14/6/19

Convolution

- Linear Systems
- Time Invariant Systems
- Linear & Time Invariant Systems
  - Convolution Sum
- Demo

Notation: n: index

$x[n]$: input signal

e.g. $x[n] = \{\underset{\substack{\uparrow \\ 0}}{4}, 0, 7, 5, 6\}$

$y[n]$: output signal

$T\{\}$: digital system

Digital System: A function or a mapping from an input sequence to a <u>desired output</u> sequence

$$y[n] = T\{x[n]\}.$$

Ex1. Find the runs given by a bowler per over at the end of 'n' overs.

$$y[n] = \frac{1}{n} \sum_{i=0}^{n-1} x[i]$$

Ex2: Find the number of runs given by a bowler after n^{th} over

$$y[n] = \sum_{i=6(n-1)}^{6n-1} x[i]$$

- <u>Linear System:</u> A system $T\{\ \}$ is said to be linear if $y_1[n] = T\{x_1[n]\}$, $y_2[n] = T\{x_2[n]\}$, and

$$T\{a x_1[n] + b x_2[n]\} = a y_1[n] + b y_2[n]. \quad -(1)$$

- <u>Time-invariant System:</u> A system $T\{\ \}$ is said to be shift invariant if $y[n] = T\{x[n]\}$ and

$$T\{x[n-n_0]\} = y[n-n_0] \quad -(2)$$

Ex 3: $y[n] = x[Mn] \qquad M > 1$

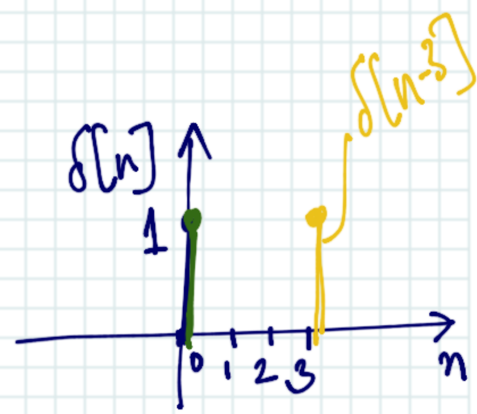
Hint: let $x_1[n] = x[n-n_0]$.

Find $T\{x_1[n]\}$ and compare with $y[n-n_0]$

- <u>Linear & Time Invariant System (LTI):</u> A system that is both linear and time invariant

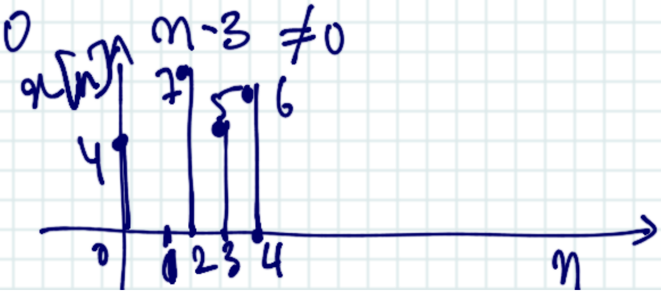
- <u>The discrete delta function:</u>

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



$$\delta[n-3] = \begin{cases} 1, & n-3 = 0, \\ 0 & n-3 \neq 0 \end{cases}$$

$$x[n] = \{\underset{\uparrow}{4}, 0, 7, 5, 6\}$$



$$x[n] = 4.\delta[n] + 0.\delta[n-1] + 7.\delta[n-2] + 5.\delta[n-3] + 6.\delta[n-4].$$

Observation: Any discrete/digital signal $x[n]$ can be expressed as

$$x[n] = \sum_{k=-\infty}^{\infty} a[k].\delta[n-k] \quad - \text{ (3)}$$

<u>LTI System response:</u>

$$y[n] = T\{x[n]\}$$

$$= T\left\{\sum_{k=-\infty}^{\infty} a[k].\delta[n-k]\right\} \quad (\text{from (3)})$$

Apply linearity property

$$= \sum_{k=-\infty}^{\infty} a[k].T\{\delta[n-k]\} \quad \text{due to (1)}$$

Let $h[n] = T\{\delta[n]\}.$

$$\boxed{y[n] = \sum_{k=-\infty}^{\infty} a[k].\,h[n-k].} \quad \text{due to (2)}$$

$\hookrightarrow$ $\underbrace{\text{Convolution}}_{\text{Same as in } \boxed{\text{CNN}}}$ sum.

Extend to 2D

$$\underbrace{h(m,n)} = T\{\delta(m,n)\}$$

$$y[m,n] = \sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} a[j,k]\cdot h[m-j,n-k]$$

2D convolution sum.

$$a = \begin{array}{ll} (0,0) & \qquad\qquad\qquad (0,3) \\ & \begin{bmatrix} 1 & 3 & 4 & 5 \\ 6 & 8 & 9 & 0 \\ 3 & 2 & 8 & 7 \\ 0 & 1 & 5 & 4 \end{bmatrix} \\ (3,0) & \qquad\qquad\qquad (3,3) \end{array} \qquad h = \begin{array}{l} (0,0) \\ \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix} \end{array}$$

$$y[2,2] = \sum_{j=0}^{3}\sum_{k=0}^{3} \underbrace{a[j,k]}\cdot\underbrace{h[2-j,2-k]}$$

h[-j,-k] is found by flipping h[j,k] along both axes

$$h[-j,-k] = \begin{array}{ll} & \qquad\qquad (0,-2) \\ & \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix} \\ (-2,0) & \end{array}$$

$$a[j,k] = \begin{bmatrix} 1 & 3 & 4 & 5 \\ 6 & 8 & 9 & 0 \\ 3 & 2 & 8 & 7 \\ 0 & 1 & 5 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1\cdot 1/9 & 3\cdot 1/9 & 4\cdot 1/9 & 5 \\ 6\cdot 1/9 & 8\cdot 1/9 & 9\cdot 1/9 & 0 \\ 3\cdot 1/9 & 2\cdot 1/9 & 8\cdot 1/9 & 7 \\ 0 & 1 & 5 & 4 \end{bmatrix}$$

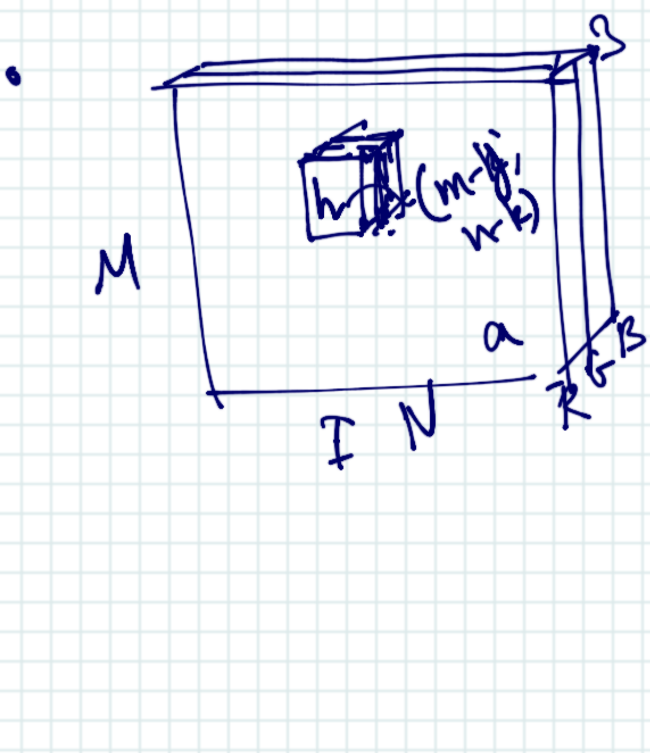
$h[2-j, 2-k]$

$a[j, k]$

$$\therefore\ y[2,2] = \frac{1}{9}\left[1+3+4+6+8+9+3+2+8\right]$$

$$= \frac{44}{9}$$

$$h[m,n] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{(HPF ex)}$$



32 5x5

end w/ learning weights of h.