Newton-Raphson Method

Let x_0 be an approximated root of the eqn $f(x) = 0$. If $x_1 = x_0 + h$ be the exact root, then $f(x_1) = 0$

$$\Rightarrow \quad f(x_0 + h) = 0$$

Expanding $f(x_0+h)$ by Taylor series,

$$f(x_0+h) = f(x_0) + h \cdot f'(x_0) + \frac{h^2}{2!} f''(x_0) + \dots = 0$$

Since h is small, neglecting h^2 & higher powers of h, we get

$$0 = f(x_0 + h) \doteq f(x_0) + h f'(x_0)$$

$$\Rightarrow \quad h = -\frac{f(x_0)}{f'(x_0)}$$

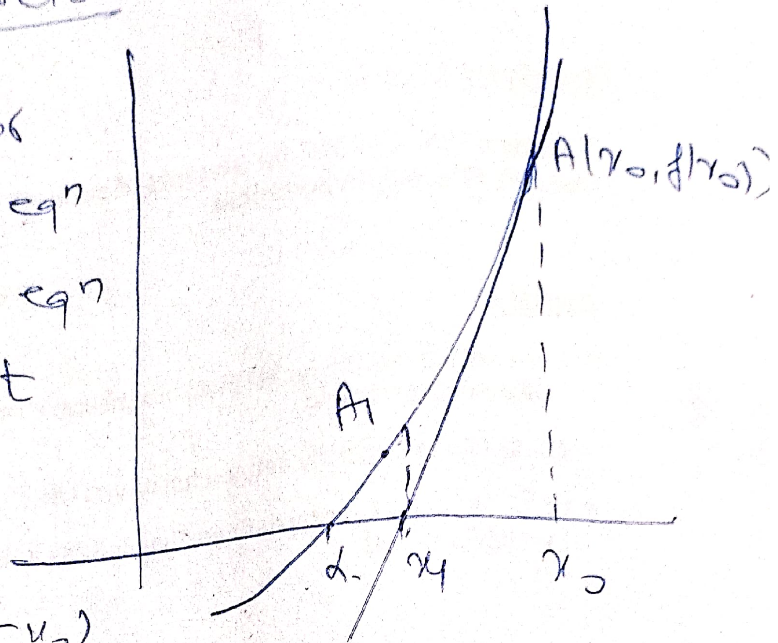
$\therefore$ A closer approximation to the root is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Similarly $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

In general, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ $(n = 0, 1, 2, \dots)$

Geometrical Interpretation



Let x_0 be a pt near the root α of the eqn $f(x) = 0$. Then, the eqn of the tangent at $A(x_0, f(x_0))$ is

$$y - f(x_0) = f'(x_0)(x - x_0)$$

It cuts the x-axis

at $$x_1 = x_0 - \frac{f(x_1)}{f(x_0)}$$

Which is a first approx^n to the root α

Q. Find the positive root of $x^4 - x = 10$ correct to 2 decimal places, using N-R method

Sol:- $f(x) = x^4 - x - 10 = 0$

$f(1) = 1 - 1 - 10 = -10 < 0$, $f(2) = 16 - 2 - 10 = 4 > 0$

$\therefore$ Root lies b/w 1 & 2

Take $x_0 = 2$.

Also, $f'(x) = 4x^3 - 1$

First approx^n is given by:-

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{4}{31} = 1.8710$$

$f(1.8710) = 0.3835$

Second approx^n is given by:-

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.8710 - \frac{0.3825}{25.1988}$$

$$= 1.8558$$

$$= 1.856$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.8558 - \frac{f(1.8558)}{f'(1.8558)}$$

$$= 1.8556$$

$$x_3 = 1.856$$

Since, $x_2 = x_3$

$\therefore$ The desired root is 1.856