13^{**} 2^{**} 2^{***}

Unit - 4

13. Solve:- Proof = $[(a*b)^{-1} = b^{-1} * a^{-1}]$

let $c = a*b$, $d = b^{-1} * a^{-1}$

[To show :- $c^{-1} = d$]
[To show :- $c*d = d*c = e$]

Consider, $c*d = a*b*(b^{-1}*a^{-1})$
$= a*(b*b^{-1})*a^{-1}$
$= a*e*a^{-1}$
$= a*a^{-1}$
$= e$

$d*c = (b^{-1}*a^{-1})*a*b$
$= a^{-1}*(b^{-1}*b)*a$
$= a^{-1}*e*a$
$= e$

$\Rightarrow c^{-1} * d$
$\Rightarrow (a+b)^{-1} = b^{-1} * a^{-1}$

14. Solved $C_1 = (2, 3\,7)$ and $C_2 = (1\,4\,3\,2)$ be cycle in S_8

find $C_1 C_2 = ?$

$$C_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 7 & 4 & 5 & 6 & 2 & 8 \end{pmatrix}$$

$$C_2 : \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 1 & 2 & 3 & 5 & 6 & 7 & 8 \end{pmatrix}$$

Now

$$C_1 C_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 1 & 3 & 7 & 5 & 6 & 2 & 8 \end{pmatrix}$$

17. Solved- Given

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 1 & 4 & 4 & 2 & 7 & 6 \end{pmatrix} \in S_7$$

then find σ^{-1}

$$\sigma^{-1} : \begin{pmatrix} 3 & 1 & 4 & 4 & 2 & 7 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix}$$

$$\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & & & 1 & & & \end{pmatrix}$$

17. Solut- Given

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 1 & 4 & 4 & 2 & 7 & 6 \end{pmatrix}$$

$$\sigma^{-1} = \begin{pmatrix} 3 & 1 & 4 & 4 & 2 & 7 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix}$$

$$\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 3 & 4 & 4 & 1 & 6 & 7 \end{pmatrix}$$

18. Given

$$\alpha = (1\,3\,2\,5)(1\,4\,3)(2\,5) \in S_5$$

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 2 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 1 & 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 3 & 4 & 2 \end{pmatrix}$$

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 3 & 2 & 5 \end{pmatrix}$$

$$\alpha^{-1} = \begin{pmatrix} 4 & 1 & 3 & 2 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

$$\alpha^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 1 & 5 \end{pmatrix}$$

21. Solved. Let e_1 and e_2 be two identity element in G

[To Show :- $e_1 = e_2$]

$a * e_1 = e_1 * a = a$ — (i)
& $a * e_2 = e_2 * a = a$ — (ii)
for all $a \in G$

$\Rightarrow$ $e_2 * e_1 = e_1 * e_2 = e_2$ — (iii)
$\Rightarrow$ $e_1 * e_2 = e_2 * e_1 = e_1$ — (iv)
$e_1 * e_2 = e_2 * e_1 = e_2 * e_1 = e_1$

$\therefore$ Identity element is unique.

Let b and c be two inverse of a

$\Rightarrow$ $a * b = b * a = e$ — (v)
and $a * c = c * a = e$ — (vi)

Consider

$b = b * c$
$= b * (a * c)$ from (6)
$= (b * a) * c$ (By associativity)
$= e * c$ (from (5))
$= c$

$\boxed{b = c}$ Pd

36. Solⁿ. $G = \{1, \omega, \omega^2\}$

Composition Table

$\times$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	$\omega^3 = 1$
ω^2	ω^2	$\omega^3 = 1$	$\omega^4 = \omega$

① Closure property

Since all enteries in the Composition table are in A. Therefore closure property is satisfied

② Associative property

$$1 \times (\omega \times \omega^2) = (1 \times \omega) \times \omega^2$$
$$1 \times \omega^3 = \omega \times \omega^2$$
$$1 = 1$$

~~then~~ It is also satisfied.

③ Identity element
Here Identity element 1 is in the set under multiplication

4. <u>Inverse property</u>

If all belonging to G there exist a, b such that $a * b = e$, then b is Called the inverse of a

- Clearly Inverse of 1 is 1
  Inverse of ω is ω^2
  Inverse of ω^2 is ω

Since all 4 properties of group are Satisfied G is a grap

43 Solu:- (a) $Z_7 = \{0, 1, 2, 3, 4, 5, 6\}$

$$a * b = \frac{ab}{n} = \frac{ab}{7}$$

<u>Composition table</u>

$*_7$	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	3
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

(b) Semi-group

(i) closure law:-
In compositional table, All the element are the element of $Z_7 = \{0, 1, 2, 3, 4, 5, 6\}$

$\therefore$ Clouser law is verified

(ii) Associativ law

$$(a*b)*c = a*(b*c)$$
$$\left(\frac{ab}{7}\right)*c = a*\left(\frac{bc}{7}\right)$$
$$\frac{abc}{7}/7 = \frac{abc}{7}/7$$
$$\frac{abc}{49} = \frac{abc}{49}$$
$$L.H.S = R.H.S$$

$\therefore$ Associative law is verified
So, the given $(Z_n, +)$ is semi group.

(44) $G = \{1, 2, 3, 4, 5, 6\}$

(a) Table

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

(b) From above table

$$2^{-1} = 4$$
$$3^{-1} = 5$$
$$6^{-1} = 6$$