Unit-2

① Sum Rule :→

If a task can be done either or in one of n_1 ways or in one of n_2 ways, where none of the set of n_1 ways is same as any of the set of n_2 ways, there are $n_1 + n_2$ ways to do the task

Ex- There are 37 ways to chose a member of maths faculty, 83 ways to chose a student who is a mathematics major. Choosing of faculty is never same as choosing of a student as both faculty & student can't be same.

So, from whole, to pick this representative, we follow, Sum rule $37+83 = 120$ possible ways to pick the representative

② Product rule

→ Suppose that a procedure can be broken down into a sequence of two tasks. If there are n_1 ways to do the first task, and for each of these ways of doing the first task, there are n_2 ways to to do the second task, then there are $n_1 n_2$ ways to to do procedure

Ex A new company with just two employees, Sanchez and Patel rents a floor of a building with 12 office

For assigning ~~these~~ offices to these two employees consist of assigning to Sanchez in 12 ways, then assigning an office to Patel as different from Sanchez, which can be done in 11 ways,

By the product rule there are $12 \cdot 11 = 132$ ways to assign offices to these two employes

(3) Permutation of n objects taking r objects at time.

If n and r are integers,

$$P(n,r) = (n)(n-1) \dots (n-r+1), \quad 1 \le r \le n$$

$$P(n,r) = \frac{n!}{(n-r)!}, \quad 0 \le r \le n$$

(4) Combinatig of n objects takg r at atme

$$C(n,r) = \frac{n!}{r!\,(n-r)!}$$

(5) Relation of permutation & combenation of n object takg r objects at atime

$$\boxed{P(n,r) = C(n,r) \cdot P(r,r)}$$

(6) BANANA

$6! = 720$

(7) circular permutation formula:

$$\frac{n!}{n} = (n-1)!$$

(8) Inclusion - Exclusion Principle

Let $A_1, A_2 \ldots\ldots A_n$ be finite sets

Then,

$$|A_1 \cup A_2 \cup A_3 \ldots \cup A_n| = \sum_{1 \le i \le n} |A_i| - \sum_{1 \le i < j \le n} |A_i \cap A_j|$$

$$+ \sum_{1 \le i < j \le k \le n} |A_i \cap A_j \cap A_k| - \ldots + (-1)^{n+1} |A_1 \cap A_2 \cap \ldots \cap A_n|$$

(9) Piegon-hole Principle

→ If K is a positive integer and $K+1$ or more objects are placed into K boxes, then there is atleast one box containg two or more of objects.

(10) Generalised Piegon-hole principle :-

If N objects are placed into K boxes, then there is at least one box containing at least $\lceil N/K \rceil$ objects.

(11) m of permutation of six objects say, A, B, C, D, E, F taken 3 at a time

$$^6P_3 = \frac{6!}{(6-3)!} = 6 \times 5 \times 4 = 120$$

(12) BENZENE

total words = 7

repeated letter = 2

$$m = \frac{7!}{3!\,2!} = \frac{7 \times 6 \times 5 \times 4^2}{2} = 420$$

(13) (a) with replacement

Each card can be chosen 52 times

$$m = 52 \cdot 52 \cdot 52 = 140608$$

(b) without replacement

first will be chosen 52 times, 2nd will be chosen 51, and 3rd will be 50

$m = 52 \times 51 \times 50 = 132600$

(14) $^4C_3 = \frac{4!}{3!(4-3)!} = \frac{4!}{3! \times 1!} = 4$ A

(15) Farmer buys 3 cows
2 pigs
4 hens

from man has 6 cows
5 pigs
8 hens

$m = {}^6C_3 \cdot {}^5C_2 \cdot {}^8C_4$

$= 20 \cdot 10 \cdot 70 = 14000$

(16) $S = \{1, 2, 3 ---- 9\}$

Combination of all sets adding to get 10 ~~upto~~ $\{9,1\}, \{2,8\}, \{7,3\}, \{6,4\}, \{5,5\}$

No. of ways $= n+1$

$= 5+1 = 6$

(17) $n = 12$

$k+1 = 3$

$k = 2$

minimum no. of student $= kn+1$

$= 12 \times 2 + 1 = 25$

(18) 7 people can arrange

(a) In a row of chairs

$m = 7!$

(b) around a circular table

$m = (n-1)! = 6!$

(19)

8 blue socks
6 red socks

(a) They can be any colour

$$^{14}C_2 = \frac{\cancel{14}^{7} \times 13}{\cancel{2}} = 91$$

(b) same colour

$$^{8}C_2 + {}^{6}C_2$$

$$= \frac{\cancel{8}^{4} \times 7}{\cancel{2}} + \frac{\cancel{6}^{3} \times 5}{\cancel{2}} = 28 + 15$$

$$= 43$$

(20) $12 \cdot {}^{11}C_4$

$$= 12 \times \frac{11 \times 10 \times \cancel{9}^{3} \times \cancel{8}^{2}}{\cancel{4} \times \cancel{3} \times \cancel{2}}$$

$$= 12 \times 330$$

$$= 3960$$

(21)

8 - male
6 - female

(a) 1 class representative

$$^{14}C_1 = 14$$

(b) 2 class representate 1 male, & 1 female

$^{8}C_{1} \cdot {}^{6}C_{1}$

$= 8 \cdot 6$

$= 48$

(c) 1 president & 1 vice preident.

$^{14}C_{1} \cdot {}^{13}C_{1}$

$14 \cdot 13$

$= 182$

140
42
(22)

22 female students

18 male students

total no. of students $= 22+18 = 40$

(23) 5 History

3 sociology.

6 Anthropology.

4 Psychology texts

(a) one of texts $\Rightarrow$ Sum rule $: 5+3+6+4 = 18$

(b) one of each text $\Rightarrow$

apply Product rule $\Rightarrow$ $5 \cdot 3 \cdot 6 \cdot 4 = 360$

(24) 12 students

first test take $^{12}C_4$

2nd test taken by $^{8}C_4$

3rd will $^{4}C_4$

$n = {}^{12}C_4 \cdot {}^{8}C_4 \cdot {}^{4}C_4$

$= \frac{\cancel{12} \times 11 \times \cancel{10} \times 9 \times \cdot}{\cancel{4} \times \cancel{3} \times \cancel{2}} \cdot \frac{\cancel{8} \times \cancel{7} \times \cancel{6} \times 5}{\cancel{4} \times \cancel{3} \times \cancel{2}}$

$= 495 \cdot 70$

$= 34650$

(25) $|A| = 12$ $|A \cap C| = 7$ $|A \cap D| = 4$

$|A \cap B| = 5$ $|B \cap C| = 16$ $|A \cap B \cap C| = 3$

$|B \cap D| = 4$ $|C \cap D| = 3$ $|A \cap B \cap C \cap D| = 2$

$|B \cap C \cap D| = 2$ $|A \cap C \cap D| = 3$

$|B| = 20$ $|C| = 20$

$|A \cap C| = 7$ $|A \cap B \cap D| = 2$

$|D| = 8$

$|A \cup (B \cup C \cup D)| =$ The no. of students taking at least one course.

$= |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C|$
$- |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D|$
$+ |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| +$
$|B \cap C \cap D| + |A \cap B \cap C \cap D|$

$N = |A \cup (B \cup C \cup D)| + 71 = 29 + 71 = 100$

(26) $|F \cup G \cup R| = |F| + |G| + |R| - |F \cap G| - |F \cap R|$
$- |G \cap R| + |F \cap G \cap R|$

$= 65 + 45 + 42 - 20 - 25 - 15 + 8$
$= 100$

$|F| = 65$
$|R| = 42$ $|G| = 45$
$|F \cap G| = 20$ $|F \cap R| = 25$
$|G \cap R| = 15$ $|F \cap G \cap R| = 8$

(27) (a) there are 5 letters

$n = 6$ (pigeonhole)

then $k + 1 = 4$
$k = 3$

$nk + 1 = 6 \times 3 + 1 = 19 < 21$

Some subtitles must have 4 consonants.

(b) L begins with vowel , $n = 51$

$K+1 = 5$

$K = 4$

So,

$Kn+1 = 4 \times 5+1 = 21$

Some have atleast 5 constants.

(28)

$n|P| = 30$ $n|B| = 14$

$n(P \cup B) = 32$

(a)

Save both

$n|P \cap B| = -n(P \cup B| + n|P| + n|B|$

$= 30+14-32$

$= 44-32$

$= 12$

(b) save only paper

$m = n(P/B) = n|P| - n(P \cap B|$

$= 30-12$

$= 18$

(c) save only bottles

$m = n(B/P) = n(B) - n(P \cap B)$

$= 14 - 12 = 2$

(29) (a) The three area are the piegonholes and the student must take five closes (piegeons) Hence, the student must take alteast two closes in one area

(b) Let each of the three area of study represent three disjoint sets, A, B, & C. Since, the sets are disjoint $n(A \cup B \cup C) = 5$
$= n(A) + n(B) + n(C)$. Since the student can take atmost two closes in any area of study, the sum of closes in any two sets, say A & B must be less then or equal to four. Hence,

$5 - \{n(A) + n(B)\} = n(C) \geq 1$

Thus the student must take atlest one close in any area.

(30) (a) 6 times so that there is exactly 3 heads and no two heads occur in a row)

→

HTHTHT, HTTHTH, HTHTTH,
THTHTH

$2n = 6$

$n = 3$

$\boxed{n+1 = 4}$

(b) $n+1$

(31) (a) QUEUE

$$P = \frac{5!}{2!2!} = \frac{5\times\cancel{4}^{2}\times3\times\cancel{2}\times1}{\cancel{2}\times\cancel{2}\times1}$$

$= 30$

(b) COMMITTEE

$$P = \frac{9!}{2!2!2!} = \frac{9\times8\times7\times6\times5\times4\times3\times\cancel{2}}{\cancel{2}\times\cancel{2}\times\cancel{2}}$$

$= 45360$

(c) PROPOSITION

$$P = \frac{11!}{2!\,3!\,2!}$$

$$= 1663200$$

(d) BASEBALL

$$P = \frac{8!}{2!\,2!\,2!}$$

$$= 5040$$

(32) 4 - red
2 blue
3 - green

total = 4+2+3
= 9

$$m = \frac{9!}{4!\,2!\,3!} = \frac{9\times 8\times 7\times \cancel{6}^{4}\times 5\times \cancel{4\times 3\times 2\times 1}}{\cancel{4\times 3\times 2\times 1}\times 2\times \cancel{3\times 2}\times 1}$$

$$= 1260$$

(33) (a) With replacement

total = 10

$$n = 10\times 10\times 10\times 10$$

$$= 10^4 = 10000$$

(b) without replacement

$n = {}^{10}P_4 = 10 \times 9 \times 8 \times 7$

$= 90 \times 56$

$= 5040$

(35)

total = 6

(a) 1 defect $= {}^6C_1 = 6$

(b) 2 defect $= {}^6C_2 = \frac{6 \times 5}{2} = 15$

(c) 3 defect $= {}^6C_3 = \frac{6 \times 5 \times 4}{3 \times 2} = 20$

(36)

9 men
3 women

12 total

(a) no restriction ${}^{12}C_4 = \frac{\cancel{12} \times 11 \times \cancel{10} \times 9}{\cancel{4} \times \cancel{3} \times \cancel{2}}$

$= 495$

(b) 2 men & 2 women $= {}^9C_2 \cdot {}^3C_2$

$= \frac{9 \times 8}{2} \times 3$

$= 108$

(c) exactly one women

$^9C_3 \cdot {^3C_1} = \frac{9\times8\times7}{3\times2} \cdot 3$

$= 36\times7$

$= 252$

(d) at least one women

$^{12}C_4 - {^9C_4} = 495 - 126$

$= 369$

(37) (a) $^{11}C_5$

(b) when not atted

$^9C_5 \cdot {^2C_0} = 126$

when atend

$^9C_5 \cdot {^2C_2}$

$= 84$

total ways $= 126+84$

$= 210$

(c) $^9C_5 + {^9C_4} \times 2$

$= 126 + 126 \cdot 2$

$= 126 + 252$

$= 378$

(38) 8 men

6 women

$^{12}C_4 + 2 \cdot {^{12}C_3}$

$= 495 + 440$

$= 935$

(39) $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$

$6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$, $10^2 = 100$

~~total~~ = 10, $n(\text{odd}) = 50$, $|A| = 50$

total m
$n(\text{odd in squ}) = 5$, $|B| = 10$

(a) odd or the square of integr $|A \cap B| = 5$

$(A \cup B) = |A| + |B| - |A \cap B|$

$= 50 + 10 - 5$

$= 55$

(b) $1^3 = 1$ $2^3 = 8$ $3^3 = 27$

$4^3 = 64$

No = 4 N (even n cube) = 2

N (even) = 50

$(A \cup B)$: $50 + 4 - 2$

$= 54 - 2$

$= 52$ A

(40) 6 blue

4 white

10 total

(a) no restrictions $^{10}C_2 = \frac{10 \times 9}{2} = 45$

(b) ~~same~~ differen clur $^6C_2 \cdot {}^4C_2 = 6 \cdot 4 = 24$

(c) Same clor = $^6C_2 + {}^4C_2 = \frac{6 \times 5}{2} + \frac{4 \times 3}{2}$

$= 21$

(41) (a) $n = {}^{11}C_8 = \frac{11 \times 10 \times 9}{8 \times 2} = 165$

(b) $n = 2 \cdot {}^{11}C_9 = \frac{2 \times 11 \times 10}{2} = 110$

(c) $n = {}^5C_3 \cdot {}^8C_7 = \frac{5\times4}{2} \cdot 8$

$= 10 \cdot 8$

$= 80$

(d) $n = {}^5C_3 \cdot {}^8C_7 + {}^5C_4 \cdot {}^8C_6 + {}^5C_5 \cdot {}^8C_5$

$= 80 + 5 \cdot \frac{8\times7}{2} + \frac{8\times7\times6}{3\times2}$

$= 80 + 140 + 56$

$= 276$

(42)

$|A \cup B| = 80$

$|A| = 24$

$|B| = 60$

(a) $|A \cap B| = 24 + 60 - 80$

$= 4$

(b) $\left|\frac{F}{B}\right| = 24 - 4 = 20$

$= n(F) - n(F \cap B)$

(c) $\left|\frac{B}{F}\right| = 60 - 4 = 56$

(43) total = ~~20~~ 30

A - 10 (1st test)

B - 9 (2nd test)

Not in any ~~grade~~ test = 15

(a) A on both test

$= 10 + 9 - 15$

$= 19 - 15$

$= 4$

(b) A on first

$10 - 4 = 6$

(c) A on sec

$9 - 4 = 5$

(44) divisible by $3 = \frac{300}{3} = 100$

——— $5 = \frac{300}{5} = 60$

——— $7 = \frac{300}{7} = 42$

——— $3 \& 5 = \frac{300}{15} = 20$

——— $3 \& 7 = \frac{300}{21} = 14$

——— $5 \& 7 = \frac{300}{35} = 8$

——— $3 \& 5 \& 7 = \frac{300}{105} = 2$

(a) at lest one

$n = 100 + 60 + 42 - 20 - 14 - 8 + 2$

$= 162$

(b) 3, ~~2~~, 5 but not 7

$n = 20 - 2 = 18$

(c) by 5 ony

$n = 60 - 20 - 8 + 2$

$= 34$

(d) by none 3, 5, 7

$n = 300 - 162$

$= 138$

(45)

(a) $|F| = 20$ $|S| = 25$ $|G| = 15$

$|F \cap S| = 8$ $|S \cap G| = 6$ $|F \cap G| = 5$

$|F \cap G \cap S| = 2$

(a) $|F \cup G \cup S| = 20 + 25 + 15 - 8 - 6 - 5 + 2$

$= 43$

$|F' \cap S' \cap G'| = 80 - |F \cup G \cup S|$

$= 80 - 43$

$= 37$ A

(b) only french

$\Rightarrow |F| - |S \cup G|$

$20 - 8 - 5 + 2 = 9$

(c) $|F - \{S \cup G\}| + |S - \{F \cup G\}| + |G - \{S \cup F\}|$

$= |20 - 8 - 5 + 2| + |25 - 8 - 6 + 2| + |15 - 6 - 5 + 2|$

$= 9 + 13 + 6 = 28$

(d) $|\{S \cap G\} - \{F\}|$

$= 6 - 2 = 4$

(e) $|(S \cap G) - \{F\}| + |F \cap G| - \{S\}| + |S \cap F) - \{G\}|$

$= |6 - 2| + |8 - 2| + |5 - 2|$

$= 4 + 6 + 3$

$= 13$ A

(46)

(1) $|A| = 50$
$|B| = 60$
$|C| = 70$
$|D| = 80$

$|A \cap B) = 20$ $(A \cap C) = 20$ $(A \cap D) = 20$
$(B \cap C) = 20$ $(B \cap D) = 20$ $(C \cap D) = 20$

$(A \cap B \cap C) = 10$ $(A \cap B \cap D) = 10$
$(B \cap C \cap D) = 10$
$(A \cap B \cap C \cap D) = 5$

$n = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + (A \cap B \cap C) + (A \cap B \cap D) + (B \cap C \cap D) + (A \cap B \cap C \cap D)$

$= 50 + 60 + 70 + 80 - (20 \times 6) + (10 \times 3) + 5$

$= 175$

(47) (a) $n = 3^3 = 27$ (each element can be placed in any of three cells.

(b) The no of elements in 3 cell can be distributed as follows

~~[3,0,0], [2,1,0], [1,1,1] ...~~

$[3,0,0]$ $[2,1,0]$ or $[1,1,1]$

$n = 1+3+1 = 5$

(48)

(a) 3 ordered cells,

$n = 3^4 = 81$ (each element can be placed in any of three cells.

(b) 3 unordered cells.

The no. of elements in three cells can be distributed as follows:

$[4,0,0]$ $[3,1,0]$ $[2,2,0]$, $[2,1,1]$

Thus, $n = 1+4+3+6 = 14$

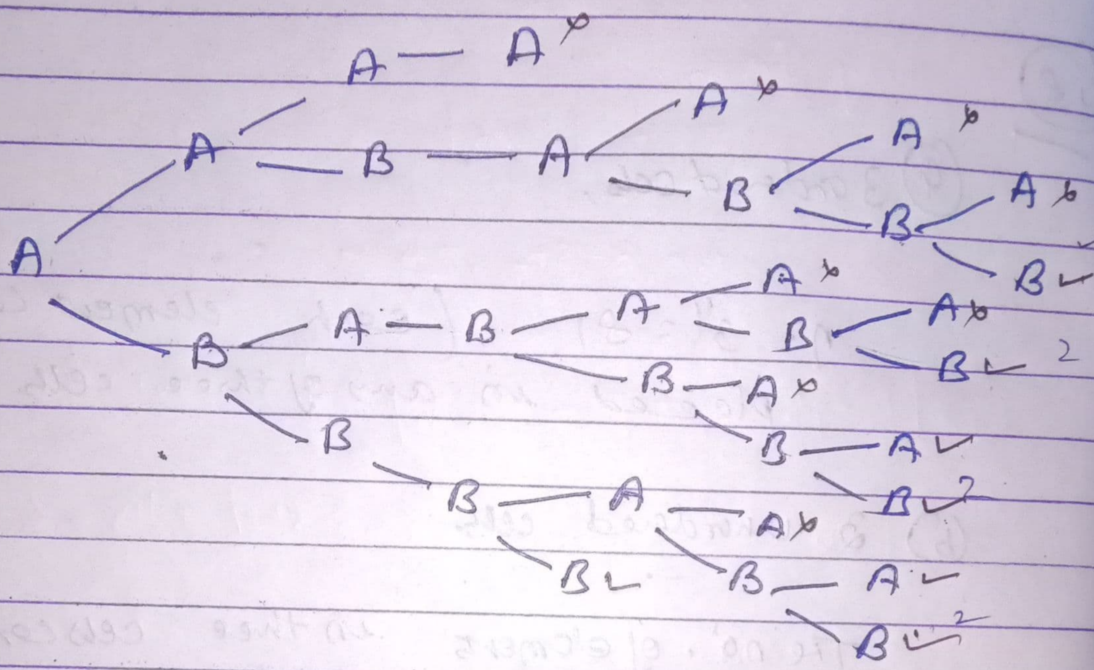
(49) (a) have no restrictions $= {}^{21}C_3 \cdot {}^5C_2 \cdot 5!$

(b) contain the letter B $= {}^{20}C_2 \cdot {}^5C_2 \cdot 5!$

(c) Contain the letter B & C $= 19 \cdot {}^5C_2 \cdot 5!$

(d) Begin with B & contain the letter C $= 19 \cdot {}^5C_2 \cdot 4!$

(50) (a) Tree diagram



(a) $n = 15$

AAAA, AABAA, AABABA,
AABABBA, ABABABA, ABABABB,
AABABBB, ABABAA, ABABBA,
ABABBB, ABABBBB, ABBBAA,
ABBBAAB, ABBBAA, ABBBB

(b) $= 6$

(c) 8