Errors:- is the diff b/w the true value & the approximate value.

(1.) **Inherent Errors:-** Errors which are already present in the statement of a problem before its soln, are called inherent errors. Such errors arise either due to the given data being approximate or due to the limitations of mathematical tables, calculators or digital computers.

e.g. $x = \frac{1}{3} = 0.3333$, $y = \pi = 3.1416$

~~then~~ the error introduced in the alg. operation between these two approximate numbers is the inherent error.

(2.) **Rounding Error:-** arise from the process of rounding off the numbers during the computation.

e.g. $x = 26.5 = 26$

$3.25378905 = 3.254$

(3.) **Truncation Errors:-** are caused by using approximate results or on replacing an infinite process by a finite one.

Ex:- Rounding off of 13.658 gives 13.66 whereas truncation gives 13.65

If $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = X$ ⓶
is replaced by $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} = X'$
then the truncation error is $X - X'$.

(4.) <u>Absolute, Relative & Percentage Errors</u>:-
If X is the true value of a quantity & X' is its approximate value, then $|X - X'| = |\text{Error}|$ is called the absolute error E_a.
The relative error is defined by

$$E_r = \left|\frac{X - X'}{X}\right| = \frac{|\text{Error}|}{|\text{True Value}|}$$

& the percentage error is

$$E_p = 100 E_r = 100 \frac{|X - X'|}{|X|}$$

Remark:- If a no. is correct to n decimal places, then the error is $\frac{1}{2} 10^{-n}$
e.g. If the no. is 3.1416 correct to 4 decimal places, then the error is $\frac{1}{2} 10^{-4} = 0.$

Rules for Estimating Errors

If the approximate value of a no X having n decimal digits is x', then

(1.) Absolute error due to truncation to K digits
$$= |X - x'| < 10^{n-K}$$

(2.) Absolute error due to rounding off to K digits
$$= |X - x'| < \frac{1}{2} 10^{n-K}$$

(3.) Relative error due to truncation to K digits
$$= \left|\frac{X - x'}{X}\right| < 10^{1-K}$$

(4.) Relative error due to rounding off to K digits
$$= \left|\frac{X - x'}{X}\right| < \frac{1}{2} 10^{1-K}$$

Remark:- (1.) If a no. is correct to n significant digits, then the maximum relative error $\leq \frac{1}{2} 10^{-n}$. If a no. is correct to d decimal places, then the absolute error $\leq \frac{1}{2} 10^{-d}$.

(2) If the first significant figure of a no. is K & the no. is correct to n significant figures, then the relative error $< \left(\frac{1}{K \times 10^{n-1}}\right)$

Q. Find the truncation error in the result of the following f^n for $f(x)$ $x = \frac{1}{5}$ when we use (a.) first 3 terms (b.) first 4 terms.

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots \frac{x^6}{6!}$$

(4)

(Q.) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{2!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} = X$

Truncation error when first 3 terms are added,

$X' = 1 + x + \frac{x^2}{2!}$

$X - X' = \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!}$

$= \frac{(0.2)^3}{3!} + \frac{(0.2)^4}{4!} + \frac{(0.2)^5}{5!} + \frac{(0.2)^6}{6!}$

$= 0.00140275556$

$= 0.140275556 \times 10^{-2}$

Q. If 1.414 is used as an approximation to $\sqrt{2}$. Find the absolute & relative error

Soln:-

$X = \sqrt{2}, \quad X' = 1.414$

Error $= X - X' = \sqrt{2} - 1.414$

$= 0.00021356$

Absolute Error $= |\text{Error}| = 0.00021356$

Relative Error $= \left|\frac{X - X'}{X}\right| = \frac{0.00021356}{\sqrt{2}}$

$= 0.000151$

$= 0.151 \times 10^{-3}$