SI Report
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Assignment 1

Question 1

Question 1

Proof for Gamma Distribution

(shape = r) and (rate = λ)

$$f(x) = \frac{\lambda^{r}}{\Gamma(r)} x^{r-1} e^{-\lambda x}.$$

where $f(x)$ is the PDF of the gamma distribution

Likelihood function.

$$L(\theta) = \prod_{i=1}^{n} f(x)$$

$$L(\theta) = \prod_{i=1}^{n} \left\{ \frac{\lambda^{r}}{\Gamma(r)} * x^{r-1} * e^{-\lambda x} \right\}$$

$$= \left\{ \frac{\lambda^{r}}{\Gamma(r)} \right\}^{n} * \prod_{i=1}^{n} x^{(r-1)} * e^{-nx\lambda}$$

explaination

~~we are taking product and hence~~

the product for $\prod_{i=1}^{n} e^{-\lambda x}$ is $e^{-nx\lambda}$.

which is because we take summation of all $\left\{ \sum_{i=1}^{n} (x_i) \right\}$

$$L(\theta) = \left(\frac{\lambda^{r}}{\Gamma(r)}\right)^{n} * \prod_{i=1}^{n} x_i^{(r-1)} * e^{-nx\lambda}$$

taking log on both sides we get.

$$\log L(\theta) = \log\left\{\left(\frac{\lambda^{r}}{\Gamma(r)}\right)^{n} * \prod_{i=1}^{n} x_i^{(r-1)} * e^{-nx\lambda}\right.$$

$$\log L(\theta) = \left\{\log \lambda^{rn} - \log(\Gamma(r))^{n} + \log\left\{\prod_{i=1}^{n} x_i^{(r-1)}\right\} + \log e^{-nx\lambda}\right\}$$

$$= rn\log(\lambda) - n\log\Gamma(r) + (r-1)\log\sum_{i=1}^{n}(x_i) - nx\lambda$$

$$= nr\log\lambda + n\log\Gamma(r) + (r-1)\overline{\log x_i} - nx\lambda.$$

$$= nr\log\lambda + n\log\Gamma(r) + (r-1)\overline{\log x_i} - \bar{x}_i\lambda$$

we took $\log \prod_{i=1}^{n} x_i^{(r-1)}$ to be.

$(\overline{\log x_i})$ and $n\lambda$ as $\bar{x}_i \lambda$.

$$\boxed{\log L(\theta) = nr \log \lambda + n \log \Gamma(r) + (r-1)\, \overline{\log x_i} - \bar{x}_i \lambda.}$$

(i) we use this log likelihood function.

for our graphs.

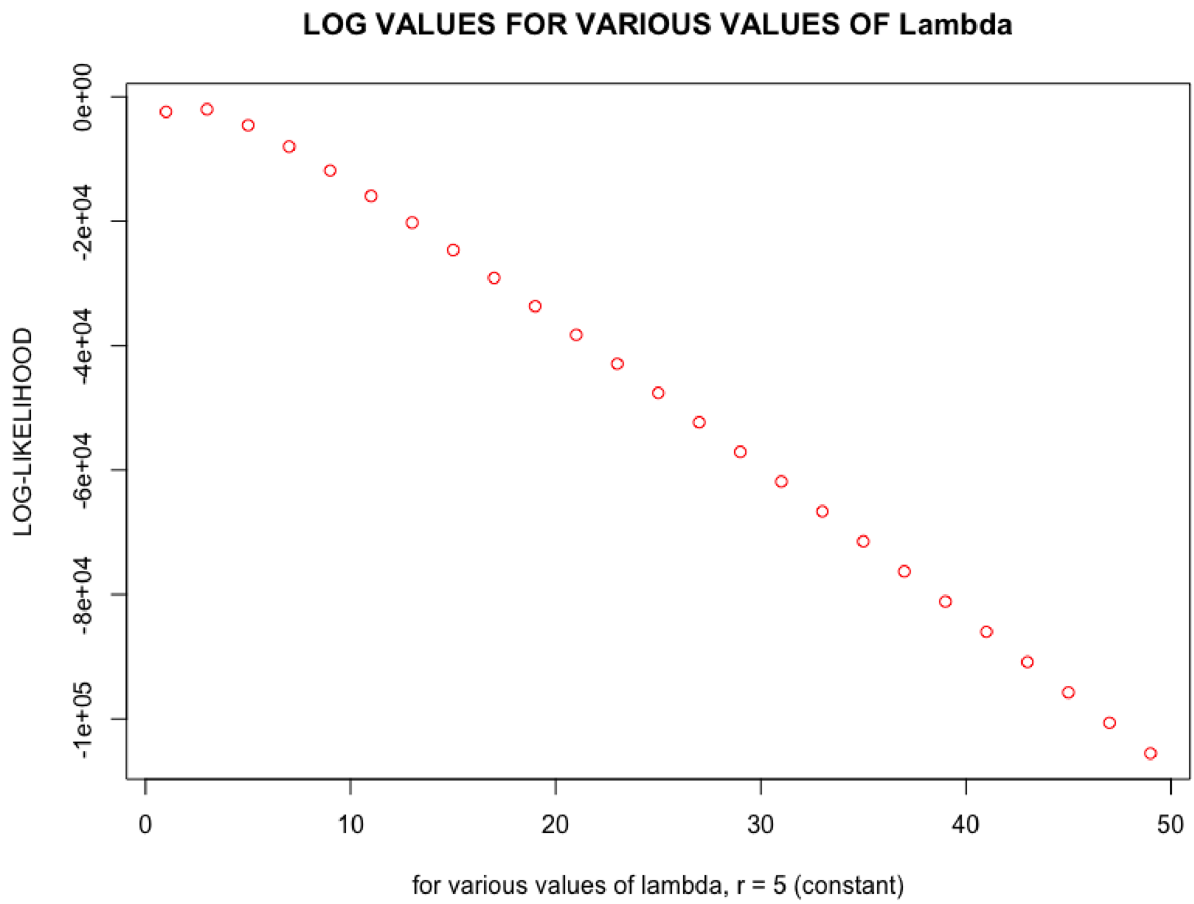
$$\boxed{r = 5}$$
$$\boxed{\lambda = 2}$$

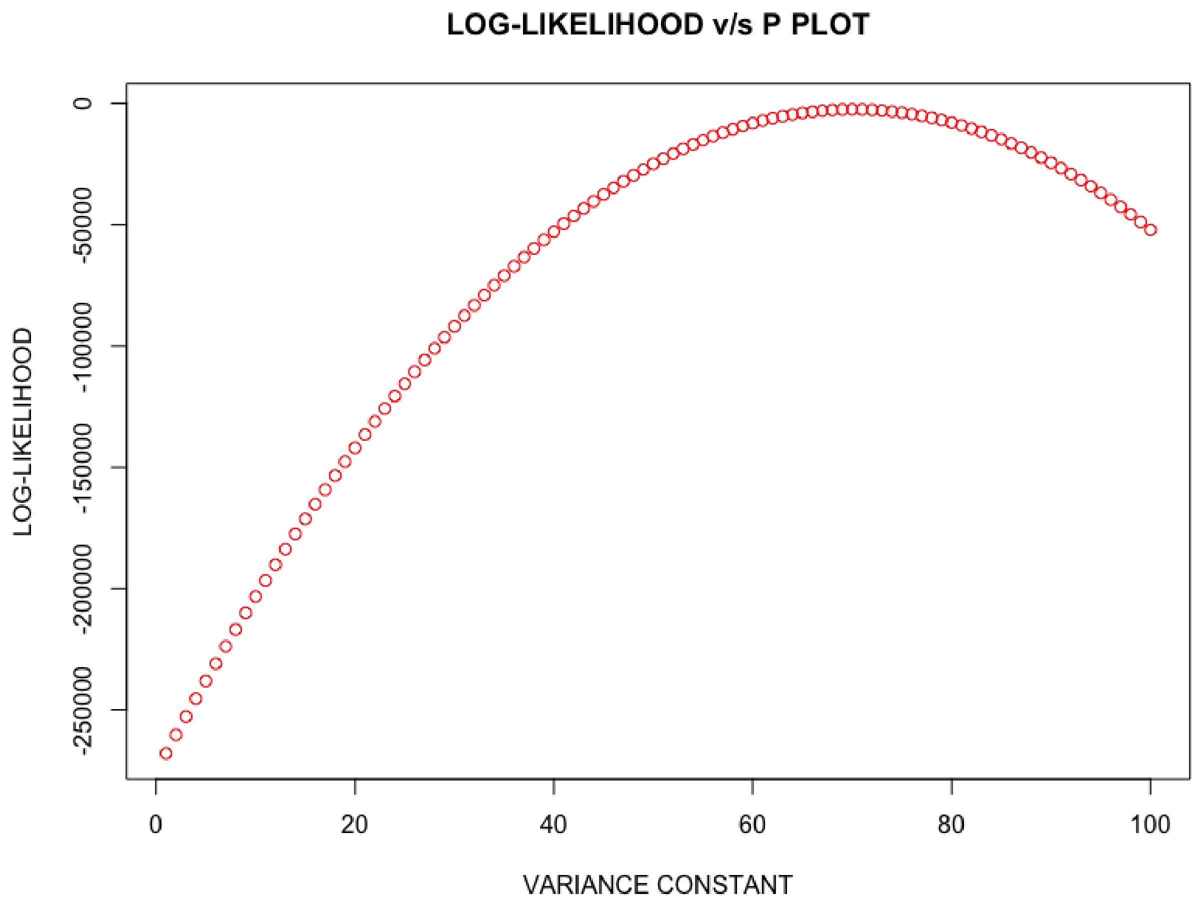
(ii)

$$\log L(\theta) = 1000 * 5 * \log(2) + 1000 * \log \Gamma(5) + (5-1) \log x_i - 1000 * x * 2$$

taking $L(\theta)$ w.r.t λ.

we get $\frac{1}{\lambda} = \frac{\bar{x}}{r}$.


LOG VALUES FOR VARIOUS VALUES OF Lambda
LOG-LIKELIHOOD
0e+00
-2e+04
-4e+04
-6e+04
-8e+04
-1e+05
0
10
20
30
40
50
for various values of lambda, r = 5 (constant)


LOG-LIKELIHOOD v/s P PLOT
LOG-LIKELIHOOD
0
-50000
-100000
-150000
-200000
-250000
0
20
40
60
80
100
VARIANCE CONSTANT

Question 2

Normal Distribution

(2) $x_i \overset{iid}{\sim} N(\mu, \sigma^2)$

$$\underset{\sim}{\theta} = (\underset{\theta_1}{\underset{\downarrow}{\mu}}, \underset{\theta_2}{\underset{\downarrow}{\sigma^2}})$$

MLE of $\underset{\sim}{\theta}$ Likelihood function

$$L(\underset{\sim}{\theta}) = \frac{1}{(2\pi\theta_2)^{n/2}} \exp\left\{-\frac{1}{2} \frac{\Sigma(x_i - \theta_1)^2}{\theta_2}\right\}$$

$$l(\theta) = -\frac{n}{2} \log(2\pi\theta_2) - \frac{1}{2} \frac{\Sigma(x_i - \theta_1)^2}{\theta_2}$$

$$= c - \frac{n}{2} \log \theta_2 - \frac{1}{2} \frac{\Sigma(x_i - \theta_1)^2}{\theta_2}$$

$$\frac{dl}{d\theta_1} = 0 \quad ; \quad \frac{dl}{d\theta_2} = 0$$

$$\frac{dl}{d\theta_1} = \frac{\Sigma(x_i - \theta_1)}{\theta_2} = 0$$

$$\frac{dl}{d\theta_2} = -\frac{n}{2} \frac{1}{\theta_1} + \frac{1}{2} \frac{\Sigma(x_i - \theta_1)^2}{\theta_2^2} = 0$$

$$\hat{\theta}_1 = \bar{x} \quad , \quad \hat{\theta}_2 = \frac{\Sigma(x_i - \hat{\theta}_1)^2}{n}$$

$$\hat{\theta}_2 = \frac{1}{n} \Sigma (x_i - \bar{x})^2$$

ⓐ $\left.\frac{d^2 l}{d\theta_1^2}\right|_{\hat{\theta}_1, \hat{\theta}_2} < 0$

& $\left.\frac{d^2 l}{d\theta_2^2}\right|_{\hat{\theta}_1, \hat{\theta}_2} < 0$

and ⓑ

$$\left|\begin{matrix} \frac{d^2 l}{d\theta_1^2} & \frac{d^2 l}{d\theta_2 d\theta_1} \\ \frac{d^2 l}{d\theta_1 d\theta_2} & \frac{d^2 l}{d\theta_2^2} \end{matrix}\right|_{\hat{\theta}_1, \hat{\theta}_2} > 0$$

$$\left.\frac{d^2 l}{d\theta_1^2}\right|_{\hat{\theta}_1, \hat{\theta}_2} = \frac{-n}{\hat{\theta}_2} = \frac{-n^2}{\Sigma (x_i - \bar{x})^2} < 0$$

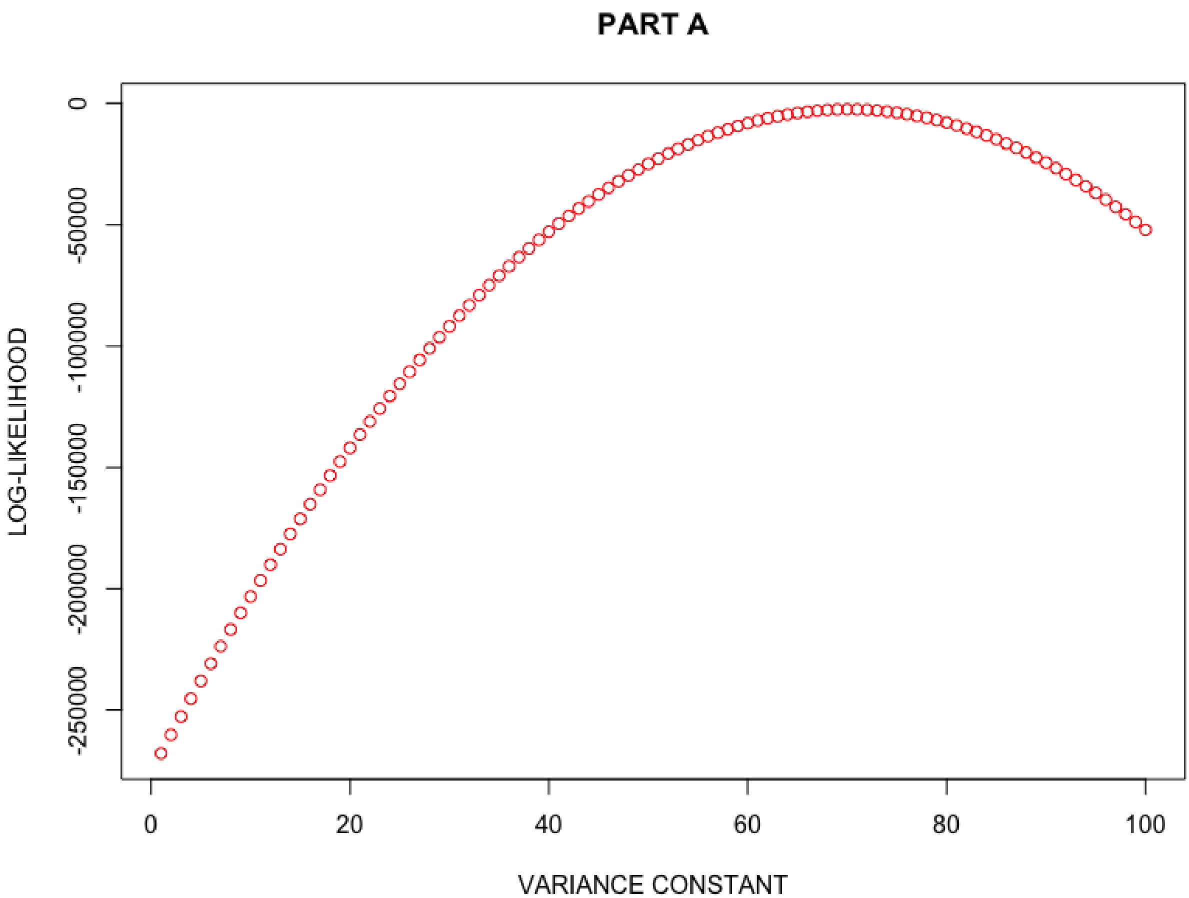
$$\left.\frac{d^2 l}{d\theta_1^2}\right|_{\hat{\theta}_1, \hat{\theta}_2} = \frac{n^3}{2\hat{\theta}_2^2} < 0$$

$$\left.\frac{d^2 l}{d\theta_1\, d\theta_2}\right|_{\hat{\theta}_1, \hat{\theta}_2} = 0$$

$[J] > 0$

thus $\hat{\theta}_1 = \bar{x}$ & $\hat{\theta}_2 = \frac{1}{n} \Sigma (x_i - \bar{x})^2$

are MLEs for θ_1 θ_2



PART A
LOG-LIKELIHOOD
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-50000
-100000
-150000
-200000
-250000
VARIANCE CONSTANT
0
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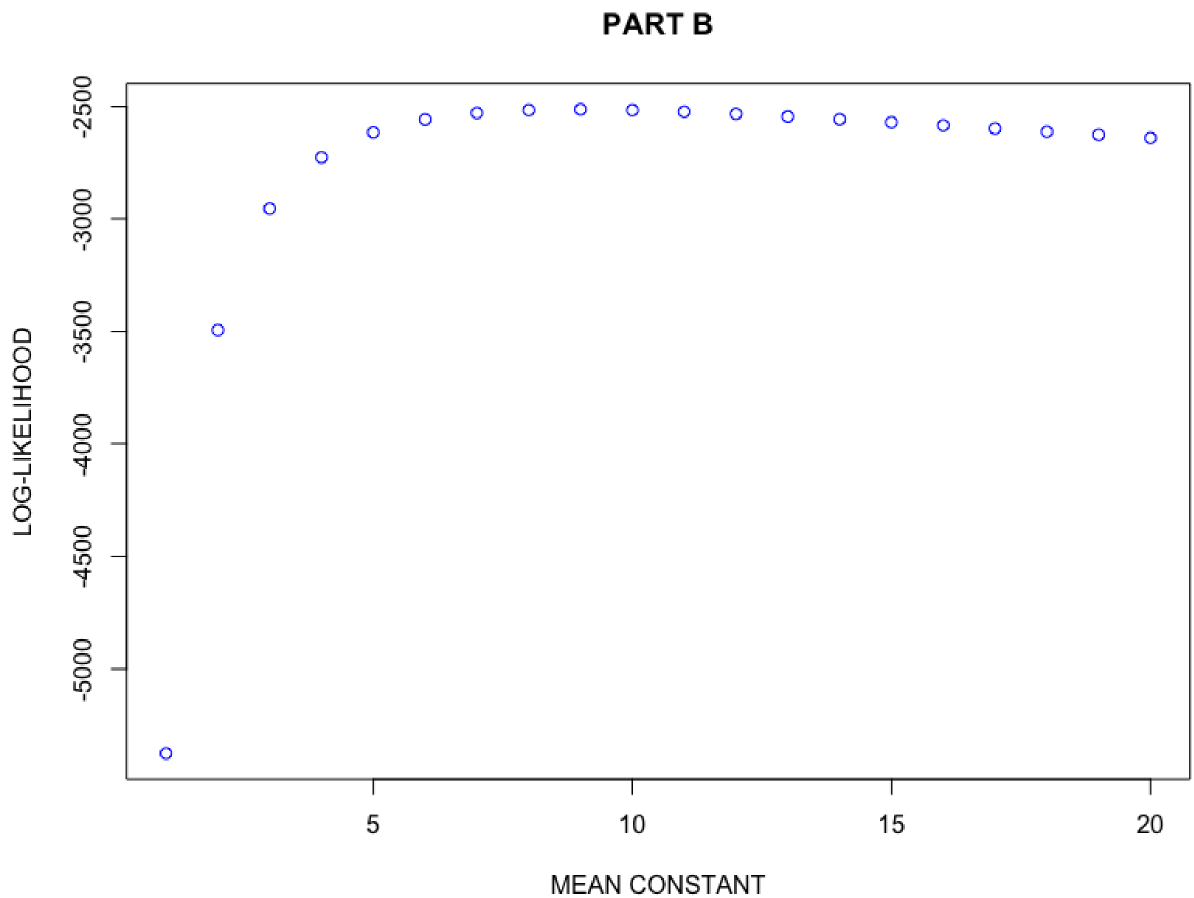
80
100


PART B
LOG-LIKELIHOOD
-2500
-3000
-3500
-4000
-4500
-5000
5
10
15
20
MEAN CONSTANT