

## Decimal to base:

(a) Decimal to binary:  $\rightarrow 2$

e.g: convert  $(12.125)_{10}$  into binary. Ans

Soln  $\Rightarrow$  for integer part:

$$\begin{array}{r} 2 | 12 \\ 2 | 6 \quad 0 \\ 2 | 3 \quad 0 \\ \hline 1 \quad 1 \end{array}$$

bottom to top

$$(12)_{10} = (1100)_2$$

for fractional part:

$$\begin{array}{l} 0.125 \times 2 = 0.25 \quad | \quad 0 \leftarrow \text{take integer part} \\ 0.25 \times 2 = 0.5 \quad | \quad 0 \quad \text{top} \\ 0.5 \times 2 = 1.0 \quad | \quad 1 \quad \text{to bottom} \end{array}$$

$$(0.125)_{10} = (0.001)_2$$

$$\text{Hence, } (12.125)_{10} = (1100.001)_2 \text{ Ans}$$

(b) Decimal to octal:  $\rightarrow 8$

e.g: convert  $(658.825)_{10}$  into octal. Ans

Soln  $\Rightarrow$  for integer part:

$$\begin{array}{r} 8 | 658 \\ 8 | 82 \quad 2 \\ 8 | 10 \quad 2 \\ \hline 1 \quad 2 \end{array}$$

$$(658)_{10} = (1222)_8$$

for fractional part:

$$\begin{array}{l} 0.825 \times 8 = 6.6 \quad | \quad 6 \leftarrow \text{take only integer} \\ 0.6 \times 8 = 4.8 \quad | \quad 4 \quad \text{top} \\ 0.8 \times 8 = 6.4 \quad | \quad 6 \quad \text{to bottom} \\ 0.4 \times 8 = 3.2 \quad | \quad 3 \end{array}$$

$$0.000000$$

$$(0.825)_{10} = (0.6463)_8$$

$$\text{Hence } (658.825)_{10} = (1222.6463)_8$$

(c) decimal to hexadecimal:

e.g: convert  $(5326.345)_{10}$  into hexadecimal.

Soln  $\Rightarrow$  for integer part

$$\begin{array}{r} 16 | 5326 \\ 16 | 336 \quad 10 \quad A \\ 16 | 21 \quad 0 \\ \hline 1 \quad 5 \end{array}$$

$$(5326)_{10} = (5A5A)_{16}$$

for fractional part:

$$\begin{array}{l} 0.345 \times 16 = 5.52 \quad | \quad 5 \\ 0.52 \times 16 = 8.32 \quad | \quad 8 \\ 0.32 \times 16 = 5.12 \quad | \quad 5 \end{array}$$

$$= (150A)_{16}$$

$$0.12 \times 16 = 1.92 \quad | \quad 1$$

$$(0.345)_{10} = (0.5851)_{16}$$

$$\text{Hence, } (5386.345)_{10} = (150A.5851)_{16}$$

Base to decimal:

(a) binary to decimal:

$$\text{eg: } (110001.1011)_2 = (\dots)_{10}$$

$$\text{soln} \Rightarrow (1 \times 2^0) + (1 \times 2^1) + (0 \times 2^2) + (0 \times 2^3) + (0 \times 2^4) + (1 \times 2^5) + (1 \times 2^6) + (0 \times 2^7) + (1 \times 2^8)$$

$$= 1 + 1 + \dots$$

$\leftarrow^{+ve} \quad \rightarrow^{-ve}$

$$\text{eg: } (110001.1011)_2 = (\dots)_{10}$$

$$\Rightarrow (1 \times 2^0) + (0 \times 2^1) + (0 \times 2^2) + (0 \times 2^3) + (1 \times 2^4) + (1 \times 2^5) + (1 \times 2^6) + (0 \times 2^7) + (1 \times 2^8) + (1 \times 2^9)$$

$$= 1 + 0 + 0 + 0 + 16 + 32 + \frac{1}{2} + 0 + \frac{1}{8} + \frac{1}{16}$$

$$= 4.9 + 0.5 + 0.125 + 0.0625$$

$$= (49.6875)_{10}$$

(b) octal to decimal:

$$\text{eg: } (62.5463)_8 = (\dots)_{10}$$

$$\text{soln} \Rightarrow (6 \times 8^1) + (2 \times 8^0) + (5 \times 8^{-1}) + (4 \times 8^{-2}) + (6 \times 8^{-3}) + (3 \times 8^{-4})$$

$$= 48 + 2 + 0.625 + 0.0625 + 0.01171 + 0.0007$$

$$= (50.7)_{10} \text{ Ans}$$

(c) Hexadecimal to decimal:

$$\text{eg: } (150A.5851)_{16} = (\dots)_{10}$$

$$\text{soln} \Rightarrow (10 \times 16^0) + (0 \times 16^1) + (5 \times 16^2) + (1 \times 16^3) + (5 \times 16^{-1}) + (8 \times 16^{-2}) + (5 \times 16^{-3}) + (1 \times 16^{-4})$$

$$= 10 + 0 + 1280 + 0.3125 + 0.03125 + 0.00122 + 1.52587$$

$$= (5386.345)_{10}$$

Eg: Determination of base number.

Eg: Determine the value of base  $x$  if  $(193)_x = (623)_8$

Soln  $\Rightarrow$  convert both the numbers into decimal system i.e.

$$6x^2 + 2x^1 + 3x^0 = (403)_{10}$$

$$\text{also } (1x^2) + (9x^1) + (3x^0) = 403$$

$$\Rightarrow x^2 + 9x + 3 - 403 = 0 \Rightarrow x^2 + 9x - 400 = 0$$

$$\Rightarrow x = 16, -25$$

negative base is not applicable, Hence  $x = 16$  Ans

Octal to Hexadecimal:

Q. Convert  $(572)_8 = (\dots\dots\dots)_{16}$

Soln  $\Rightarrow$  method 1<sup>st</sup> we find octal to decimal

$$(2 \times 8^0) + (7 \times 8^1) + (5 \times 8^2) = 2 + 56 + 320 = (378)_{10}$$

method 2<sup>nd</sup> and then decimal to hexadecimal

$$\begin{array}{r} 16 \Big| 378 \\ 16 \quad | 23 \\ \quad | 10 \quad 2A \\ \quad | \quad | 7 \\ \quad \quad | \end{array} \quad (378)_{10} = (17A)_{16}$$

\* Binary addition :

Rules: sum carry

$$1+1=0 \quad 1$$

$$1+0=1 \quad 0$$

$$0+0=0 \quad 0$$

$$1+1+1=1 \quad 1$$

Eg: Add  $(11001100)_2$  and  $(11011010)_2$

$$\begin{array}{r} 11001100 \\ 11011010 \\ \hline 00100110 \end{array}$$

$$\begin{array}{r} 11001100 \\ 11011010 \\ \hline 110010110 \end{array}$$

$$\begin{array}{r} 11001100 \\ 11011010 \\ \hline 101001110 \end{array}$$

## Octal addition

### Octal addition :

eg: Add  $(167)_8$  and  $(325)_8$

Soln  $\Rightarrow$

$$\begin{array}{r}
 & 11 \rightarrow \text{carry} \\
 (167)_8 & \\
 + (325)_8 & \\
 \hline
 (514)_8 &
 \end{array}
 \quad \text{rough} \quad
 \begin{array}{r}
 1 \\
 167 \\
 - 325 \\
 \hline
 514
 \end{array}
 \quad
 \begin{array}{l}
 5(9-8)(12-8) \rightarrow \text{subtract} \\
 \text{and } 2, 4 \\
 \text{carry } 1 \leftarrow \text{left}
 \end{array}$$

Rules:

if result is greater than base then divide the result by base.  
quotient = carry  
remainder = sum

## Hexadecimal addition :

Rules:

- result or sum is greater than 16. Then divide the result by 16.  
 quotient = carry  
 remainder = sum

eg: Add  $(3F8)_{16}$  and  $(5B3)_{16}$

Soln  $\Rightarrow$

$$\begin{array}{r}
 1 \rightarrow \text{carry} \\
 3 F 8 \\
 + 5 B 3 \\
 \hline
 (9 A B)_{16}
 \end{array}$$

$$\begin{array}{r}
 1 \quad \text{rough} \\
 3 F 8 \\
 + 5 B 3 \\
 \hline
 9 A B
 \end{array}
 \quad
 \begin{array}{l}
 11 \rightarrow B \\
 \downarrow
 \end{array}$$

0 - 9
10 - A
11 - B
12 - C
13 - D
14 - E
15 - F
16 <del>&gt; 16</del>

$$F = 15$$

$$B = 11$$

$$26 - 16 = 10$$

Subtract  $A$   
 hog a to  
 carry  $1 \leftarrow \text{left}$

## binary subtraction :

eg: perform  $(11101100)_2 - (00110010)_2$

soln ⇒

$$\begin{array}{r}
 & 1 & 0 & 0 & 2 \\
 & 1 & + & 1 & 0 & 1 & + & 0 & 0 & 2 \\
 - & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
 \hline
 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0
 \end{array}$$

$(10111010)_2$

## Octal subtraction :

eg: (i) compute  $(516)_8 - (413)_8$

soln ⇒

$$\begin{array}{r}
 (516)_8 \\
 - (413)_8 \\
 \hline
 (103)_8
 \end{array}$$

(ii) compute  $(506)_8 - (413)_8$

$$\begin{array}{r}
 4.8 \\
 \overline{-} 506 \\
 - 413 \\
 \hline
 (073)_8
 \end{array}$$

## Hexadecimal subtraction :

eg: (i) compute  $(CB2)_{16} - (972)_{16}$

soln ⇒

$$\begin{array}{r}
 CB2 \\
 - 972 \\
 \hline
 (-340)_{16}
 \end{array}$$

(ii) compute  $(CB2)_{16} - (9C2)_{16}$

$$\begin{array}{r}
 C.27 \\
 - 9C2 \\
 \hline
 2F0
 \end{array}$$

## \* method to find complement :

- (i)  $(245)_8$  find 7's complement and 8's complement.
- (ii)  $(785)_{16}$  find F's complement and 16's complement.
- (iii)  $(789)_{10}$  find 9's complement and 10's complement.

(i) soln  $\Rightarrow$

7 7 7

2 4 5

5 3 2

$\rightarrow$  7's complement

+ 1

5 3 3

$\rightarrow$  8's complement

(iii)

9 9 9

7 8 9

2 1 0

$\rightarrow$  9's complement

+ 1

2 1 1

$\rightarrow$  10's complement

(ii)

F F F

7 8 5

8 7 A

$\rightarrow$  15's complement

+ 1

8 7 B

$\rightarrow$  16's complement

(iv) Find 7's complement of  $(2456)_8$ .

soln  $\Rightarrow$

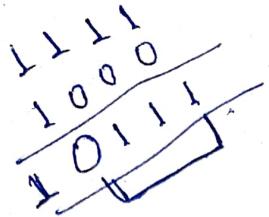
7 7 7 7

2 4 5 6

5 3 2 1

$\rightarrow$  7's complement.

(0), 1



8

?

Trick:

For 9's complement जितनी digit होंगी उसकी times 9  
लिखना है 1 and उसकी given number का subtract  
करें।

e.g.: find 9's complement of  $(65270)_{10}$

9 9 9 9 9

6 5 2 7 0

3 4 7 2 9 → 9's complement

and

+1

3 4 7 2 A → 10's complement

e.g.: find 2's complement of  $(10101)_2$

Soln ⇒

1 1 1 1 1  
- 1 0 1 0 1

0 1 0 1 0 → 1's complement

+1

0 1 0 1 1 → 2's complement

Q.) Subtract  $(57232)_{10} - (03550)_{10}$  using 9's complement

Soln ⇒ 10's complement of  $(03550)_{10}$

= 9 9 9 9 9  
- 0 3 5 5 0

+ 9 6 4 4 9 → 9's complement

+1

+ 9 6 4 5 0 → 10's complement

then  $57232$

+ 9 6 4 5 0

153682

↓  
discard

$$(57232)_{10} - (03550)_{10} = (53682)_{10}$$

Note: complement +ve number ko -ve karta hai & -ve number ko +ve karta hai.

$$\begin{array}{r} \underline{2} \mid \underline{8} \quad 0 \\ \underline{2} \mid \underline{4} \quad 0 \\ \underline{2} \mid \underline{2} \quad 0 \\ \hline & 1 \end{array} \qquad \begin{array}{r} \underline{2} \mid \underline{15} \quad 1 \\ \underline{2} \mid \underline{7} \quad 1 \\ \underline{2} \mid \underline{3} \quad 1 \\ \hline & 3 \end{array}$$

8) Subtract  $(3550)_{10} - (57232)_{10}$

Soln  $\Rightarrow$  10's complement of  $(57232)_{10}$

$$\begin{array}{r}
 \underline{9\ 9\ 9\ 9\ 9} \\
 - 5\ 7\ 2\ 3\ 2 \\
 \hline
 4\ 2\ 7\ 6\ 7 \rightarrow 9\text{'s complement} \\
 + 1 \\
 \hline
 4\ 2\ 7\ 6\ 8 \rightarrow 10\text{'s complement}
 \end{array}$$

$$\begin{array}{r} 03550 \\ + 42768 \\ \hline 46318 \end{array}$$

NO end carry generates.

$\therefore$  10's complement of

$(4G3L8)_{10}$  is with ~~the~~ (-ve sign)

$$\begin{array}{r} \underline{\phantom{0}9\phantom{0}9\phantom{0}9\phantom{0}9\phantom{0}9} \\ 46\phantom{0}3\phantom{0}18 \\ \hline 53\phantom{0}6\phantom{0}8\phantom{0}1 + 1 = 0(53682) \rightarrow 10\text{'s complement} \end{array}$$

$$\therefore (3550)_{10} - (57232)_{10} = -(53682)_{10}$$

Q) perform  $(15)_{10} - (8)_{10}$  using 4 bit 2's complement representation.

$$\text{Soln} \Rightarrow (15)_{10} = (1111)_2$$

$$(8)_{10} = (1000)_2$$

2's complement of  $(1000)_2$  is ~~(1111)~~

$$\begin{array}{r} \text{i.e. } \underline{\begin{array}{r} 1111 \\ 1000 \\ \hline 0111 \end{array}} \rightarrow \text{i's complement} \\ \qquad\qquad\qquad + \\ \qquad\qquad\qquad \underline{\underline{1111}} \end{array}$$

1000 → 2's complement

## Binary codes :

(i) Weighted binary code

\* (a) BCD and binary code

\* (ii) Non-weighted binary code

\* (a) Excess 3 code

\* (b) Gray code.

\* weighted code : In this, each digit position of the number represents a specific weight.

	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
ob.	32	16	8	4	2	1

decimal to binary

decimal binary (4 bit)

5 0101

8 1000

9 1001

13 1101

$2^5$   $2^4$   $2^3$   $2^2$   $2^1$   $2^0$

32 16 8 4 2 1

0 0 0 1 0 1  $\rightarrow$  5

1 0 0 0  $\rightarrow$  8

1 0 0 1  $\rightarrow$  9

1 1 0 1  $\rightarrow$  13

$\rightarrow$  binary coded decimal

## BCD code :

Here, each digit of decimal number is represented by a separate group of 4-bits with weightage (8-4-2-1).

eg: BCD code of  $(58)_{10} = (0101 \quad 1000)_{BCD}$

(a) Excess 3 code : In a simple BCD code, add 3(0011) to get the excess-3 code.

Note: BC<sub>D</sub> code only we write for decimal number  
 Since we have given any no. in octal or hexadecimal form then 1st we change them in decimal.

Q. Find the excess-3 code for following decimal number

- (a) 592 (b) 403

Soln  $\Rightarrow$  (a) 592  $\Rightarrow$  0101 1001 0010 → add each pair of 4 digit by 3 i.e (0011)  

$$\begin{array}{r} + \quad 0011 \quad 0011 \quad 0011 \\ - \quad 1000 \quad 1100 \quad 0101 \end{array}$$

(b) Gray code: It is also called reflected code.  
 In this, the number is changes by only one bit as it proceeds from one number to the next.

Two<sup>bit</sup> binary code      change in gray code

2 - 1	$0 \rightarrow 0$	$1 \rightarrow 1$	binary code	mirror	0 0 0 1 1 1 1 0	gray code
0 0						
0 1						
1 0	$\rightarrow 2$					
1 1	$\rightarrow 3$					

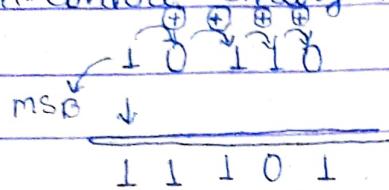
3 bit gray code

0 0 0
0 0 1
0 1 1
0 1 0
1 1 0
1 1 1
1 0 1
1 0 0

MSB  $\rightarrow$  most significant bit

### \* Binary to gray code conversion :

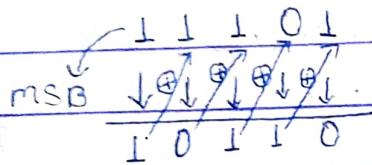
e.g. (i) convert binary number  $(10110)_2$  to gray code.



⊕ - module 2 addition

means (addition without carry)

(ii) convert  $(11101)_\text{gray}$  to binary equivalent .



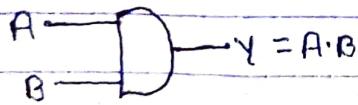
### \* LOGIC GATES :

(i) Basic Gates (AND, OR, NOT)

(ii) Universal Gates (NAND, NOR)

(iii) Exclusive gates (EX-OR, EX-NOR)

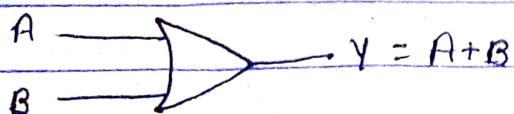
#### AND Gate:



Truth table

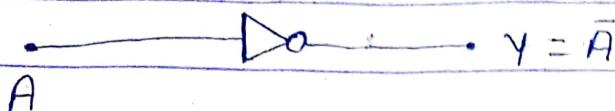
input		Output
A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

#### OR Gate:



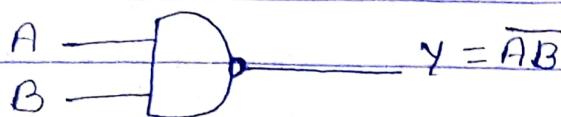
Input		Output
A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

## NOT Gate (Inverter) :



Input	Output
A	$Y = \bar{A}$
0	1
1	0

## NAND Gate (universal gate) :



Input	Output
A   B	$Y = \bar{A}\bar{B}$
0   0	1
0   1	1
1   0	1
1   1	0

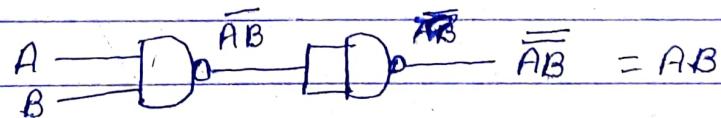
NOTE : by using NAND gate

we can find rest gate like NOT, AND, OR Gate  
for example.

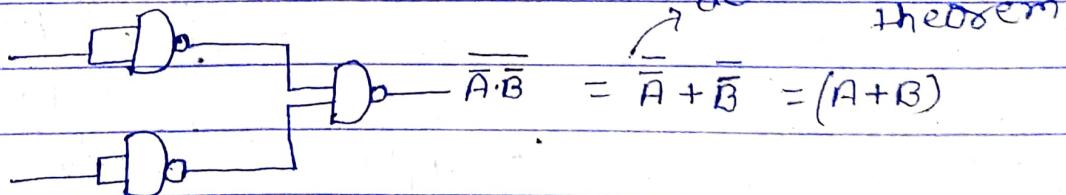
(a) NOT :



(b) AND :



(c) OR :



## Demorgan's theorem:

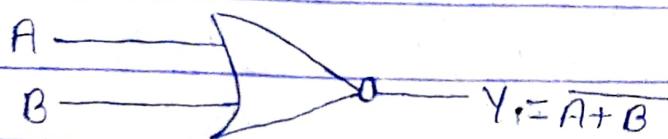
→ change the sign, break the line.

$$(i) \bar{A} \cdot \bar{B} = \bar{A} + \bar{B}$$

$$(ii) \bar{A} + \bar{B} = \bar{A} \cdot \bar{B}$$

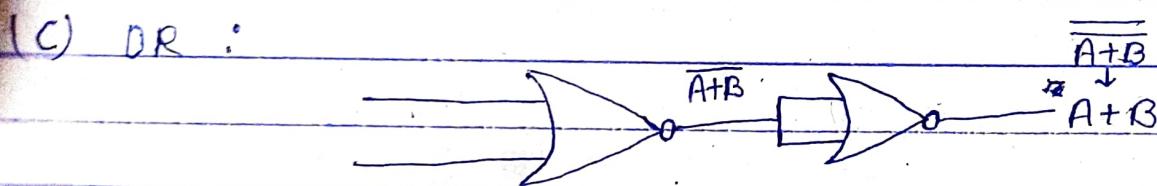
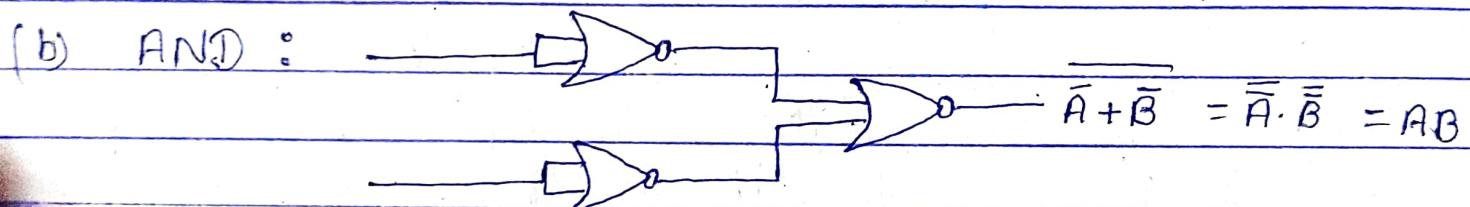
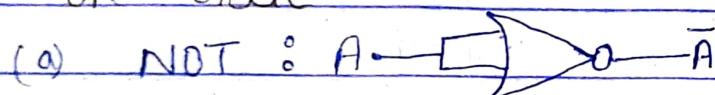
Note : we can say NAND gate as a universal gate because by using this gate we can <sup>design</sup> define other gate.

## NOR Gate (universal gate) :

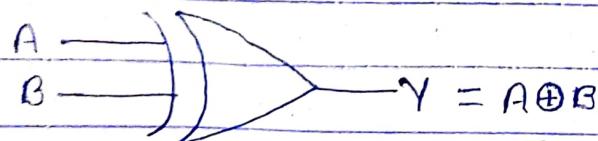


Input		Output	$Y = \bar{A} + \bar{B}$
A	B		
0	0	1	
0	1	0	
1	0	0	
1	1	0	

→ by using NOR Gate we can design NOT, AND, OR gate.



## Ex-OR Gate (exclusive OR)

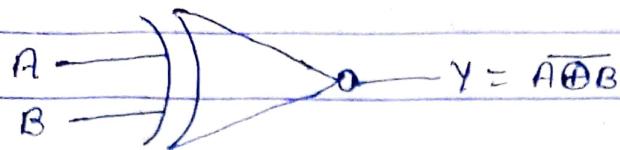


$$= A\bar{B} + \bar{A}B$$

Input		Output
A	B	$Y = A\bar{B} + \bar{A}B$
0	0	0
0	1	1
1	0	1
1	1	0

Note: Ex-OR Gate is a comparator. means this is use for compare input. If both input are same then output is zero and if both input are different then output is one.

## EX - NOR Gate (exclusive NOR)



$$Y = A \oplus B$$

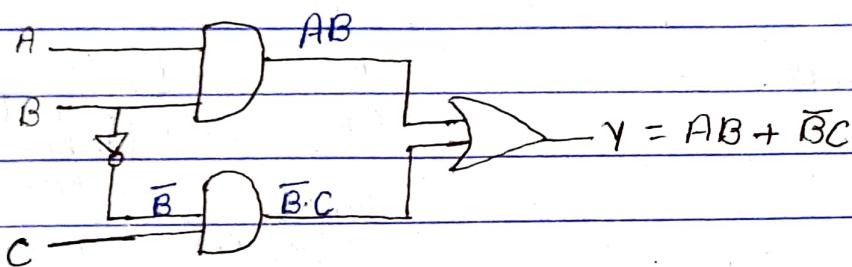
$$= AB + \bar{A}\bar{B}$$

Input		Output
A	B	$Y = \bar{A} \oplus \bar{B}$
0	0	1
0	1	0
1	0	0
1	1	1

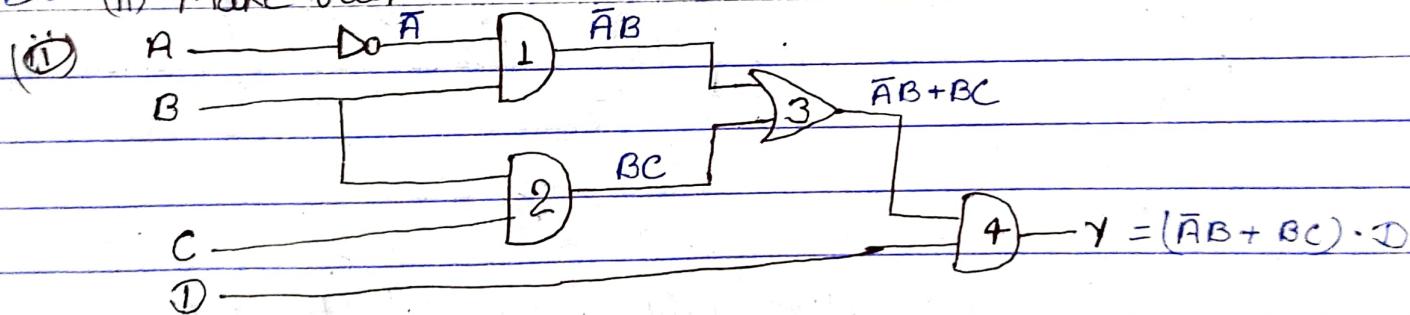
Q. Function realization using basic gates :

$$(i) Y = AB + \bar{B}C$$

Soln  $\Rightarrow$



~~(ii)~~ (ii) make output at each gate.



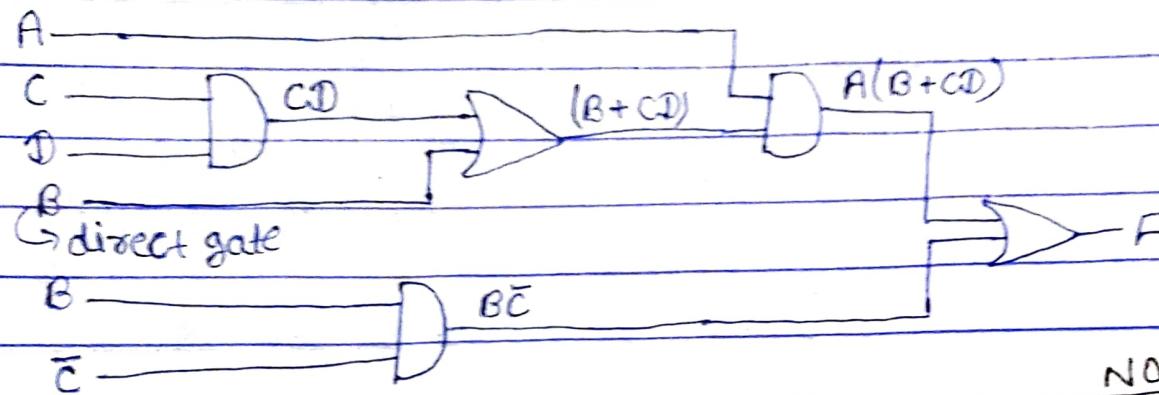
Soln  $\Rightarrow$  1 gate output =  $\bar{A}B$ , 2 gate output =  $BC$

3 gate output = 1+2 =  $\bar{A}B + BC$

4 gate output i.e.  $Y = 3 \cdot D = (\bar{A}B + BC) \cdot D$

Q. Implement the following circuit using NAND and NOR gates only. •  $F = A(B + CD) + BC$

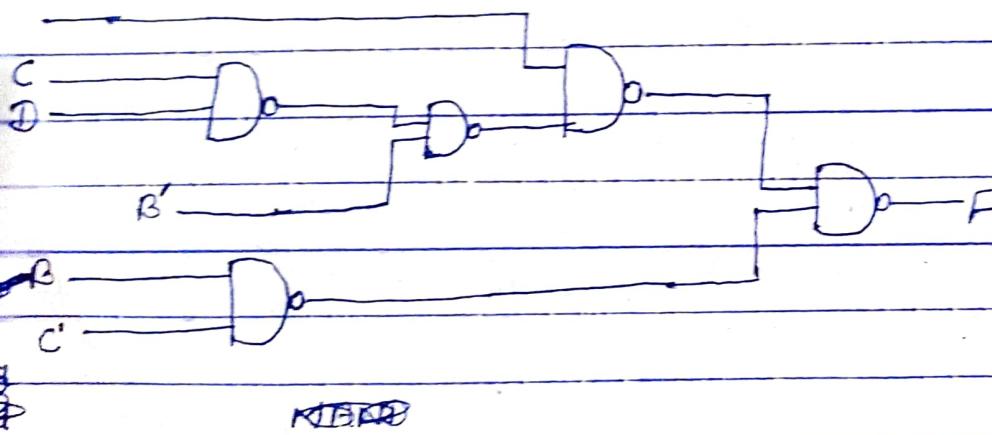
Soln  $\Rightarrow$  Step 1: AND/OR implementation



Step 2: NAND Implementation

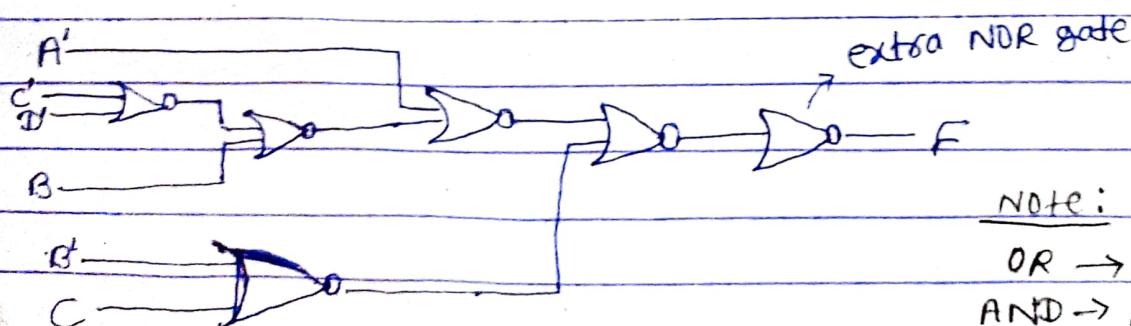
$AND \rightarrow NAND$   
 $OR \rightarrow NAND$  (with direct input complemented)

Here  
complemented means if output is A then take its complement which is  $A'$  or  $\bar{A}$



also we solve by using NOR gate 1st step is as it is

Step 2:



Note:

$OR \rightarrow NOR$

$AND \rightarrow NOR$  (with direct input complemented)

Note: Whenever we change any gate into NOR gate then we use one extra NOR gate at end to getting final answer.

\* simplification of boolean function using boolean algebra.

\* boolean laws:

$$A + 0 = A$$

$$A + B = B + A \quad (\text{commutative})$$

$$A(B+C) = AB+AC \quad (\text{distributive})$$

$$A + \bar{A} = 1 \quad (\text{complement})$$

$$A + A = A$$

$$A + 1 = 1$$

$$A + AB = A$$

$$A + \bar{A}B = A + B$$

$$A \cdot 0 = 0$$

$$A \cdot B = B \cdot A$$

$$A \cdot BC = (A+B)(A+C)$$

$$A \cdot \bar{A} = 0$$

$$A \cdot A = A$$

$$A \cdot 0 = 0$$

$$A(A+B) = A$$

$$A \cdot (\bar{A} + B) = AB$$

\* DE-MORGAN'S THEOREM:

→ change the sign, break the line.

$$(i) \overline{A \cdot B} = \bar{A} + \bar{B}$$

$$(ii) \overline{A+B} = \bar{A} \cdot \bar{B}$$

Q. (i) Simplify  $\bar{A}BC\bar{D} + BCD\bar{D} + B\bar{C}\bar{D} + B\bar{C}D$

$$\text{Soln} \Rightarrow \bar{A}BC\bar{D} + BCD\bar{D} + B\bar{C}\bar{D} + B\bar{C}D$$

$$= BCD(\bar{A} + 1) + B\bar{C}\bar{D} + B\bar{C}D$$

$$= BCD + B\bar{C}\bar{D} + B\bar{C}D$$

$$= B\bar{D}(C + \bar{C}) + B\bar{C}D$$

$$= B\bar{D} + B\bar{C}D$$

$$= B(\bar{D} + \bar{C}D)$$

$$= B(\bar{D} + \bar{C})$$

(1+97)

SCP  $\rightarrow$  standard canonical product

iii) Simplify  $\bar{A}B + \bar{A} + AB$

Soln  $\Rightarrow$  Lct F =  $\bar{A}B + \bar{A} + AB$

$$= \bar{A}B \cdot \bar{A} \cdot \bar{A}B \quad [ \because A \cdot \bar{A} = 0 ]$$

$$= AB \cdot A \cdot \bar{A}B$$

$$= 0$$

iv) Simplify  $F(w, x, y, z) = x + xyz + \bar{x}yz + w\bar{z} + \bar{w}x + \bar{x}y$

$$\text{Soln} \Rightarrow F(w, x, y, z) = x(1 + yz + w + \bar{w}) + \bar{x}(yz + y)$$

$$= x + \bar{x}(y)(1+z)$$

$$= x + \bar{x}y$$

$$= x + y$$

\* SOP (sum of product) and POS (product of sum) :

(i)  $F(A, B, C) = AC + AB + BC \xrightarrow{\text{SOP}}$  (SOP)

(ii)  $F(A, B, C) = (A+B)(B+C)(A+C) \xrightarrow{\text{POS}}$  (POS)

\* Canonical SOP (sum of product) :

$\rightarrow$  In the SOP (sum of product) form, many times all the individual terms do not involve all literals.

For example, in expression  $AB + ABC$  the first product term do not contain literal C.

$\rightarrow$  If each term in SOP form contain all the literals then the SOP form is known as standard or canonical SOP form.

Eg: Convert the given expression in standard SOP form.

~~SOP~~  $\Rightarrow$   $F(A, B, C) = AC + AB + BC$

$$\text{Soln} \Rightarrow F = AC(B+\bar{B}) + AB(C+\bar{C}) + BC(A+\bar{A})$$

$$= ABC + A\bar{B}C + AB\bar{C} + ABC + A\bar{B}C + A\bar{B}\bar{C}$$

$$\therefore F(A, B, C) = ABC + A\bar{B}C + AB\bar{C} + \bar{A}BC \quad [ \because A + \bar{A} = A ]$$

are present.  $\rightarrow$  In every term three literals are present. Since this is SCP form in

replacing  $A/B/C$  by  $\sum m_7, m_5, m_6, m_3$  &  $\bar{A}/\bar{B}/\bar{C}$  by  $0$ .

for minterm:  $F(A,B,C) = \sum m_7 + \sum m_5 + \sum m_6 + \sum m_3$   
 $= \Sigma(7, 5, 6, 3)$

Q. convert the given expression in standard SOP form

$$F(A,B,C) = A + ABC$$

Soln  $\Rightarrow F = A(B + \bar{B})(C + \bar{C}) + ABC$   
 $= (AB + A\bar{B})(C + \bar{C}) + ABC$   
 $= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + ABC$   
 $= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C}$

for minterm  $\Rightarrow F(A,B,C) = \sum m_7, \sum m_5, \sum m_6, \sum m_4$   
 $= \Sigma(7, 6, 5, 4)$

\* Canonical POS (product of sum):

In the POS (product of sum) form, many times all the individual terms do not involve all literals for example, in expression  $(A+B) \cdot (B+C)$  the literal is missing in both the terms of sum.

→ If each term in POS form contain all the literals then the POS term is known as standard POS or canonical POS.

eg: convert the given expression in standard POS form.

(i)  $Y = A(A+B+C)$

solt $\Rightarrow$   $Y = A(A+B+C)$

$$= (A + B \cdot \bar{B} + C \cdot \bar{C})(A+B+C)$$

$$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+\bar{B}+C)$$

$$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(A+\bar{B}+\bar{C})$$

for max term: replace  $A/B/C$  by 0 and  $\bar{A}/\bar{B}/\bar{C}$  by 1.

$$\text{then } Y = \pi(000, 001, 010, 011)$$

$$= \pi(0, 1, 2, 3) \text{ Ans}$$

(ii)  $f(A,B,C) = (A+B)(B+C)(A+C)$

solt $\Rightarrow$   $f(A,B,C) = (A+B+C \cdot \bar{C})(B+C+A \cdot \bar{A})(A+C+B \cdot \bar{B})$

$$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+C+\bar{B})$$

for max term  $\Rightarrow$  Replace  $A/B/C$  by 0 and  $\bar{A}/\bar{B}/\bar{C}$  by 1.

$$f(A,B,C) = \pi(000, 001, 010, 011, 001, 000, 010)$$

$$= \pi(0, 1, 2, 3)$$

$$f(A,B,C) = (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+\bar{B}+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})$$

$$= (A+B+C)(A+B+\bar{C})(\bar{A}+\bar{B}+C)(A+\bar{B}+C)$$

for max term  $\Rightarrow$  Replace  $A/B/C$  by 0 and  $\bar{A}/\bar{B}/\bar{C}$  by 1

$$\text{then } Y = \pi(000, 001, 100, 010)$$

$$= \pi(0, 1, 4, 2)$$

$$= \pi(0, 1, 2, 4)$$

0<sup>x</sup>2<sup>1</sup>  
2<sup>0</sup>1<sup>0</sup>

## # Simplification of boolean function by k-map.

- (i) 2-variable k-map
- (ii) 3-variable k-map
- (iii) 4-variable k-map.

K-map (Karnaugh map) : The method first proposed by Veitch and modified by Karnaugh, hence it is known as the Veitch diagram or the Karnaugh map.

### Steps in k-map :

- (i) Representation
- (ii) placing
- (iii) Grouping
- (iv) simplified expression.

### (i) Representation :

#### (a) 2 variable k-map :

→ In 2 variables we have  $2^n = 2^2 = 4$  combinations are available.

A\B	0	1
0	0	1
1	2	3

2 variable k-map

$$\begin{array}{c} \xrightarrow{AB} \\ \begin{matrix} 0 & 0 \\ 1 & 0 \end{matrix} = 0 \\ 1 & 0 = 2 \end{array}$$

Decimal	A	B
0	0	0
1	0	1
2	1	0
3	1	1

### b) 3 variable K-map:

there are 8 combination ( $2^3$ )

A	BC	00	01	11	10
0		0	1	2	3
1		4	5	6	7

(cells or block)

Decimal	A	B	C
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

### c) 4 variable K-map : there are $2^4$ (16) combination.

AB	CD	00	01	11	10	Gray code
00		0	1	3	2	
01		4	5	7	6	
11		12	13	15	14	
10		8	9	11	10	

decimal	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

minterm  $\rightarrow \Sigma$   
maxterm  $\rightarrow \times$

### (ii) placing (filling) :

- For minterms we place '1' on the ~~representation~~ respective block.  
→ For maxterms we place '0' on the respective block.

### (iii) Grouping:

It is done in adjacent cell and in power of 2.

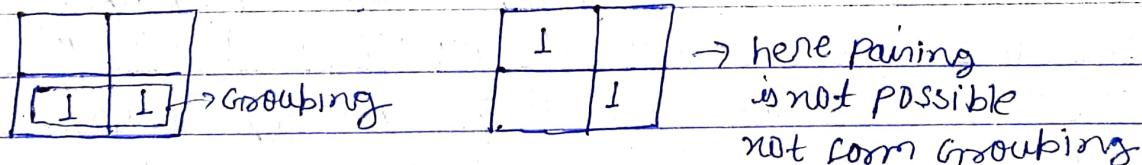
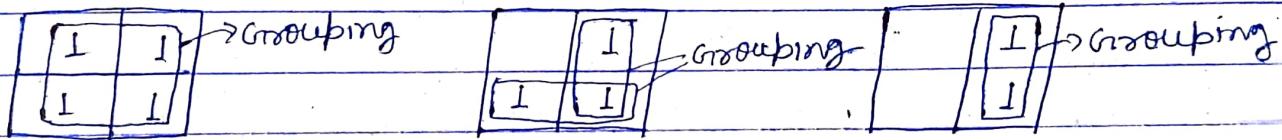
for eg:  $2^0 = 1$  (single not paired)

$2^1 = 2$  (double)

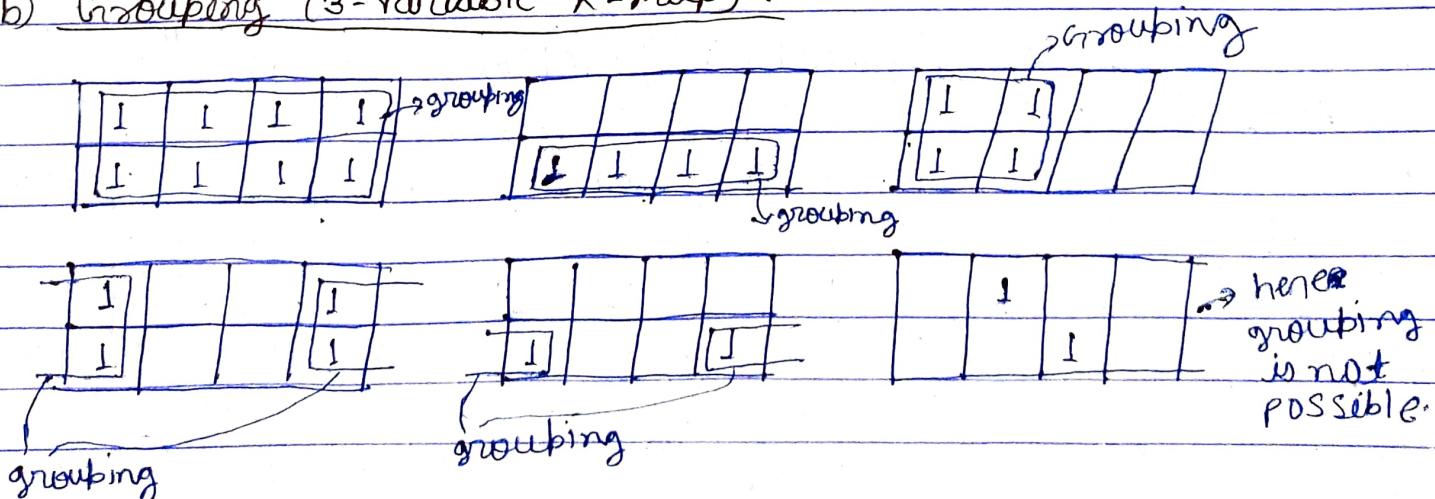
$2^2 = 4$  (quad),  $2^3 = 8$  (oct)

$2^4 = 16$  (Hex)

#### (a) Grouping in (2-variable k-map):



#### (b) Grouping (3-variable k-map):



## Grouping (4-variable K-map) :

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

grouping

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

grouping

1			
1			
1			
1			

1		
1	1	1
1	1	1

	1	1
1		1
1	1	

1	1
1	1
1	1
1	1

grouping .

110 111 001

Eg: (i) Reduce  $Y = ABC\bar{C} + ABC + \bar{A}\bar{B}\bar{C}$  using K-map.

Soln  $\Rightarrow$  It is in SOP form. since replacing A/B/C by 1 or  $\bar{A}/\bar{B}/\bar{C}$  by 0.

1st find minterm by converting the expression into its standard form if required.

$$\therefore Y = \Sigma(110, 111, 001) = \Sigma(6, 7, 1)$$

since it is 3-variable function

$$\begin{cases} 4^{\frac{2-1}{2}} = 6 \\ 110 = 6 \\ 111 = 7 \\ 001 = 1 \end{cases}$$

A	BC	00	01	11	C <sub>1</sub> T <sub>1</sub> T <sub>0</sub>	B = 1
$\bar{A} \leftarrow 0$		1				
A $\leftarrow 1$		1	1		grouping	

$$Y = \bar{A}\bar{B}\bar{C} + AB$$