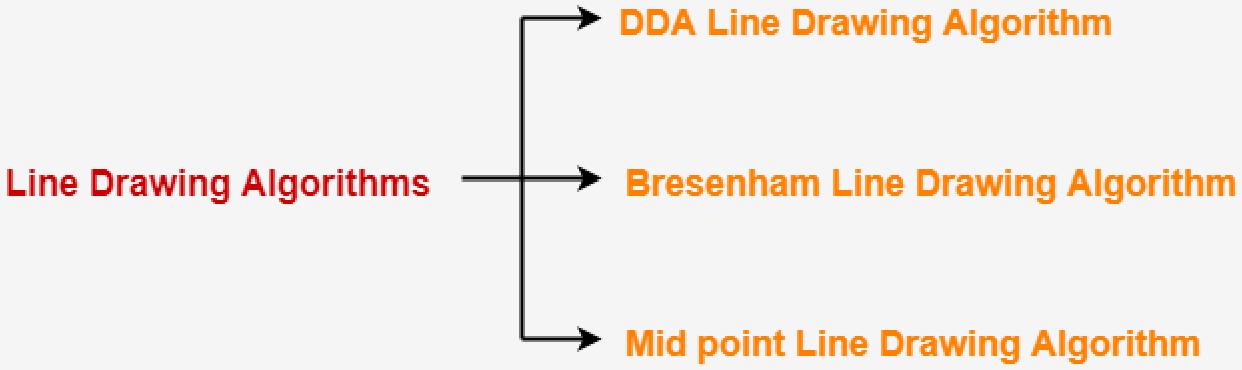
DDA Algorithm | Line Drawing Algorithms

Computer Graphics

Line Drawing Algorithms-

In computer graphics, popular algorithms used to generate lines are-


Line Drawing Algorithms
DDA Line Drawing Algorithm
Bresenham Line Drawing Algorithm
Mid point Line Drawing Algorithm

1. Digital Differential Analyzer (DDA) Line Drawing Algorithm
2. Bresenham Line Drawing Algorithm
3. Mid Point Line Drawing Algorithm

DDA Algorithm-

DDA Algorithm is the simplest line drawing algorithm.

> Given the starting and ending coordinates of a line,
> DDA Algorithm attempts to generate the points between the starting and ending coordinates.

Procedure-

Given-

- Starting coordinates = (X_0, Y_0)
- Ending coordinates = (X_n, Y_n)

The points generation using DDA Algorithm involves the following steps-

Step-01:

Calculate ΔX, ΔY and M from the given input.
These parameters are calculated as-

- $\Delta X = X_n – X_0$
- $\Delta Y =Y_n – Y_0$
- $M = \Delta Y / \Delta X$

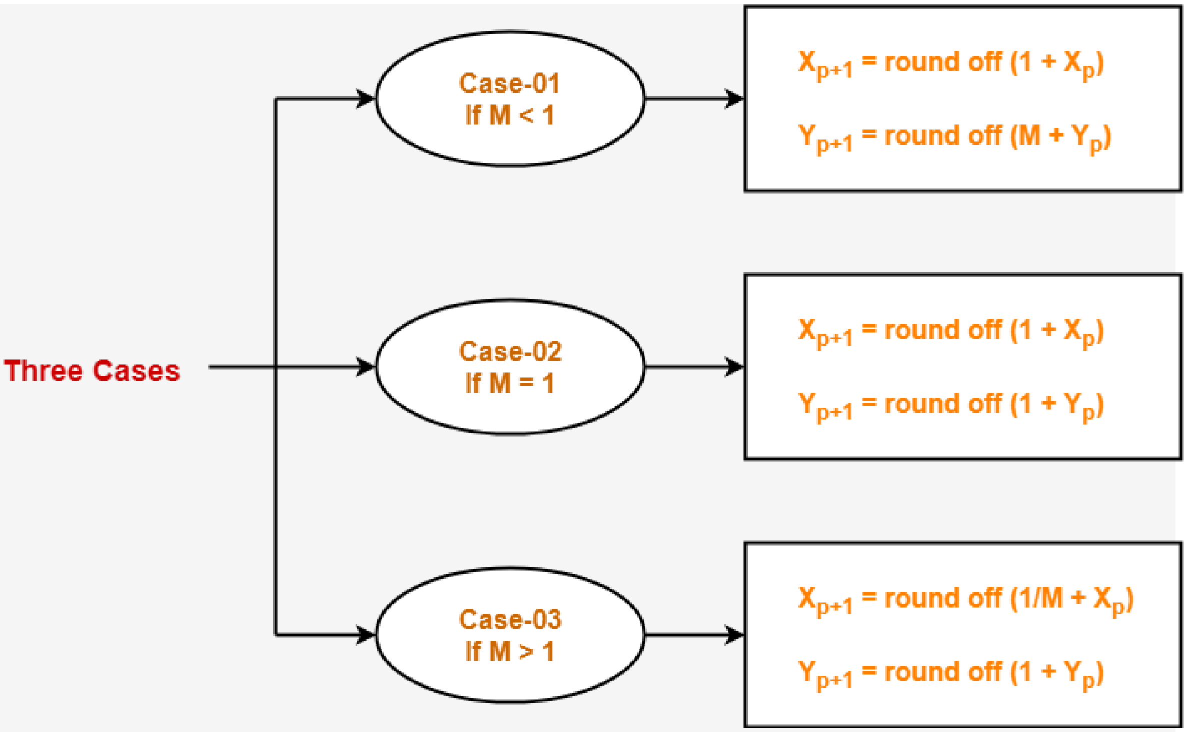
Step-02:

Find the number of steps or points in between the starting and ending coordinates.

```
if (absolute (ΔX) > absolute (ΔY))
Steps = absolute (ΔX);
else
Steps = absolute (ΔY);
```

Step-03:

Suppose the current point is (X_p, Y_p) and the next point is (X_{p+1}, Y_{p+1}).
Find the next point by following the below three cases-


Three Cases
Case-01
If M < 1
X_{p+1} = round off (1 + X_p)
Y_{p+1} = round off (M + Y_p)
Case-02
If M = 1
X_{p+1} = round off (1 + X_p)
Y_{p+1} = round off (1 + Y_p)
Case-03
If M > 1
X_{p+1} = round off (1/M + X_p)
Y_{p+1} = round off (1 + Y_p)

Step-04:

Keep repeating Step-03 until the end point is reached or the number of generated new points (including the starting and ending points) equals to the steps count.

PRACTICE PROBLEMS BASED ON DDA ALGORITHM-

Problem-01:

Calculate the points between the starting point (5, 6) and ending point (8, 12).

Solution-

Given-

- Starting coordinates = $(X_0, Y_0) = (5, 6)$
- Ending coordinates = $(X_n, Y_n) = (8, 12)$

Step-01:

Calculate ΔX, ΔY and M from the given input.

- $\Delta X = X_n – X_0 = 8 – 5 = 3$
- $\Delta Y = Y_n – Y_0 = 12 – 6 = 6$
- $M = \Delta Y / \Delta X = 6 / 3 = 2$

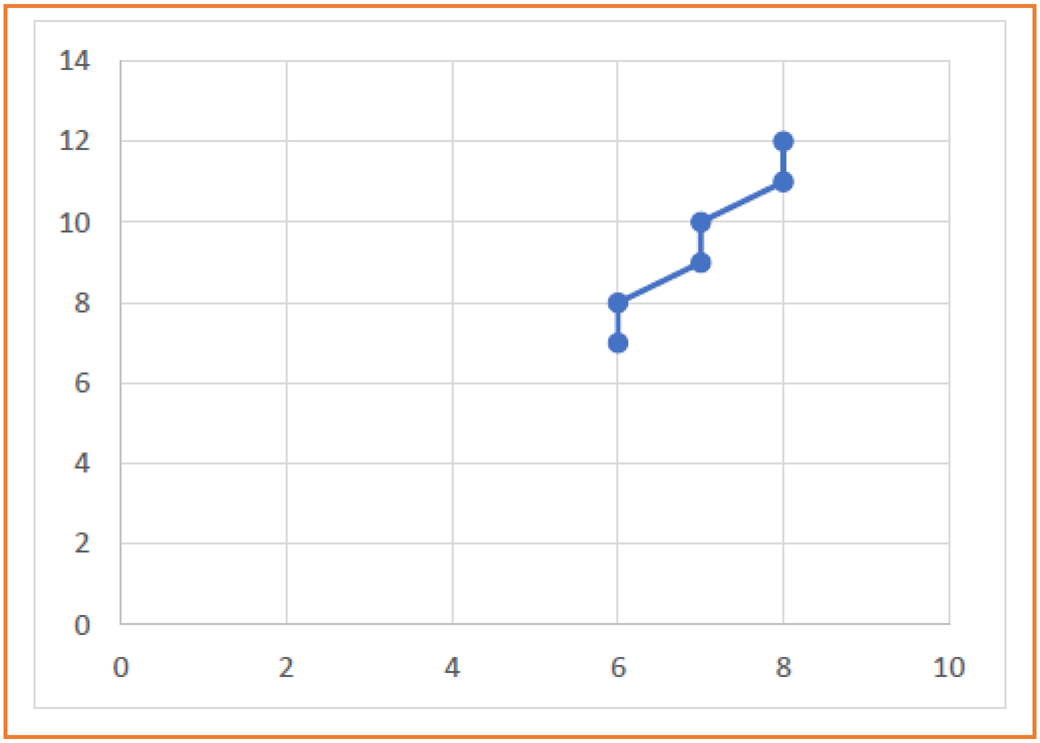
Step-02:

Calculate the number of steps.
As $|\Delta X| < |\Delta Y| = 3 < 6$, so number of steps = $\Delta Y = 6$

Step-03:

As M > 1, so case-03 is satisfied.
Now, Step-03 is executed until Step-04 is satisfied.

X_p	Y_p	X_{p+1}	Y_{p+1}	Round off (X_{p+1}, Y_{p+1})
5	6	5.5	7	(6, 7)
		6	8	(6, 8)
		6.5	9	(7, 9)
		7	10	(7, 10)
		7.5	11	(8, 11)
		8	12	(8, 12)


14
12
10
8
6
4
2
0
0
2
4
6
8
10

Problem-02:

Calculate the points between the starting point (5, 6) and ending point (13, 10).

Solution-

Given-

- Starting coordinates = (X_0, Y_0) = (5, 6)
- Ending coordinates = (X_n, Y_n) = (13, 10)

Step-01:

Calculate ΔX, ΔY and M from the given input.

- $\Delta X = X_n – X_0 = 13 – 5 = 8$
- $\Delta Y = Y_n – Y_0 = 10 – 6 = 4$
- $M = \Delta Y / \Delta X = 4 / 8 = 0.50$

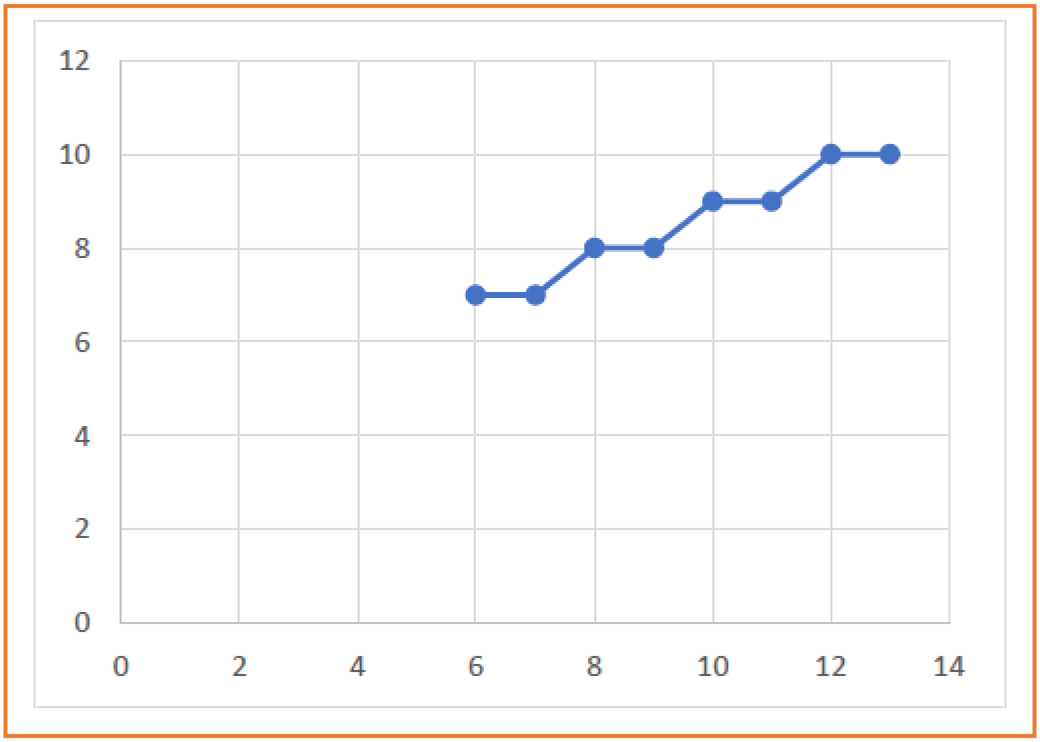
Step-02:

Calculate the number of steps.
As $|\Delta X| > |\Delta Y| = 8 > 4$, so number of steps = $\Delta X = 8$

Step-03:

As $M < 1$, so case-01 is satisfied.
Now, Step-03 is executed until Step-04 is satisfied.

X_p	Y_p	X_{p+1}	Y_{p+1}	Round off (X_{p+1}, Y_{p+1})
5	6	6	6.5	(6, 7)
		7	7	(7, 7)
		8	7.5	(8, 8)
		9	8	(9, 8)
		10	8.5	(10, 9)
		11	9	(11, 9)
		12	9.5	(12, 10)
		13	10	(13, 10)


12
10
8
6
4
2
0
0
2
4
6
8
10
12
14

Problem-03:

Calculate the points between the starting point (1, 7) and ending point (11, 17).

Solution-

Given-

- Starting coordinates = (X_0, Y_0) = (1, 7)
- Ending coordinates = (X_n, Y_n) = (11, 17)

Step-01:

Calculate ΔX, ΔY and M from the given input.

- $\Delta X = X_n – X_0 = 11 – 1 = 10$
- $\Delta Y = Y_n – Y_0 = 17 – 7 = 10$
- $M = \Delta Y / \Delta X = 10 / 10 = 1$

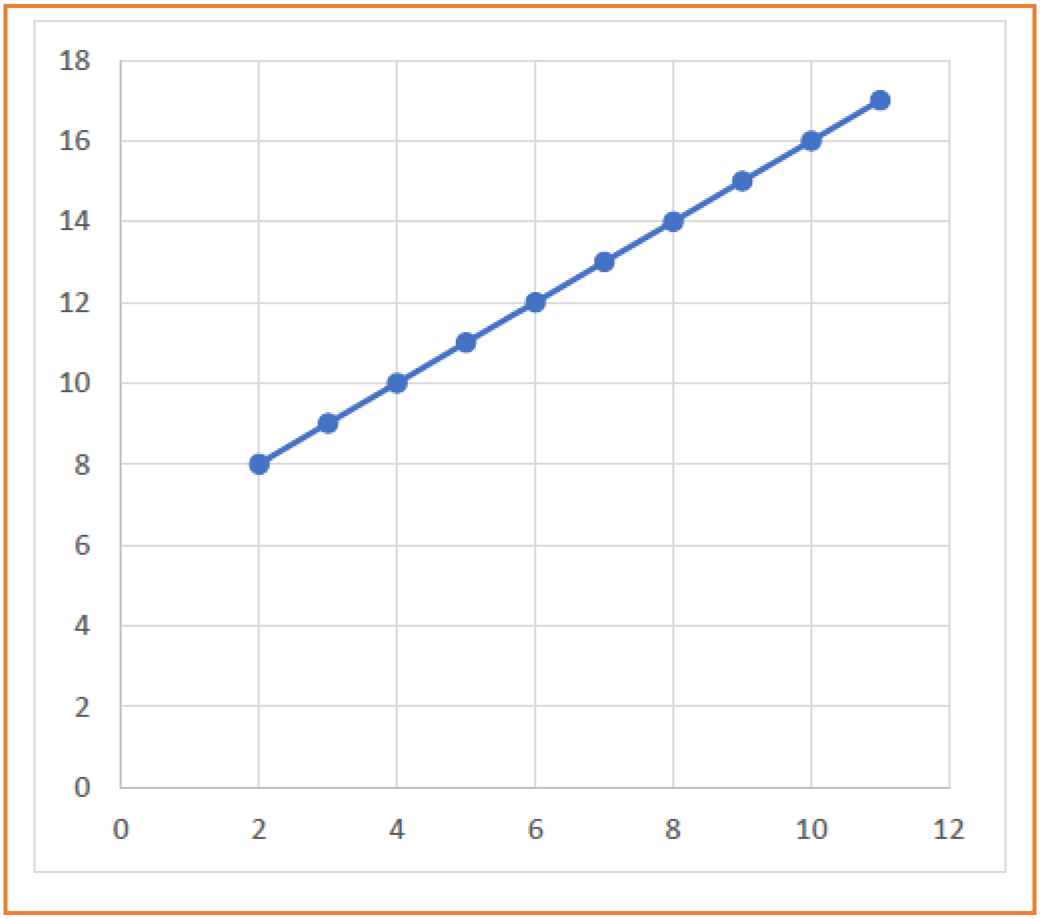
Step-02:

Calculate the number of steps.
As |ΔX| = |ΔY| = 10 = 10, so number of steps = ΔX = ΔY = 10

Step-03:

As M = 1, so case-02 is satisfied.
Now, Step-03 is executed until Step-04 is satisfied.

X_p	Y_p	X_{p+1}	Y_{p+1}	Round off (X_{p+1}, Y_{p+1})
1	7	2	8	(2, 8)
		3	9	(3, 9)
		4	10	(4, 10)
		5	11	(5, 11)
		6	12	(6, 12)
		7	13	(7, 13)
		8	14	(8, 14)
		9	15	(9, 15)
		10	16	(10, 16)
		11	17	(11, 17)



Advantages of DDA Algorithm-

The advantages of DDA Algorithm are-

- It is a simple algorithm.
- It is easy to implement.
- It avoids using the multiplication operation which is costly in terms of time complexity.

Disadvantages of DDA Algorithm-

The disadvantages of DDA Algorithm are-

- There is an extra overhead of using round off( ) function.
- Using round off( ) function increases time complexity of the algorithm.
- Resulted lines are not smooth because of round off( ) function.
- The points generated by this algorithm are not accurate.

Bresenham Line Drawing Algorithm-

> Given the starting and ending coordinates of a line,
> Bresenham Line Drawing Algorithm attempts to generate the points between the starting and ending coordinates.

Procedure-

Given-

- Starting coordinates = (X_0, Y_0)
- Ending coordinates = (X_n, Y_n)

The points generation using Bresenham Line Drawing Algorithm involves the following steps-

Step-01:

Calculate ΔX and ΔY from the given input.
These parameters are calculated as-

- $\Delta X = X_n - X_0$
- $\Delta Y = Y_n - Y_0$

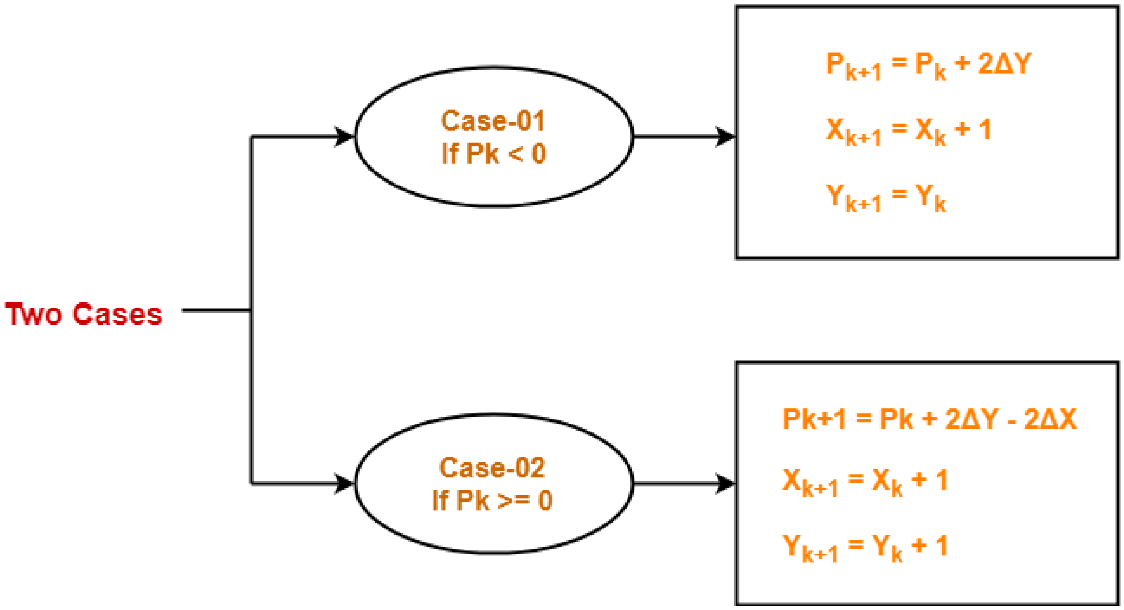
Step-02:

Calculate the decision parameter P_k.
It is calculated as-

$$P_k = 2\Delta Y - \Delta X$$

Step-03:

Suppose the current point is (X_k, Y_k) and the next point is (X_{k+1}, Y_{k+1}).
Find the next point depending on the value of decision parameter P_k.
Follow the below two cases-



Step-04:

Keep repeating Step-03 until the end point is reached or number of iterations equals to (ΔX-1) times.

PRACTICE PROBLEMS BASED ON BRESENHAM LINE DRAWING ALGORITHM-

Problem-01:

Calculate the points between the starting coordinates (9, 18) and ending coordinates (14, 22).

Solution-

Given-

- Starting coordinates = (X_0, Y_0) = (9, 18)
- Ending coordinates = (X_n, Y_n) = (14, 22)

Step-01:

Calculate ΔX and ΔY from the given input.

- $\Delta X = X_n – X_0 = 14 – 9 = 5$
- $\Delta Y = Y_n – Y_0 = 22 – 18 = 4$

Step-02:

Calculate the decision parameter.

P_k
$= 2\Delta Y – \Delta X$
$= 2 \text{ x } 4 – 5$
$= 3$
So, decision parameter $P_k = 3$

Step-03:

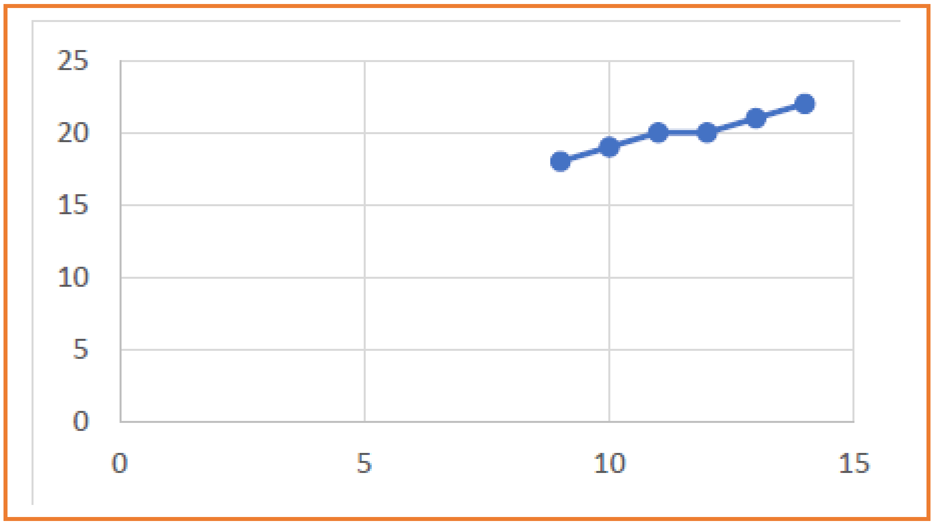
As $P_k >= 0$, so case-02 is satisfied.

Thus,

- $P_{k+1} = P_k + 2\Delta Y – 2\Delta X = 3 + (2 \text{ x } 4) – (2 \text{ x } 5) = 1$
- $X_{k+1} = X_k + 1 = 9 + 1 = 10$
- $Y_{k+1} = Y_k + 1 = 18 + 1 = 19$

Similarly, Step-03 is executed until the end point is reached or number of iterations equals to 4 times.
(Number of iterations = ΔX – 1 = 5 – 1 = 4)

P_k	P_{k+1}	X_{k+1}	Y_{k+1}
		9	18
3	1	10	19
1	-1	11	20
-1	7	12	20
7	5	13	21
5	3	14	22


25
20
15
10
5
0
0
5
10
15

Problem-02:

Calculate the points between the starting coordinates (20, 10) and ending coordinates (30, 18).

Solution-

Given-

- Starting coordinates = (X_0, Y_0) = (20, 10)
- Ending coordinates = (X_n, Y_n) = (30, 18)

Step-01:

Calculate ΔX and ΔY from the given input.

- $\Delta X = X_n – X_0 = 30 – 20 = 10$
- $\Delta Y = Y_n – Y_0 = 18 – 10 = 8$

Step-02:

Calculate the decision parameter.
P_k
$= 2\Delta Y – \Delta X$
$= 2 \text{ x } 8 – 10$
$= 6$
So, decision parameter $P_k = 6$

Step-03:

As $P_k >= 0$, so case-02 is satisfied.

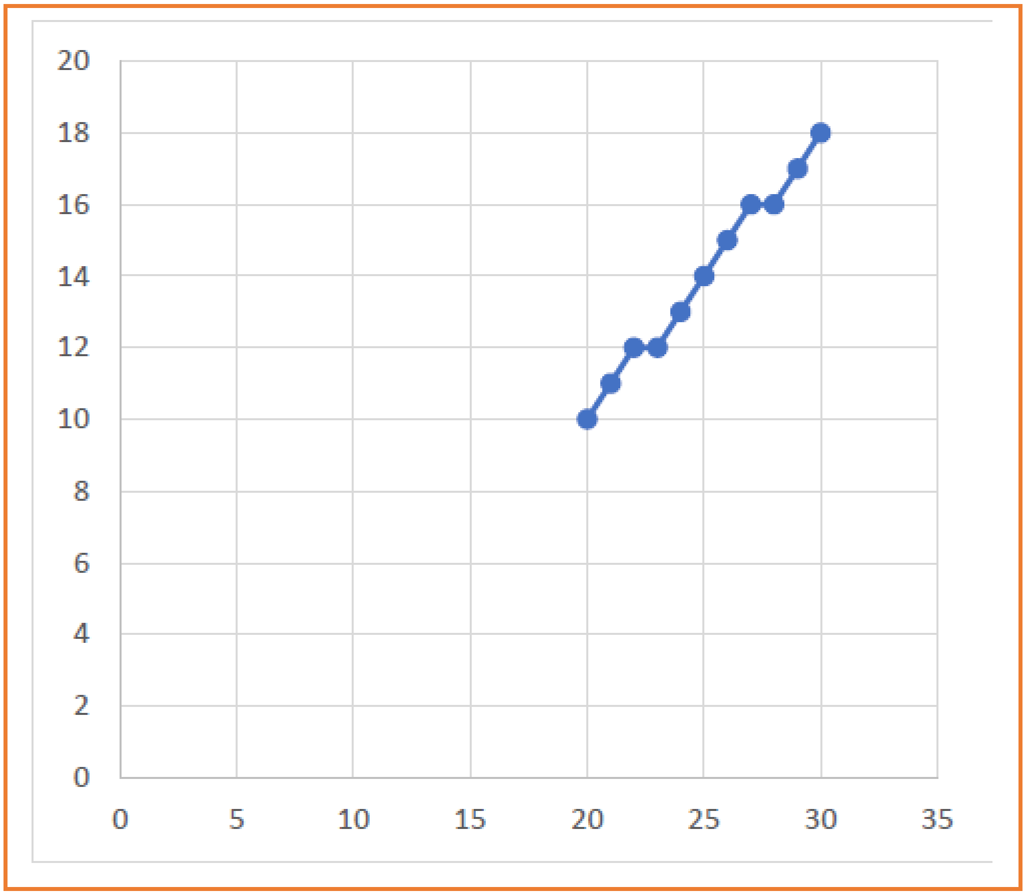
Thus,

- $P_{k+1} = P_k + 2\Delta Y – 2\Delta X = 6 + (2 \text{ x } 8) – (2 \text{ x } 10) = 2$
- $X_{k+1} = X_k + 1 = 20 + 1 = 21$
- $Y_{k+1} = Y_k + 1 = 10 + 1 = 11$

Similarly, Step-03 is executed until the end point is reached or number of iterations equals to 9 times.
(Number of iterations = ΔX – 1 = 10 – 1 = 9)

P_k	P_{k+1}	X_{k+1}	Y_{k+1}
		20	10
6	2	21	11
2	-2	22	12
-2	14	23	12
14	10	24	13
10	6	25	14

6	2	26	15
2	-2	27	16
-2	14	28	16
14	10	29	17
10	6	30	18



Advantages of Bresenham Line Drawing Algorithm-

The advantages of Bresenham Line Drawing Algorithm are-

- It is easy to implement.
- It is fast and incremental.
- It executes fast but less faster than DDA Algorithm.
- The points generated by this algorithm are more accurate than DDA Algorithm.
- It uses fixed points only.

Disadvantages of Bresenham Line Drawing Algorithm-

The disadvantages of Bresenham Line Drawing Algorithm are-

- Though it improves the accuracy of generated points but still the resulted line is not smooth.
- This algorithm is for the basic line drawing.
- It can not handle diminishing jaggies.

Mid Point Line Drawing Algorithm-

> Given the starting and ending coordinates of a line,
> Mid Point Line Drawing Algorithm attempts to generate the points between the starting and ending coordinates.

Procedure-

Given-

- Starting coordinates = (X_0, Y_0)
- Ending coordinates = (X_n, Y_n)

The points generation using Mid Point Line Drawing Algorithm involves the following steps-

Step-01:

Calculate ΔX and ΔY from the given input.
These parameters are calculated as-

- $\Delta X = X_n - X_0$
- $\Delta Y = Y_n - Y_0$

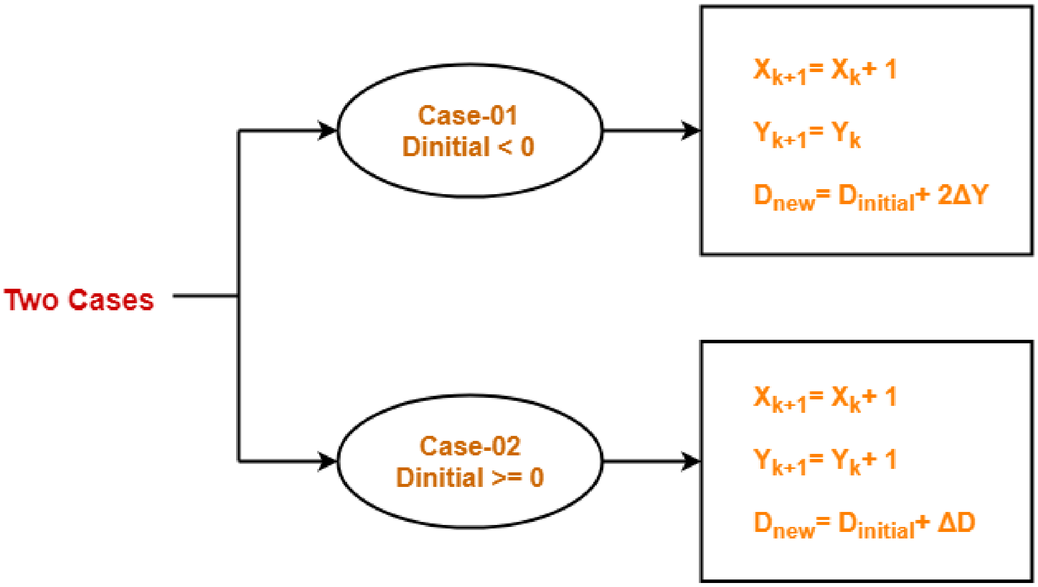
Step-02:

Calculate the value of initial decision parameter and ΔD.
These parameters are calculated as-

- $D_{initial} = 2\Delta Y - \Delta X$
- $\Delta D = 2(\Delta Y - \Delta X)$

Step-03:

The decision whether to increment X or Y coordinate depends upon the flowing values of $D_{initial}$.
Follow the below two cases-



Step-04:

Keep repeating Step-03 until the end point is reached.
For each D_{new} value, follow the above cases to find the next coordinates.

PRACTICE PROBLEMS BASED ON MID POINT LINE DRAWING ALGORITHM-

Problem-01:

Calculate the points between the starting coordinates (20, 10) and ending coordinates (30, 18).

Solution-

Given-

- Starting coordinates = $(X_0, Y_0) = (20, 10)$
- Ending coordinates = $(X_n, Y_n) = (30, 18)$

Step-01:

Calculate ΔX and ΔY from the given input.

- $\Delta X = X_n – X_0 = 30 – 20 = 10$
- $\Delta Y = Y_n – Y_0 = 18 – 10 = 8$

Step-02:

Calculate $D_{initial}$ and ΔD as-

- $D_{initial} = 2\Delta Y – \Delta X = 2 \text{ x } 8 – 10 = 6$
- $\Delta D = 2(\Delta Y – \Delta X) = 2 \text{ x } (8 – 10) = -4$

Step-03:

As $D_{initial} >= 0$, so case-02 is satisfied.

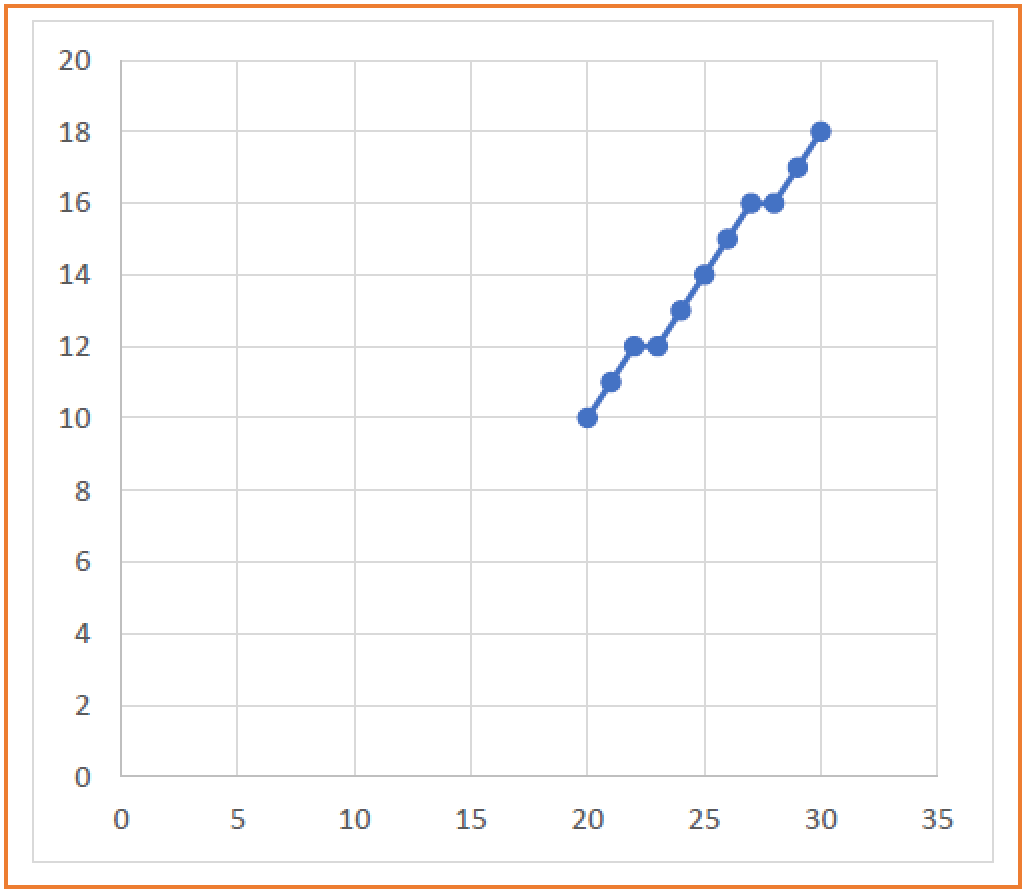
Thus,

- $X_{k+1} = X_k + 1 = 20 + 1 = 21$
- $Y_{k+1} = Y_k + 1 = 10 + 1 = 11$
- $D_{new} = D_{initial} + \Delta D = 6 + (-4) = 2$

Similarly, Step-03 is executed until the end point is reached.

$D_{initial}$	D_{new}	X_{k+1}	Y_{k+1}
		20	10
6	2	21	11
2	-2	22	12
-2	14	23	12
14	10	24	13
10	6	25	14
6	2	26	15

2	-2	27	16
-2	14	28	16
14	10	29	17
10		30	18



Problem-02:

Calculate the points between the starting coordinates (5, 9) and ending coordinates (12, 16).

Solution-

Given-

- Starting coordinates = (X_0, Y_0) = (5, 9)
- Ending coordinates = (X_n, Y_n) = (12, 16)

Step-01:

Calculate ΔX and ΔY from the given input.

- $\Delta X = X_n – X_0 = 12 – 5 = 7$
- $\Delta Y = Y_n – Y_0 = 16 – 9 = 7$

Step-02:

Calculate $D_{initial}$ and ΔD as-

- $D_{initial} = 2\Delta Y – \Delta X = 2 \times 7 – 7 = 7$
- $\Delta D = 2(\Delta Y – \Delta X) = 2 \times (7 – 7) = 0$

Step-03:

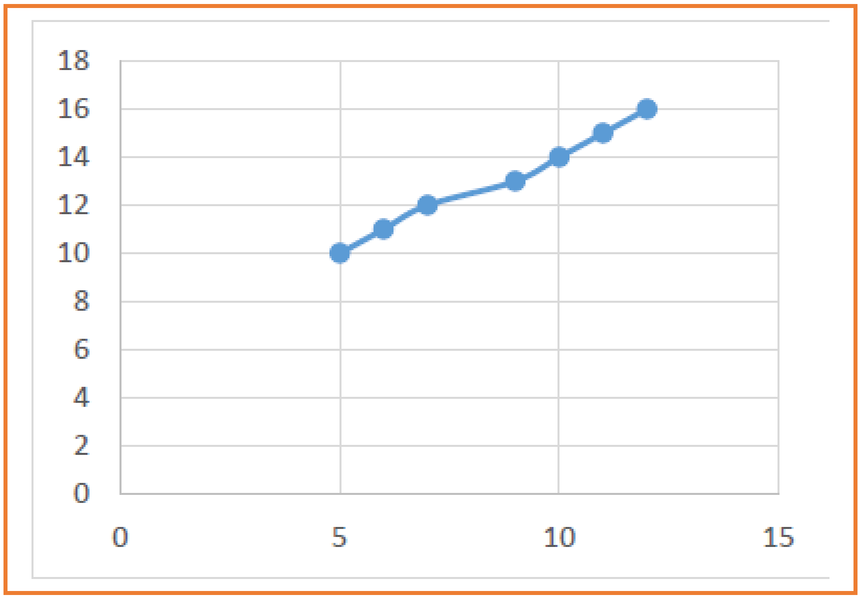
As $D_{initial} >= 0$, so case-02 is satisfied.

Thus,

- $X_{k+1} = X_k + 1 = 5 + 1 = 6$
- $Y_{k+1} = Y_k + 1 = 9 + 1 = 10$
- $D_{new} = D_{initial} + \Delta D = 7 + 0 = 7$

Similarly, Step-03 is executed until the end point is reached.

$D_{initial}$	**D_{new}**	**X_{k+1}**	**Y_{k+1}**
		5	9
7	7	6	10
7	7	7	11
7	7	8	12
7	7	9	13
7	7	10	14
7	7	11	15
7		12	16



Advantages of Mid Point Line Drawing Algorithm-

The advantages of Mid Point Line Drawing Algorithm are-

- Accuracy of finding points is a key feature of this algorithm.
- It is simple to implement.
- It uses basic arithmetic operations.
- It takes less time for computation.
- The resulted line is smooth as compared to other line drawing algorithms.

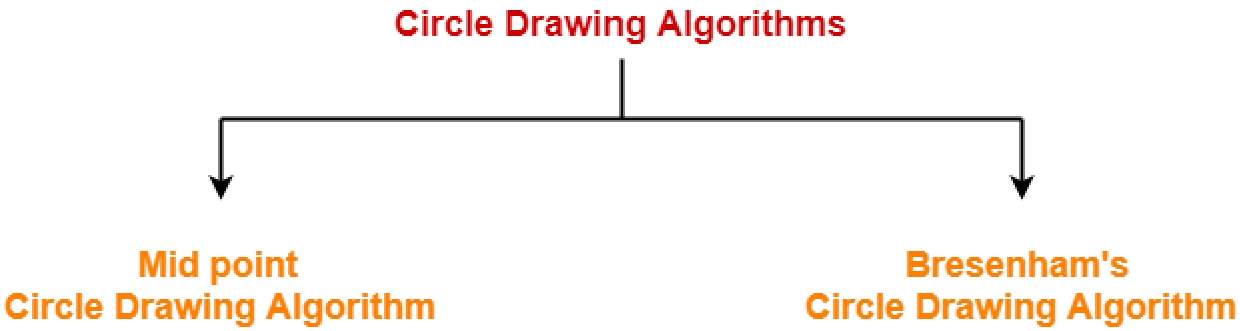
Disadvantages of Mid Point Line Drawing Algorithm-

The disadvantages of Mid Point Line Drawing Algorithm are-

- This algorithm may not be an ideal choice for complex graphics and images.
- In terms of accuracy of finding points, improvement is still needed.
- There is no any remarkable improvement made by this algorithm.

Circle Drawing Algorithms-

In computer graphics, popular algorithms used to generate circle are-



1. Mid Point Circle Drawing Algorithm
2. Bresenham’s Circle Drawing Algorithm

In this article, we will discuss about Mid Point Circle Drawing Algorithm.

Mid Point Circle Drawing Algorithm-

> Given the centre point and radius of circle,
> Mid Point Circle Drawing Algorithm attempts to generate the points of one octant.

The points for other octacts are generated using the eight symmetry property.

Procedure-

Given-

- Centre point of Circle = (X_0, Y_0)
- Radius of Circle = R

The points generation using Mid Point Circle Drawing Algorithm involves the following steps-

Step-01:

Assign the starting point coordinates (X_0, Y_0) as-

- $X_0 = 0$
- $Y_0 = R$

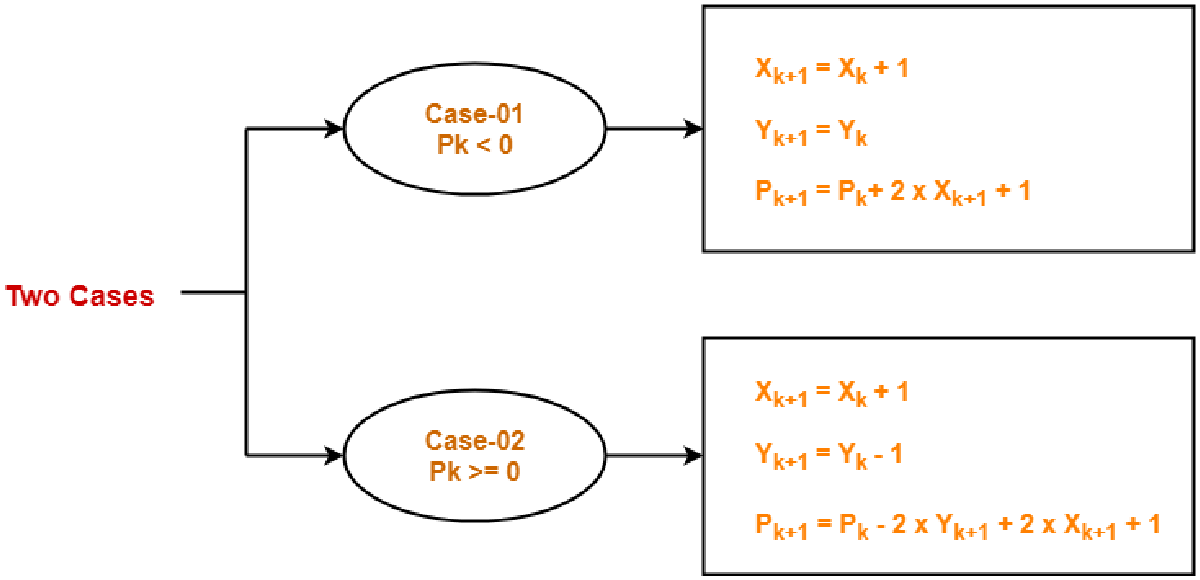
Step-02:

Calculate the value of initial decision parameter P_0 as-

$$P_0 = 1 – R$$

Step-03:

Suppose the current point is (X_k, Y_k) and the next point is (X_{k+1}, Y_{k+1}).
Find the next point of the first octant depending on the value of decision parameter P_k.
Follow the below two cases-



Step-04:

If the given centre point (X_0, Y_0) is not (0, 0), then do the following and plot the point-

- $X_{plot} = X_c + X_0$
- $Y_{plot} = Y_c + Y_0$

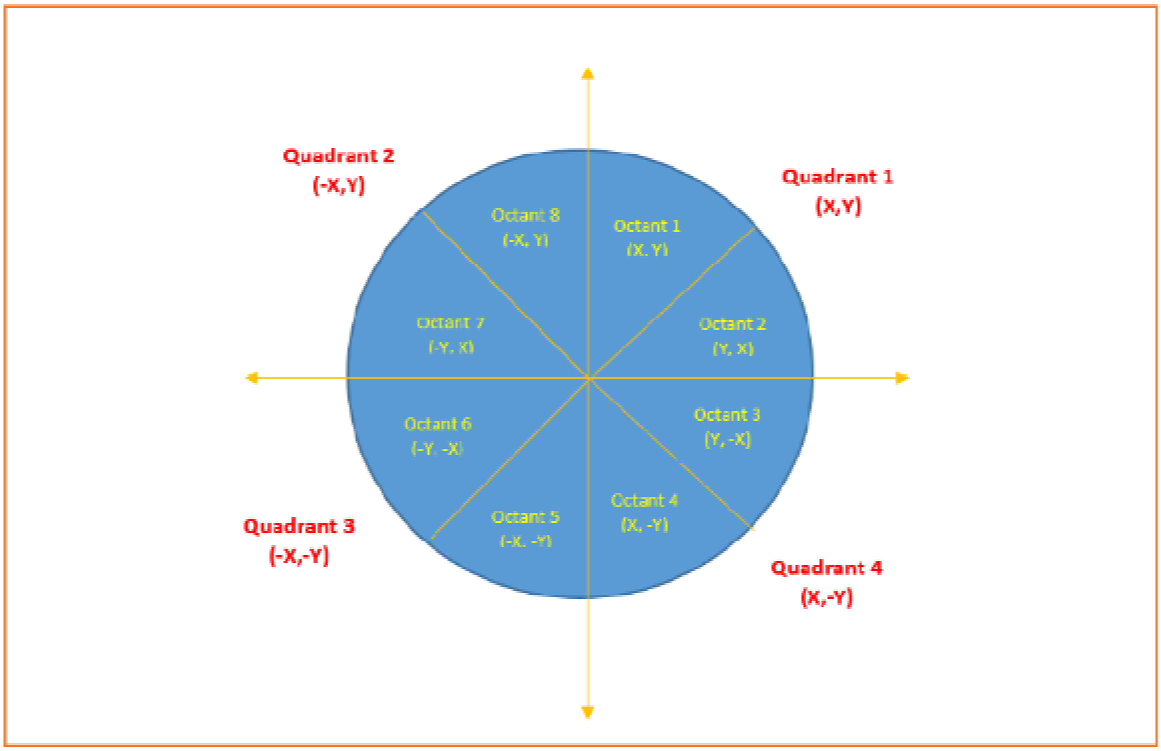
Here, (X_c, Y_c) denotes the current value of X and Y coordinates.

Step-05:

Keep repeating Step-03 and Step-04 until $X_{plot} >= Y_{plot}$.

Step-06:

Step-05 generates all the points for one octant.
To find the points for other seven octants, follow the eight symmetry property of circle.
This is depicted by the following figure-



PRACTICE PROBLEMS BASED ON MID POINT CIRCLE DRAWING ALGORITHM-

Problem-01:

Given the centre point coordinates (0, 0) and radius as 10, generate all the points to form a circle.

Solution-

Given-

- Centre Coordinates of Circle $(X_0, Y_0) = (0, 0)$
- Radius of Circle = 10

Step-01:

Assign the starting point coordinates (X_0, Y_0) as-

- $X_0 = 0$
- $Y_0 = R = 10$

Step-02:

Calculate the value of initial decision parameter P_0 as-

$P_0 = 1 – R$

$P_0 = 1 – 10$

$P_0 = -9$

Step-03:

As $P_{initial} < 0$, so case-01 is satisfied.

Thus,

- $X_{k+1} = X_k + 1 = 0 + 1 = 1$
- $Y_{k+1} = Y_k = 10$
- $P_{k+1} = P_k + 2 \text{ x } X_{k+1} + 1 = -9 + (2 \text{ x } 1) + 1 = -6$

Step-04:

This step is not applicable here as the given centre point coordinates is (0, 0).

Step-05:

Step-03 is executed similarly until $X_{k+1} >= Y_{k+1}$ as follows-

P_k	P_{k+1}	(X_{k+1}, Y_{k+1})
		(0, 10)
-9	-6	(1, 10)
-6	-1	(2, 10)

-1	6	(3, 10)
6	-3	(4, 9)
-3	8	(5, 9)
8	5	(6, 8)
Algorithm Terminates **These are all points for Octant-1.**		

Algorithm calculates all the points of octant-1 and terminates.
Now, the points of octant-2 are obtained using the mirror effect by swapping X and Y coordinates.

Octant-1 Points	**Octant-2 Points**
(0, 10)	(8, 6)
(1, 10)	(9, 5)
(2, 10)	(9, 4)
(3, 10)	(10, 3)
(4, 9)	(10, 2)
(5, 9)	(10, 1)
(6, 8)	(10, 0)
These are all points for Quadrant-1.	

Now, the points for rest of the part are generated by following the signs of other quadrants.
The other points can also be generated by calculating each octant separately.

Here, all the points have been generated with respect to quadrant-1-

Quadrant-1 (X,Y)	**Quadrant-2 (-X,Y)**	**Quadrant-3 (-X,-Y)**	**Quadrant-4 (X,-Y)**
(0, 10)	(0, 10)	(0, -10)	(0, -10)
(1, 10)	(-1, 10)	(-1, -10)	(1, -10)

(2, 10)	(-2, 10)	(-2, -10)	(2, -10)
(3, 10)	(-3, 10)	(-3, -10)	(3, -10)
(4, 9)	(-4, 9)	(-4, -9)	(4, -9)
(5, 9)	(-5, 9)	(-5, -9)	(5, -9)
(6, 8)	(-6, 8)	(-6, -8)	(6, -8)
(8, 6)	(-8, 6)	(-8, -6)	(8, -6)
(9, 5)	(-9, 5)	(-9, -5)	(9, -5)
(9, 4)	(-9, 4)	(-9, -4)	(9, -4)
(10, 3)	(-10, 3)	(-10, -3)	(10, -3)
(10, 2)	(-10, 2)	(-10, -2)	(10, -2)
(10, 1)	(-10, 1)	(-10, -1)	(10, -1)
(10, 0)	(-10, 0)	(-10, 0)	(10, 0)
These are all points of the Circle.			

Problem-02:

Given the centre point coordinates (4, -4) and radius as 10, generate all the points to form a circle.

Solution-

Given-

- Centre Coordinates of Circle $(X_0, Y_0) = (4, -4)$
- Radius of Circle = 10

As stated in the algorithm,

- We first calculate the points assuming the centre coordinates is (0, 0).
- At the end, we translate the circle.

Step-01, Step-02 and Step-03 are already completed in Problem-01.
Now, we find the values of X_{plot} and Y_{plot} using the formula given in Step-04 of the main algorithm.

The following table shows the generation of points for Quadrant-1-

- $X_{plot} = X_c + X_0 = 4 + X_0$
- $Y_{plot} = Y_c + Y_0 = 4 + Y_0$

(X_{k+1}, Y_{k+1})	**(X_{plot}, Y_{plot})**
(0, 10)	(4, 14)
(1, 10)	(5, 14)
(2, 10)	(6, 14)
(3, 10)	(7, 14)
(4, 9)	(8, 13)
(5, 9)	(9, 13)
(6, 8)	(10, 12)
(8, 6)	(12, 10)
(9, 5)	(13, 9)
(9, 4)	(13, 8)
(10, 3)	(14, 7)
(10, 2)	(14, 6)

(10, 1)	(14, 5)
(10, 0)	(14, 4)
These are all points for Quadrant-1.	

The following table shows the points for all the quadrants-

Quadrant-1 (X,Y)	Quadrant-2 (-X,Y)	Quadrant-3 (-X,-Y)	Quadrant-4 (X,-Y)
(4, 14)	(4, 14)	(4, -6)	(4, -6)
(5, 14)	(3, 14)	(3, -6)	(5, -6)
(6, 14)	(2, 14)	(2, -6)	(6, -6)
(7, 14)	(1, 14)	(1, -6)	(7, -6)
(8, 13)	(0, 13)	(0, -5)	(8, -5)
(9, 13)	(-1, 13)	(-1, -5)	(9, -5)
(10, 12)	(-2, 12)	(-2, -4)	(10, -4)
(12, 10)	(-4, 10)	(-4, -2)	(12, -2)
(13, 9)	(-5, 9)	(-5, -1)	(13, -1)
(13, 8)	(-5, 8)	(-5, 0)	(13, 0)
(14, 7)	(-6, 7)	(-6, 1)	(14, 1)
(14, 6)	(-6, 6)	(-6, 2)	(14, 2)
(14, 5)	(-6, 5)	(-6, 3)	(14, 3)
(14, 4)	(-6, 4)	(-6, 4)	(14, 4)
These are all points of the Circle.			

Advantages of Mid Point Circle Drawing Algorithm-

The advantages of Mid Point Circle Drawing Algorithm are-

- It is a powerful and efficient algorithm.
- The entire algorithm is based on the simple equation of circle $X^2 + Y^2 = R^2$.
- It is easy to implement from the programmer's perspective.
- This algorithm is used to generate curves on raster displays.

Disadvantages of Mid Point Circle Drawing Algorithm-

The disadvantages of Mid Point Circle Drawing Algorithm are-

- Accuracy of the generating points is an issue in this algorithm.
- The circle generated by this algorithm is not smooth.
- This algorithm is time consuming.

Important Points

- Circle drawing algorithms take the advantage of 8 symmetry property of circle.
- Every circle has 8 octants and the circle drawing algorithm generates all the points for one octant.
- The points for other 7 octants are generated by changing the sign towards X and Y coordinates.
- To take the advantage of 8 symmetry property, the circle must be formed assuming that the centre point coordinates is (0, 0).
- If the centre coordinates are other than (0, 0), then we add the X and Y coordinate values with each point of circle with the coordinate values generated by assuming (0, 0) as centre point.

Bresenham Circle Drawing Algorithm-

Given the centre point and radius of circle,
Bresenham Circle Drawing Algorithm attempts to generate the points of one octant.

The points for other octacts are generated using the eight symmetry property.

Procedure-

Given-

- Centre point of Circle = (X_0, Y_0)
- Radius of Circle = R

The points generation using Bresenham Circle Drawing Algorithm involves the following steps-

Step-01:

Assign the starting point coordinates (X_0, Y_0) as-

- $X_0 = 0$
- $Y_0 = R$

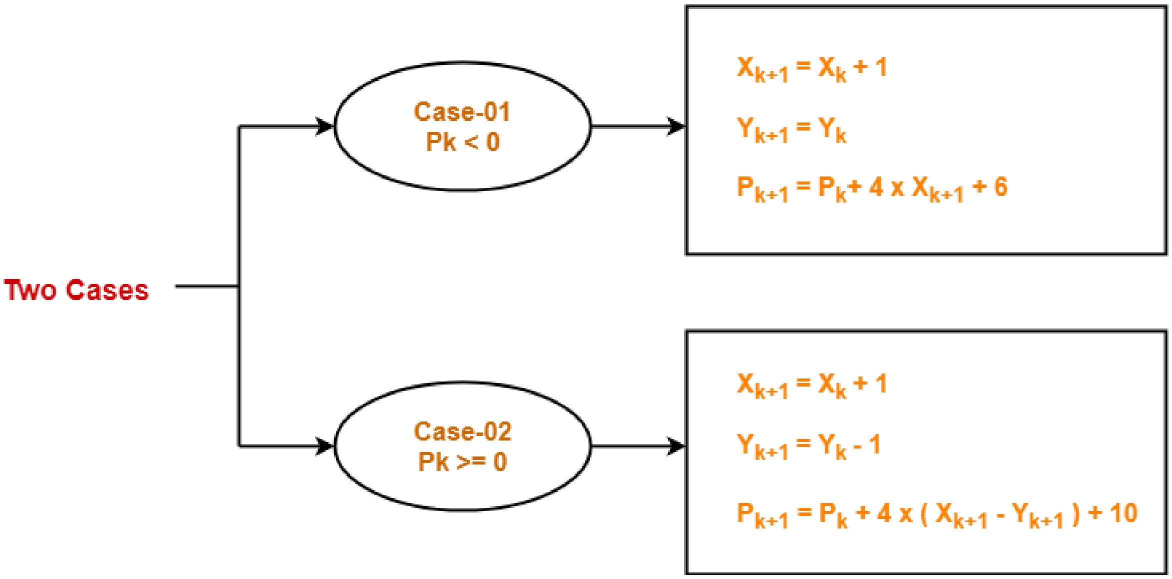
Step-02:

Calculate the value of initial decision parameter P_0 as-

$$P_0 = 3 – 2 \text{ x } R$$

Step-03:

Suppose the current point is (X_k, Y_k) and the next point is (X_{k+1}, Y_{k+1}).
Find the next point of the first octant depending on the value of decision parameter P_k.
Follow the below two cases-



Step-04:

If the given centre point (X_0, Y_0) is not (0, 0), then do the following and plot the point-

- $X_{plot} = X_c + X_0$
- $Y_{plot} = Y_c + Y_0$

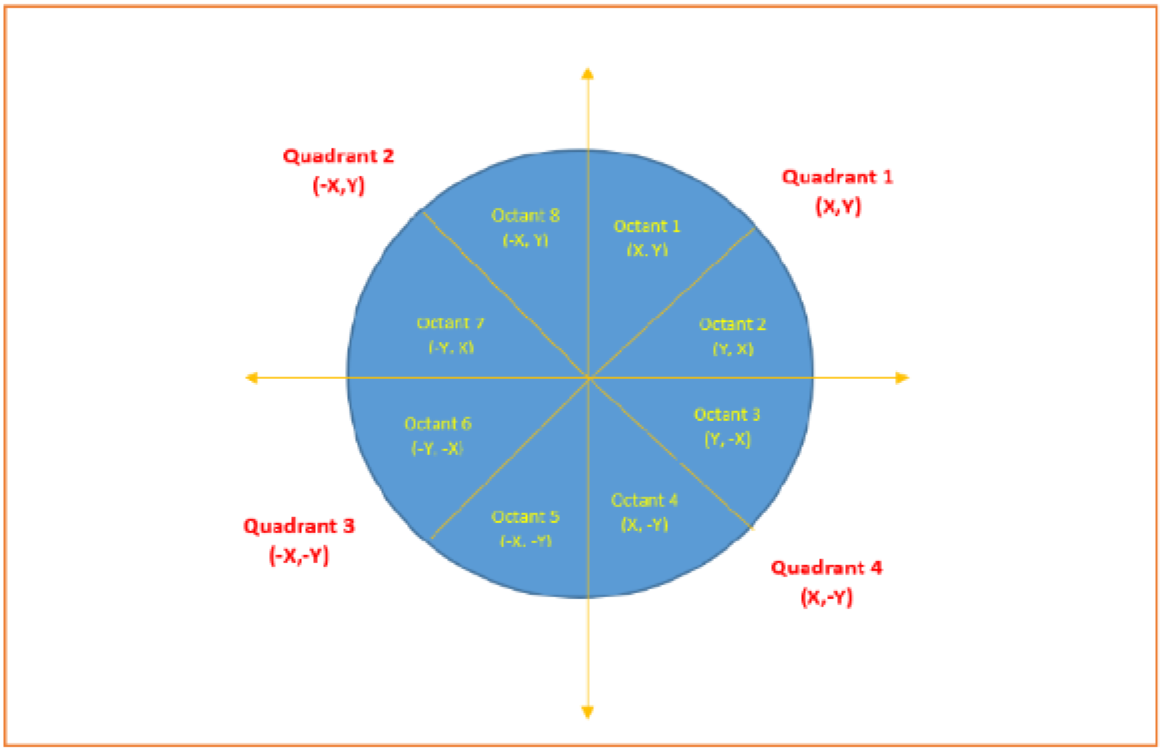
Here, (X_c, Y_c) denotes the current value of X and Y coordinates.

Step-05:

Keep repeating Step-03 and Step-04 until X_{plot} => Y_{plot}.

Step-06:

Step-05 generates all the points for one octant.
To find the points for other seven octants, follow the eight symmetry property of circle.
This is depicted by the following figure-



PRACTICE PROBLEMS BASED ON BRESENHAM CIRCLE DRAWING ALGORITHM-

Problem-01:

Given the centre point coordinates (0, 0) and radius as 8, generate all the points to form a circle.

Solution-

Given-

- Centre Coordinates of Circle $(X_0, Y_0) = (0, 0)$
- Radius of Circle = 8

Step-01:

Assign the starting point coordinates (X_0, Y_0) as-

- $X_0 = 0$
- $Y_0 = R = 8$

Step-02:

Calculate the value of initial decision parameter P_0 as-
$P_0 = 3 – 2 \text{ x } R$
$P_0 = 3 – 2 \text{ x } 8$
$P_0 = -13$

Step-03:

As $P_{initial} < 0$, so case-01 is satisfied.

Thus,

- $X_{k+1} = X_k + 1 = 0 + 1 = 1$
- $Y_{k+1} = Y_k = 8$
- $P_{k+1} = P_k + 4 \text{ x } X_{k+1} + 6 = -13 + (4 \text{ x } 1) + 6 = -3$

Step-04:

This step is not applicable here as the given centre point coordinates is (0, 0).

Step-05:

Step-03 is executed similarly until $X_{k+1} >= Y_{k+1}$ as follows-

P_k	P_{k+1}	(X_{k+1}, Y_{k+1})
		(0, 8)
-13	-3	(1, 8)
-3	11	(2, 8)

11	5	(3, 7)
5	7	(4, 6)
7		(5, 5)
Algorithm Terminates **These are all points for Octant-1.**		

Algorithm calculates all the points of octant-1 and terminates.
Now, the points of octant-2 are obtained using the mirror effect by swapping X and Y coordinates.

Octant-1 Points	Octant-2 Points
(0, 8)	(5, 5)
(1, 8)	(6, 4)
(2, 8)	(7, 3)
(3, 7)	(8, 2)
(4, 6)	(8, 1)
(5, 5)	(8, 0)
These are all points for Quadrant-1.	

Now, the points for rest of the part are generated by following the signs of other quadrants.
The other points can also be generated by calculating each octant separately.

Here, all the points have been generated with respect to quadrant-1-

Quadrant-1 (X,Y)	Quadrant-2 (-X,Y)	Quadrant-3 (-X,-Y)	Quadrant-4 (X,-Y)
(0, 8)	(0, 8)	(0, -8)	(0, -8)
(1, 8)	(-1, 8)	(-1, -8)	(1, -8)
(2, 8)	(-2, 8)	(-2, -8)	(2, -8)
(3, 7)	(-3, 7)	(-3, -7)	(3, -7)

(4, 6)	(-4, 6)	(-4, -6)	(4, -6)
(5, 5)	(-5, 5)	(-5, -5)	(5, -5)
(6, 4)	(-6, 4)	(-6, -4)	(6, -4)
(7, 3)	(-7, 3)	(-7, -3)	(7, -3)
(8, 2)	(-8, 2)	(-8, -2)	(8, -2)
(8, 1)	(-8, 1)	(-8, -1)	(8, -1)
(8, 0)	(-8, 0)	(-8, 0)	(8, 0)
These are all points of the Circle.			

Problem-02:

Given the centre point coordinates (10, 10) and radius as 10, generate all the points to form a circle.

Solution-

Given-

- Centre Coordinates of Circle (X_0, Y_0) = (10, 10)
- Radius of Circle = 10

Step-01:

Assign the starting point coordinates (X_0, Y_0) as-

- $X_0 = 0$
- $Y_0 = R = 10$

Step-02:

Calculate the value of initial decision parameter P_0 as-
$P_0 = 3 – 2 \text{ x } R$
$P_0 = 3 – 2 \text{ x } 10$
$P_0 = -17$

Step-03:

As $P_{initial} < 0$, so case-01 is satisfied.

Thus,

- $X_{k+1} = X_k + 1 = 0 + 1 = 1$
- $Y_{k+1} = Y_k = 10$
- $P_{k+1} = P_k + 4 \text{ x } X_{k+1} + 6 = -17 + (4 \text{ x } 1) + 6 = -7$

Step-04:

This step is applicable here as the given centre point coordinates is (10, 10).

$X_{plot} = X_c + X_0 = 1 + 10 = 11$
$Y_{plot} = Y_c + Y_0 = 10 + 10 = 20$

Step-05:

Step-03 and Step-04 are executed similarly until X_{plot} => Y_{plot} as follows-

P_k	P_{k+1}	(X_{k+1}, Y_{k+1})	(X_{plot}, Y_{plot})
		(0, 10)	(10, 20)
-17	-7	(1, 10)	(11, 20)

-7	7	(2, 10)	(12, 20)
7	-7	(3, 9)	(13, 19)
-7	15	(4, 9)	(14, 19)
15	13	(5, 8)	(15, 18)
13	19	(6, 7)	(16, 17)
Algorithm Terminates **These are all points for Octant-1.**			

Algorithm calculates all the points of octant-1 and terminates.
Now, the points of octant-2 are obtained using the mirror effect by swapping X and Y coordinates.

Octant-1 Points	**Octant-2 Points**
(10, 20)	(17, 16)
(11, 20)	(18, 15)
(12, 20)	(19, 14)
(13, 19)	(19, 13)
(14, 19)	(20, 12)
(15, 18)	(20, 11)
(16, 17)	(20, 10)
These are all points for Quadrant-1.	

Now, the points for rest of the part are generated by following the signs of other quadrants.
The other points can also be generated by calculating each octant separately.

Here, all the points have been generated with respect to quadrant-1-

Quadrant-1 (X,Y)	**Quadrant-2 (-X,Y)**	**Quadrant-3 (-X,-Y)**	**Quadrant-4 (X,-Y)**

(10, 20)	(10, 20)	(10, 0)	(10, 0)
(11, 20)	(9, 20)	(9, 0)	(11, 0)
(12, 20)	(8, 20)	(8, 0)	(12, 0)
(13, 19)	(7, 19)	(7, 1)	(13, 1)
(14, 19)	(6, 19)	(6, 1)	(14, 1)
(15, 18)	(5, 18)	(5, 2)	(15, 2)
(16, 17)	(4, 17)	(4, 3)	(16, 3)
(17, 16)	(3, 16)	(3, 4)	(17, 4)
(18, 15)	(2, 15)	(2, 5)	(18, 5)
(19, 14)	(1, 14)	(1, 6)	(19, 6)
(19, 13)	(1, 13)	(1, 7)	(19, 7)
(20, 12)	(0, 12)	(0, 8)	(20, 8)
(20, 11)	(0, 11)	(0, 9)	(20, 9)
(20, 10)	(0, 10)	(0, 10)	(20, 10)
These are all points of the Circle.			

Advantages of Bresenham Circle Drawing Algorithm-

The advantages of Bresenham Circle Drawing Algorithm are-

- The entire algorithm is based on the simple equation of circle $X^2 + Y^2 = R^2$.
- It is easy to implement.

Disadvantages of Bresenham Circle Drawing Algorithm-

The disadvantages of Bresenham Circle Drawing Algorithm are-

- Like Mid Point Algorithm, accuracy of the generating points is an issue in this algorithm.
- This algorithm suffers when used to generate complex and high graphical images.
- There is no significant enhancement with respect to performance.