GALGOTIAS UNIVERSITY

CAT I Semester III, V, VII, IX All Programs
Answer uploading Template

Admission No. of Student	21SCSE1011615	Name of Course	Numerical methods
Name of Student	Abhinav kumar choudhary	Course Code	MATH2300
Program	B.Tech CSE	Date of Examination	29/09/2022
Semester	Third	Time	11:00-12:300
Signature of Student	Abhinav kumar chy		

Student shall start writing from below:

Ans 1) Inherent error :-

Errors which are already present in the statement of a problem before its solution are called inherent errors. Such errors arise either due to the given data being approximate or due to the limitations of mathematical tables, calculator or digital computers.

e.g — $x = \frac{1}{3} = 0.3333$, $y = \pi = 3.1416$

Ans 2) Interpolation polynomial is a method of estimating values between known data points of lowest possible degree that passes through the points of the dataset. Given set of $(n+1)$ data points $(x_0, y_0), \ldots\ldots, (x_n, y_n)$, with no two x_j the same polynomial function $P(x)$ is said to interpolate the data if $P(x_j) = y_j$ for each $j \in \{0, 1, \ldots n\}$

3) $f(x) = x^3 - x^2 - 1$

$f'(x) = 3x^2 - 2x - 0 = 0$

Now Take $x = 0$ $f(0) = -1$

$x = 1$ $f(1) = -1$

$x = 2$ $f(2) = 3$

Roots will lies in b/w $(1,2)$

Now Taking $x_0 = 2$

$f(2) = 3,$ $f'(2) = 8$

First approximation, so $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{3}{8} = 1.625$

$f(x_1) = f(1.625) = 0.6503$

$f'(x_1) = f'(1.625) = 4.6718$

Second approximation ;, $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.625 - \frac{0.6503}{4.6718}$

$= 1.4858$

$f(1.4858) = 0.0725$

$f'(1.4858) = 3.5612$

3rd approximation

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.4858 - \frac{0.0725}{3.5612}$

$= 1.4659$ Ans

④ 1955 lies in (1951, ~~2001~~ 1961). Hence we use Newton's backward interpolation formula

Here $x = 1995$, $x = 2001$, $h = 10$

$$1955 = x + nh$$

$$1995 = \cancel{2001}\ 1961 + 10n$$

$$n = \frac{(1955 - \cancel{2001}\ 1961)}{10}$$

$$n = -0.6$$

The backward difference Table is given below :-

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1921	46				
		20	[illegible]		
1931	66		-5		
		15		2	
1941	81		-3		$\boxed{-3}$
		12		$\boxed{-1}$	
1951	93		$\boxed{-4}$		
		$\boxed{8}$			
1961 ~~2001~~	$\boxed{101}$				

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^2 y_n + \cdots$$

~~$p = x_1 - x_0 = 19$~~

$$y = 101 + \frac{(-0.6)}{\lfloor 1}(8) + \frac{(-0.6)(-0.6+1)}{\lfloor 2}(-4)$$

$$+ \frac{(-0.6)(-0.6+1)(-0.6+2)}{\lfloor 3}(-1) + \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)}{\lfloor 4}(-3)$$

$$y = 101 - 4.8 + 0.48 + 0.056 + 0.1008$$

$$y = 96.8368$$

Hence, the population for the year 1955 is 96.837 thousands

5) $x+y+z=9$

$2x-3y+4z=13$

$3x+4y+5z=40$

$AX=B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 13 \\ 40 \end{bmatrix}$$

Augmented matrix $[A:B]$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & -3 & 4 & 13 \\ 3 & 4 & 5 & 40 \end{array}\right]$$ ~~apply $R_2 \to R_2-2R_1$ & $R_3 \to R_3-3R_1$~~

apply $R_2 \to R_2-2R_1$ & $R_3 \to R_3-3R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 1 & 2 & 13 \end{array}\right]$$

$R_2 \to R_2 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -6 & 0 & -18 \\ 0 & 1 & 2 & 13 \end{array}\right]$$ Apply $R_2 \to R_2/6$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -1 & 0 & -3 \\ 0 & 1 & 2 & 13 \end{array}\right]$$ Apply $R_1 \to R_1+R_2$ & $R_3 \to R_3+R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 6 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 2 & 10 \end{array}\right]$$ Apply $(R_2 \to (-1)R_2)$ $(R_1 \to R_1 - \frac{R_3}{2})$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 2 & 10 \end{array}\right] \quad \text{Apply } R_3 \to R_3/2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\left.\begin{array}{l} x = 1 \\ y = 3 \\ z = 5 \end{array}\right\} \text{Ans}$$

6)

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
10	43	$\frac{a-43}{15-10} = \frac{a-43}{5}$	$\frac{29-a-a+43}{5}$, $\frac{72-2a}{5}$	$\frac{2a+b-87-144+4a}{10}$, $\frac{6a+b-231}{10}$
15	a	$\frac{29-a}{20-15} = \frac{29-a}{5}$	$\frac{b-29-58+2a}{10}$, $\frac{2a+b-87}{10}$	$\frac{183-3b-2a-b-87}{10}$, $\frac{270-2a-4b}{10}$
20	29	$\frac{b-29}{30-20} = \frac{b-29}{10}$	$\frac{154-2b-b+29}{10}$, $\frac{183-3b}{10}$	
30	b	$\frac{77-b}{35-30} = \frac{77-b}{5}$		
35	77			

$$\Rightarrow \frac{6a+b-231}{10} = 0 \Rightarrow b = 231 - 6a$$

$$\frac{270 - 4b - 2a}{10} = 0$$

$\Rightarrow 270 - 4(231 - 6a) - 2a$

$\Rightarrow 270 - 924 + 24a - 2a$

$-654 + 22a = 0$

$a = \dfrac{654}{22}$

$a = 29.72$

$b = 231 - 6 \times 29.72$

$= 231 - 178.32$

$= 52.68$

$$\boxed{\begin{aligned} a &= 29.72 \\ b &= 52.68 \end{aligned}}$$