Logical Equivalences :-

1. Identity Laws :- $p \wedge T \equiv p$, $p \vee F \equiv p$
2. Domination Laws :- $p \vee T \equiv T$, $p \vee F \equiv F$
3. Idempotent Laws :- $p \vee p \equiv p$ & $p \wedge p \equiv p$
4. Double negation law :- $\neg(\neg p) \equiv p$
5. Commutative Laws :- $p \vee q \equiv q \vee p$

   $p \wedge q \equiv q \wedge p$

6. Associative Laws :- $(p \vee q) \vee r \equiv p \vee (q \vee r)$

   $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

7. Distributive Laws :- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

   Imp

   $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

8. De Morgan's Law :- $\neg(p \wedge q) \equiv \neg p \vee \neg q$

   $\neg(p \vee q) \equiv \neg p \wedge \neg q$

9. Absorption Laws :- $p \vee (p \wedge q) \equiv p$

   $p \wedge (p \vee q) \equiv p$

10. Negation Laws :- $p \vee \neg p \equiv T$

    $p \wedge \neg p \equiv F$

Q. Use De-Morgan's law to find negation of each of the following statements :-

(i) $p \wedge q$ Jasbir is rich & happy

By De-Morgan's Law $\neg(p \wedge q) = \neg p \vee \neg q$

$p \rightarrow$ Jasbir is rich

$q \rightarrow$ " " happy

Q. Show that $p \to q$ & $\neg p \vee q$ are logically equivalent

Q. Show that $(p \wedge q) \to (p \vee q)$ is a tautology

Soln:- $(p \wedge q) \to (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$

$\equiv (\neg p \vee \neg q) \vee (p \vee q)$

(by first De-Morgan's law)

$\equiv (\neg p \vee p) \vee (\neg q \vee q)$

(By ass. laws)

$\equiv T \vee T \equiv T.$

Hence, $(p \wedge q) \to (p \vee q)$ is a tautology.

Predicates & Quantifiers

The statement "x is greater than 3" has 2 part. The first part, the variable x, is the subject & the second part - the predicate "is greater than 3" → is refers to a property that the subject of the statement can have.

We can denote the statement "x > 3" by $P(x)$.

Where P → predicate, x → variable.

P(x) is said to be the value of the propositional fn P at x.

Once a value has been assigned to the variable x, the statement $P(x)$ becomes a proposition & has a truth value.

Ex: Let $P(x)$ denote the statement "$x>3$"
What are the truth values of $P(4)$ & $P(2)$?

Soln:- $P(4)$ is the statement "$4>3$" → true
$P(2)$ " " " "$2>3$" → false.

Ex:- Let $Q(x,y)$ be the statement "$x=y+3$"
What are truth values of the propn
$Q(1,2)$, $Q(3,0)$

Soln:- $Q(1,2)$ is the statement "$1=2+3$" → (F)
$Q(3,0)$ " " " "$3=0+3$" → (T)

When the variables ~~are~~ in a propositional fn are assigned values, the resulting statement becomes a propn with a certain truth value.

However, there is another imp. way to create a proposition from a propositional function, using some quantity words like all, some, many, none & few etc.
The words are called quantifiers & the process is called quantification.

We will focus on two types of quantification:-

(1) Universal quantification:- which tells us that a predicate is true for all elements under consideration.

(2) Existential quantification:- which tells us that there is one or more element under consideration for which predicate is T.

Many mathematical statements assert that a property is true for all values of a variable in a particular domain, called the domain of discourse (or domain)

Defn:- The universal quantification of $P(x)$ is the statement

"$P(x)$ is true for all values of x in the domain"

The notation $\forall x\, P(x)$ denotes the universal quantification of $P(x)$ for all x $P(x)$ or for every x $P(x)$.

An element for which $P(x)$ is false is called a counterexample of $\forall x\, P(x)$.

Ex:- $P(x) \to$ "$x+1 > x$". Truth value of the quan: $\forall x\, P(x)$, where domain consists of all real nos

Soln:- $\forall x\, P(x)$ for all x, $x+1 > x$

$\therefore$ The quan: ~~is~~ $\forall x\, P(x)$ is true

→ for all → for every, all of, for each, given any, forarb, for ~~ex~~ any.

Ex:- $Q(x)$ → "$x < 3$".

Value of quan. $\forall x\, Q(x)$, D → all real nos.

$Q(x)$ is not true for every real no, because for example, $Q(3)$ is false.

That is, $x = 3$ is a counter example for the statement $\forall x\, Q(x)$. Thus, $\forall x\, Q(x)$ is false.

Statement	When true	When false
$\forall x\, P(x)$	$P(x)$ is true for all x.	There is an x for which $P(x)$ is false.
$\exists x\, P(x)$	There is an x for which $P(x)$ is true	$P(x)$ is false for every x.

→ When all elts in the domain can be listed, say $x_1, x_2, \ldots, x_n$ it follows that the universal quantification $\forall P(x)$ is ~~true~~ the same as the conjn

$$P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \cdots \wedge P(x_n)$$ because this conjn is true iff $P(x_1), P(x_2), \ldots P(x_n)$ are all true.

Defn:- The existential quan. of $P(x)$ is the propn "There exists an elt x in the domain s.t $P(x)$"

~~the~~ Notation is $\exists x\, P(x)$

$\exists$ → existential quantifier

→ The existential quantification of $P(x)$ is the proposition "There exists an element x in the domain such that $P(x)$"

Notation is $\exists x P(x)$

↓

existential quantifier

- A domain must be specified when a statement $\exists x\, P(x)$ is used.

Some other words for existential Quf. is for some, for at least one, there is

e.g $P(x)$: "$x > 3$". The existential quantification $\exists x\, P(x)$ is there is an element x for which $x > 3$. So, this is a true statement.

When all elements in the domain can be listed — say $x_1, x_2, \ldots, x_n$ — the existential quantification $\exists x\, P(x)$ is same as the disjunction $P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)$

because this disjunction is true iff at least one of $P(x_i)$ is T.