

Instructions:

Name your script `hw7.py` and submit it on CCLE. Add comments to each function.

Problem 1:

The Pearson correlation coefficient, r , is a commonly used measure of linear correlation between two variables \vec{x} and \vec{y} , taking the forms of two sample datasets

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \text{ and } \vec{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}. \text{ The usual definition does not use linear algebra and looks like:}$$

$$r = \frac{\sum_{i=1}^n (x_i - \mu_x)(y_i - \mu_y)}{\sqrt{\sum_{i=1}^n (x_i - \mu_x)^2} \sqrt{\sum_{i=1}^n (y_i - \mu_y)^2}}.$$

If we “centralize” the vectors \vec{x} and \vec{y} around the means, or in other words, consider the

$$\text{deviation vectors } \vec{x}_c = \begin{pmatrix} x_1 - \mu_x \\ x_2 - \mu_x \\ \vdots \\ x_n - \mu_x \end{pmatrix} \text{ and } \vec{y}_c = \begin{pmatrix} y_1 - \mu_y \\ y_2 - \mu_y \\ \vdots \\ y_n - \mu_y \end{pmatrix} \text{ then we have}$$

$$r = \cos(\theta) = \frac{\vec{x}_c \cdot \vec{y}_c}{\|\vec{x}_c\|_2 \times \|\vec{y}_c\|_2}.$$

This corresponds to thinking of the centralized datasets as two vectors in n -dimensional space, where each dimension corresponds to an observation. These two vectors are correlated if they point in the same direction, i.e. if a positive deviation from the mean of one vector in some direction results in a positive deviation of the other vector. (And same for negative deviations.) The two variables are anti-correlated if they point in opposite directions. Remember that the cosine of an angle is 1 when the angle is 0, and -1 when the angle is 180° .

Write a function named `corrcoeff(x,y)` that takes two vectors of length `n` as input (representing two variables at `n` observations), and outputs the correlation coefficient between them.

Test case:

```
x = np.array([3,4,6,1,2,3])
y = np.array([2,3,1,4,1,2])
corrcoeff(x,y) should return -0.51318
```

Problem 2:

Use integration in SymPy to write a function named `normalcurve(a,b)` that takes as input two boundaries a , b , and returns the probability that the a standard normal random variable falls in the interval between a and b .

Problem 3:

Write a function named `balance(eq)` that balances chemical equations. So, it takes as input strings of the form "H2+O2=H2O" into "2H2+O2=2H2O". This function does not need to account for parenthesis like Pb(OH)4 or Pb(SO4)2.