SECANT METHOD

Taking x_0, x_1 as the initial limits of the interval s.t. $f(x_0) \cdot f(x_1) < 0$.

Eq^n of the chord joining $A(x_0, f(x_0))$ & $B(x_1, f(x_1))$ is

$$y - f(x_0) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \cdot (x - x_0)$$

Putting $y = 0$,

$$-f(x_0) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} (x - x_0)$$

$$\Rightarrow \quad x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

In general,

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}.$$

Q. Find a root of the eqⁿ $x^3 - 2x - 5 = 0$ using Secant method correct to three decimal places.

Soln:

$$f(x) = x^3 - 2x - 5 = 0$$

$$f(2) = -1 < 0, \quad f(3) = 16 > 0$$

Let $x_0 = 2, \quad x_1 = 3$

First approxⁿ to the root is

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{2 \times 16 - 3 \times (-1)}{16 - (-1)} = \frac{35}{17} = 2.0588$$

$$f(x_2) = -0.3911$$

Next approxⁿ is

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{3 \times (-0.3911) - 2.0588 \times 16}{-0.3911 - 16}$$

$$= 2.0813$$

$$f(x_3) = -0.1468$$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{2.0588 \times (-0.1468) - 2.0813 \times (-0.3911)}{-0.1468 + 0.3911}$$

$$= 2.0948$$

$f(2.0948) = 0.0028$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{2.0813 \times 0.0028 - 2.0948 \times (-0.1461)}{0.0028 - (-0.1468)}$$

$$= 2.0945$$

Hence, the root is 2.094

Q. Find the root of the eqn $xe^x = \cos x$ using the secant method correct to 4 decimal places.

(Ans. 0.5177)