Optimization

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Optimization

Problems in finance and economics are often concerned with the behavior of agents who are considered to be utility maximizers. Utility functions are thought to be monotonic in their variables—more of a utility enhancing variable is better than less—and are thought to obey diminishing returns as the variables scale. Additionally, real-life constraints ensure that variables are bounded. Hence optimization of functions under constraints forms an important discipline in the study of such subjects.

After linear optimization, quadratic optimization problems are the simplest since they can be captured in *quadratic forms* and can be represented via symmetric matrices with special properties. A distinguishing feature of quadratic optimization is the presence of first and second order conditions that characterize the nature of the extreme point.

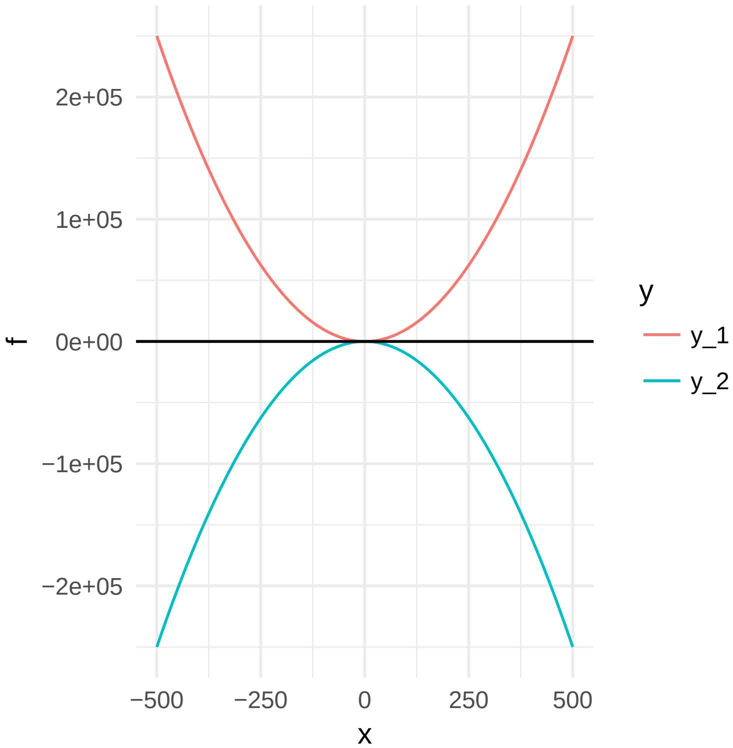
Quadratic Optimization

In one dimension, $x \in \mathbb{R}$ the simplest quadratic objective functions can be $f(x) = \{x^2, -x^2\}$ with the first attaining a global minimum and the second a global maximum at $x = 0$.

```
x <- -500:500
y_1 <- x^2
y_2 <- -x^2
data_l <- cbind(x, y_1, y_2) %>%
  dplyr::as_tibble() %>% #wide format
  tidyr::gather(.,
                y_1:y_2,
```

```r
                 key = 'y',
                 value = 'f') #long format

ggplot(data_l, aes(x, f, color = y)) +
  geom_line() +
  geom_hline(yintercept = 0) +
  theme_minimal()
```



Quadratic Forms

For the case when $x = (x_1, x_2) \in \mathbb{R}^2$, a general quadratic form is the following:

$$Q(x) = a_1 x_1^2 + a_2 x_2^2 + 2a_{12} x_1 x_2$$

which may be represented as a matrix:

$$Q(x) = [x_1, x_2] \begin{bmatrix} a_1 & a_{12} \\ a_{12} & a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$