Binary Morphology

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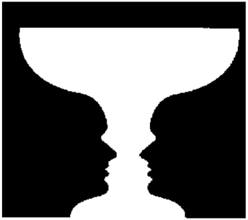
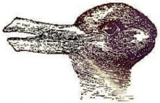
Binary Images – Illusions



Man Playing Horn... Or Woman Silhouette?
(hint: woman’s right eye is the black speck in front of horn handle)



A Rabbit.... Or A Duck?
hint: the duck is looking left, the rabbit is looking right

Binary Images – Definition

- Digital image is an array of sampled and quantized values
- For gray scale images, scale defined by K levels and B bits where $K = 2^B$
- For binary images, $K = 2$ levels and $B = 1$ bit

Binary Images – Interpretation

Common binary image meanings:

- Intensity differentiator: low vs high
- Presence or absence of an object
- Presence or absence of a property

Why work with binary images?

- Contain useful information: shape, structure, form
- Compression (application dependent): B-fold reduction, efficient compression algorithms exist

Binary Images – Generation

Several ways to generate binary images:

- Specialized inputs: stylus based (light pen), tablet etc
- Gray level thresholding
  - Simple thresholding: pick a threshold T and make a binary decision
  - For an image $I(i,j)$ with K levels, pick $0 \leq T \leq K-1$
  - Binary image
    $J(i,j) = 1, \text{ } if \text{ } I(i,j) \geq T, J(i,j) = 0 \text{ } if \text{ } I(i,j) < T$

Binary Images – Threshold Selection

Why is threshold selection important?

- Quality of binary image directly dependent on threshold
- Different thresholds may give different insights
- Some images may not produce useful binary images for any threshold

A couple of questions:

- Is thresholding useful/possible?
- How to pick theshold T?

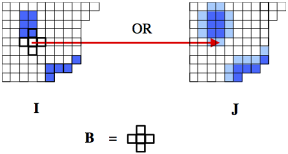
Binary Images – Gray Level Image Histogram

- Histogram H_I of image I is a graph of the gray-level frequency – like a probability mass function
- A one-dimensional function defined on the gray scale i.e., $0 \leq k \leq K-1$
- $H_I(k) = n$ means the gray level k occurs n times in the image
- Histograms reveal a lot about images – examples on board

Binary Images – Histogram Types

- Modal: histograms with distinct peaks or modes
- Bimodal: two peaks or modes
  - Images with a distinct light and dark region
  - Choosing T to lie between modes may produce good results
  - Exact location of T hard to guess
- Multimodal: multiple peaks or modes
  - Images with multiple distinct light and dark regions
  - Varying T produces different results
- Flat: uniform or flat intensity distribution
  - Images with greater complexity, non-uniform background etc
  - Choosing a threshold hard

Binary Morphology



We have a binary image, now what? Let's process it. How?

- Morphology: the study of form and structure
- Mathematical morphology: tool for extracting image components for describing shapes like boundaries, skeletons, convex hulls
- Binary morphology: a class of binary image operators

Binary Morphology

Morophological operations:

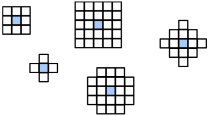
- affect the shapes of objects and regions of binary images
- operate on a local basis i.e., on local neighborhoods

Morophological operators:

- expland or **dilate** objects
- shrink or **erode** objects
- smooth object boundaries
- eliminate holes
- fill gaps and eliminate convex hulls
- are local logical operations

Binary Morphology – Structuring Element

Definition: A structuring element defines a geometric relationship between a pixel and its neighbors.

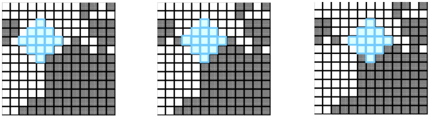


Window:

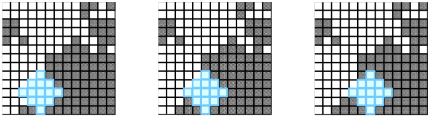
- is a method of collecting pixels according to a geometric rule
- is a structuring element
- almost always contains odd number of elements along each dimension – why?

Binary Morphology – Windows

Using a window to perform local operations over an image.



...



Binary Morphology – Windows

- Definition: A window $\mathbf{B}$ is set of coordinate shifts $\mathbf{B_i} = (p_i, q_i)$ centered around (0, 0) i.e., $\mathbf{B} = \{\mathbf{B_1}, \mathbf{B_2}, \ldots, \mathbf{B_{2P+1}}\}$ $= \{(p_1, q_1)$, (p_2, q_2), . . ., $(p_{2P+1}, q_{2P+1})\}$
- Examples:
  - $\mathbf{B}$ = ROW(2P + 1) = {(0, -P), . . ., (0, P)}
  - $\mathbf{B}$ = COL(2P + 1) = {(-P, 0), . . ., (P, 0)}
  - $\mathbf{B}$ = CROSS(2P + 1) = ROW(2P + 1) **U** COL(2P + 1)

(a) 1-D windows (b) 2-D windows

Binary Morphology – The Windowed Set

- Definition: for binary image $\mathbf{I}$ and window $\mathbf{B}$, the **windowed set** at point (i,j) is defined as
  $\mathbf{B} \diamond \mathbf{I}(i,j) = \{\mathbf{I}(i-p, j-q); (p,q) \in \mathbf{B}\}$
- Interpreted as the set of pixels covered by $\mathbf{B}$ centered at (i,j)
- Helps make simple and flexible definitions of binary filters
- Examples:
  - $\mathbf{B} = \mathrm{ROW}(3)$:
    $\mathbf{B} \diamond \mathbf{I}(i,j) = \{\mathbf{I}(i,j-1), \mathbf{I}(i,j), \mathbf{I}(i,j+1)\}$
  - $\mathbf{B} = \mathrm{SQUARE}(9)$:
    $$\begin{aligned}\mathbf{B} \diamond \mathbf{I}(i,j) = \\ \{&\mathbf{I}(i-1,j-1), \mathbf{I}(i-1,j), \mathbf{I}(i-1,j+1), \\ &\mathbf{I}(i,j-1), \quad \mathbf{I}(i,j), \quad \mathbf{I}(i,j+1), \\ &\mathbf{I}(i+1,j-1), \mathbf{I}(i+1,j), \mathbf{I}(i+1,j+1)\}\end{aligned}$$

Binary Morphology – General Binary Filter

- Notation: A binary operator $\mathbf{G}$ on a windowed set $\mathbf{B} \diamond \mathbf{I}(i, j)$ is denoted as
  $\mathbf{J}(i, j) = \mathbf{G}\{\mathbf{B} \diamond \mathbf{I}(i, j)\} = \mathbf{G}\{\mathbf{I}(i - p, j - q); (p, q) \in \mathbf{B}\}$
- Performing the operation at every pixel gives the filtered image
  $\mathbf{J} = \mathbf{G}[\mathbf{I}, \mathbf{B}] = [\mathbf{J}(i, j);\ 0 \leq i \leq N - 1,\ 0 \leq j \leq M - 1]$
- How about image boundary?

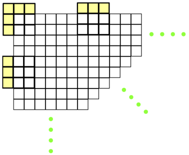


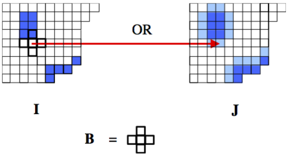
Figure: Replication: use nearest neighbor to fill empty slots

Binary Morphology – Dilation and Erosion Filters

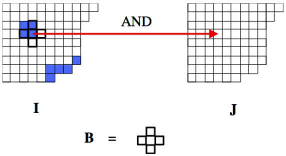
- **Dilation:** Given a window $\mathbf{B}$ and a binary image $\mathbf{I}$,
  $\mathbf{J} = \text{DILATE}(\mathbf{I}, \mathbf{B})$ if
  $\mathbf{J}(i,j) = \text{OR}\{\mathbf{B} \diamond \mathbf{I}(i,j)\} = \text{OR}\{\mathbf{I}(i-p, j-q); (p,q) \in \mathbf{B}\}$
- **Erosion:** Given a window $\mathbf{B}$ and a binary image $\mathbf{I}$,
  $\mathbf{J} = \text{ERODE}(\mathbf{I}, \mathbf{B})$ if
  $\mathbf{J}(i,j) = \text{AND}\{\mathbf{B} \diamond \mathbf{I}(i,j)\} = \text{AND}\{\mathbf{I}(i-p, j-q); (p,q) \in \mathbf{B}\}$

Binary Morphology – Dilation and Erosion Filters

- Change in convention: for ease of illustration, let logical 1 be denoted by a dark pixel
- **Dilation** increases the size of logical 1 objects



- **Erosion** decreases the size of logical 1 objects



Binary Morphology – Interpreting Dilation

(a) Window rolling along/outside black object edges

(b) Center of window traces out a set of paths

(c) The path forms the dilated image boundary

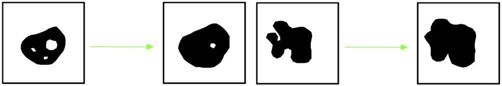
Binary Morphology – Interpreting Erosion

(a) Window rolling inside black object edges

(b) Center of window traces out a set of paths

(c) The path forms the eroded image boundary

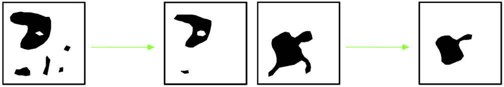
Binary Morphology – Properties of Dilation and Erosion



(a) Removes small holes

(b) Removes small gaps or bays

Figure: Dilation properties



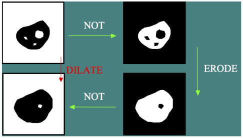
(a) Removes small objects

(b) Removes narrow peninsulas

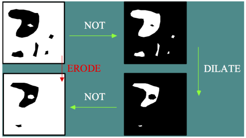
Figure: Erosion properties

Binary Morphology – Relating Dilation and Erosion

Dilation and Erosion are dual operations w.r.t complementation



(a) Dilation directly equivalent to complement erosion and complement



(b) Erosion directly equivalent to complement dilation and complement

Figure: Duality of dilation and erosion

Binary Morphology – Relating Dilation and Erosion

- Dilation and Erosion are approximate inverses of one another
- Dilating an eroded image rarely yields the original
  - Peninsulas eliminated by erosion cannot be recreated
  - Small objects eliminated by erosion cannot be recreated
- Eroding a dilated image rarely yields the original
  - Holes filled by dilation cannot be unfilled
  - Gaps or bays filled by dilation cannot be recreated

Binary Morphology – Median (Majority) Filter

Definition: Given a window $\mathbf{B}$ and a binary image $\mathbf{I}$,
$\mathbf{J} = \text{MEDIAN}(\mathbf{I}, \mathbf{B})$ if
$\mathbf{J}(i,j) = \text{MAJ}\{\mathbf{B} \diamond \mathbf{I}(i,j)\} = \text{MAJ}\{\mathbf{I}(i-p, j-q); (p,q) \in \mathbf{B}\}$
Properties:

- A special case of the gray scale median filter
- Properties of erosion and dilation but usually doesn't change object size

Binary Morphology – Properties of Median Filter

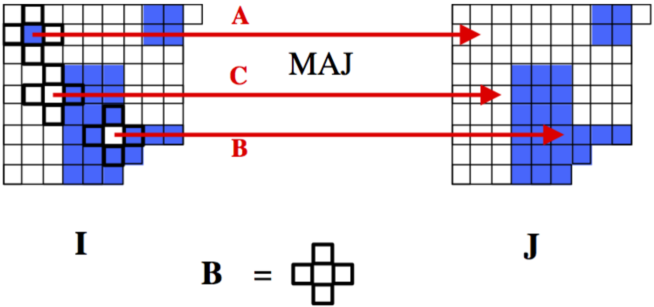


Figure: Small object A and small hole B removed but size of C unchanged

Binary Morphology – Properties of Median Filter

- Generally does not change object size (boundary), but alters them
- Median is its own dual w.r.t complementation i.e., $\text{MEDIAN}[\text{NOT}[\mathbf{I}]] = \text{NOT}[\text{MEDIAN}[\mathbf{I}]]$
- Shape smoother

Binary Morphology – Example of 3-D Median Filter

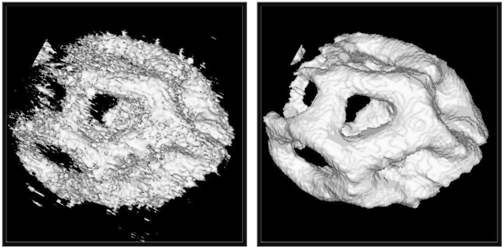
Consider a 3-D image of a pollen grain taken with a Laser Scanning Confocal Microscope (LSCM)

(a) CUBE(125)

(b) CROSS(13)

Figure: 3-D windows

Binary Morphology – Example of 3-D Median Filter



(a) LSCM pollen image

(b) Median filtered using CUBE(125)

Figure: Example of 3-D median filtering

Binary Morphology – OPEN and CLOSE operators

- Define new operators by applying basic operators in sequence
- Given a binary image $\mathbf{I}$ and a window $\mathbf{B}$,
  $\text{OPEN}(\mathbf{I}, \mathbf{B}) = \text{DILATE}[\text{ERODE}(\mathbf{I}, \mathbf{B}),\ \mathbf{B}]$
  $\text{CLOSE}(\mathbf{I}, \mathbf{B}) = \text{ERODE}[\text{DILATE}(\mathbf{I}, \mathbf{B}),\ \mathbf{B}]$
- Similar to MEDIAN filter
- OPEN removes small objects better than MEDIAN but not holes, gaps or bays
- CLOSE removes small holes and gaps better than MEDIAN but not small objects
- In general, OPEN and CLOSE do not affect object size

Binary Morphology – OPEN and CLOSE versus MEDIAN

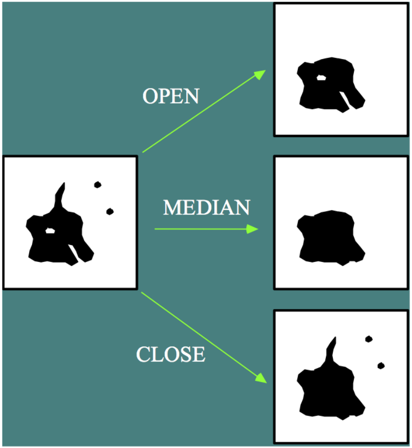


Figure: Comparing OPEN and CLOSE with MEDIAN filter

Binary Morphology – OPEN-CLOSE, CLOSE-OPEN

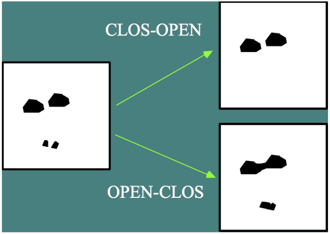
Continuing to cascade basic operators:

- OPEN-CLOS($\mathbf{I}, \mathbf{B}$) = OPEN[CLOSE($\mathbf{I}, \mathbf{B}$), $\mathbf{B}$]
- CLOS-OPEN($\mathbf{I}, \mathbf{B}$) = CLOSE[OPEN($\mathbf{I}, \mathbf{B}$), $\mathbf{B}$]

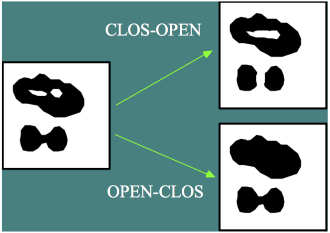
Properties:

- Good smoothing operators
- Remove small objects without affecting size
- Similar to median filter but more smoothing
- OPEN-CLOS tends to link neighboring objects together
- CLOS-OPEN tends to link neighboring holes together

Binary Morphology – Example



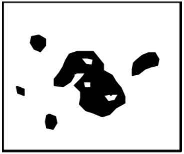
(a) OPEN-CLOS connecting objects



(b) CLOS-OPEN connecting holes

Figure: Examples of OPEN-CLOS and CLOS-OPEN operations

Binary Images – Connected Components



The connected components algorithm:

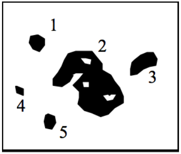
- Also called "region labeling" or "blob coloring"
- Why?
  - Thresholding results in imperfect binary images
  - Extraneous blobs or holes either due to noise or low-interest regions
- Blob coloring is an algorithm for indexing/labeling/coloring objects

Binary Images – Connected Components Algorithm

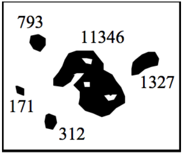
- For binary image I, define a "region color" array R, where, $R(i,j)$ is region number of pixel $I(i,j)$
- Set $R = 0$ (all zeros) and $k = 1$ ($k =$ region number counter)
- While scanning the image from left to right and top to bottom, do the following
  - if $I(i,j) = 0$ and $I(i,j-1) = 1$ and $I(i-1,j) = 1$ then set $R(i,j) = k$ and $k = k + 1$;
  - if $I(i,j) = 0$ and $I(i,j-1) = 1$ and $I(i-1,j) = 0$ then set $R(i,j) = R(i-1,j)$;
  - if $I(i,j) = 0$ and $I(i,j-1) = 0$ and $I(i-1,j) = 1$ then set $R(i,j) = R(i,j-1)$;
  - if $I(i,j) = 0$ and $I(i,j-1) = 0$ and $I(i-1,j) = 0$ then set $R(i,j) = R(i-1,j)$; if $R(i,j-1) \neq R(i-1,j)$ then set $R(i,j-1) = R(i-1,j)$

Binary Images – Connected Components Example

Blob coloring result:

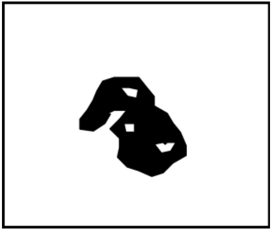


Blob counting result (counting number of pixels in each color):



Binary Images – Minor Blob Removal

- Let m be label of largest region
- While scanning image left to right and top to bottom if $I(i,j) = 0$ and $R(i,j) \neq m$, set $I(i,j) = 1$



Binary Images – Minor Blob Removal

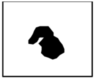
To clean up blob:

- Complement

- Count blobs

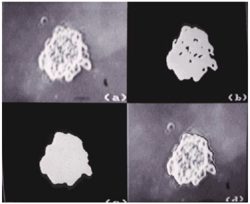
- Minor blob removal

- Complement



Binary Morphology – Practical Application

Measuring cell area:



- Binarize image after thresholding
- Applying region correction
  - Blob coloring
  - Minor blob removal
  - CLOS-OPEN
- Display result for verifying operator
- Count pixels for cell area calculation
- True cell area computed using perspective projection

Binary Morphology – Summary

- Binary images are a very useful class of digital images
- Binary morphology provides techniques for accomplishing several useful tasks