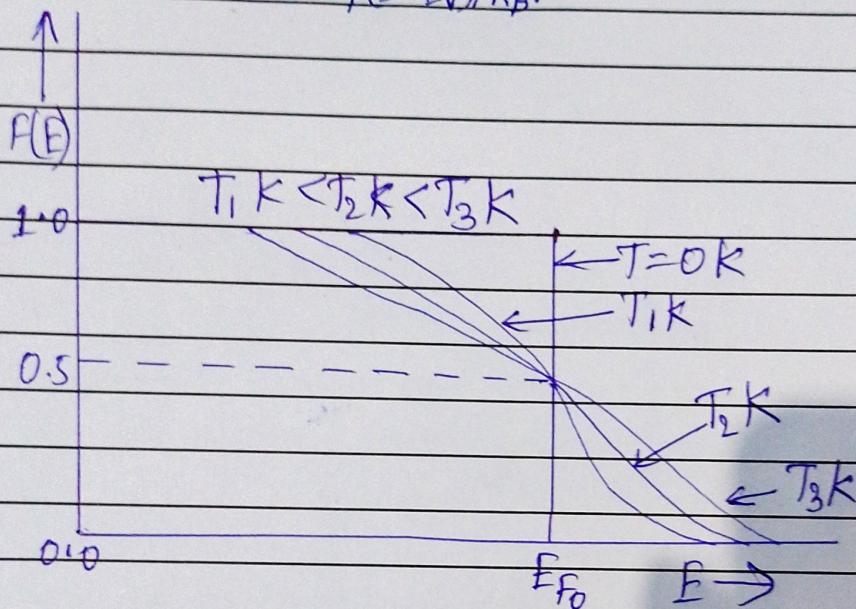


1) Write the formula of Fermi-Dirac distribution function and plot it for two different temperatures ($T_2 > T_1$).

Ans → The formula for Fermi-Dirac distribution is

$$F(E) = \frac{1}{1 + \exp(E - E_F)/k_B T}$$

Where k_B = Boltzmann Constant

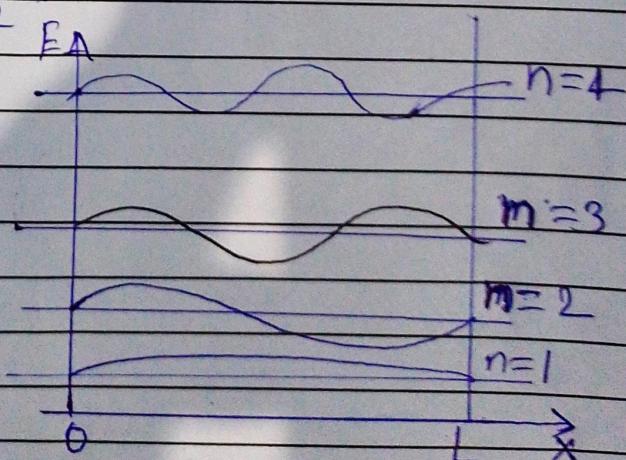


2) Write the expression of Eigen value and wave function for a free particle moving one dimensionally (1-D) in a potential well.

Ans → Expression of Eigen value:-

$$E_n = \frac{n^2 \pi^2 h^2}{8 m L^2}$$

Wave function:-



$$E_1 = \frac{\hbar^2}{8 m L^2} = E_0$$

3) Explain the idea of wave function for a quantum particle. What are the basic characteristics of well behaved wave function?

Ans) Wave function for a quantum particles describes variable quantity that mathematically describes the wave characteristics of a particle. Basic characteristics are:-

It is single valued function.

It has infinite value.

The wave function should be continuous.

The derivative of wave function is continuous.

4) Define Fermi energy. Write its expression.

It is the energy difference between highest and lowest occupied single particle states in a quantum system of non-interacting fermions at absolute zero temperature.

$$E_F = \frac{\hbar^2}{2m} \left(\frac{3\pi^2 N}{V} \right)^{2/3}$$

N = No. of particles

m = Rest mass of each fermions

V = Volume of system

\hbar = Reduced Plank constant

5) If the Fermi energy is 10eV, what is the mean energy of electron at 0K?

$$E_e = \frac{3}{5} E_F = \frac{3}{5} (10\text{eV}) = 6\text{eV}$$

6) What would be the band structure if the barrier strength is extremely negligible? Justify the answer with a diagram.

Ans \rightarrow

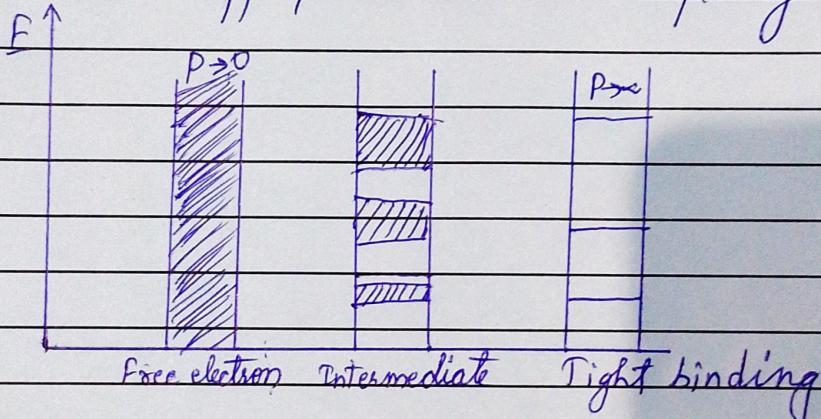
$$\cos \alpha a = \cos ka \quad i.e. \alpha = k \text{ or } \alpha^2 = k^2$$

$$\frac{2mF}{\hbar^2} = k^2$$

$$\frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m} \quad \left[\text{As } k = \frac{2\pi}{\lambda}, \lambda = \frac{\hbar}{p} \right]$$

$$F = \frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m}, \quad \left[\text{As } k = \frac{2\pi}{\lambda}, \lambda = \frac{\hbar}{p} \right]$$

which is appropriate to the completely free particle.



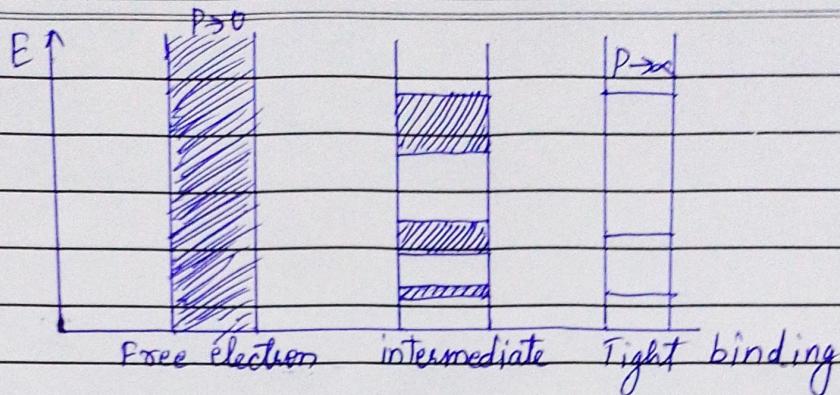
7) What would be the band structure if the barrier strength is extremely high? Justify your answer with diagram.

Ans \rightarrow $P \sin \alpha a + \cos \alpha a = \cos \alpha a$

$$\text{where } P = \frac{mv_e a}{2}$$

$$\alpha a = 0 \quad \text{or} \quad \alpha a = n\pi$$

$$E = \frac{\pi^2}{2ma^2} n^2$$



8) Based on band theory of solid, distinguish between conductors, semiconductors and insulators.

Conductor: In a conductor there are no bond gaps between the valence & conduction band in some metals the conduction & valence band overlaps partially. This means that electrons can now move freely between the valence band & the conduction band.

Insulator: An insulator has a large gap b/w the valence band and the conduction band. The valence band is full as no electrons can move upto the conduction band is empty. i.e. -7eV

Semiconductor: The gap between the valence band and the conduction band is in b/w the conductors & insulators. i.e 1.1eV

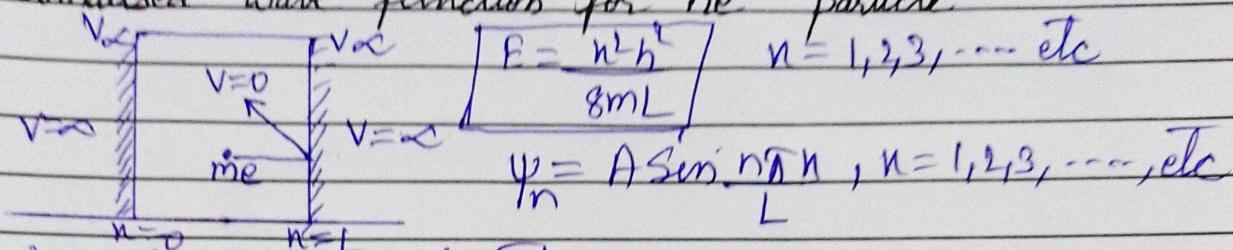
9) Define the density of energy state in a solid. Find the expression for density of states.

DOS: The density of states (DOS) is essentially the number of different states at a particular energy level that electrons are allowed to occupy, i.e. the number of electron state per unit volume per unit energy.

$$D(E) = \frac{1}{2\pi^2} \left(\frac{2m}{\hbar}\right)^{3/2} E^{1/2}$$

10)

A particle is in motion along a line b/w $x=0$ & $x=L$ with 0 potential energy. At points for which $x \leq 0$ & $x \geq L$, the potential energy is ∞ . At points for which $x \leq 0$ is infinite. Solving Schrödinger's eqn, obtain energy Eigen values & normalised wave function for the particle:



Normalised Wave Function

Any wave function is said to be normalized. If probability of finding the particle is unity.

$$\text{i.e. } \int_0^L [\psi]^2 dx = 1$$

$$\Rightarrow A^2 \int_0^L \sin^2 \frac{n\pi x}{L} dx = 1$$

$$\Rightarrow A^2 \int_0^L \sin^2 \frac{n\pi x}{L} dx = 1 \Rightarrow A^2 \int_0^L \frac{1 - \cos \frac{2n\pi x}{L}}{2} dx = 1$$

$$\Rightarrow A^2 \int_0^L \left[\frac{1 - \cos \frac{2n\pi x}{L}}{2} \right] dx = 1 \Rightarrow A^2 \left[\frac{\sin \frac{2n\pi x}{L}}{2n\pi} \right]_0^L = 1$$

$$\frac{A^2}{2} [0 - 0] - \frac{A^2}{2} \left[\frac{\sin \frac{2n\pi L}{L}}{2n\pi} \right]_0^L = 1$$

$$= \frac{A^2}{2} [L - 0] - \frac{A^2}{2} [0 - 0] = 1$$

$$\frac{A^2 L}{2} = 1$$

$$\boxed{A = \sqrt{\frac{2}{L}}}$$