Qsn bank - 5

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Ans 1) $A(x,y)\frac{\delta^2 y}{\delta x^2} + B(x,y)\frac{\delta^2 y}{\delta x \delta y} + C(x,y)\frac{\delta^2 y}{\delta y^2}$

$+ D(x,y)\frac{\delta y}{\delta x} + E(x,y)\frac{\delta y}{\delta y} + F(x,y) = 0.$

$A = 1, B = 0, C = 3.$

$B^2 - 4AC = 0$

$0 - 4 \times 1 \times 3$

-12

$-12 < 0$ so Elliptic.

2) $A(x,y)\frac{\delta^2 y}{\delta x^2} + B(x,y)\frac{\delta^2 y}{\delta x \delta y} + C(x,y)\frac{\delta^2 y}{\delta y^2}$

$+ D(x,y)\frac{\delta y}{\delta x} + E(x,y)\frac{\delta y}{\delta y} + F(x,y) = 0$

$A = 2, B = 3, C = -4$

$B^2 - 4AC = 0$

$9 - 4 \times 2 \times -4$

$9 + 32$

$= 41$

$41 > 0$ Hyperbolic

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(3) $A = -4,\ B = -2,\ C = -1$

$B^2 - 4AC = 0$

$4 - 4 \times -4 \times -1$

$4 - 16$

$= -12$

$-12 < 0$

Elliptic.

(4) $A(x,y)\frac{\delta^2 y}{\delta x^2} + B(x,y)\frac{\delta^2 y}{\delta x \delta y} + C(x,y)\frac{\delta^2 y}{\delta y^2} +$
$D(x,y)\frac{\delta y}{\delta x} + E(x,y)\frac{\delta y}{\delta y} +$
$F(x,y) = 0.$

$A = \cancel{3}\,2,\ B = 2,\ C = 2$

$B^2 - 4AC$

$4 - 4 \times 2 \times 2$

$4 - 16 = -12$

$-12 < 0$ Elliptic.

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(9) $A=-5,\ B=-5,\ C=-5$

$B^2-4AC>0$

$25-4\times-5\times-5>0$

$25-100$

$=-75$

$-75<0$

Elliptic.

(8) $A=100,\ B=10,\ C=10$

$B^2-4AC=0$

$100-4\times100\times10$

$100-4000$

$=-3900$

$-3900<0$ Elliptic

(8) $B^2-4AC=0$, $A=y^3, B=x^2, C=y^2$

$x^4-4\times y^3\times y^2$

x^4-4y^5

(2)

$A = x^2, \quad B = x^2y, \quad C = x^3$

$B^2 - 4AC = 0$

$(x^2y)^2 - 4 \times x^2 \times x^3$

$x^4y^2 - 4x^5$

$x^4(\cancel{y^2 - 4x})$

(1) case(1)

$\cancel{x > 0} \quad y > 0$

~~PDE is Elliptic~~

(2) ~~$x > 0, y < 0$ or $x < 0, y > 0$~~

~~$x = 1, \quad y = -1$~~

$x^2y^2 - 4x^5$

(1) case no - 1

$\underline{B^2 - 4AC < 0}$

$x^2y^2 - 4x^5 < 0$

$x^2y^2 < 4x^5$

$y^2 < 4x^3$

then Elleptic

(2) case no - 2

$y^2 > 4x^3$

then Hyperbolic.

(3) case no - 3 $\quad y^2 = 4x^3$ then parabolic.

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(8) $A = x^2 y, \; B = y^2, \; C = x^5$

$B^2 - 4AC$

$y^4 - 4 \times x^2 y \times x^5$

$y^4 - 4x^7 y$

case no - 1

$b^2 - 4AC < 0$

$y^4 - 4x^7 y < 0$

$y(y^3 - 4x^7) < 0$

$y^3 < 4x^7$ Elliptic

(2) case - 2

$b^2 - 4AC > 0$

$y^3 > 4x^7$ Hyperbolic.

case 3 $b^2 - 4AC = 0$

$y^3 = 4x^7$

Parabolic.

(9) $B^2 - 4AC = 0$

$x^4 - 4y^5 = 0$

(1) $x^4 < 4y^5$ Elliptic

(2) $x^4 > 4y^5$ Hyperbolic

(3) $x^4 = 4y^5$ Parabolic.

x^{-3} $-3x^{-4}$ $-12x$ nx^{n-1}

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(10) $\cancel{x^{-3}(x,y)\delta}$

$\cancel{-x^{3}(x,y)\frac{\delta^2 y}{\delta x^2} + (xy)^{-3/2}\frac{\delta^2 y}{\delta x \delta y} + c = y^3 \delta}$

(11) $x^{-3} u_{xx} + (xy)^{-3/2} u_{xy} + y^{-3} u_{yy} +$

$xy^2 u_x + y^2 u_y + y^2 = 0$

$B^2 - 4AC \neq 0$

$(xy)^{-3/2 \times 2}$ $-4 \times x^{-3} \times y^{-3}$

$x^{-3}y^{-3}$ $-4x^{-3}y^{-3}$

$\cancel{x^{-3}y^{-3}}$ $-3x^{-3}y^{-3}$

$-12x^{-5}$ +

(10), (11), (12), (13)

$c^2 4 x x$

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(13) $\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2}$

$C = 1, \quad A = c^2, \quad B = 0.$

$B^2 - 4AC$

$0 - 4 \times c^2 \times 1$

$-4c^2$

~~Elliptic~~

(1) $c > 0$

Elliptic

(2) $c < 0$

hyperbolic

(3) $c = 0$

parabolic.

$c^2(x,y) \frac{\delta^2 y}{\delta x^2} + B(x,y) \frac{\delta^2 y}{\delta x \delta y}$

$+ C(x,y) \frac{\delta^2 y}{\delta t^2}$

$A = c^2, \; B = 0, \quad C = 1$

$-4x^2y$
$-4\times1\times1$

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(14) $(x^2-y^2)u_{xx}+2(x^2+y^2)u_{xy}+(x^2-y^2)u_{yy}=0$

$x>0,\ y>0$

$A=(x^2-y^2);\ B=2(x^2+y^2),\ C=(x^2-y^2)$

B^2-4AC

$\{2(x^2+y^2)\}^2-4(x^2-y^2)(x^2-y^2)$

$4(x^4+y^4+2x^2y^2)-4(x^4+y^4-2x^2y^2)$

$\cancel{4x^4}+\cancel{4y^4}+8x^2y^2\cancel{-4x^4}\cancel{-4y^4}+8x^2y^2$

$16x^2y^2$

When $x>0,\ y>0$

$16x^2y^2>0$

$b^2-4AC>0$ Hyperbollic,

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(15) $(x^2)_{4xx} - x(y^2-1)_{4xy} + y(x^2-y^2)_{4yy} = 0$

$A = x^2, \quad B = -x(y^2-1) \quad x = y(x^2-y^2)$

$B^2 - 4AC$

$\{-x(y^2-1)\}^2 - 4x^2 x y(x^2-y^2)$

$x^2y^4 + x^2 - 2x^2y^2 - 4x^4y + 4x^2y^3$

$x^2y^4 + x^2 - 2x^2y^2 - 4x^4y + 4x^2y^3 > 0$

$x^2 > 2x^2y^2 + 4x^4y + 4x^2y^3 - x^2y^4$

$2y^2 + 4y + 4y^3 - y^4 > 1$

$x > 0, \ y > 0$

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(12)

$$\frac{\delta y}{\delta t} = c^2 \frac{\delta^2 y}{\delta x^2}$$

Assume $y(t, x)$ is a soln of (1) which is seperable.

$$y(t, x) = T(t) \cdot X(x)$$

Diff y w.r.t t once ~~&~~ w.r.t x twice

$$y(t) = \frac{dT}{dt} X(x) \quad , \quad y_{xx} = T(t) \cdot \frac{d^2 X}{dx^2}$$

$$= \dot{T} X \quad , \quad = X'' T$$

from eqn :

$$\dot{T} X = c^2 X'' T$$

$$\frac{\dot{T}}{T} = c^2 \frac{X''}{X} = k \text{ (constant)}$$

$$\frac{\dot{T}}{T} = k \quad \Big| \quad c^2 \frac{X''}{X} = k$$

$$\dot{T} - Tk = 0 \quad \Big| \quad c^2 X'' = kX$$

$$\frac{dT}{dt} - TK$$

$A.E = m - K$

$m = K$

C.F $T(t) = ce^{Kt}$

$$c^2 x'' - Kx = 0$$

$$c^2 m^2 - K = 0$$

$$m^2 = \frac{K}{c^2}$$

$$m = \pm \frac{\sqrt{K}}{c}$$

$$C.F = x_{(x)} = A \cdot e^{\frac{\sqrt{K}}{c} x} + B e^{-\frac{\sqrt{K}}{c} x}$$

(17) $$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2}$$

Assume $y(t, x)$ is a solⁿ of (1) which is seprable.

$$y(t, x) = T(t) \cdot X(x)$$

Diff y w.r.t t twice & w.r.t. x twice

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$$u_{tt} = \frac{d^2 t}{dt^2} \cdot X_x \quad , \quad u_{xx} = T_{(t)} \cdot \frac{d^2 x}{dx^2}$$

$$= T'' \cdot x \quad , \quad = x'' T$$

From eq^n =

$$T'' x = c^2 x'' T$$

$$\frac{T''}{T} = \frac{c^2 x''}{x} = K \text{ (constan}$$

$$\frac{T''}{T} = K$$

$$T'' - TK = 0$$

A.E $\quad m^2 - K = 0$

$$m^2 = K$$

$$m = \pm\sqrt{K}$$

$$\frac{c^2 x''}{x} = K$$

$$c^2 x'' - Kx = 0$$

A.E $= c^2 m^2 - K = 0$

$$m^2 = \frac{K}{c^2}$$

$$m = \pm\frac{\sqrt{K}}{c}$$

C.F =

$$T(t) = Ae^{\sqrt{K} x} + Be^{-\sqrt{K} x}$$

$$X(x) = Ae^{\frac{\sqrt{K}}{c} x} + Be^{-\frac{\sqrt{K}}{c} x}$$

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(18) $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0$.

Assume $u(x,y)$ is a sol^n ① which is separable.

$$u(x,y) = X(x) \cdot Y(y)$$

Diff u. w.r.t x twice & w.r.t y twice

$$u_{xx} = \frac{d^2 X}{dx^2} Y(y) \quad , \quad u_{yy} = X(x) \cdot \frac{d^2 Y}{dy^2}$$

$$= X''Y \quad , \quad = XY''$$

From eq^n -

$$X''Y + XY'' = 0.$$

$$X''Y = -XY''$$

$$\frac{X''}{X} = -\frac{Y''}{Y} = k \text{ (constant)}$$

$$X'' - kX = 0$$

A·E

$$m^2 - k = 0$$

$$m^2 = k$$

$$m = \pm k$$

$$X(x) = Ae^{kx} + Be^{-kx}$$

$$Y'' + Yk = 0$$

A·E = $m^2 + k = 0$

$$m^2 = -k$$

$$m = \sqrt{-k}$$

$$m = \pm ki$$

$Y(y) = Ae$ ~~ki~~ $\cos$ + Be ~~cos~~

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(12) $$\frac{\delta^2 u}{\delta x^2} = 5\frac{\delta u}{\delta x} + \frac{\delta u}{\delta y}$$

Assume $u(x,y)$ is a solⁿ of (1) which is separable.

$$u(x,y) = X(x) \cdot Y(y)$$

diff u. w.r.t. x twice & w.r.t. y once, and diff w.r.t. x once.

$$u_{xx} = \frac{d^2 X}{dx^2} Y_y \quad , \quad u_y = X \cdot \dot{Y}$$

$$u_x = \dot{X} \cdot Y$$

$$= X''Y \qquad = X\dot{Y} \quad , = \dot{X}Y$$

from eqⁿ

$$X''Y = 5\dot{X}Y + X\dot{Y} = K$$

$$\frac{X''}{X} - \frac{\dot{Y}}{Y} = 5\frac{\dot{X}}{X} - \frac{\dot{Y}}{Y} = K$$

$$\frac{X''}{X} = \frac{-\dot{Y}}{Y} = 5\frac{\dot{X}}{X} = \frac{-Y'}{Y} = K$$

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$\frac{X''}{X} = K$ | $\frac{-\dot{Y}}{Y} = K$ | $5\frac{\dot{X}}{X} = K$

$X'' - KX = 0$ | $-\dot{Y} - YK = 0$, $\dot{Y} + YK = 0$ | $5\frac{\dot{X}}{X} = K$, $5\dot{X} - KX = 0$

A.E $m^2 = K$, $m = \pm K$ | $m + K = 0$, $m = -K$ | $5m - K = 0$, $m = K/5$

$X_X = Ae^{KX} + Be^{-KX}$ | $Y_Y = Ce^{-KX}$ | $X_X = De^{-KX}$

(20) $\frac{du}{dt} = \frac{\delta u}{\delta x}$, $u(0, x) = 2e^{-3x}$

$\frac{du}{dt} - \frac{\delta u}{\delta x} = 0 \longrightarrow (1)$

Assume $u(t, x)$ is a sol^n of (1) which is separable.

$u(t, x) = T(t) \cdot X(x)$

diff u.w.r.t t once & w.r.t x once.

$u(t) = \frac{\delta T}{\delta t} X$, $u_x = T(t) \cdot \dot{X}$

$T'X - T\dot{X} = 0$

$\frac{T'}{T} = \frac{\dot{X}}{X} = K$

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$\frac{x''}{x} = K$	$\frac{-\dot{y}}{y} = K$	$\frac{5\dot{x}}{x} = K$
$x'' - Kx = 0$	$-\dot{y} - yK = 0$ $\dot{y} + yK = 0$	$\frac{5\dot{x}}{x} = K$ $5\dot{x} - Kx = 0$
A.E $m^2 = K$ $m = \pm K$	$m + K = 0$ $m = -K$	$5m - K = 0$ $m = K/5$
$x_x = Ae^{Kx} + Be^{-Kx}$	$y_y = Ae^{-Kx}$	$x_x = De^{-Kx}$

(20) $\frac{du}{dt} = \frac{\delta u}{\delta x}$, $u(0,x) = 2e^{-3x}$

$$\frac{du}{dt} - \frac{\delta u}{\delta x} = 0 \rightarrow (1)$$

Assume $u(t,x)$ is a solⁿ of (1) which is separable.

$$u(t,x) = T(t)\cdot X(x)$$

diff u w.r.t. t once & w.r.t x once.

$$u(t) = \frac{\delta T}{\delta t} x \quad , \quad u_x = T(t)\cdot \dot{x}$$

$$T'x - T\dot{x} = 0$$

$$\frac{T'}{T} = \frac{\dot{x}}{x} = K$$

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$T' - TK = 0$	$\frac{X'}{X} = K$
A.E $m - K = 0$	$X' - XK = 0$
$m = K$	A.E $= m - K = 0$
	$m = K$

$T(t) = Ae^{Kt}$ $\quad X(x) = Be^{Kx}$

$u(T, x) = Ae^{Kt} \cdot C_2 e^{Kx} = C_3 e^{K(x+t)}$

$u(0, x) = 2e^{-3x}$

$C_3 e^{Kx} = 2e^{-3x}$

$C_3 = 2, \quad K = -3$

$u(t, x) = 2e^{-3(x+t)}$

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(22)

$$u_t - u_{xx} = 0,$$

$$\frac{\delta u}{\delta t} - 1\frac{\delta^2 u}{\delta x^2} = 0 \rightarrow (1)$$

(i) $0 < x < \pi$, $t > 0$,

(ii) $u(0,t) = 0$

(iii) $u(\pi, t) = 0$.

(iv) $u(x, 0) = 3\sin 2x$

we know that most suitable sol^n of heat eq^n

$$\frac{\delta u}{\delta t} = c^2\frac{\delta^2 u}{\delta x^2} \rightarrow (2)$$

$$u(x,t) = (c_1\cos\lambda x + c_2\sin\lambda x)e^{-c^2\lambda^2 t} \rightarrow (3)$$

comparing (1) & (2) then $c^2 = 1$

$$u(x,t) = (c_1\cos\lambda x + c_2\sin\lambda x)e^{-\lambda^2 t} \rightarrow (4)$$

Apply boundary condition $u(0,t) = 0$

(i) $u(0,t) = c_1 e^{-\lambda^2 t} = 0$

$$0 = c_1 e^{-\lambda^2 t}$$

$$c_1 = 0, \quad e^{-\lambda^2 t} \neq 0$$

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from (4)

$u(\pi, t) = 0$

$= \cancel{c_1 \cos\lambda\pi} + c_2 \sin$

from (4)

$$u(x,t) = c_2 \sin\lambda x \, e^{-\lambda^2 t} \rightarrow (5)$$

(ii) $u(\pi, t) = 0$

$$c_2 \sin\lambda\pi \, e^{-\lambda^2 t} = 0.$$

$$c_2 \neq 0, \quad e^{-\lambda^2 t} \neq 0$$

when both two are zero the trivial sol^n.

$$\sin\lambda\pi = 0$$

$$\sin\lambda\pi = \sin n\pi$$

$$\lambda = \frac{n\pi}{\pi}$$

$$\boxed{\lambda = n}$$

from (5)

$$u(x,t) = c_2 \sin\lambda x \, e^{-\lambda^2 t} \rightarrow (6)$$

$\lambda = n$

$n = 1, 2, 3 \ldots$

$$u_n(x,t) = b_n \sin nx \, e^{-n^2 t}$$

$$u_n(x,t) = b_n \sin nx \, e^{-n^2 t}$$

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t)$$

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$u(x,t) = \sum b_n \sin nx\, e^{-n^2 t}$

$b_n = \frac{2}{a}\int_0^a f(x) \cancel{\sin\left(\frac{n\pi x}{a}\right)} dx$ $\sin nx \cdot e^{-n^2 t}$

$\cancel{u(x,0) = 3\sin 2x}$

$\cancel{= \frac{2}{a}\int_0^1 3\sin 2x \cdot \sin}$

$\cancel{f(x) = \sum b_n \sin nx}$

$u(x,0) = f(x)$

$f(x) = \sum b_n \sin nx$

$f(x) \sin nx = \sum b_n \sin^2 nx$

Integrating w.r.t x, 0 to 1

$\int_0^1 f(x) \sin nx\, dx = b_n \int_0^1 \sin nx\, dx$

$\int_0^1 3 \sin 2x \cdot \sin nx\, dx =$

$3\int_0^1 \sin 2x \sin nx\, dx.$

$\cos n = 1$

$\cos n, \cos 0$

$$\int_0^1 f(x)\sin nx\,dx = -b_n[\cos nx]_0^1$$

$$\int_0^1 f(x)\sin nx\,dx = -b_n[+\cos n0 - \cos n]$$

$$\int_0^1 f(x)\sin nx\,dx = -b_n[1-\cos n]$$

$$\int_0^1 f(x)\sin nx\,dx = -b_n(1-\cos n)$$

$$\boxed{b_n = \frac{1}{\cos n - 1}\int_0^1 3\sin 2x \cdot \sin nx\,dx}$$

$$b_n = \frac{2}{\cancel{a}}\int_0^1$$

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$$\frac{\delta u}{\delta t} = \frac{\delta^2 u}{\delta x^2}$$

(2) $u(x,t)$ is separable

$u(x,t) = X(x) \cdot T(t)$ → (2)

diff u w.r.t t once, x twice.

$u_t = X\dot{T}$

$u_{xx} = TX''$

$X\dot{T} = TX''$

$$\frac{\dot{T}}{T} = \frac{X''}{X} = -p^2$$

$\frac{\dot{T}}{T} = -p^2$	$\frac{X''}{X} = -p^2$
$\dot{T} + Tp^2$	$X'' + p^2 X$
AE = $m + p^2 = 0$	$m^2 + p^2 = 0$
$m = -p^2$	$m^2 = -p^2$
	$m = \sqrt{-p^2}$
$T(t) = c e^{-p^2 t}$	$X = A\cos px + B\sin px$

G.S of (1) is

$u(x,t) = (A\cos px + B\sin px) \cdot c e^{-p^2 t}$

~~$u(0,t) = 0$~~

(i) $u(0,t) = 0$

(i) $u(0,t) = Ae^{-p^2t}$

$0 = Ae^{-p^2t}$

$A = 0$

use (ii) $u(\pi,t) = 0$

$u(\pi,t) = (A\cos p\pi + B\sin p\pi)\cdot e^{-p^2t}$

$0 = B\sin p\pi\, e^{-p^2t}$

$\sin p\pi = 0$

$\sin p\pi = \sin n\pi$

$p\cancel{\pi} = n\cancel{\pi}$

$\boxed{p = n}$ where $n = 1, 2, 3.$

∴ The most G.S. of ① with bounde condition is

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \boxed{B_n \sin nx\, e^{-n^2 t}}$$

use

$u(x,0) = 3\sin 2x.$

$3\sin 2x = \sum B_n \sin nx$

$$B_n = \frac{2}{l}\int_0^l 3\sin 2x \cdot \sin nx\,dx$$

$$= 2\int_0^{\pi} 3\sin 2x \cdot \sin nx\,dx.$$

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$$\int_0^{1} \sin nx \cdot \sin nx \, dx$$

$$\left[\frac{\sin((n-2)x)}{2(n-2)} - \frac{\sin((n+2)x)}{2(n+2)}\right]_0^{1} + c$$

$$\left[\frac{\sin(n-2)}{2(n-2)} - \frac{\sin(n+2)}{2(n+2)}\right],$$

$\therefore$ The ~~req~~ sol^n is

$$u(x,t) = \frac{4\pi}{\pi} \sin x \, e^{-}$$

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(29) $u_{tt} - 4u_{xx} = 0$

~~$u(x,0) = \sin x$~~

Assume $u(x,t)$ is separable

$u(x,t) = X_{(x)} \cdot T_{(t)}$

diff u.w.r.t t & x twice.

$u_{xx} = T X''$

$u_{tt} = X \ddot{T}$

$X\ddot{T} - 4TX'' = 0.$

$X\ddot{T} = 4TX''$

$\frac{1}{4}\frac{\ddot{T}}{T} = \frac{X''}{X} = k$

$\frac{1}{4}\frac{\ddot{T}}{T} = k$

$\ddot{T} - 4Tk = 0$

AE $= m^2 - 4k = 0$

$m^2 = 4k$

$m = \pm 2\sqrt{k}$

$\frac{X''}{X} = k$

$X'' - kX = 0$

A.E $m^2 - k = 0$

$m^2 = \sqrt{k}$

$m = \pm\sqrt{k}$

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$$K = -p^2$$

$m = \pm 2\sqrt{-p^2}$ | $m = \pm\sqrt{-p^2}$

$T = c_3 \cos 2pt + c_4 \sin 2pt$ | $c_1 \cos px + c_2 \sin px$

G.S. $u(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos 2pt + c_4 \sin 2pt)$

~~(ii) $u(0,t) = \sin x$~~

(i) $u(x,0) = \sin x$

$u(x,0) = (c_1 \cos px + c_2 \sin px)\, c_3$

$\sin x = c_3 X(x)$

$c_3 = \sin x$

(ii) u_x

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Q.4) (i) $4\frac{\delta u}{\delta t} + \frac{\delta u}{\delta y} = u, \quad u(0,y) = 4e^{-3y}$

$$\frac{\delta u}{\delta y} = -4\frac{\delta u}{\delta t} + u$$

Assume $u(y,T)$ is separable.

$$u(y,T) = y(y)T(t)$$

diff wrt x & y once.

$$\cancel{x'y = -4xT' + xT}$$

$$(x'-x) \quad y'T = yT$$

$$y'T = -4yT' + yT$$

$$(y'-y)T = -4yT'$$

$$\frac{y'-y}{y} = -4\frac{T'}{T}$$

Solving

$$\frac{y'-y}{y} = K$$

$y' - y = ky$

$y' = ky + y$

$y' = y(k+1)$

$\frac{y'}{y} - 1 = k$

$\frac{y'}{y} = k+1$

suppose $k+1 = 2$

$\frac{y'}{y} = 2$

$y' - y2 = 0$

AE

$m - 2 = 0$

$m = 2$

$m = (k+1)$

$g(y) = c_1 e^{(k+1)y}$

$-4\frac{T'}{T} = k$

$-4T' = Tk$

$4T' + Tk = 0$

A.E $4m + k = 0$

$4m = -k$

$m = -k/4$

$T(t) = c_2 e^{-k/4 t}$

$u(x, y) = c_1 e^{(k+1)y} \cdot c_2 e^{-k/4 t}$

$u(0, y) = c_1 e^{(k+1)y} \cdot c_2$

$4e^{-3y} = c_1 c_2 e^{(k+1)y}$

$c_1 c_2 = 4$, $k+1 = -3$

$k = -4$

$$u(y,t) = 4e^{(-(k+1)y + \frac{k}{4}t)}$$

$$= 4e^{-3y+t}$$

$$u(y,t) = 4e^{-3y+t}$$

(ii)

$$u(0,y) = c_1c_2e^{(k+1)y}$$

$$4e^{-3y} - 2e^{y} + 5e^{-2y} = c_1c_2e^{(k+1)y}$$

$$e^{y}(4e^{-2y} - 2 + 5e^{-y}) = c_1c_2e^{(k+1)y}$$

$$\cancel{e^{y}}(4e^{-2y} - 2 + 5e^{-y}) = c_1c_2 \cdot e^{ky} \cdot \cancel{e^{y}}$$

$$4e^{-2y} + 5e^{-y} - 2 = c_1c_2 \cdot e^{ky}$$

$$e^{y}(4e^{-y} + 5e^{0} - 2) = c_1c_2e^{ky}$$

$$\cancel{e^{y}}(4e^{-y} + 5 - 2) = c_1c_2e^{k\cancel{y}}$$

$$4e^{-y} + 3 = c_1c_2e^{k}$$

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(ii) $u(0,y) = c_1c_2 e^{(k+1)y}$

$4e^{-3y} - 2e^{y} + 8e^{-2y} = c_1c_2e^{(k+1)y}$

$e^{-2y}\left(4e^{-y} - 2e^{y} + 5\right) = c_1c_2e^{ky}e^{y}$

$4e^{-3y} + 5e^{-2y} = c_1c_2e^{(k+1)y} + 2e^{y}$

$= c_1c_2e^{ky}e^{y} + 2e^{y}$

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(iii) $u(0,y) = c_1c_2 e^{(k+1)y}$

$$4e^{-3y} - 2e^{y} + 5e^{-2y} = c_1c_2 e^{(k+1)y}$$

$$e^{-2y}\left(4e^{-y} - 2e^{y} + 5\right) = c_1c_2 e^{ky} \cdot e^{y}$$

$$4e^{-3y} + 5e^{-2y} = c_1c_2 e^{(k+1)y} + 2e^{y}$$

$$= c_1c_2 e^{ky} \cdot e^{y} + 2e^{y}$$

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(25)
$$\frac{\delta^2 u}{\delta t^2} = c^2 \frac{\delta^2 u}{\delta x^2}$$

$0 < x < l \ , \ t > 0$

$u(0,t) = u(l,t) = 0$

$u(x,0) = x(l-x) \ , \ \frac{\delta u}{\delta t}(x,0) = 0$

Assume u(t, x) is separable.

$u(t,x) = T(t) \cdot X(x)$

diff w.r.t t and x twice.

$u_{tt} = T'' \cdot X$

$u_{xx} = \ddot{X} T$

$T'' X = c^2 \ddot{X} T$ = k constant

$$\frac{T''}{T} = c^2 \frac{\ddot{X}}{X} = K$$

$T'' - TK = 0$

A.E = $m^2 - K = 0$

$m^2 = k$

$m = \pm \sqrt{K}$

$c^2 \ddot{X} - kX = 0$

A.E = $c^2 m^2 - k = 0$

$m^2 = \frac{k}{c^2}$

$m = \pm \frac{\sqrt{k}}{c}$

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$\sqrt{-p^2}$ pi $T \to$ R

~~$T_T = A e^{\sqrt{-1}p^2 t} + B e^{-\sqrt{kc} t}$~~ | ~~$x_x = C e^{\sqrt{k}x} + D e^{-\sqrt{k}x}$~~

$i^2 = -1$

$\sqrt{-p^2}$
$\sqrt{p^2 i^2}$
$= \pm pi$

$k = -p^2$

$\frac{x''}{x} = -p^2$
$m^2 + p^2 = 0$
$m^2 = -p^2$
$m = \pm\sqrt{-p^2}$

$m = \pm\sqrt{-p^2}$

$T_T = A\cos pct + B\sin pct$ | $x_x = C\cos pcx +$

$A\cos px + B\sin$

$\frac{x''}{x}$

$x'' + p^2 x = 0$

$x'' + p^2 = 0$

$m^2 + p^2 = 0$

$T'' x = c^2 x'' T$

$\frac{x''}{x} = \frac{1}{c^2}\frac{T''}{T} = k$

$x'' - kx = 0$

$m = \pm\sqrt{k}$

$T'' - c^2 kT = 0$

$m^2 - c^2 k = 0$

$m = \pm c\sqrt{k}$

take $k = -p^2$

$x'' + p^2 x = 0$ & $T'' + c^2 p^2 T = 0$

$x_x = c_1\cos px + c_2\sin px$ & $T_t = c_3\cos pct + c_4\sin pct$

GS $= u(x,t) = (c_1\cos px + c_2\sin px)\cdot(c_3\cos pct + c_4\sin pct)$

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$u(0,t) = c_1(c_3\cos pct + c_4\sin pct)$

$\boxed{0 = c_1}$

Trivial soln

$u(l,t) = (c_1\cos pl + c_2\sin pl)(c_3\cos pct + c_4\sin pct)$

$u_n(x,t)$ $\quad 0 = c_2\sin pl\,(c_3\cos pct + c_4\sin pct)$

$\sin pl = 0$

$\sin pl = \sin n\pi$

$pl = n\pi$

$p = \frac{n\pi}{l}$ $\quad$ ~~n=~~ $n = 1, 2, 3 \ldots$

hence

$u_n(x,t) = c_2\sin\frac{n\pi}{l}\cancel{x}\,(c_3\cos pct + c_4\sin pct)$

$u_n(x,t) = c_2\sin\frac{n\pi}{l}(c_3\cos pct + c_4\sin pct)$

by superposition principal -

$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t)$

$u(x,t) = \sum_{n=1}^{\infty}\left(B_n\cos\frac{n\pi ct}{l} + B_n^*\sin\frac{n\pi ct}{l}\right)\sin nx$

Use $u(x,0)$

$\sin x = cB_1^* \sin x$

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$$x(l-x) = \sum_{n=1}^{\infty} B_n \sin n\pi$$

$$B_n = x(l-x)$$

$$\therefore \quad u(x,t) = \sum \left(B_n^* \sin \frac{n\pi ct}{l}\right) \sin n\pi x$$

use. 2nd I.C $\quad \frac{\partial u}{\partial t}(x,0) = 0$

$$\frac{\partial u}{\partial t} = \sum_{n=1}^{\infty} \left(B_n^* \cos \frac{n\pi ct}{l} \times \frac{n\pi}{l} c\right) \sin n\pi$$

$$0 = \sum_{n=1}^{\infty} \frac{n\pi}{l} c B_n^* \sin n\pi$$

$$0 = cB_1^* \sin x + 2cB_2^* \sin 2x + \dots$$

$$\sin x = cB_1^* \sin x + 2cB_2^* \sin 2x \dots$$

$$B_1^* = 1/c, \quad B_2^* = 0, \quad B_3^* = 0$$

$$u(x,t) = \frac{1}{c} \sin ct \sin x.$$

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(2) $\frac{\delta u}{\delta t} = c^2 \frac{\delta^2 u}{\delta x^2}$

Assume $u(x,t)$ is separable.

~~Diff $u(x,t)$~~

$u(x,t) = x_x \cdot T(t)$

~~diff~~ diff u w.r.t x twice and t once

$u_{xx} = X''T \quad , \quad u_t = X\dot{T}$

$X\dot{T} = c^2 X''T$

$\frac{1}{c^2}\frac{\dot{T}}{T} = \frac{X''}{X}$

$\frac{X''}{X} = K$

$X'' = KX = 0$

A.E $m^2 - K = 0$

$m^2 = K$

$m = \pm\sqrt{K}$

$K = -\sqrt{P^2}$

$m = \pm\sqrt{-p^2}$

$X(x) = A\cos px + B\sin px$

$\frac{1}{c^2}\frac{\dot{T}}{T} = K$

$\dot{T} - c^2TK = 0$

A.E $m - c^2K = 0$

$m = -c^2p^2$

$K = -p^2$

~~$T = C\cos c^2pt + D\sin c^2pt$~~

$T(t) = Ce^{-c^2p^2t}$

e^{-c^2p}

G.S: $u(x,t) = (A\cos px + B\sin px)\,(\cancel{C\cos c^2pt + D\sin c^2pt})$

$\cancel{u(0,t) = A(C\cos c^2pt + D\sin c^2pt)}$

$\cancel{0 = A(\cos c^2pt + D\sin c^2pt)}$

$\boxed{\cancel{A = 0}}$

$0 = ACe^{-c^2p^2t}$

$\boxed{A = 0}$

$\cancel{u(\pi,t) = 0}$

$\cancel{u(\pi,t) = -A \cdot C}$

$0 = -AC$

$AC = 0$

$\boxed{C = 0}$

$u(\pi,t) = 0$.

$0 = B\sin p\pi \cdot Ce^{-p^2c^2t}$

$\sin p\pi = 0$

$\sin p\pi = \sin n\pi$

$\boxed{p = n}$, $n = 1, 2, 3, 4, \ldots\ldots$

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G.S. of ① with Bound condition is

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t).$$

$$= \sum_{n=1}^{\infty} B_n \sin nx\, e^{-n^2c^2t}$$

$u(x,0) = T$

$$T = \sum B_n \sin nx$$

$$B_n = \frac{2}{\pi}\int_0^{\pi} T \sin nx\, dx$$

$$B_n = \frac{2}{\pi} T \int_0^{\pi} \sin nx\, dx$$

$$= \frac{2}{\pi} T \left[-\frac{\cos nx}{n}\right]_0^{\pi}$$

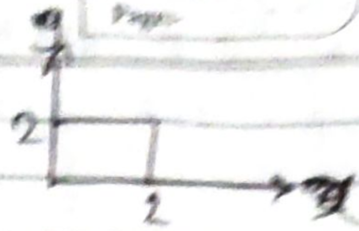
$$= \frac{2}{n\pi} T \left[1 - \cos n\pi\right]$$

$$= \frac{2T}{n\pi}\left[1-(-1)^n\right]$$

$\therefore$ The soln is

$$u(x,t) = \frac{4T}{\pi}\sin x\, e^{-c^2t} + \frac{4T}{3\pi}\sin 3x\, e^{-9c^2t}$$

(2) $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0$



$u(x,0) = 0$

$u(0,y) = 0$

$u(2,y) = 0$

$u(x,2) = 100 \sin \frac{\pi}{2} x$

Assume that u is separable.

$u(x,y) = X_x \cdot Y_y$

Diff u w.r.t to x & y twice.

$u_{xx} = x'' \cdot y$

$u_{yy} = x \cdot y''$

$x''y + xy'' = 0$

$x''y = -xy'' = k$

$\frac{x''}{x} = k$

$x'' - kx$

$m^2 - k = 0$

$m^2 = k$

$m = \pm\sqrt{k}$

$m = \pm\sqrt{p^2}$

$X_x = (A\cos px + B\sin px)$

$\frac{y''}{y} = -k$

$y'' + ky = 0$

AE: $m^2 + k = 0$

$m^2 = -k$

$m^2 = \sqrt{-k}$

$m^2 = +p^2$

$m = \pm p$

$Y_y = Ce^{py} + De^{-py}$

G.S. $u(x,y) = (A\cos px + B\sin px) \cdot (Ce^{py} + De^{-py})$

use. $u(x, 0)$

$0 = (A\cos px + B\sin px) \cdot (C + D)$

$C + D = 0$

$\boxed{C = -D}$ or $\boxed{D = -C}$

$u(0, y)$

(ii) $u(0, y) = (A\cos p0 + B\sin p0)(Ce^{py} + De^{-py})$

$0 = A \cdot (Ce^{py} + De^{-py})$

$\boxed{A = 0}$

(iii) $u(2, y) = 0$

$u(2, y) = B\sin p \times 2$

$0 = B\sin p2$

$p2 = n\pi$

$p = n\pi/2$, $n = 1, 2, 3, 4 \ldots$

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$\therefore$ the solⁿ is $u(x,y) = B \sin \frac{n\pi}{2} x (c e^{py} - c e^{-py})$

$$u_n(x,y) = B_n \sin \frac{n\pi}{2} x \sinh \frac{n\pi}{2} y$$

$$\therefore \quad u(x,y) = \sum_{n=1}^{\infty} B_n \sinh \frac{n\pi}{2} y \cdot \sin \frac{n\pi}{2} x$$

(iv) use $u(x,2) = 1000 \sin \frac{\pi}{2} x$

$$1000 \sin \frac{\pi}{2} x = \sum_{n=1}^{\infty} B_n \sinh n\pi \cdot \sin \frac{n\pi}{2} x$$

$$= B_1 \sinh \pi \sin \frac{\pi}{2} x + B_2 \sinh 2\pi \sin \pi x$$

On comparing

$$B_1 \sinh \pi = 1000, \; B_2 = 0, \; B_3 = 0$$

$$B_1 = \frac{1000}{\sinh \pi}$$

$$u(x,y) = \frac{1000}{\sinh \pi} \sinh \frac{\pi}{2} y \sin \frac{\pi}{2} x.$$

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(28) $\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2}$

Assume $y(x,t)$ is separable.

$y(x,t) = X_x T_t$

Diff y w.r.t x & t twice.

$y_{xx} = X'' T \quad , \quad y_{yy} = T'' X$

$T'' X = c^2 X'' T$

$\frac{1}{c} \frac{T''}{T} = k$

$T'' - cTk = 0$

A.E $m^2 - c^2 k = 0$

$m^2 = c k$

$m = \sqrt{c^2 k}$

$m = \pm c\sqrt{k}$

$k = -p^2$

$m = \pm c\sqrt{-p^2}$

$T = c\cos pct + D\sin pcT$

$\frac{X''}{X} = k$

$X'' - kX = 0$

A.E $= m^2 - k = 0$

$m^2 = k$

$m = \pm\sqrt{k}$

$m = \pm\sqrt{-p^2}$

$X = A\cos px + B\sin px$

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G.S: $u(x,t) = (A\cos px + B\sin px)(C\cos pct + D\sin pct)$

$u(0,t) = A(C\cos pcT + D\sin pcT)$

$0 = A(C\cos pcT + D\sin pcT)$

$\boxed{A=0}$

$u(l,t) = 0$

$u(l,t) = B\sin pl\,(C\cos pcT + D\sin pcT)$

$0 = B\sin pl\,(C\cos pcT + D\sin pcT)$

$\sin pl = 0$

$\sin pl = \sin n\pi$

$pl = n\pi$

$p = \frac{n\pi}{l}$

The mo of G.S of (1) with Bound cond.

$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t)$

$\cancel{\sum_{n=1}^{\infty} B_n \cos \frac{n\pi}{l}}$

$u_n(x,t) = \cancel{B\sin} \; B\sin \frac{n\pi x}{l}\left(C\cos \frac{n\pi}{l} cT + D \sin \frac{n\pi c T}{l}\right)$

$\sin\frac{3\pi x}{l} - \sin\frac{\pi x}{l}$

$\sin$

$\frac{\sin 3\pi x}{l}$

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~~~~

$$u(x,t) = \sum_{n=1}^{\infty}\left(B_n\cos\frac{n\pi}{l}cT + B_n^{*}\sin\frac{n\pi}{l}cT\right)\sin\frac{n\pi}{l}x$$

$u(x,0)$

$$u(x,0) = \cancel{B_n\cos\frac{n\pi}{l}}\; B_n\sin\frac{n\pi}{l}x$$

$$\sin\frac{\pi x}{l} - 5\sin\frac{3\pi x}{l} = B_n\sin\frac{n\pi}{l}x$$

$$\cancel{} - 5\sin\frac{3\pi x}{l} = B_n\sin\frac{n\pi x}{l} - \sin\frac{\pi x}{l}$$

$$\frac{\sin\pi x - 5\sin 3\pi x}{\cancel{l}} = B_n\sin\frac{n\pi x}{\cancel{l}}$$

$\sin\pi x - \sin 3\pi x$

$\sin \pi x$

$$\sin\pi - 5\sin 3\pi = B_n\sin n\pi$$

$$\boxed{B_n = 0}$$

$u(\cancel{x,t}) =$

$$\frac{du}{dt}(x,0) = 0.$$
~~~~

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$$\cancel{\sin x} = \frac{dy}{dt} = \sum_{n=1}^{\infty} \left(B_n^* \sin \frac{n\pi}{l} ct \right) \sin \frac{n\pi x}{l}$$

$$\frac{dy}{dt} = \sum_{n=1}^{\infty} B_n^* \cos \frac{n\pi}{l} ct \times \frac{n\pi}{l} c \Big) \sin \frac{n\pi x}{l}$$

$$\frac{dy}{dt}(x, 0) = \sum_{n=1}^{\infty} B_n^* \sin \frac{n\pi x}{l}$$

$$0 = \sum_{n=1}^{\infty} B_n^* \sin \frac{n\pi x}{l}$$

$$0 = B_1^* \sin \frac{\pi x}{l} + B_2^* \sin \frac{2\pi x}{l}$$

$$B_1^* = 0 \quad , \quad B_2^* = 0 \quad \ldots$$

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(29)
$$\frac{\delta u}{\delta t} = c^2 \frac{\delta^2 u}{\delta x^2}$$

$0 < x < l,\ t > 0,\ u(0,t) = u(l,t) = 0,$
$u(x,0) = x$

$\Rightarrow$ Assume $u(x,t)$ is separable -

$u(x,t) = X_x\, T_t$

$u_{xx} = X'' T \quad , \quad u_t = T' X$

$T' X = c^2 X'' T$

$$\frac{1}{c^2}\frac{T'}{T} = \frac{X''}{X}$$

$\frac{1}{c^2}\frac{T'}{T} = K$ | $\frac{X''}{X} = K$

| $X'' - KX = 0$

A.E = $T' - c^2 T K = 0$ | A.E $m^2 - K = 0$

$m - c^2 K = 0$ | $m^2 = K$

$m = c^2 K$ | $m = \pm\sqrt{K}$

$u_t = e^{c^2 K t}$ | $u_x = A\cos px + B\sin px$

$u(x,t) = (A\cos px + B\sin px)\, e^{c^2 p^2 t}$

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$u(0,t) = A \cdot c\, e^{c^2p^2t}$

$0 = A \cdot c\, e^{c^2p^2t}$

$\boxed{A=0}$

$u(l,t) = B \sin pl \cdot c\, e^{c^2p^2t}$

$0 = B \sin pl \cdot c\, e^{c^2p^2t}$

$\sin pl = \sin n\pi$

$pl = n\pi$

$p = \frac{n\pi}{l}$

The most G.S of (1) with bond condition.

$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t)$

$= \sum_{n=1}^{\infty} B_n \sin \frac{n\pi}{l} x \cdot e^{c^2\frac{n^2\pi^2}{l^2}T}$

$u(x,0) = x$ constant.

$$x = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi}{l} x$$

$$B_n = \frac{2}{l} \int_0^l x \cdot \sin \frac{n\pi}{l} x$$

$$= \frac{2}{l} x \int_0^l \sin \frac{n\pi}{l} x$$

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$$= \frac{2}{\cancel{l}} x \frac{\cancel{l}}{n\pi}\left[-\cos \frac{n\pi}{l} x\right]_0^l$$

$$B_n = +\frac{2x}{n\pi}\left[1 - \cos n\pi\right]$$

$$B_n = \frac{2x}{n\pi}\left(1-(-1)^n\right)$$

$B_n = n = 1, 2 \ldots$

C.S $= \frac{4x}{\pi} \sin x l \cdot e^{-\frac{c^2\pi^2 t}{l^2}} + \frac{4x}{3\pi} \sin 3x l \; e^{-\frac{9c^2\pi^2}{l}}$

(5)

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2}, \cancel{0 < x < l,\ t > 0}$$

Assume $y(x,t)$ is separable -

$$y(x,t) = XT$$

$$y_{x} = X''T, \quad y_{TT} = T''X$$

$$y(x,t) = (A\cos px + B\sin px)(C\cos pcT + D\sin pct)$$

$$y(0,t) = A.(C\cos pct + D\sin pct)$$

$$0 = A.(C\cos pct + D\sin pct)$$

$$\boxed{A = 0}$$

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$$y(l,t) = B\sin pl\,(C\cos pct + D\sin pct)$$

$$0 = B\sin pl\,(C\cos pct + D\sin pct)$$

$$\sin pl = \sin n\pi$$

$$pl = n\pi$$

$$\boxed{p = \frac{n\pi}{l}}$$

$$y_n(x,t) = \sin\frac{n\pi}{l}x\left(B_n\cos\frac{n\pi}{l}ct + B_n^*\sin\frac{n\pi}{l}ct\right)$$

$$y(x,0) = \sin\frac{n\pi}{l}x\,(B_n)$$

$$y(x,0) = B_n\sin\frac{n\pi}{l}x$$

$$x(l-x) = B_n\sin\frac{n\pi}{l}x$$

$$B_n = x(l-x)$$

$$y(x,t) = \sum\left(B_n^*\sin\frac{n\pi ct}{l}\right)\sin\frac{n\pi}{l}x$$

$$\frac{dy}{dt} = \sum_{n=1}^{\infty}\left(B_n^*\frac{n\pi}{l}\cos\frac{n\pi}{l}ct\right)\sin\frac{n\pi}{l}x$$

$$0 = \sum_{n=1}^{\infty}\left(B_n^*\frac{nc\pi}{l}\cdot\cos\frac{n\pi}{l}ct\right)\sin\frac{n\pi}{l}x$$

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$0 = \frac{c\pi}{l} B_1^* \sin\frac{\pi x}{l} + 2\frac{c\pi}{l} B_2^* \sin\frac{2\pi x}{l}.$

$\frac{c\pi}{l} B_1^* \sin\frac{\pi x}{l} = \sin\frac{\pi x}{l}$

$B_1^* = \frac{l}{\pi c} \cdot \frac{\sin\pi}{\sin\frac{\pi x}{l}}$

(34) $\frac{dy}{dt} = c^2 \frac{\delta^2 y}{\delta x^2}$

Assume $y(x,t)$ is separable.

$y(x,t) = X, T$

$y_t = XT'$, $y_{xx} = X''T$

$XT' = c^2 X'' T$

$\frac{1}{c^2} \cdot \frac{T'}{T} = k$	$\frac{X''}{X} = k$
$T' - c^2 T k = 0$	$X'' - kX = 0$
$m = c^2 k$	$m^2 - k = 0$
$m = -c^2 p^2$	$m^2 = k$
	$m = \pm\sqrt{k}$
	$m = \pm\sqrt{-p^2}$

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$G.S = (A\cos px + B\sin px) \cdot ce^{-c^2p^2t}$

$u(x,t) = (A\cos px + B\sin px) \cdot ce^{-c^2p^2t}$

$u(0,t) = A \cdot ce^{-c^2p^2t}$

$0 = A \cdot ce^{-c^2p^2t}$

$\boxed{A = 0}$

$u(\pi, t) = 0$

$u(\pi, t) = \cancel{A\cos} \; B\sin p\pi \cdot ce^{-c^2p^2t}$

$0 = B\sin p\pi \cdot ce^{-c^2p^2t}$

$\sin p\pi = 0$

$\sin p\pi = \sin n\pi$

$p\cancel{\pi} = n\cancel{\pi}$

$\boxed{p = n}$

$u(x,t) = \sum_{n=1}^{\infty} B_n \sin nx \cdot ce^{-c^2p^2t}$

$u(x,0) = \sum_{n=1}^{\infty} B_n \sin nx \cdot ce^{-c^2p^2t}$

$T = \sum_{n=1}^{\infty} cB_n \sin nx$

$\cancel{B_n = Tc\int_0^{\pi} \sin x}$

$B_n = Tc\left[\cos x\right]_0^{\pi}$

$B_n = Tc[\cos\pi - \cos 0]$

$B_n = Tc[-1-1]$

$B_n = -2Tc$

$$B_n = \frac{2}{\pi}\int_0^{\pi} T \sin n\alpha$$

$$B_n = \frac{2T}{\pi}\int_0^{\pi} \sin nx$$

$$= \frac{2T}{\pi}\left[-\cos n\pi + \cos 0\right]$$

$$B_n = \frac{2T}{\pi}(1-\cos n\pi)$$

$n = 1$

$$B_1 = \frac{4T}{\pi}, \quad B_3 = \frac{4T}{3\pi}$$

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(18) $u(0,t) = 0$, $u(\pi,t) = 0$ and
$u(x,0) = \cos x \sin 5x$

$$u_t = u_{xx}$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Assume $u(x,t)$ is separable.

$$u(x,t) = X_x \cdot T_t$$

$$u_t = XT', \quad u_{xx} = X''T$$

$$XT' = X''T$$

$$\frac{X''}{X} = k \qquad \frac{T'}{T} = k$$

$$m^2 - k = 0 \qquad m - k = 0$$

$$m = \pm\sqrt{k} \qquad m = k$$

$$m = \pm\sqrt{-p^2} \qquad m = -p^2$$

$X_x = A\cos px + B\sin px$ | $T_t = Ce^{-p^2 t}$

$$u(x,t) = (A\cos px + B\sin px)\, Ce^{-p^2 t}.$$

$$u(0,t) = A \cdot Ce^{-p^2 t}$$

$$\boxed{A = 0}$$