Linear Algebra

① Determinants and matrices, in linear algebra, are used to solve linear equations by applying Cramer's rule to a set of non-homogeneous equations which are in linear form. Determinants are calculated for square matrices only.

②

Matrix	Determinant
• Matrix is representation of number in row & column format.	• Determinant is number associated with a matrix
• Matrix can be any order	• Determinant is only possible for a square matrix.
• Scalar multiplied to matrix If a number is multiplied to matrix, it is multiplied to each element of the matrix.	• Scalar multiplied to determinant If a number is multiplied to determinant It is multiplied to either one row, or one column.

③ Calculate $3A + 0.5B$

$$= 3\begin{bmatrix} 0 & 2 & 4 \\ 6 & 5 & 5 \\ 1 & 0 & -3 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 5 & 2 \\ 5 & 3 & 4 \\ -2 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 6 & 12 \\ 18 & 15 & 15 \\ 3 & 0 & -9 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & 1 \\ \frac{5}{2} & \frac{3}{2} & 2 \\ -1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 0+0 & 6+\frac{5}{2} & 12+1 \\ 18+\frac{5}{2} & 15+\frac{3}{2} & 15+2 \\ 3-1 & 0+2 & -9-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 17/2 & 13 \\ 41/2 & 33/2 & 17 \\ 2 & 2 & -11 \end{bmatrix}$$

$\frac{12+5}{2} = \frac{17}{2}$

$\frac{36+5}{2} = \frac{41}{2}$

$30\ \frac{33}{2}$

Maturity is
when you realise that
Money can actually buy happiness

$[\,\vdots\,] \cdot [\,\vdots\,]$

$\boxed{A_{2\times 3} = A^T_{3\times 2}}$

(4) Calculate the following expression:

$$A = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix},\ B = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix},\ C = \begin{bmatrix} 0 & 1 \\ 3 & 2 \\ -2 & 0 \end{bmatrix},\ a = \begin{bmatrix} 1 & -2 & 0 \end{bmatrix},\ b = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$$

* AB =

$$AB = \begin{bmatrix} 4+6+0 & -12-2+0 & 0+0-6 \\ -2-3+0 & 6+1+0 & 0+0-12 \\ 1-6+0 & -3+2+0 & 0+0-4 \end{bmatrix} = \begin{bmatrix} 10 & -14 & -6 \\ -5 & 7 & -12 \\ -5 & -1 & -4 \end{bmatrix}$$

$$* \quad Aa^T = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4+0 \\ -2-2+0 \\ 1-4+0 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ -3 \end{bmatrix}$$

(5) Example of a system of 3 Homogeneous linear equations in 4 unknown.

$$\begin{aligned} w + 2x - y + z &= 0 \\ 2w + 3x + 2y - z &= 0 \\ 3w + x + 4y + 3z &= 0 \end{aligned}$$

A system of m homogeneous linear eq^n in 'n' unknown $(x_1, x_2, x_3, \ldots x_n)$ is respented

Condition for consistency Non-Homogeneous.

Case-I If $R(A) = R(A:B) = n$ (no. of unknowns) then, the system of linear eq^n has a unique solution.

Case-II If $R(A) = R(A:B) < n$ then the system of linear eq^n has many solution.

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 3 & 2 & -1 \\ 3 & 1 & 4 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$$

5 6

7 $\begin{bmatrix} \lambda & 1 & 2 \\ 0 & -1 & 5 \\ 2 & 0 & 1 \end{bmatrix}$

$$= \lambda \begin{vmatrix} -1 & 5 \\ 0 & 1 \end{vmatrix} - 1 \begin{vmatrix} 0 & 5 \\ 2 & 1 \end{vmatrix} + 2 \begin{vmatrix} 0 & -1 \\ 2 & 0 \end{vmatrix}$$

$= \lambda(-1-0) - 1(0-10) + 2(0+2) = 0$

$= -\lambda + 10 + 4 = 0$

$= \cancel{-}\lambda = \cancel{-}14$

8 $A = \begin{bmatrix} -1 & 0 & 3 & 0 \\ 1 & 3 & 2 & 9 \\ -9 & 7 & -5 & 7 \end{bmatrix} \quad R_3 \to 4R_3$

$= \begin{bmatrix} -1 & 0 & 3 & 0 \\ 1 & 3 & 2 & 9 \\ -36 & 28 & -20 & 28 \end{bmatrix} \quad R_1 \to 14R_1 + R_2$

$= \begin{bmatrix} -13 & 3 & 44 & 9 \\ 1 & 3 & 2 & 9 \\ -36 & 28 & -20 & 28 \end{bmatrix}$ ~~intercha~~ $R_3 \leftrightarrow R_2$ $\begin{bmatrix} -13 & 3 & 44 & 9 \\ -36 & 28 & -20 & 28 \\ 1 & 3 & 2 & 9 \end{bmatrix}$

(9)

$$A = \begin{bmatrix} \lambda & 1 & 2 \\ 0 & -1 & 5 \\ 2 & 0 & 1 \end{bmatrix}$$

$r(A) = 2 <$ no. of unknowns

$\Rightarrow |A| = 0$

$\Rightarrow \lambda(-1-0) - 1(0-10) + 2(0+2) = 0$

$\Rightarrow -\lambda + 10 + 4 = 0$

$\Rightarrow \cancel{-}\lambda = \cancel{-}14$

(10) (10)

(11)

$$A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix}$$

$r(A) = 2 <$ no. of unknown

$\Rightarrow |A| = 0$

$\Rightarrow 1(30-26) - 5(0-2b) + 4(0-3b) = 0$

$\Rightarrow 4 + 10b - 12b = 0$

$\Rightarrow 4 + 2b = 0$

$\Rightarrow 2b = -4 \Rightarrow b = -2.$

(12)

$$A = \begin{bmatrix} 0 & -4 & 1 \\ 2 & \lambda & -3 \\ 1 & 2 & -1 \end{bmatrix}$$

$|A| = 0(\cancel{4}-\lambda+6) + 4(-2+3) + 1(4-\lambda) = 0$

$= \cancel{(4+4-\lambda = 0)}\ 8-\lambda$

$= \cancel{8}\ 8$

$4 \cdot 2$

$-2+3$

$A_{11} = (-\lambda+6)$ $A_{21} = 2$ $A_{31} = 12-\lambda$

$A_{12} = 1$ $A_{22} = -1$ $A_{32} = -2$

$A_{13} = 4-\lambda$ $A_{23} = 4$ $A_{33} = 4$

12

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj}A$$

$$= \frac{1}{8-\lambda}\begin{bmatrix} (-\lambda+6) & 1 & 4-\lambda \\ 2 & -1 & 4 \\ 12-\lambda & -2 & 4 \end{bmatrix}$$

13

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$A \cdot A^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Ans 13 we know that in an orthogonal matrix

$$|A| = \pm 1$$

$$A^T = A^{-1}$$

Given $(A \cdot A^T)^{-1}$

$$= (A \cdot A^{-1})^{-1}$$

$$= I^{-1} = I$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(14) Find the product of the Matrices $(PQ)^{-1}P$

→ Properties of matrices:

$(PQ)^{-1} = B^{-1}A^{-1}$ -- (1)

$PP^{-1} = P^{-1}P = I$ -- (2)

where I is identity matrix

$(PQ)^{-1}P$

using (1)

$P^{-1} \cdot Q^{-1} \cdot P$

using (2)

$Q^{-1}(P^{-1} \cdot P)$

$Q^{-1}I = Q^{-1}$

(15)

$$A = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} \text{ and } B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1n} \end{bmatrix}$$

be two non-zero column and row matrices respectively

$$\begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{11}b_{13} & \cdots & a_{11}b_{1n} \\ a_{21}b_{21} & a_{21}b_{12} & a_{21}b_{13} & \cdots & a_{21}b_{1n} \\ \vdots & \vdots & \vdots & & \cdots \\ a_{m1}b_{11} & a_{m1}b_{12} & a_{m1}b_{13} & \cdots & a_{m1}b_{1n} \end{bmatrix}$$

Since A,B are non-zero matrixes:

∴ matrix AB will be a non-zero matrix.

∴ rank of AB = 1

(16) Given $A_{5\times3}$ and $(AB)_{5\times7}$

$A = [a_{ij}]_{n\times m}$ ~~and~~

Multiplication of the matrices

$[B]_{p\times n} \times [A]_{n\times m} = [BA]_{p\times m}$

$\Rightarrow [B]_{p\times n} \times [A]_{5\times3} = [BA]_{5\times7}$

The $[B]_{5\times5}$

we know that

Given $[B]_{n\times p} \times [A]_{m\times n} = [BA]_{m\times p}$

$= [B]_{n\times p} \times [A]_{5\times3} = [BA]_{5\times7}$

$= [B]_{3\times7}$.

(17) $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

we know that

$|A\cdot(\text{adj}A)| = |A|^n$

Here n is order of matrix $= 2$

$|A| = 12 - 2 = 10$

Now:- $A\cdot(\text{adj}A) = 10^2 = 100$.

Date ______ $| \oplus = T \quad T \quad = F$

(18) $\det(B^{-1}AB)$

where A is a square matrix and B is non-~~sig~~ singular matrix, so B is invertible and a square matrix.

$\det(B^{-1}AB)$

$\Rightarrow \det(B^{-1}BA)$

$\Rightarrow \det(I_n A)$ (where I is an identity matrix of order n)

$= \det(A)$

(19) (a) 3 Number of non-zero rows = 2

(b) 3 Hence the rank of matrix $A, B, C, D = 3$.

(c) 3

(d) 3

$\frac{3}{6}, \frac{-3}{6}$

$\frac{-0.1}{60} \quad \frac{2}{6}$

(20) Let $A = \begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{1}{2} & a \\ 0 & b \end{bmatrix}$

$A_{12} = a = -\frac{1}{2}$

$A_{22} = b = \frac{1}{3}$

$(a+b) = \frac{7}{20}$

$-\frac{1}{2} + \frac{1}{3}$

$\frac{-3+2}{6} = \frac{-1}{6}$

$\frac{1}{10} + \frac{1}{2}$

20

$A \cdot A^{-1} = I$

$\begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & a \\ 0 & b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 2a - 0.1b \\ 0 & 3b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore 3b = 1$

$b = \frac{1}{3}$

$2a - \frac{0.1}{3} = 0$

$\Rightarrow \frac{6a - 0.1}{3} = 0$

$\Rightarrow 6a = 0.1$

$\Rightarrow a = \frac{0.1}{6}$

$a + b = \frac{7}{20}$

$\Rightarrow \frac{0.1}{6} + \frac{1}{3} = \frac{7}{20}$

$= \frac{0.1 + 2}{6} = \frac{2.1}{6}$

$A \cdot B$ = A
$[2] B = 3$
$\boxed{B = \frac{3}{1}}$

(21)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Non-zero row = 2
Rank of matrix A = 2
$R(A) = 2$.

(22)

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix} \quad R_2 \to R_2 - 3R_1,\ R_3 \to R_3 - R_1$$

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

Non-zero row = 2
Rank of matrix A = 2.

(23) $A \cdot A'$ is a symmetric matrix.

→ let $P = A \cdot A'$

$P' = (A \cdot A')'$

$P = \cancel{B}(A')' A'' \quad [\because (AB)' = \cancel{A} B'A']$

$\cancel{A} = A \cdot A'$

(24) $\cancel{BA \cdot A^{-1} \Rightarrow B}$ $A^{-1} \cdot AB = B$ (24)

(25) Any square matrix A is called involutory if $A^2 = I$

$$A = \begin{bmatrix} 4 & -1 \\ 15 & -4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & -1 \\ 15 & -4 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 15 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 16-15 & -4+4 \\ 60-60 & -15+16 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(26) $(3K-8)x + 3y + 3Z = 0$

$3x + (3K-8)y + 3Z = 0$

$3x + 3y + (3K-8)Z = 0.$

$$A = \begin{bmatrix} 3K-8 & 3 & 3 \\ 3 & 3K-8 & 3 \\ 3 & 3 & 3K-8 \end{bmatrix} R_1 \to R_1 - R_3$$

$$\begin{bmatrix} 3K-11 & 0 & 11-3K \\ 3 & 3K-8 & 3 \\ 3 & 3 & 3K-8 \end{bmatrix} = 0$$

$\Rightarrow (3K-11)\left[(3K-8)^2 - 9\right] + (11-3K)(9-9K+24) = 0$

$\Rightarrow (3K-11)(9K^2 + 55 - 48K) + (11-3K)(33-9K) = 0$

$\Rightarrow 27K^3 + 165K - 144K^2 - 99K^2 - 605 + 528K + 363 - 99K - 99K + 27K^2 \doteq 0$

$= 27K^3 - 216K^2 + 495K$

(24) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} ? \end{bmatrix}$, $A \cdot B = 0$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$a + 2c = 0$ — (1) $\Rightarrow a = -2c$

$b + 2d = 0$ — (2)

$3a + 4c = 0$ — (3)

$3b + 4d = 0$ — (4)

$\cancel{\ }$

$a = -2c$ put in eq^n (3)

$3(-2c) + 4c = 0$

$\Rightarrow -6c + 4c = 0$

$\Rightarrow -2c = 0$

(27) $A = \begin{bmatrix} 0 & 2 & 2 \\ 7 & 4 & 8 \\ -7 & 0 & -4 \end{bmatrix} R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 7 & 4 & 8 \\ 0 & 2 & 2 \\ -7 & 0 & -4 \end{bmatrix} R_3 \to R_3 + R_1$$

$$\begin{bmatrix} 7 & 4 & 8 \\ 0 & 2 & 2 \\ 0 & 4 & 4 \end{bmatrix} R_3 \to R_3 - 2R_2$$

$$\begin{bmatrix} 7 & 4 & 8 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank(A) = 2.

(28) $A = \begin{bmatrix} \cancel{0} & \cancel{2} & \cancel{2} \\ \cancel{7} & \cancel{4} & \cancel{8} \\ \cancel{-7} & \cancel{0} & \cancel{4} \end{bmatrix}$

$A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$

$|A| = 2(x) - 4(3x-2) + 2(-1)$

$= 2x - 12x + 8 - 2$

$\Rightarrow 6 - 10x$

$A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$ $R_1 \leftrightarrow R_3$ $\frac{}{2}$

$\begin{bmatrix} 1 & 0 & x \\ 3 & 1 & 2 \\ 2 & 4 & 2 \end{bmatrix}$ $R_2 \to R_2 - 3R_1$

$\begin{bmatrix} 1 & 0 & x \\ 0 & 1 & 2-3x \\ & & \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$ $R_2 \to R_2 - 3R_1$
$R_3 \to R_3 - R_1$

$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -5 & -1 \\ 0 & -2 & x-1 \end{bmatrix}$

28

2-3

(29) A.T.Q

$C_{3\times3} = AB = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}_{3\times1}$

$[b_1 \; b_2 \; b_3]_{1\times3}$

$C_{3\times3} = \begin{bmatrix} a_1b_1 & a_1b_2 & a_1b_3 \\ a_2b_1 & a_2b_2 & a_2b_3 \\ a_3b_1 & a_3b_2 & a_3b_3 \end{bmatrix}_{3\times3}$

$|C| = a_1b_1(\cancel{a_2b_2a_3b_3 - a_3b_2a_2b_3}) - a_1b_2(\cancel{a_2b_1a_3b_3 - a_3b_1a_2b_3})$
$- a_1b_3(\cancel{a_2b_1a_3b_2 - a_3b_1a_2b_2})$

$|C| = 0$

2x2 1^{st} :- $a_1b_1\, a_2b_2 - a_1b_2\, a_2b_2$
$= 0$

Every matrix = (2x2 group) = 0.

but $D \neq 0$
$\therefore$ Rank of $C > 0$
solving like this
we get
Rank of $C = 1$

(30) $A\begin{bmatrix}1\\-1\end{bmatrix} = -\begin{bmatrix}1\\-1\end{bmatrix}$ & $A\begin{bmatrix}1\\-2\end{bmatrix} = -2\begin{bmatrix}1\\-2\end{bmatrix}$

Doubt

(31) $4x+2y=7$
$2x+y=6$

$$\begin{bmatrix}4 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}7\\6\end{bmatrix}$$

$C = [A:B]$

$$C = \left[\begin{array}{cc:c}4 & 2 & 7\\2 & 1 & 6\end{array}\right]$$

Rank (A) $\neq$ Rank (C) —— No solution.

$$C = \left[\begin{array}{cc:c}4 & 2 & 7\\2 & 1 & 6\end{array}\right]$$

$R_2 \to 2R_2 - R_1$

$$C = \left[\begin{array}{cc:c}4 & 2 & 7\\0 & 0 & 5\end{array}\right]$$

Rank (C) = 2
Rank (A) = 1 . Rank (C) $\neq$ Rank (A)
Thus the system of equations has no solution,

(32)
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 6 \\ 2 & 4 & 5 \end{bmatrix}$$

$A = IA$

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 6 \\ 2 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 3R_1,\ R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -3 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

$-3+6$

$R_1 \to R_1 + 2R_2,\ R_2 \to R_2 - 3R_3$

0-

1-6

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -5 & 2 & 0 \\ 3 & 1 & -3 \\ -2 & 0 & 1 \end{bmatrix} A$$

$-5+6$

$R_1 \to R_1 + 3R_3,\ R_2 \to -R_2,\ R_3 \to -R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 1 & 3 \\ 2 & 0 & -1 \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 1 & 3 \\ 2 & 0 & -1 \end{bmatrix}$$

(33) find the rank

(a)
$$\begin{bmatrix} 4 & -2 & 6 \\ -2 & 1 & -3 \end{bmatrix}$$

$\cancel{R_2 \to \frac{R_1}{4}}$ $R_2 \to 2R_2 + R_1$

$\frac{-1}{4}\ \frac{-1}{2}$

$\frac{6}{4}\ \frac{3}{2}$

$$\cancel{\begin{bmatrix} 1 & -1/4 & 3/2 \\ -2 & 1 & -3 \end{bmatrix}}_R$$

$$\begin{bmatrix} 4 & -2 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank (A) = 1.

(b) $A = \begin{bmatrix} 8 & 0 & 4 & 0 \\ 0 & 2 & 0 & 4 \\ 4 & 0 & 2 & 0 \end{bmatrix}$.

$R_3 \to 2R_3 - R_1$

$\begin{bmatrix} 8 & 0 & 4 & 0 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Rank $(A) = 2$

(c) $\begin{bmatrix} 9 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $R_4 \to R_4 - R_2$

$\begin{bmatrix} 9 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Rank $(A) = 3$.

34 (a) $3x_1 - 2x_2 + x_3 = 6$
$x_1 + 10x_2 - x_3 = 2$
$-3x_1 - 2x_2 + x_3 = 0$

$\begin{bmatrix} 3 & -2 & 1 \\ 1 & 10 & -1 \\ -3 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix}$

$C = [A : B] = \begin{bmatrix} 3 & -2 & 1 & : & 6 \\ 1 & 10 & -1 & : & 2 \\ -3 & -2 & 1 & : & 0 \end{bmatrix}$

$R_3 \to R_3 - R_1$

$C = \begin{bmatrix} 3 & -2 & 1 & : & 6 \\ 1 & 10 & -1 & : & 2 \\ -6 & 0 & 0 & : & -6 \end{bmatrix}$ $R_2 \to 3R_2 - R_1$

$\begin{bmatrix} 3 & -2 & 1 & : & 6 \\ 0 & 28 & -4 & : & 0 \\ -6 & 0 & 0 & : & -6 \end{bmatrix}$

continued.

R(C) = 3   R(C) = R(A) = n
R(A) = 3   unique soln

R(C) = R(A) < 3
infinite soln

$A X = B$ ← constant
A: coefficient, X: unknown

$-3 \quad +2 \quad 1$
$-3 \quad -2 \quad 1$

$x_1 + 10x_2 + x_3 = 2$ — (1)
$-32x_2 - 2x_3 = 16$ — (2)
$28x_2 + 4x_3 = 6$ — (3)

(34) (a) $3x_1 - 2x_2 + x_3 = 6$
$x_1 + 10x_2 + x_3 = 2$
$-3x_1 - 2x_2 + x_3 = 0$

$AX = B$

$$\begin{bmatrix} 3 & -2 & 1 \\ 1 & 10 & 1 \\ -3 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$C = (A:B)$

$$C = \left[\begin{array}{ccc|c} 1 & 10 & 1 & 2 \\ -3 & -2 & 1 & 6 \\ -3 & -2 & 1 & 0 \end{array}\right] \quad R_3 \to R_3 + R_2$$

$C = [A:B]$

$$C = \left[\begin{array}{ccc|c} 3 & -2 & 1 & 6 \\ 1 & 10 & -1 & 2 \\ -3 & 2 & 1 & 0 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$C = \left[\begin{array}{ccc|c} 3 & -2 & -1 & 6 \\ 1 & 10 & -1 & 2 \\ -6 & 0 & 0 & -6 \end{array}\right]$$

$R_2 \to 3R_2 - R_1$

$$C = \left[\begin{array}{ccc|c} 3 & -2 & 1 & 6 \\ 0 & 28 & -4 & 0 \\ -6 & 0 & 0 & -6 \end{array}\right]$$

$R_2 \to 3R_2 - R_1$

$$\left[\begin{array}{cccc} 3 & -2 & 1 & 6 \\ 0 & 32 & 2 & 0 \end{array}\right]$$

$R_3 \to R_3 + R_1$

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & 6 \\ 1 & 10 & 1 & 2 \\ 0 & -4 & 2 & 6 \end{array}\right]$$

$R_2 \to 3R_2 - R_1$

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & 6 \\ 0 & 32 & 2 & 0 \\ 0 & -4 & 2 & 6 \end{array}\right] \quad R_3 \to 8R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & 6 \\ 0 & 32 & 2 & 0 \\ 0 & 0 & 18 & 48 \end{array}\right]$$

Rank(A) = 3
Rank(C) = 3
No. of unknown (n) = 3

$R(A) = R(C) = n$

$3x_1 - 2x_2 + x_3 = 6$ — (1)

$32x_2 + 2x_3 = 0$ — (2)

$18x_3 = 48$ — (3)

from eqn (3)

$$x_3 = \frac{48}{18} = \frac{8}{3}$$

from eqn (2)

$$32x_2 + 2 \times \frac{8}{3} = 0$$

$$32x_2 + \frac{16}{3} = 0$$

$$\Rightarrow \cancel{32x}\ 96x_2 = -16$$

$$\Rightarrow x_2 = \frac{-1}{6}$$

from eqn (1)

$$3x_1 - 2\left(\frac{-1}{6}\right) + \frac{8}{3} = 6$$

$$\Rightarrow 3x_1 + \frac{1}{3} + \frac{8}{3} = 6$$

$$\Rightarrow 9x_1 + 9 = 18$$

$$\Rightarrow 9x_1 = 9$$

$$\Rightarrow x_1 = 1$$

~~(b) $3x_2 - 4x$.~~

(e)

$$x_1 + x_2 + 10x_3 = 0$$

$$x_1 - x_2 + 3x_3 - x_4 + 4x_5 = 0$$

$$2x_1 - 2x_2 + x_3 + x_4 = 0$$

$$x_1 - 2x_3 + x_5 = 0$$

$$x_3 + x_4 - x_5 = 0$$

$$AX = B$$

$$\begin{bmatrix} 1 & 2 & 10 & 0 & 0 \\ 1 & -1 & 3 & -1 & 4 \\ 2 & -2 & 1 & 1 & 0 \\ 1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$C = [A:B]$

$$C = \left[\begin{array}{ccccc|c} 1 & 2 & 10 & 0 & 0 & 0 \\ 1 & -1 & 3 & -1 & 4 & 0 \\ 2 & -2 & 1 & 1 & 0 & 0 \\ 1 & 0 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array}\right] \quad R_2 \to R_2 - R_1,\ R_3 \to R_3 - 2R_2,\ R_4 \to R_4 - R_2$$

$$C = \left[\begin{array}{ccccc|c} 1 & 2 & 10 & 0 & 0 & 0 \\ 0 & -3 & -7 & -1 & 4 & 0 \\ 0 & 0 & -5 & 3 & -8 & 0 \\ 0 & 1 & -5 & 1 & -3 & 0 \end{array}\right]$$

continued

(35)

$x - ky + z = 0$
$kx + 3y - kz = 0$
$3x + y - z = 0$

$AX = B$

$$\begin{bmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$C = [A:B]$

$$C = \left[\begin{array}{ccc|c} 1 & -k & 1 & 0 \\ k & 3 & -k & 0 \\ 3 & 1 & -1 & 0 \end{array}\right] \quad R_3 \leftrightarrow R_3 - 3R_1$$

For unique solⁿ and only trivial solution.

$R(A) = \text{Rank}(C) = \text{No. of unknown}$

$$C = \left[\begin{array}{ccc|c} k & 3 & -k & 0 \\ 1 & -k & 1 & 0 \\ 3 & 1 & -1 & 0 \end{array}\right] \quad R_3 \to R_3 - 3R_2$$

$$\begin{bmatrix} k & 3 & -k \\ 1 & -k & 1 \end{bmatrix}$$

(35) $x - ky + z = 0$

$kx + 3y - kz = 0$

$3x + y - z = 0$

System of eq^n

$$AX = 0$$

$$A = \begin{bmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{bmatrix}$$

(ii) Non trival solution.

$$\Rightarrow \begin{bmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{bmatrix} \neq 0$$

$\Rightarrow 1(-3+k) + k(-k+3k) + 1(k-9) \neq 0$

$\Rightarrow -3 + k + 2k^2 + k - 9 \neq 0$

$\Rightarrow 2k^2 + 2k - 12 \neq 0$

$\Rightarrow k^2 + k - 6 \neq 0$

$\Rightarrow (k+3)(k-2) \neq 0$

$\therefore k \neq 2, -3$

$k \in R - \{2, -3\}$

(iii) trival solution.

$$\begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} = 0$$

$\Rightarrow (k+3)(k-2) = 0$

$\Rightarrow k = -3$ and $k = 2$

Hence $k = 2, -3$.

(36) $x + 2y + z = 6$

$x + 4y + 3z = 10$

$x + 4y + \lambda z = \mu$

System of equation

$$Ax = B$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 4 & 3 \\ 1 & 4 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

The augmented matrix

$$C = [A : B]$$

$$C = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 1 & 4 & 3 & 10 \\ 1 & 4 & \lambda & \mu \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

Case-I, $\lambda - 3 = 0$ and $\mu - 10 \neq 0$

i.e $\lambda = 3$ and $\mu \neq 10$

Rank of A, $\rho(A) = 2$

Rank of A:B, $\rho(A:B) = 3$

The given system is inconsistent but has no solution.

~~the~~.

case-II $\lambda - 3 \neq 0$ and $\mu \in R$

i.e $\lambda \neq 3$

$\rho(A) \neq \rho(A,B) = 3$

The given system is consistent and has unique solution.

case-III: $\lambda = 3$ and $\mu = 10$

$\rho(A) = \rho(A,B) = 2 <$ number of unknown.

The given system is consistent but has an infinite no. of sol^n.

(38) $x_1 - x_2 + 3x_3 - x_4 = 1$

$x_2 - 3x_3 + 5x_4 = 2$

$x_1 - x_3 + x_4 = 0$

$x_1 + 2x_3 - x_4 = -5$

{ For unique solution
R(A) = R(C) = no. of unknown

$AX = B$

$$\begin{bmatrix} 1 & -1 & 3 & -1 \\ 0 & 1 & -3 & 5 \\ 1 & 0 & -1 & 1 \\ 1 & 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -5 \end{bmatrix}$$

The augmended matrix

$$C = \left[\begin{array}{cccc|c} 1 & -1 & 3 & -1 & 1 \\ 0 & 1 & -3 & 5 & 2 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 2 & -1 & 5 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$R_3 \to R_3 - R_1$

$$C = \left[\begin{array}{cccc|c} 1 & -1 & 3 & -1 & 1 \\ 0 & 1 & -3 & 5 & 2 \\ 0 & 1 & -4 & 2 & -1 \\ 1 & 0 & 2 & -1 & 5 \end{array}\right] R_3 \to R_3 - R_2,\ R_4 \to R_4 - R_1$$

$$C = \left[\begin{array}{cccc|c} 1 & -1 & 3 & -1 & 1 \\ 0 & 1 & -3 & 5 & 2 \\ 0 & 0 & -1 & -3 & -3 \\ 0 & 1 & -1 & 0 & 4 \end{array}\right]$$

Incomplete

(37) (a) $\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

~~$A = IA$~~

$$C = \left[\begin{array}{cc|cc} -1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right] R_2 \to R_2 + 2R_1$$

$$\left[\begin{array}{cc|cc} -1 & 2 & 1 & 0 \\ 0 & 5 & 2 & 1 \end{array}\right] R_2 \to \frac{R_2}{5}$$

$$1 - \frac{4}{5} \qquad \frac{5-4}{5} = \frac{1}{5} \qquad \frac{-2}{5}$$

$$\left[\begin{array}{cc|cc} -1 & 2 & 1 & 0 \\ 0 & 1 & \frac{2}{5} & \frac{1}{5} \end{array}\right] R_1 \to R_1 - 2R_2,\ R \to -1R_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{5} & \frac{2}{5} \\ 0 & 1 & \frac{2}{5} & \frac{1}{5} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} \frac{-1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

$-3+6$

(d)

$$A = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 3 & 5 & 6 & 0 & 1 & 0 \\ 2 & 4 & 5 & 0 & 0 & 1 \end{array}\right] \quad R_2 \to R_2 - 3R_1,\ R_3 \to R_3 - 2R_1$$

$$A = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -3 & -3 & 1 & 0 \\ 0 & 0 & -1 & -2 & 0 & 1 \end{array}\right] \quad R_1 \to R_1 + 2R_2,\ R_2 \to R_2 - 3R_3$$

$1-1$

$3-6$ $\qquad -5+6$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & -5 & 2 & 0 \\ 0 & -1 & 0 & 3 & 1 & -3 \\ 0 & 0 & -1 & -2 & 0 & 1 \end{array}\right] \quad R_1 \to R_1 - 3R_3$$

$R_2 \to -1R_2,\ R_3 \to -1R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & -3 \\ 0 & 1 & 0 & -3 & 1 & 3 \\ 0 & 0 & 1 & 2 & 0 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 2 & -3 \\ -3 & 1 & 3 \\ 2 & 0 & -1 \end{bmatrix}$$

(39) (ii)

$$\left[\begin{array}{cccc|cccc} 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 2 & 2 & 2 & 3 & 0 & 0 & 1 & 0 \\ 2 & 3 & 3 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 \\ 2 & 2 & 2 & 3 & 0 & 0 & 1 & 0 \\ 2 & 3 & 3 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \quad R_3 \to R_3 - 2R_1$$

$R_4 \to R_4 - 2R_1$

$2-4$
$3-6$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & -3 & 0 & -2 & 1 & 0 \\ 0 & 1 & -1 & -2 & 0 & -2 & 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -2 & -3 \\ 0 & 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix} \quad R_4 \to R_4 - R_2$$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & -3 & -4 \end{bmatrix}$$

Incomplete

(40) (a) $4x - 2y + 6z = 8$

$x + y - 3z = -1$

$15 - 3y + 9z = 21$

Here the no of unknowns $= 3$

The matrix form of the system is $AX = B$ where

$$\begin{bmatrix} 4 & -2 & 6 \\ 1 & 1 & -3 \\ 15 & -3 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \\ 21 \end{bmatrix}$$

$C = (A:B)$

$$C = \left[\begin{array}{ccc|c} 4 & -2 & 6 & 8 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & -1 \\ 4 & -2 & 6 & 8 \\ 15 & -3 & 9 & 21 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 15R_1 \end{array}$$

2+4 15

9-45

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & -1 \\ 0 & -6 & -6 & 12 \\ 0 & -18 & -36 & 36 \end{array}\right]$$

-2-4

6-12

-3-15

$R_3 \to R_3 - 3R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & -1 \\ 0 & -6 & -6 & 12 \\ 0 & 0 & -18 & 0 \end{array}\right]$$

$\rho(A) = 3 =$ no. of unknown

$x + y + z = -1$ — (1)

$-6y - 6z = 12$ — (2)

$-18z = 0$ — (3)

from eqn (3)

$z = 0$

From eqn (2) $-6y = 12$

$y = -2$

From eqn (1)

$x - 2 = -1$

$x = 1$.

$x = 1, \; y = -2, \; z = 0$.

(41)

$$A=\begin{bmatrix}1 & 3 & 4\\ 3 & K & 6\\ -1 & 5 & 1\end{bmatrix}$$

$K \neq \frac{3}{5}$

$$A^{-1} = \frac{1}{8}\begin{bmatrix}-29 & 17 & 14\\ -9 & 5 & 6\\ 16 & -8 & -8\end{bmatrix}$$ Ans

$|A| = 1(K-30)-3(3+6)+4(15+K) \neq 0$

$= K-30 \; -27 + 60 + 4K \neq 0$

$\Rightarrow 5K+3 \neq 0$

$\Rightarrow K \neq \frac{-3}{5}$

For $k=1$

$|A| \Rightarrow 5k+3 = 8$

$A_{11} = -29$	$A_{21} = 17$	$A_{31} = 14$
$A_{12} = -9$	$A_{22} = 5$	$A_{32} = 6$
$A_{13} = 16$	$A_{23} = -8$	$A_{33} = -8$

$$A^{-1} = \frac{1}{|A|}\,\text{adj}(A)$$

$$= \frac{1}{8}\begin{bmatrix}-29 & 17 & 14\\ -9 & 5 & 6\\ 16 & -8 & -8\end{bmatrix}$$

(44)

$$A=\begin{bmatrix}1 & 2 & 1\\ 3 & 2 & 3\\ 1 & 1 & 2\end{bmatrix}$$

elementary transformation.

$$A=\begin{bmatrix}1 & 2 & 1\\ 3 & 2 & 3\\ 1 & 1 & 2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$$

Not solved

(45) $x+y+z=5$
$x+3y+3z=9$
$x+2y+\alpha z=\beta$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 3 \\ 1 & 2 & \alpha \end{vmatrix}$$

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & \alpha-1 \end{vmatrix}$$

$[2(\alpha-1)-2]$

$D \Rightarrow 2\alpha-2-2 = 2\alpha-4$

$D=0$

$\Rightarrow 2\alpha=4$

$\Rightarrow \alpha=2$

$D_x = 0$

$$\begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0 \quad \begin{vmatrix} 5 & 1 & 1 \\ 9 & 3 & 3 \\ \beta & 2 & 2 \end{vmatrix} = 0$$

$D_x = 5(6-6) - 1(18-3\beta) + 1(18-3\beta) = 0$

$= -18+3\beta+18-3\beta$

Incomplete

(46) $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

(i) $(A^{-1})^T = (A^T)^{-1}$

LHS

$A^{-1} = \begin{bmatrix} -2-3 & & \\ -2-3 & -2-5 & \\ 1+2 & 3-4 & \end{bmatrix}$

$A_{11} =$
$A_{12} =$
$A_{13} =$

$|A| = 1(35-1) + 2(-10-1) + 1(-2-3)$

$= 34 - 22 - 5$

$= 7$

$|A| = 1(15-1) + 2(-10-1) + 1(-2-3)$

$= 14-22-5$

$= -13$

$A^{-1} = \frac{-1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix} = \begin{matrix} \frac{-14}{13} & \frac{-11}{13} & \frac{5}{13} \\ \frac{-11}{13} & \frac{-4}{13} & \frac{3}{13} \\ \frac{5}{13} & & \end{matrix}$

-5

$(A^{-1})^T = \frac{-1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$

RHS =

$$A^T = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$$\begin{aligned} A^T &= 1(15-1)+2(-10-1)+1(-2-3) \\ &= 14 - 22 - 5 \\ &= -13 \end{aligned}$$

$$(A^T)^{-1} = \frac{1}{-13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

Hence LHS = RHS proved.

(47) $(AB)^{-1} = B^{-1}A^{-1}$

→ If x, y are two matrices

$xy = yx = I$

Then $x = y^{-1}$

Given $(AB)^{-1} = B^{-1}A^{-1}$

⇒ multiplying with AB to $B^{-1}A^{-1}$

$$ABB^{-1}A^{-1} = A(BB^{-1})A^{-1} = AA^{-1} = I$$

Multiplying with $B^{-1}A^{-1}$ to AB

$$B^{-1}A^{-1}AB = B^{-1}IB = B^{-1}B = I$$

$$AB(B^{-1}A^{-1}) = (B^{-1}A^{-1})AB = I$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

There is passion in love
but there is peace in self-love.

(49) $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 3 & 10 \end{bmatrix}$. $R_3 \to R_3 - 2R_1$

★ Rest easy hi hai.!!

$\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 0 & -7 & 2 \end{bmatrix} R_3 \to R_3 - R_2$

$-3-10$

$\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 0 & -10 & 0 \end{bmatrix}$

$\rho(A) = 3.$

(50) $x + 2y + 3z = \lambda x$
$3x + y + 2z = \lambda y$
$2x + 3y + z = \lambda z$

$(0,0,0)$
$\downarrow$
trivial sol^n

$\Rightarrow (1-\lambda)x + 2y + 3z = 0$
$\Rightarrow 3x + (1-\lambda)y + 2z = 0$
$2x + 3y + (1-\lambda)z = 0$

For non-trivial sol^n
$\Delta = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

$= (1-\lambda)\left[(1-\lambda)^2 - 6\right] - 2(3 - 3\lambda - 4) + 3(9 - 2 + 2\lambda) = 0$

$= (1-\lambda)^3 - 6 + 6\lambda + 2 + 6\lambda + 21 + 6\lambda = 0$

$= 1 - 3\lambda + 3\lambda^2 - \lambda^3 + 17 + 18\lambda = 0$

$= -\lambda^3 + 3\lambda^2 + 15\lambda + 18 = 0$

$= \lambda^3 - 3\lambda^2 - 15\lambda - 18 = 0$

$\lambda = 6$

(51) $2x + 3y + z = 9$

$4x + y \cancel{+0y} = 7$

$x - 3y - 7z = 6$

$$\begin{vmatrix} 2 & 3 & 1 \\ 4 & 1 & 0 \\ 1 & -3 & -7 \end{vmatrix}$$

$|D| = 2(-7) - 3(-28) + 1(-12-2)$

$= -14 + 84 - 14$

$= 56$

To find the value of x, y, and z :-

$$x = \left|\frac{Dx}{D}\right|, \quad y = \frac{|D_y|}{|D|}, \quad z = \frac{|D_z|}{|D|}$$

To find x

$$Dx = \begin{vmatrix} 9 & 3 & 1 \\ 7 & 1 & 0 \\ 6 & -3 & 7 \end{vmatrix}$$

$Dx = 9(7) - 3(49) + 1(-21-6)$

$= 63 - 147 - 27$

$= -111$

To find y

$$Dy = \begin{vmatrix} 2 & 9 & 1 \\ 4 & 7 & 0 \\ 1 & 6 & 7 \end{vmatrix}$$

$Dy = 2(49) - 9(28) + 1(24-7)$

$= 98 - 252 + 17$

$= -137$

$$Dz = \begin{vmatrix} 2 & 3 & 9 \\ 4 & 1 & 7 \\ 1 & -3 & 6 \end{vmatrix}$$

$= 2(6+21) - 3(24-7) + 9(-12-1)$

$= 54 - 51 - 108$

$= -105$

$x = \frac{-111}{56}$

$y = \frac{-137}{56}$

$z = \frac{-105}{56}$

* Rest question similar.