② $x = [x_1\ x_2\ x_3]$

$\mu = [5, -5, 6]$ mean vector.

covariance matrix $K_x = \begin{bmatrix} 5 & 2 & -1 \\ 2 & 5 & 0 \\ -1 & 0 & 4 \end{bmatrix}$

$A = A(2, -1, 2)^T$

$B = 5$

mean of $y = A^T x + B$.

$E(y) = A^T E(x) + B$

$= [2, -1, 2] \begin{bmatrix} 5 \\ -5 \\ 6 \end{bmatrix} + 5$

$= 10 + 5 + 12 + 5$

$= \underline{\underline{32}}$

Scanned with CamScanner

Question 1

(a) $P(\omega_1) = \frac{1}{4}$ $\quad P(\cancel{\omega}\, x|\omega_1) = N(2,1)$

$P(\omega_2) = \frac{3}{4}$ $\quad P(x|\omega_2) = N(5,1)$

$N(a,b)$ = normal distribution with mean a and variance b.

for zero one loss.

$$P(\omega_1|x) = P(\omega_2|x)$$

$$\frac{P(x|\omega_1)\, P(\omega_1)}{\cancel{P(x)}} = \frac{P(x|\omega_2)\, P(\omega_2)}{\cancel{P(x)}}$$

$$P(x|\omega_1)\, P(\omega_1) = P(x|\omega_2)\, P(\omega_2)$$

~~$\frac{1}{\sqrt{2\pi}}$ & we know~~

~~$P(x|\omega_i) = \frac{1}{2b} e^{-|\frac{x-a_i}{b}|}$ for $i = 1, 2$~~

for Gaussian Distribution

$$P(x|\omega_i) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

here μ = mean

σ^2 = variance.

$\cancel{\text{Ex}}$ $x \sim N(\mu, \sigma^2)$

$\cancel{f(x)}$ x is random variable from gaussian distribution $P(x|w_i)$

$$P(x|w_i) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-2}{1}\right)^2}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-2)^2}$$

$$\text{Illy } P(x|w_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-5)^2}$$

$$P(x|w_1) \cdot P(w_1) = P(x|w_2) \cdot P(w_2)$$

$$\frac{1}{\cancel{\sqrt{2\pi}}} e^{-\frac{1}{2}(x-2)^2} * \frac{1}{\cancel{4}} = \frac{1}{\cancel{\sqrt{2\pi}}} e^{-\frac{1}{2}(x-5)^2} \cdot \frac{3}{\cancel{4}}$$

long on both sides

$$\Rightarrow -\frac{1}{2}(x-2)^2 = \log(3) - \frac{1}{2}(x-5)^2$$

$$\frac{1}{2}\left((x-5)^2 - (x-2)^2\right) = \log(3)$$

$$\frac{1}{2}\left(\cancel{x^2} + 25 - 10x - \cancel{x^2} - 4 + 4x\right) = \log(x)$$

$$21 - 6x = 2\log(x)$$

$$x = \frac{21 - 2\log(3)}{6}$$

taking natural log instead of log

$$x = \frac{21 - 2 \times \ln(3)}{6}$$

$$\underline{\underline{x = 3.133}}$$

1 (b) $\lambda_{12} = 2$ $\lambda_{22} = 0$

$\lambda_{21} = 3$

$\lambda_{11} = 0$

here $(\lambda_{12} - \lambda_{11})\, P(x|\omega_1) P(\omega_1) = (\lambda_{12} - \lambda_{22})\, P(x|\omega_2) P(\omega_2)$

$$\cancel{3}* \frac{1}{\cancel{\sqrt{2\pi}}} e^{-\frac{1}{2}(x-2)^2} * \frac{1}{\cancel{3}} = 2 * \frac{1}{\cancel{\sqrt{2\pi}}} * e^{-\frac{1}{2}(x-5)^2} * \frac{\cancel{3}}{\cancel{3}}$$

$$-\frac{1}{2}(x-2)^2 = \ln 2 * -\frac{1}{2}(x-5)^2$$

$$\frac{1}{2}\left\{(x-5)^2 - (x-2)^2\right\} = \ln 2$$

$$\left\{\cancel{x^2} + 25 - 10x - \cancel{x^2} - 4 + 4x\right\} = 2\ln 2$$

$$21 - 6x = 2\ln 2$$

$$x = \frac{21 - 2\ln 2}{6}$$

$$x = 3.2689$$

(a) (a) $$P(a) = \theta^{a}(1-\theta)^{1-a}$$

$$P(a=1) = \theta$$

$$P(b) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{b-\mu}{\sigma}\right)^2}$$

$$P(x) = p(a) * p(b)$$

$$= \theta^{a}(1-\theta)^{1-a} * \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{b-\mu}{\sigma}\right)^2}$$

Scanned with CamScanner

(b) $q(x) = \prod_{i=1}^{M} p(x_i)$.

$$= \prod_{i=1}^{M} \theta^{a_i} \cdot (1-\theta)^{1-a_i} \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{b_i - M}{\sigma}\right)^2}$$

$\ln(q(x))$

$$= \sum_{i=1}^{M} \left\{ a_i \ln\theta + (1-a_i)\ln(1-\theta) + \ln\frac{1}{\sqrt{2\pi\sigma^2}} - \frac{1}{2}\left(\frac{b_i - M}{\sigma}\right)^2 \right\} \rightarrow ①$$

trying to maximise $\ln q(x)$

calculating $\ln q(x)$ w.r.t θ to be 0.

$$\frac{d \ln q(x)}{d\theta} = 0$$

{ we differentiate to maximise. $\ln(q(x))$, and find θ value correspondingly.

① ①

let $h(x) = \ln(qx)$.

differentiating $h(x)$ we have.

$$\frac{d(h(x))}{d\theta} = \sum_{i=1}^{M} \left\{ \frac{a_i}{\theta} + \frac{(1-a_i)(-1)}{(1-\theta)} + 0 + 0 \right\}$$

therefore we have

$$\frac{dh(x)}{d\theta} = 0$$

$$\sum_{i=1}^{N} \left\{ \frac{a_i}{\theta} - \frac{(1-a_i)}{(1-\theta)} \right\} = 0$$

$$\sum_{i=1}^{N} \left\{ \frac{a_i(1-\theta) - \theta(1-a_i)}{\theta(1-\theta)} \right\} = 0.$$

$$\sum_{i=1}^{N} \left\{ a_i(1-\theta) - \theta(1-a_i) \right\} = 0$$

$$\sum_{i=1}^{N} \left\{ a_i - \cancel{\theta a_i} - \theta + \cancel{a_i \theta} \right\} = 0$$

$$\sum_{i=1}^{N} \left\{ a_i - \theta \right\} = 0.$$

$$\sum_{i=1}^{N} a_i = \sum_{i=1}^{N} \theta$$

$$\sum_{i=1}^{n} a_i = n\theta$$

$$\cancel{\theta = \frac{1}{n} \sum_{i=1}^{n}} \qquad \cancel{\theta = \left(\frac{\sum^{n} a_i}{n} \right)}$$

$$\theta = \frac{1}{n} \sum_{i=1}^{N} a_i$$

Question 3

3 (a)

$$P(x/\omega_i) = \frac{1}{\pi b} \frac{1}{1 + \left(\frac{x-a_i}{b}\right)^2} \qquad i = 1, 2.$$

Decision Boundary

$$P(\omega_1 | x) = P(\omega_2 / x)$$

$$\frac{P(x|\omega_1) \cancel{P(\omega_1)}}{\cancel{P(x)}} = \frac{P(x|\omega_2) \cancel{P(\omega_2)}}{\cancel{P(x)}}$$

$\because \quad P(\omega_1) = P(\omega_2) = 1/2.$

$$\frac{1}{\pi b} \frac{1}{1 + \left(\frac{x-a_i}{b}\right)^2} = \frac{1}{\pi b} \frac{1}{1 + \left(\frac{x-a_2}{b}\right)^2}$$

$$(x - a_1)^2 = (x - a_2)^2$$

$$\cancel{x^2} + a_1^2 - 2xa_1 = \cancel{x^2} + a^2 - 2xa_2$$

$$(a_1 - a_2)(a_1 + a_2) = x(2a_1 - 2a_2)$$

$$\cancel{(a_1 - a_2)}(a_1 + a_2) = 2x\,\cancel{(a_1 - a_2)}$$

$$x = \frac{(a_1 + a_2)}{2}$$

as $a_2 - a_1 \neq 0.$

$\underline{x = 0}$

3 (c)

$$P(\text{error}) = \int_{-\infty}^{\infty} P(\text{error} \mid x) \cdot P(x)\, dx$$

$$= \int_{-\infty}^{\infty} \min \left[P(x|\omega_1) \cdot P(\omega_1) , P(x|\omega_2) \cdot P(\omega_2) \right] dx$$

$$= \int_{-\infty}^{4} P(x|\omega_2) \cdot P(\omega_2) \cdot dx + \int_{4}^{\infty} P(x|\omega_1) \, P(\omega_1) \, dx$$

$$= \left\{ \frac{1}{2} \int_{-\infty}^{4} \frac{1}{\pi} \cdot \frac{1}{1+(x-5)^2} dx + \frac{1}{2} \int_{4}^{\infty} \frac{1}{\pi} \cdot \frac{1}{1+(x-3)^2} dx \right\}$$

~~Algebraic~~ trigonometric Identity

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C.$$

$$= \frac{1}{2\pi} \left\{ \int_{-\infty}^{4} \frac{1}{1+(x-5)^2} dx + \int_{4}^{\infty} \frac{1}{1+(x-3)^2} dx \right\}$$

$$= \frac{1}{2\pi} \left\{ \left[\tan^{-1} (x-5) \right]_{-\infty}^{4} + \left[\tan^{-1} (x-3) \right]_{4}^{\infty} \right\}$$

(cont)

$$P(\text{error}) = \frac{1}{2\pi}\left[\tan^{-1}(-1) - \left(\frac{-\pi}{2}\right) + \frac{\pi}{2} - \tan^{-1}(1)\right]$$

$$= \frac{1}{2\pi}\left[\cancel{-\frac{\pi}{4}} + \pi - \cancel{\frac{\pi}{4}}\right]$$

$$= \frac{1}{2\pi} * \left\{\pi - \frac{2\pi}{4}\right\}$$

$$= \frac{1}{2\cancel{\pi}} * \left(\cancel{\pi} - \frac{\pi}{2}\right)$$

$$P(\text{error}) = \frac{1}{4}$$

1 (c)

~~we~~ No.

we ~~sh~~ should not choose to use zero-one loss for a ~~real~~ real world dataset like for example of cancer prediction.

This is because for real world case; the data is unbalanced

for example if someone has cancer and it gives prediction as no (false negative).
this might be worse and pose as a risk