(iii) Every clever student is successful. $\forall x\,(C(x) \rightarrow S(x))$

(iv) There are some successful students who are not clever. $\forall x\,(S(x) \wedge \neg C(x))$

(v) Some students are clever and successful. $\exists x\,(C(x) \wedge S(x))$

Question Bank :—

① (a) $2+3=5$ (True implies proposition).
(b) Answer this question. (not declarative implies not a proposition)

② well formed formula (wff)

③ well formed formula

p	q	$p \rightarrow q$
T	T	T
T	F	~~F~~ T
F	F	T
F	T	F

④ Biconditional are true or false
(a) $2+2=4$ if and only if $1+1=2$.
$\Rightarrow$ T and T = T
(b) $1+1=2$ if and only if $2+3=4$
$\Rightarrow$ T and f = f

⑤ (a) $1+1=3$ if and only if monkey can fly
$\Rightarrow$ f and f = T
(b) $0>1$ if and only if $2>1$
$\Rightarrow$ f and T = f

⑥ Conditional statement is true or false
(a) If $1+1=2$, then $2+2=5$
$\Rightarrow$ T and f = F
(b) If $1+1=3$, then $2+2=4$
$\Rightarrow$ F and T = T

⑦ (a) If $1+1=3$, then $2+2=5$
$\Rightarrow$ F and F = T
(b) If monkey can fly then $1+1=3$
$\Rightarrow$ F and F = T

Date

(8) State the converse, contrapositive and inverse
· If it snows today, then I will ski tomorrow.

Converse: → If I will ski tomorrow, then it snows today.
Contra positive: → If I will not ski tomorrow, then it does not snow today.
Inverse: → If it does not snow today, then I will not ski tomorrow.

(9) I come to class whenever if there is going to be a quiz.

Converse: → If there is going to be a quiz, then I come to class.
Contra positive: → If there is not going to be a quiz, then I'm not come to class
Inverse: → · If I'm not come to class then there is not going to be a quiz.

(10) A positive integer is a prime only if it has no divisors other than 1 and itself.

Converse: If it has no divisors other than 1 and itself, then a positive integer is a prime.
Contra positive: → If it has divisor other than 1 and itself, then a positive integer is not a prime.
Inverse: → If a positive integer is not a prime then it has divisor other than 1 and itself.

(11) Sol. $P(x) : x \leq 4$ (given)

Then $P(4) : 4 \leq 4$ --- (True)
$P(6) : 6 \leq 4$ --- (False).

(12) Sol: $P(x) : x = x^2$. (given)
Then $P(0) = 0 = 0$ (True)
~~P(x)~~

(b) $\forall x\, P(x)$

→ Since $p(x)$ is only true for 0 and 1 this implies $\forall x\, p(x)$ is false.

(13) $p(x) : x = x^2$ (given.

Then $p(1) : 1 = 1^2$ (True)

(b) $\exists$ existantential quantifier

$\exists x\, p(x)$ is true since it is true for 0 and 1.

(14) (15) (16) (17) (18)

(19) Given p = It is raining

q = It is cold

r = It is pleasant

(a) $(\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$ • option a is correct.

(20)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

(c) $(\neg p \wedge q) \vee (p \wedge \neg q)$

(21) ~~$(\neg p) \rightarrow (\neg q)$~~ (d) always True when q is True

p	q	r	$(p \rightarrow q)$	$(p \rightarrow q) \rightarrow r$	$r \rightarrow p$	$(r \rightarrow p) \rightarrow q$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	F	T	F
T	F	F	F	T	T	F
F	T	T	T	T	F	T
F	T	F	T	T	T	T
F	F	T	T	T	F	T
F	F	F	T	T	T	F

$\boxed{\leftrightarrow \; = \wedge \; = \text{And}}$

Date ________

(22)

p	q	$\neg p$	$\neg q$	$(\neg p) \rightarrow (\neg q)$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(I.) $p \rightarrow q$	II. $q \rightarrow p$	III $(\neg q) \vee p$	IV. $(\neg p) \vee q$
T	T	T	T
F	T	T	F
T	F	F	T
T	T	T	T

(d) II and III only.

(23) (c) If a people is kind, he is not known to be corrupt.

(24) Given $p: x \in \{8, 9, 10, 11, 12\}$

q: x is a composite number. $\{4, 6, 8, 9, 10\}$

r: x is a perfect square. $\{4, 9, 16, 25, 36\}$

s: x is a prime number. $\{2, 3, 5, 7, 11\}$

(25) (26) (27)

(28)

p	q	$\neg p$	$\neg q$	$(p \rightarrow q)$	$(q \rightarrow p)$	$\neg q \rightarrow \neg p$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	T
F	F	T	T	T	T	T

(28) (a) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$ | (b) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

(a)	(b)
T	T
T	T
T	F
T	T

Mathematics

(23) (c) If a person is kind, he is not known to be corrupt.

(24) $p: x \in \{8,9,10,11,12\}$
$q: x \in \{8,9,10,12\}$
$r: x \in \{9\} \Rightarrow \neg r: x \in \{8,10,11,12\}$
$s: x \in \{11\} \Rightarrow \neg s: x \in \{8,9,10,12\}$

Now, $(p \to q) = \{8,9,10,12\}$
then $(\neg r \vee \neg s) =$ Union $= \{8,9,10,11,12\}$

$(p \to q) \wedge (\neg r \vee \neg s) =$ intersection of comon number $= \{8,9,10,12\}$

$\neg((p \to q) \wedge (\neg r \vee \neg s)) = 11$ An.

(25) (26)

(27) (d) $\exists x$: glitters$(x) \wedge \neg$gold(x)

(29)

p	q	$p \vee q$	$p \oplus q$	$(p \vee q) \to (p \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	F

Boolean algebra
$\oplus \to$ दोनो T hoga to F agar ek T hoga to True agar dono F hoga to F.

(30) (a) p = "Jasbir is rich", q = "Jasbir is happy"
Then proposition Jasbir is rich and happy can be written as $p \wedge q$. Then, according to De Morgan's Law
$\neg(p \wedge q) = \neg p \vee \neg q$. Or, by words.
Jasbir is not rich or not happy or both.

(b) Rajan will bicycle or run tomorrow.

p= "Rajan will bicycle", q= "Rajan will run tomorrow"

Then proposition Rajan will ~~not~~ bicycle or ~~not~~ run tomorrow. can be written as $p \vee q$. Then according to De-morgan's law

$\neg(p \vee q) = \neg p \wedge \neg q$ in words.

Rajan will neither bicycle nor run tomorrow.

(27) (a)

p	q	$\neg p$	$\neg q$	$\neg(p \leftrightarrow q)$	$p \leftrightarrow \neg q$	$\neg p \leftrightarrow q$	$\neg p \leftrightarrow q$
T	T	F	F	F	F	T	F
T	F	F	T	T	T	F	T
F	T	T	F	T	T	F	T
F	F	T	T	F	F	T	F

($\leftrightarrow$) or $\Leftrightarrow$ denote usually the equivalence, commonly known also as "NXOR", "if and only if" or "iff" for short.

$p \leftrightarrow q$ is equal to $(p \rightarrow q) \wedge (q \rightarrow p)$ or $(p \wedge q) \vee (\neg p \wedge \neg q)$

and truth table is

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

(27) (a) logically equivalent

(b) logically equivalent

(c) logically equivalent.

(30) Two compound proposition p and q are logically equivalent if and only if p logically implies q and q logically implies p. In other words: two compound proposition are logically equivalent if and only

if they have same truth ~~value~~ table. Using truth table to prove that $(p \rightarrow q)$ and $\neg p \vee q$ are logically equivalent.

32) (c)

$(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$
F
F
F
F

* Dural

33) (a) $p \wedge \neg q \wedge \neg r$ (b) $(p \wedge q \wedge r) \vee s$ (c) $(p \vee F) \wedge (q \vee T)$

f(a, b, c, ..., AND, OR, T, F

⇓ DUAL

f(a, b, c, ..., OR, AND, F, T

(a) → $p \vee \neg q \vee \neg r$ (b) $(p \vee q \vee r) \wedge s$ (c) $(p \wedge T) \vee (q \wedge F)$

(34) Disjunctive Normal Form (DNF) ~~p(vq~~ $(p \vee q) \rightarrow \neg r$

$= (p \vee q) \longrightarrow \neg r$

$= \neg(p \vee q) \vee \neg r$

$= (\neg p \wedge \neg q) \vee \neg r$

34
35
36

(35) $(\neg p \rightarrow r) \wedge (p \leftrightarrow q)$

→ $(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$

$\wedge \vee (\wedge)$

$\boxed{p \rightarrow q \Rightarrow \neg p \vee q}$

$\boxed{p \leftrightarrow q \Rightarrow (p \wedge q) \vee (\neg p \wedge \neg q)}$

$(\lor) \land (\lor)$

$D = (\lor)$
$C = (\land)$

(37) Conjunctive Normal form

$\neg(p \to q) \lor (r \to p)$

$\Rightarrow \neg(\neg p \lor q) \lor (\neg r \lor p)$

$\Rightarrow (p \lor \neg q) \lor (\neg r \lor p)$

37, 38, 39

(40) (a) No one is perfect. $\Rightarrow \neg \exists x (P(x))$

(b) Not everyone is perfect $\Rightarrow \neg \forall x (P(x)$

(c) All your friends are perfect. $\Rightarrow \forall x (F(x) \to P(x))$

(d) At least one of your friend is perfect. $\Rightarrow \exists x (F(x) \land P(x))$

(e) Everyone is your friend and is perfect. $\Rightarrow \forall x (F(x) \land \forall x P(x)$

(f) Not everybody is your friend or some one is not perfect $\Rightarrow \neg \forall x f(x) \lor \exists x \neg P(x)$

(41) Let, p = "socrates is human"

q = "socrates is mortal".

$p \to q$

$\underline{p}$

$\therefore q$

This argument is valid because it is constructed by modus ponens which is a valid argument form. Thus, can conclude that a conclusion is true when the premises are true.

(42) p = "It is ~~not~~ sunny this afternoon" ~~and it~~

q = It is colder than yesterday.

r = We will go swimming only if it is sunny.

s = ~~If we do not go swimming~~ we take a canoe trip

t = we will be home by sunset.

$\neg p \land q$ premises simplification

$\neg p$

$r \to p$ or $\neg p \to r$ premises by modus ponens.

$\neg r$

$\neg r \rightarrow s$ premise
s by modus ponens

$s \rightarrow t$ premise
t result

Premises:- $\neg p \wedge q$, $r \rightarrow p$, $\neg r \rightarrow s$, $s \rightarrow t$
conclusion: t

(43) p = If you inves in the stock market.
q = you will get rich.
r = you will happy

⇒ Premises: $p \rightarrow q$, $q \rightarrow r$
Conclusion $p \rightarrow r$

proof: $p \rightarrow q$, $q \rightarrow r$ premises.
$p \rightarrow r$ Result
Conclusion is valid.

(44) Premises: p = Randy works hard
q = He is a dull boy
r = He will get job

= Premises: p, $p \rightarrow q$, $q \rightarrow \neg r$
Conclusion: $\neg r$

Proof:
p premises
$p \rightarrow q$ premises
q by modus pones
$q \rightarrow \neg r$ premise
$\neg r$ result
∴ Thus, the conclusion is valid

C = represent class.

(45) Let x = represents "is a student in this class".

W = know how to write program in JAVA

J - get a high-paying job.

Then the premises are: C(Danish), W(Danish), $\forall x\,(W(x) \to J(x))$

conclusion: Someone in the class get a high paying job

Proof: C(Danish), W(Danish), $\forall x\,(W(x) \to J(x))$ premises

From 3rd premise W(Danish) $\to$ J(Danish) by universal instantiation

Now from 2nd premise and the above result J(Danish) by modus ponens

Now from 1st premise and the above result C(Danish) $\wedge$ J(Danish) by conjuction

$\exists x\,(C(x) \wedge J(x))$ by existential generalization

conclusion is valid.

(46) (47) (48) (49) (50)

(51) Let p = $3n+2$ is odd, q = n is odd.

Assume that $\neg(p \to q)$ is true OR $(p \wedge \neg q)$ is true.

It means that both p AND $\neg q$ must be true.

$\neg q$ = n is not odd = n is even

$n = 2K$

$3n+2 = 6K+2$

$= 2(3K+1) = 2t$ is even

$\neg p$ is true

Thus, p and $\neg p$ are both true.