

# Financial Time Series

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## Background

Financial prices, indices, returns etc. are sequences of real numbers indexed by time. The study of their mathematical and statistical properties is vital for those aspiring to write papers in empirical finance.

As an illustration we produce the daily time series for the closing value of the Bombay Stock Exchange index (“Sensex”).

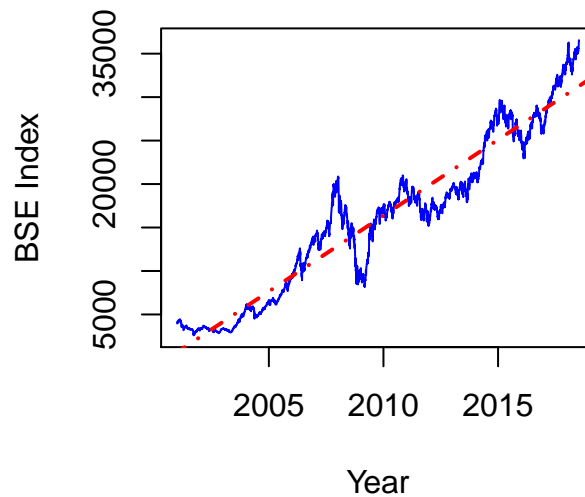
```
file_bse <- "SENSEX.csv"
index_bse <- readr::read_csv(file_bse)
index_bse$Date <- as.Date(index_bse$Date,
                           format = "%d-%B-%Y"
                           )

plot(index_bse$Date,
     index_bse$Close,
     type = "l",
     col = "blue",
     xlab = "Year",
     ylab = "BSE Index",
     main = "Indian stock market performance"
)

fit_lm <- lm(Close ~ Date,
             data = index_bse) #fit linear model
abline(fit_lm, #plot linear model line
       lty = "dotdash",
       col = "red",
```

```
lwd = 2  
)
```

## Indian stock market performance



```
# via ggplot  
  
ggplot(data = index_bse,  
       aes(Date, Close)  
       ) +  
  geom_line(lwd = 0.3,  
            color = "blue"  
            ) +  
  geom_smooth(method = "lm",  
              lty = "dotdash",  
              lwd = 0.6,  
              color = "red",  
              se = F) +  
  theme_minimal() +  
  labs(x = "Years",  
       y = "BSE Sensex",  
       title = "Indian stock market performance"  
       )
```



It seems that the level of the series is rising and the fluctuations are sometimes high and sometimes low.

This index series is an example of a *non-stationary* time series. This roughly means that the mean and the variance of such a series are functions of time.

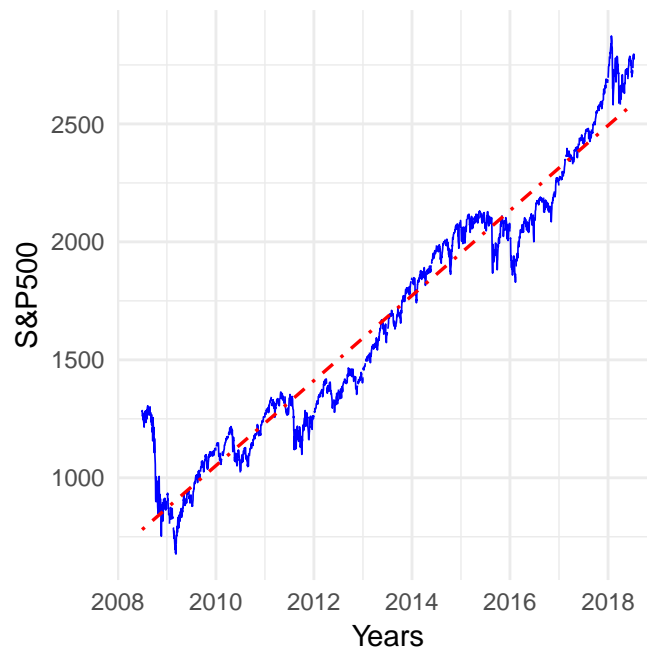
```
file_sp500 <- "SP500.csv"
ind_sp500 <- readr::read_csv(file_sp500,
                             col_types = cols(Date = col_date(),
                                                SP500 = col_double())
                             )

ggplot(ind_sp500,
       aes(Date, SP500))
  ) +
  geom_line(lwd = 0.3,
            color = "blue") +
  geom_smooth(method = "lm",
             lty = "dotdash",
             lwd = 0.6,
```

```

        se = F,
        color = "red"
    ) +
  theme_minimal() +
  labs(x = "Years",
       y = "S&P500"
  )

```



```

file_nikkei <- "NIKKEI225.csv"
ind_nikkei <- readr::read_csv(file_nikkei,
                              col_types = cols(Date = col_date(),
                                                NIKKEI225 = col_double())
                              )

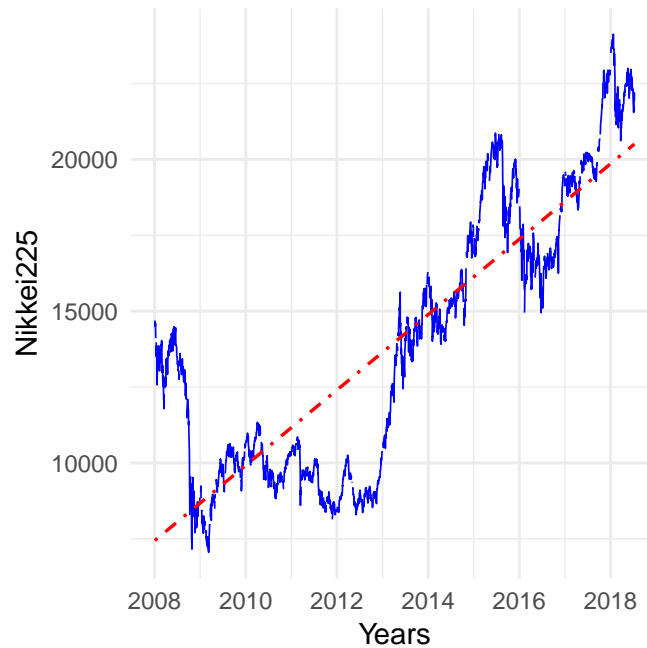
ggplot(ind_nikkei,
       aes(Date, NIKKEI225))
  ) +
  geom_line(lwd = 0.3,
            color = "blue") +

```

```

geom_smooth(method = "lm",
            lty = "dotdash",
            lwd = 0.6,
            se = F,
            color = "red"
            ) +
theme_minimal() +
labs(x = "Years",
     y = "Nikkei225"
     )

```



## Returns

We observe prices in the financial markets empirically. However, due to their non-stationary nature, they are hard to analyze. Hence they are converted to return series which are usually stationary. There are many ways to construct different notions of returns from the same underlying price sequence. We discuss some prominent ones below.

## One-Period Simple Return

The simple one period return for holding some asset whose price is given by the sequence  $\{p_t\}_{t=1}^n$  is:

$$r_t := \frac{p_t - p_{t-1}}{p_{t-1}} = \frac{p_t}{p_{t-1}} - 1$$

## Multi-period Simple Return

$$\begin{aligned} r_t[k] &:= \frac{p_t - p_{t-k}}{p_{t-k}} = \frac{p_t}{p_{t-k}} - 1 \\ r_t[k] &:= \frac{p_t}{p_{t-k}} - 1 = \frac{p_t}{p_{t-1}} \frac{p_{t-1}}{p_{t-2}} \dots \frac{p_{t-k+1}}{p_{t-k}} - 1 \\ r_t[k] &:= (1 + r_t)(1 + r_{t-1}) \dots (1 + r_{t-k+1}) - 1 \end{aligned}$$

Multi-period returns are used to convert high frequency returns to low frequency returns—i.e., daily to monthly; or monthly to yearly etc.

For example to convert monthly returns to annual returns:

$$r_t[12] = \left[ \prod_{j=0}^{12-1} (1 + r_{t-j}) \right]^{1/12} - 1$$

Often (when working with daily returns especially)  $1 + r_t \approx e^{r_t}$ , in which case:

$$r_t[k] \approx \sum_{j=0}^{k-1} r_{t-j}$$

Additionally, the simple returns for a portfolio with fractional weights  $w_1, \dots, w_n$  are:

$$r_{p,t} = \sum_{i=1}^n w_i r_{i,t}$$

## Log Returns

We know that to compute yearly returns from say, monthly returns we use the following formula:

$$r_t[12] = \left[ \prod_{j=0}^{12-1} (1 + r_{t-j}) \right]^{1/12} - 1$$

In general, if a bank pays an annual interest of  $r_t^m$   $m$  times a year, the interest rate for unit investment is  $r_t^m/m$  and after one year the value of the deposit is  $(1 + \frac{r_t^m}{m})^m$ . If there is continuous compounding,  $m \rightarrow \infty$ , in which case the value of the investment becomes:

$$\lim_{m \rightarrow \infty} (1 + \frac{r_t^m}{m})^m = e^{r_t}$$

Hence it must be that:

$$R_t = \log p_t - \log p_{t-1} = \log \frac{p_t}{p_{t-1}}$$

A particular advantage of log-returns are that multiperiod log returns are merely the sum of one period log returns:

$$R_t[k] = \log p_t - \log p_{t-k} = \log p_t - \log p_{t-1} + \log p_{t-1} - \dots - \log p_{t-k}$$

$$R_t[k] = R_t[1] + R_t[2] + \dots + R_t[k-1]$$

## Stylized Facts

Stock prices, commodity prices, exchange rates etc. in empirical financial markets display many striking regularities discussed in Cont (2001).

1. **Fat Tails:** Unconditional return distributions have tails fatter than those of normal distribution. Conditional return distributions are also non-normal.
2. **Asymmetry:** Unconditional return distributions are negatively skewed.
3. **Aggregated Normality:** Lower frequency returns resemble normal distributions more than higher frequency returns.

4. **No Autocorrelation:** Except at high frequencies, returns generally do not display autocorrelation.
5. **Volatility Clustering:** Return volatility is autocorrelated.
6. **Time-Varying Cross Correlation:** Correlation between assets returns tends to be higher during high volatility periods especially during market crashes.

## References

- Cont, Rama. 2001. “Empirical Properties of Asset Returns: Stylized Facts and Statistical Issues.” *Quantitative Finance* 1 (2): 223–36.
- Jondeau, Eric, Ser-Huang Poon, and Michael Rockinger. 2007. *Financial Modeling Under Non-Gaussian Distributions*. Springer Finance.
- Tsay, Ruey S. 2010. *Analysis of Financial Time Series*. Third Edition. John Wiley; Sons.