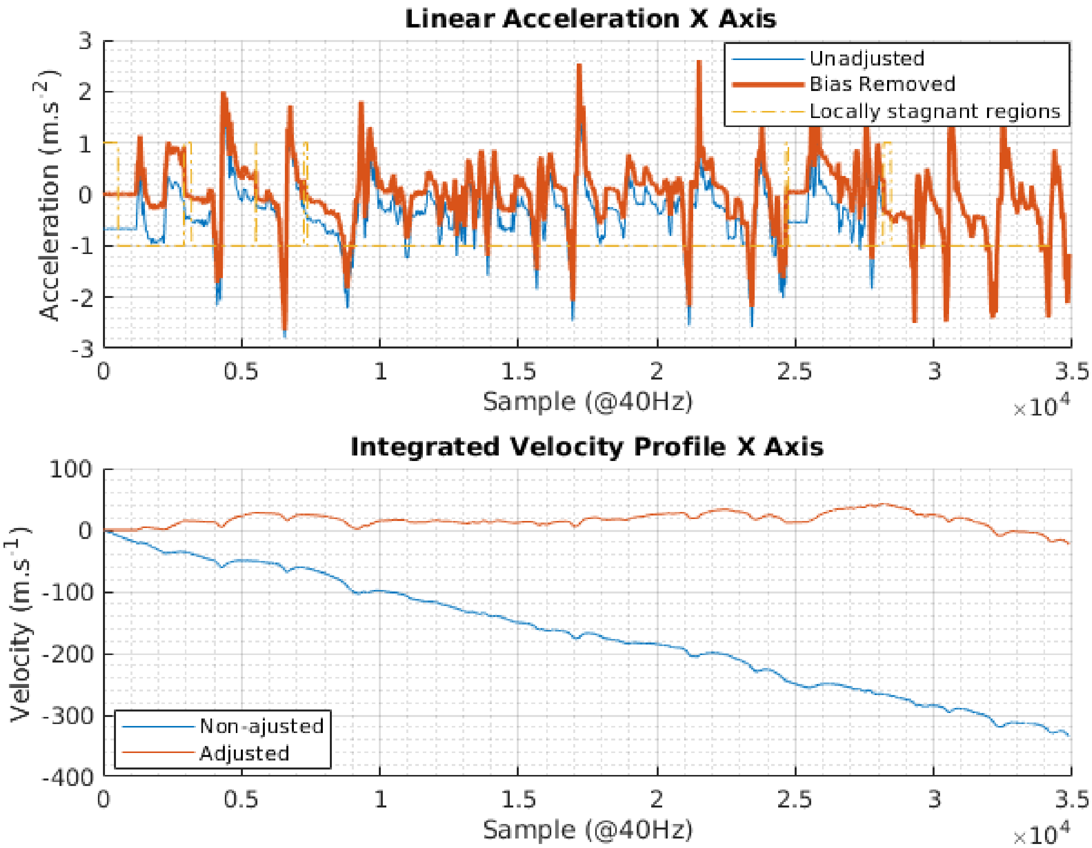

Figure 5. Comparison of acceleration and velocity profiles (Unadjusted vs Bias removed, both filtered)

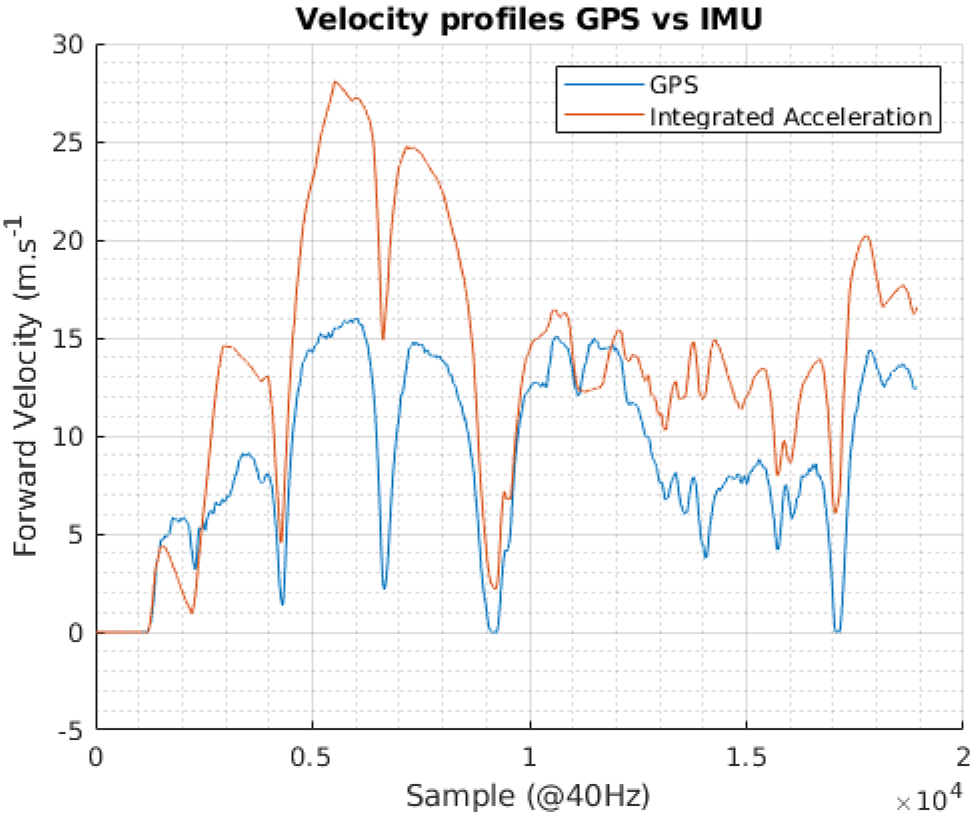

Figure 6. Velocity profiles: GPS and IMU

Part B3) Dead Reckoning

For dead reckoning with the IMU, the yaw angle time series is used in tandem with the forward acceleration measurements over the second phase of data collection. The linear acceleration is resolved into two components based on the yaw angle. The yaw angle's reference axis is aligned with the magnetic north. And as such, the acceleration is resolved into components along the magnetic North (which in this case is assumed to be close to the true North), and along the geographic East. The components are thus = $a_e = \alpha.sin(\theta_Z)$, and $\alpha_n = \alpha.cos(\theta_Z)$, where α is the bias corrected, lowpass IIR filtered forward acceleration measurements in the X-axis.

Double integrating the two components individually allows for displacements along the Northing and Easting directions, which can be directly used as coordinates in meters. Fig. 7 shows the doubly integrated resolved components as a scatter plot with reference axis along the Northing and Easting directions.

The trajectory formed by the integration of forward acceleration components is oriented along the start of the GPS data and plotted together. Additionally, the magnetometer yaw estimates are further used to generate a trajectory.

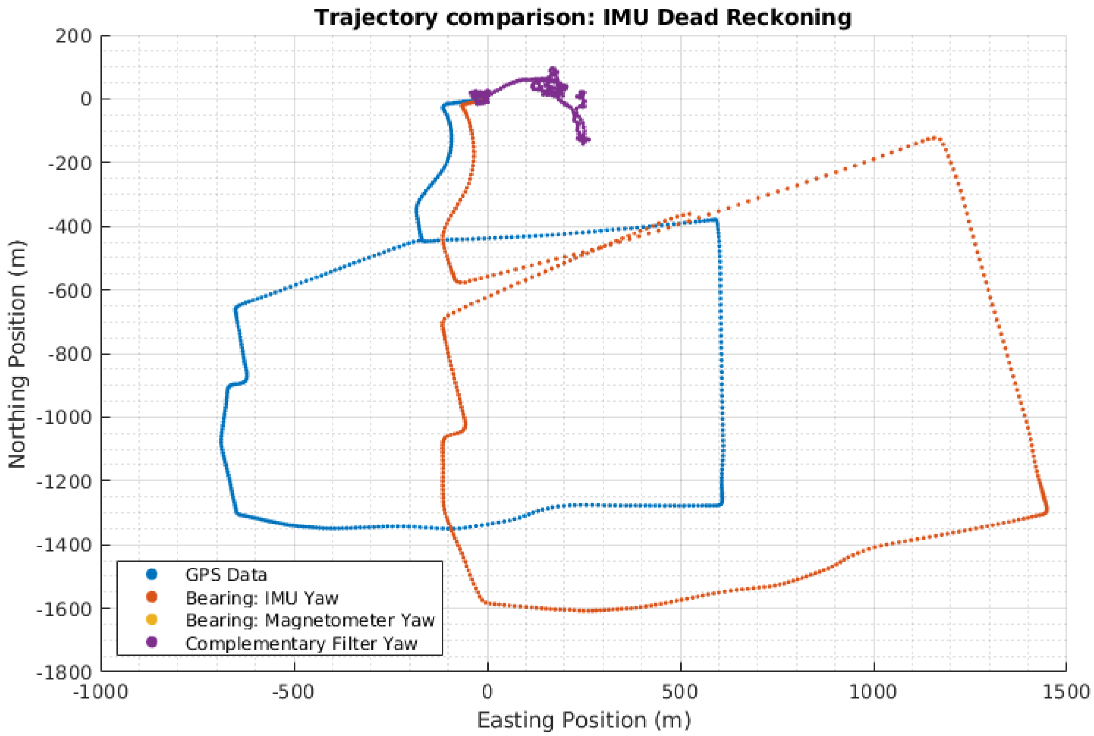

Figure 7. Comparison of dead reckoning performance using the IMU

Part C2) Comparing Lateral Acceleration values

Integrating α once provides the linear velocity. The product of Yaw and linear velocity is plotted in Fig. 8 alongside that of α_y.

It is observed that the observed lateral acceleration closely follows the estimated lateral acceleration, with some additive noise effects distorting the estimated values. This validates the assumption that the body centric lateral acceleration is $\ddot{Y} \approx 0$ when the assumption $x_c \approx 0$ holds to a reasonable extent

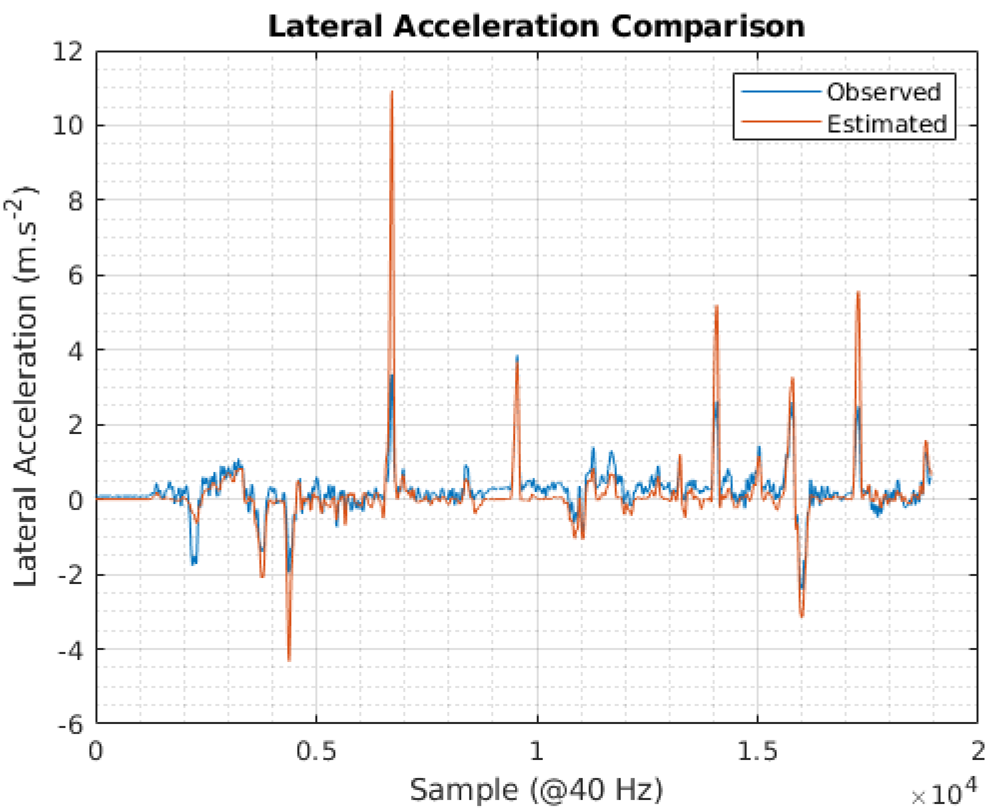

Figure 8. Comparison of lateral acceleration values

Part C3) Estimating IMU offset distance from body Centric origin

The equations defining the observed components of the acceleration for the IMU are:

$$\ddot{x}_{obs} = \ddot{X} - \omega\dot{Y} - \omega^2 x_c \tag{1}$$

$$\ddot{y}_{obs} = \ddot{Y} + \omega\dot{X} + \dot{\omega}x_c \tag{2}$$

To estimate x_c, we assume that for forward acceleration, x_c is 0 (since the observed acceleration in the forward direction is what should ideally be picked by the IMU), and $\dot{Y}$ =0 (assuming no lateral slip in the body centric frame) which results in $\ddot{X} = \ddot{x}\_obs$

Using this above result and Eqn. (2), where $\ddot{Y} = 0$ since $\dot{Y} = 0$ (no slippage in the body frame) we obtain:

$$x_c = \frac{(\ddot{y}_{obs} - \ddot{Y} - \omega\dot{X})}{\dot{\omega}} = \frac{(\ddot{y}_{obs} - \omega\dot{X})}{\dot{\omega}} \tag{3}$$

From the use of Eq. (3) over the entire time series and discarding NaN values for $\dot{\omega} = 0$, we find $x_{c=}\ -0.2965\text{m}$

CONCLUDING REMARKS

As expected, the dead reckoning data is scaled differently with variations in the observed yaw angle causing it to drift with each turn taken. The unremoved acceleration bias results in a quadratic error accumulation which can be seen growing larger in each segment. Overall, the trajectory's nature from dead reckoning closely resembles that of the GPS trajectory, but due to the unreliability of acceleration bias estimation and errors in yaw angle data, IMU based dead reckoning is generally unscalable and not recommended.

REFERENCES

[1] Colton, Shane; The Balance Filter (2007)

[2] https://robotics.stackexchange.com/questions/1717/how-to-determine-the-parameter-of-a-complementary-filter