Q. Suppose T is a L.T. from $\mathbb{R}^2$ to P_2 s.t.

$$T(1,1) = 2-3x+x^2 \text{ and } T(2,3) = 1-x^2$$

Find $T(-1,2)$ and $T(a,b)$.

$\{(1,1), (2,3)\}$ is a basis for $\mathbb{R}^2$, so every vector in $\mathbb{R}^2$ can be written as linear combination of these two.

$$(-1,2) = c(1,1) + d(2,3)$$

$$\Rightarrow (c+2d, c+3d) = (-1,2)$$

$$\Rightarrow \begin{array}{l} c+2d = -1 \\ c+3d = 2 \end{array} \Rightarrow d = 3 \quad \therefore c = -7.$$

$$\therefore (-1,2) = -7(1,1) + 3(2,3).$$

$$T(-1,2) = T[-7(1,1)+3(2,3)]$$

$$= -7T(1,1) + 3T(2,3) \quad [\because T \text{ is L.T.}]$$

$$= -7(2-3x+x^2) + 3(1-x^2)$$

$$= -14+21x-7x^2+3-3x^2 = \underline{-11+21x-10x^2}.$$

$$(a,b) = c_1(1,1) + c_2(2,3)$$

$$\Rightarrow c_1+2c_2 = a, \quad c_1+3c_2 = b$$

$$\Rightarrow c_2 = b-a \;\&\; c_1 = 3a-2b$$

$$\therefore (a,b) = (3a-2b)(1,1) + (b-a)(2,3)$$

$$T(a,b) = T[(3a-2b)(1,1) + (b-a)(2,3)]$$

$$= (3a-2b)T(1,1) + (b-a)T(2,3) \quad [\because T \text{ is L.T.}]$$

$$= (3a-2b)(2-3x+x^2) + (b-a)(1-x^2)$$

$$= (6a-4b+b-a) + (-9a+6b)x + (3a-2b-b+a)x^2$$

$$= (5a-3b) + (-9a+6b)x + (4a-3b)x^2.$$