# Financial Time Series

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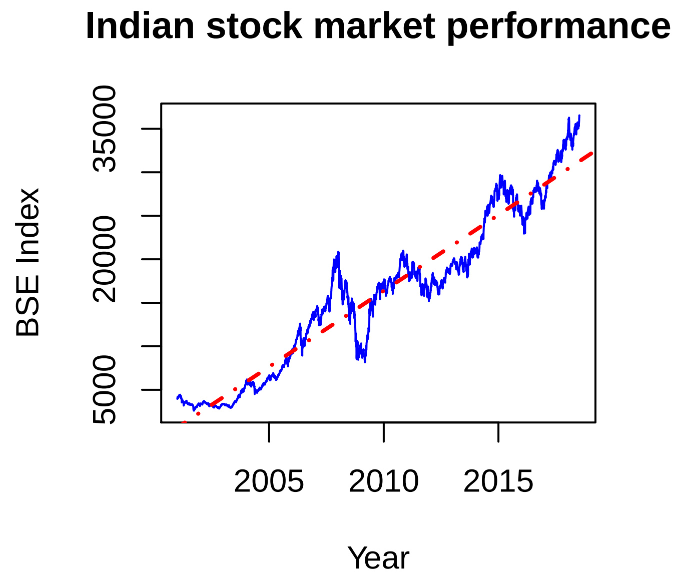
## Background

Financial prices, indices, returns etc. are sequences of real numbers indexed by time. The study of their mathematical and statistical properties is vital for those aspiring to write papers in empirical finance.

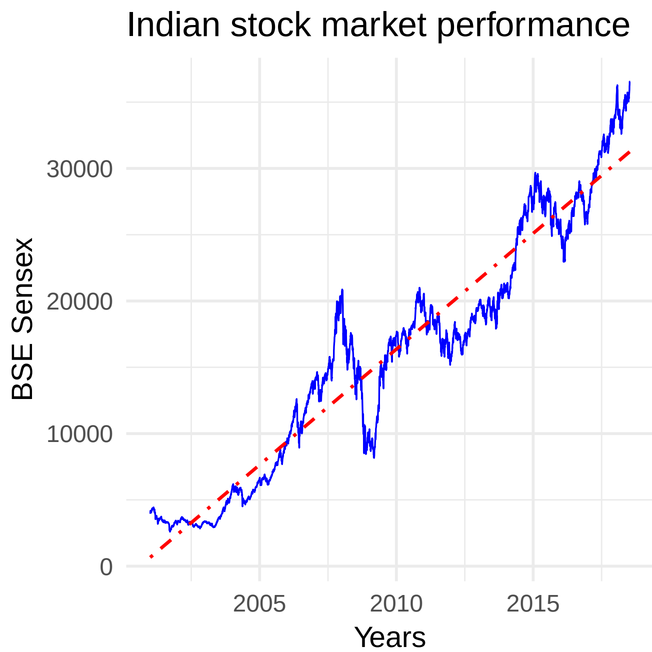
As an illustration we produce the daily time series for the closing value of the Bombay Stock Exchange index ("Sensex").

```r
file_bse <- "SENSEX.csv"
index_bse <- readr::read_csv(file_bse)
index_bse$Date <- as.Date(index_bse$Date,
                          format = "%d-%B-%Y"
                          )


plot(index_bse$Date,
     index_bse$Close,
     type = "l",
     col = "blue",
     xlab = "Year",
     ylab = "BSE Index",
     main = "Indian stock market performance"
     )
fit_lm <- lm(Close ~ Date,
             data = index_bse) #fit linear model
abline(fit_lm, #plot linear model line
       lty = "dotdash",
       col = "red",
```

```
       lwd = 2
       )
```




```
# via ggplot

ggplot(data = index_bse,
       aes(Date, Close)
       ) +
  geom_line(lwd = 0.3,
            color = "blue"
            ) +
  geom_smooth(method = "lm",
              lty = "dotdash",
              lwd = 0.6,
              color = "red",
              se = F) +
  theme_minimal() +
  labs(x = "Years",
       y = "BSE Sensex",
       title = "Indian stock market performance"
       )
```

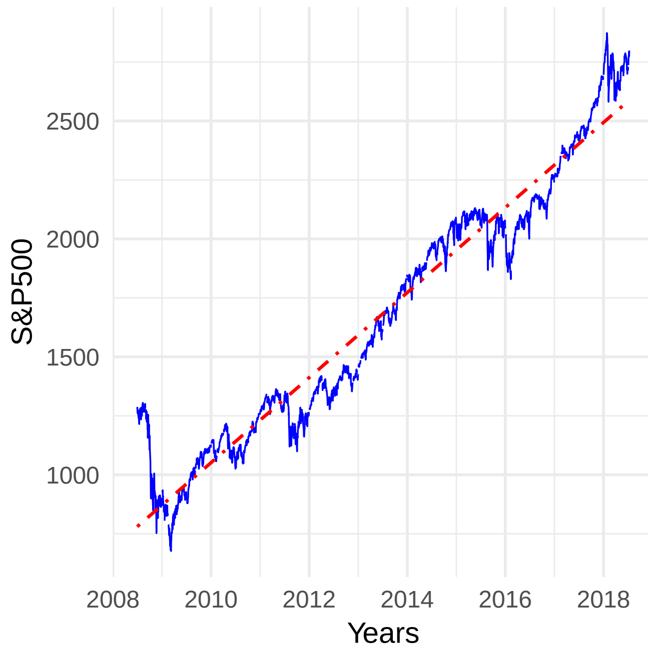



It seems that the level of the series is rising and the fluctuations are sometimes high and sometimes low.

This index series is an example of a *non-stationary* time series. This roughly means that the mean and the variance of such a series are functions of time.

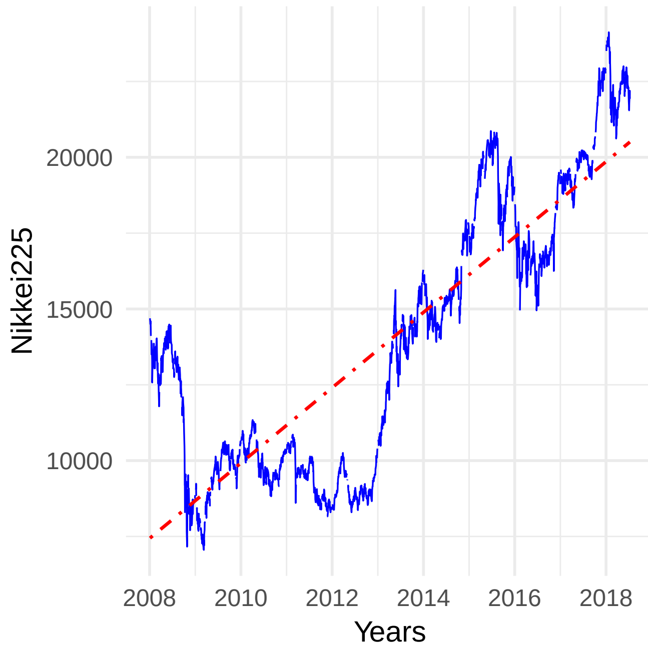
```
file_sp500 <- "SP500.csv" #S&P 500
ind_sp500 <- readr::read_csv(file_sp500,
                             col_types = cols(DATE = col_date(),
                                              SP500 = col_double())
                             )


ggplot(ind_sp500,
       aes(DATE, SP500)
       ) +
  geom_line(lwd = 0.3,
            color = "blue") +
  geom_smooth(method = "lm",
              lty = "dotdash",
              lwd = 0.6,
```

```
                se = F,
                color = "red"
                ) +
  theme_minimal() +
  labs(x = "Years",
       y = "S&P500"
       )
```




```
file_nikkei <- "NIKKEI225.csv" #Nikkei 225
ind_nikkei <- readr::read_csv(file_nikkei,
                              col_types = cols(DATE = col_date(),
                                               NIKKEI225 = col_double())
                              )

ggplot(ind_nikkei,
       aes(DATE, NIKKEI225)
       ) +
  geom_line(lwd = 0.3,
            color = "blue") +
```

```r
  geom_smooth(method = "lm",
              lty = "dotdash",
              lwd = 0.6,
              se = F,
              color = "red"
              ) +
  theme_minimal() +
  labs(x = "Years",
       y = "Nikkei225"
       )
```




## Returns

We observe prices in the financial markets empirically. However, due to their non-stationary nature, they are hard to analyze. Hence they are converted to return series which are usually stationary. There are many ways to construct different notions of returns from the same underlying price sequence. We discuss some prominent ones below.

## One-Period Simple Return

The simple one period return for holding some asset whose price is given by the sequence $\{p_t\}_{t=1}^n$ is:

$$r_t := \frac{p_t - p_{t-1}}{p_{t-1}} = \frac{p_t}{p_{t-1}} - 1$$

## Multi-period Simple Return

$$r_t[k] := \frac{p_t - p_{t-k}}{p_{t-k}} = \frac{p_t}{p_{t-k}} - 1$$
$$r_t[k] := \frac{p_t}{p_{t-k}} - 1 = \frac{p_t}{p_{t-1}} \frac{p_{t-1}}{p_{t-2}} \dots \frac{p_{t-k+1}}{p_t - k} - 1$$
$$r_t[k] := (1 + r_t)(1 + r_{t-1}) \dots (1 + r_{t-k+1}) - 1$$

Multi-period returns are used to convert high frequency returns to low frequency returns—i.e., daily to monthly; or monthly to yearly etc.

For example to convert monthly returns to annual returns:

$$r_t[12] = [\prod_{j=0}^{12-1} (1 + r_{t-j})]^{1/12} - 1$$

Often (when working with daily returns especially) $1 + r_t \approx r_t$, in which case:

$$r_t[k] \approx \sum_{j=0}^{k-1} r_{t-j}$$

Additionally, the simple returns for a portfolio with fractional weights $w_1, \dots, w_n$ are:

$$r_{p,t} = \sum_{i=1}^{n} w_i r_{i,t}$$

### Log Returns

We know that to compute yearly returns from say, monthly returns we use the following formula:

$$r_t[12] = [\prod_{j=0}^{12-1}(1 + r_{t-j})]^{1/12} - 1$$

In general, if a bank pays an annual interest of $r_t^m$ $m$ times a year, the interest rate for unit investment is $r_t^m/m$ and after one year the value of the deposit is $(1 + \frac{r_t^m}{m})^m$. If there is continous compounding, $m \to \infty$, in which case the value of the investment becomes:

$$\lim_{m\to\infty}(1 + \frac{r_t^m}{m})^m = e^{r_t}$$

Hence it must be that:

$$R_t = \log p_t - \log p_{t-1} = \log \frac{p_t}{p_{t-1}}$$

A particular advantage of log-returns are that multiperiod log returns are merely the sum of one period log returns:

$$R_t[k] = \log p_t - \log p_{t-k} = \log p_t - \log p_{t-1} + \log p_{t-1} - \ldots \log p_{t-k}$$
$$R_t[k] = R_t[1] + R_t[2] + \ldots + R_t[k-1]$$

### Computational Examples

To construct return series from prices, we write a function that accepts as input a price (or index) sequence and returns a sequence of simple one period returns by using the formula $r_t = \frac{p_t - p_{t-1}}{p_{t-1}}$. Since there is no return for the first entry of the price sequence, we append an `NA` at the beginning of the returns.

```
func_pr_to_ret <- function(price_vec)
{
  # This function takes a vector of prices and
  # returns a vector of simple one period returns
```

```
  l <- length(price_vec)
  ret_num <- diff(price_vec) #numerator = change in prices
  ret_den <- price_vec[-l] #denominator = price series

  return(c(NA, ret_num/ret_den)) #first return is NA
}

price_vec_1 <- seq(from = 1, to = 10, by = 2)
func_pr_to_ret(price_vec_1)
```

```
## [1]        NA 2.0000000 0.6666667 0.4000000 0.2857143
```

Similarly log-returns can be calculated using the formuala $R_t = \log(p_t) - \log(p_{t-1})$.

```
func_pr_to_logret <- function(p_vec)
{
  # This function takes a vector of prices and
  # returns a vector of log returns
  log_price <- log(p_vec)
  log_ret <- diff(log_price) #log ret = delta log p

  return(c(NA, log_ret)) #first return is NA
}

p_vec <- seq(from = 1, to = 10, by = 2)
func_pr_to_logret(p_vec)
```

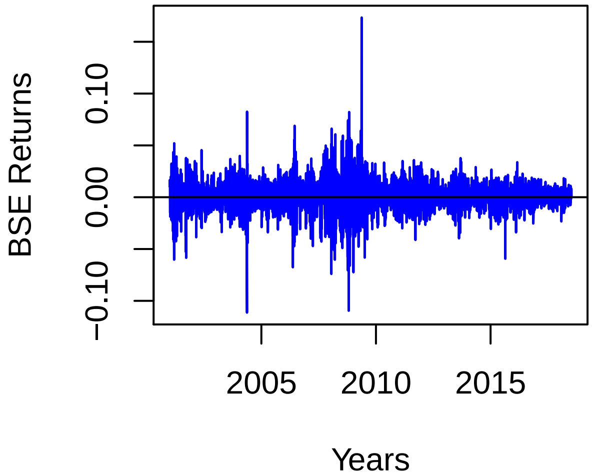
```
## [1]        NA 1.0986123 0.5108256 0.3364722 0.2513144
```

### Return Series for Market Indices

We can compute returns and log-returns for the Bombay Stock Exchange Index, S&P500 and the Nikkei 225 as follows:

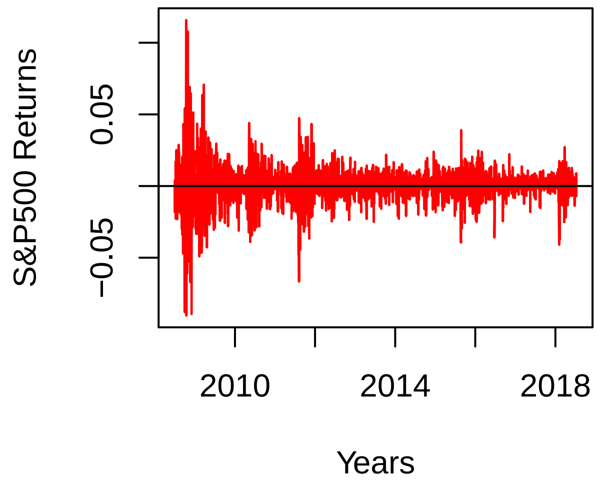
```
ret_BSE <- func_pr_to_ret(index_bse$Close)
plot(index_bse$Date, #x variable
```

```
      ret_BSE, #y variable
      type = "l",
      lwd = 1.2,
      col = "blue",
      xlab = "Years",
      ylab = "BSE Returns"
      )
abline(h = 0)
```

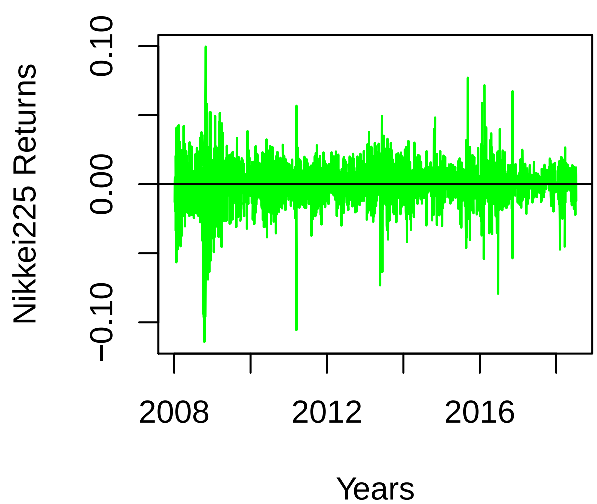



We can do the same for the S&P500 and the Nikkei225 series:

```
ret_sp500 <- func_pr_to_ret(ind_sp500$SP500)
plot(ind_sp500$DATE, #x variable
      ret_sp500, #y variable
      type = "l",
      lwd = 1.2,
      col = "red",
      xlab = "Years",
      ylab = "S&P500 Returns"
      )
abline(h = 0)
```

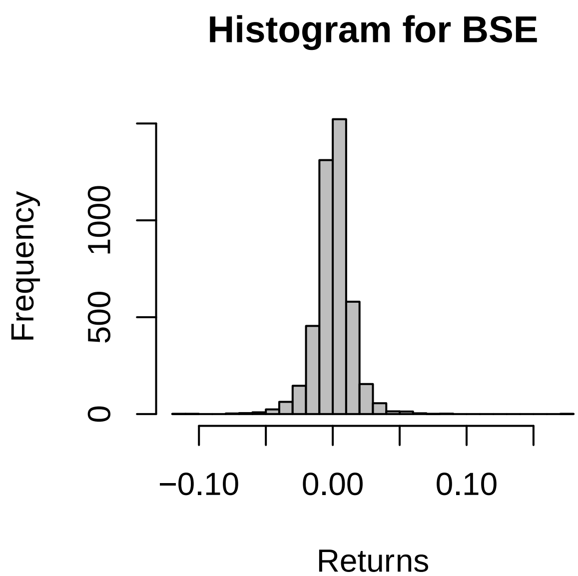



```
ret_nikkei <- func_pr_to_ret(ind_nikkei$NIKKEI225)
plot(ind_nikkei$DATE, #x variable
     ret_nikkei, #y variable
     type = "l",
     lwd = 1.2,
     col = "green",
     xlab = "Years",
     ylab = "Nikkei225 Returns"
     )
abline(h = 0)
```

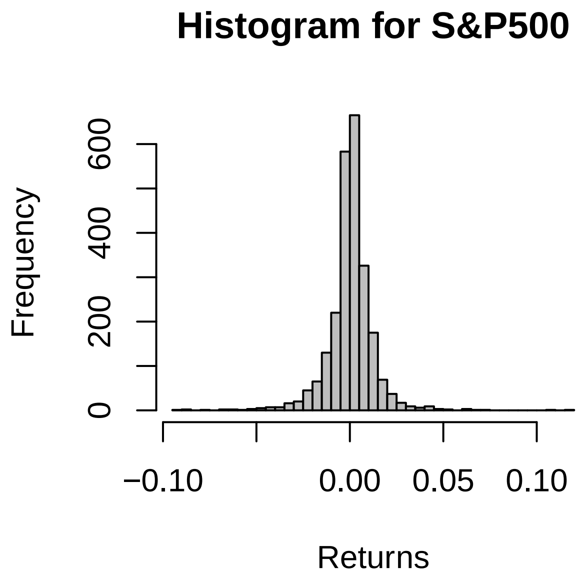


### Histograms for returns for each index

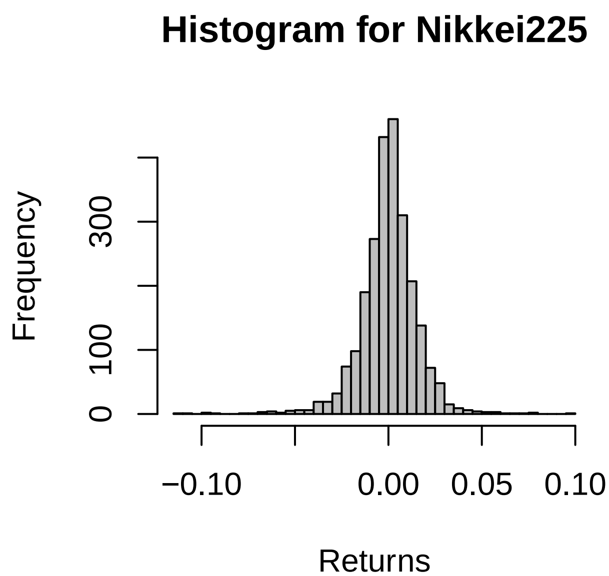
```r
hist(ret_BSE,
     breaks = 40,
     col = "grey",
     xlab = "Returns",
     ylab = "Frequency",
     main = "Histogram for BSE"
     )
```




```r
hist(ret_sp500,
     breaks = 40,
     col = "grey",
     xlab = "Returns",
     ylab = "Frequency",
     main = "Histogram for S&P500"
     )
```




```r
hist(ret_nikkei,
     breaks = 40,
     col = "grey",
     xlab = "Returns",
     ylab = "Frequency",
     main = "Histogram for Nikkei225"
     )
```



## Stylized Facts

Stock prices, commodity prices, exchange rates etc. in empirical financial markets display many striking regularities discussed in Cont (2001).

1. **Fat Tails:** Unconditional return distributions have tails fatter than those of normal distribution. Conditional return distributions are also non-normal.
2. **Asymmetry:** Unconditional return distributions are negatively skewed.
3. **Aggregated Normality:** Lower frequency returns resemble normal distributions more than higher frequency returns.
4. **No Autocorrelation:** Except at high frequencies, returns generally do not display autocorrelation.
5. **Volatility Clustering:** Return volatility is autocorrelated.
6. **Time-Varying Cross Correlation:** Correlation between assets returns tends to be higher during high volatility periods especially during market crashes.

### Summary Statistics of Empirical Returns

min, max, mean, median, mode, std, variance, iqr, skewness, kurtosis

#### Aggregated Time Series Summary Stats

## References

Cont, Rama. 2001. "Empirical Properties of Asset Returns: Stylized Facts and Statistical Issues." *Quantitative Finance* 1 (2): 223–36.

Jondeau, Eric, Ser-Huang Poon, and Michael Rockinger. 2007. *Financial Modeling Under Non-Gaussian Distributions.* Springer Finance.

Tsay, Ruey S. 2010. *Analysis of Financial Time Series.* Third Edition. John Wiley; Sons.