(2)

Given :- prior is $\theta^{\top} e^{-\theta}$

to find :- $P(x|\theta)$. in multi variable Bernoulli;

$$P(x|\theta) = x_i^{\theta} (1-x_i)^{1-\theta}$$

$$\theta_{MAP} = p(x|\theta) * p(\theta).$$

$$F(\theta) = \sum_{i=1}^{N} P(x|\theta) * P(\theta)$$

$$\ln \{ F(\theta) = \sum_{i=1}^{N} \ln P(x|\theta) + \ln (P(\theta))$$

$$= \sum_{i=1}^{N} x_i \ln(\theta) + \ln P(\theta)$$

$$+ (1-x) \ln(1-\theta)$$

$$\frac{d \ln F(\theta)}{d\theta} = 0$$

$$\Rightarrow \sum_{i=1}^{N} \frac{x_i}{\theta} = \frac{1-x_i}{1-\theta} + \frac{d \ln P(\theta)}{d\theta} = 0$$

— (1)

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$$\Rightarrow \sum_{i=1}^{N} \frac{x_i - \theta}{\theta(1-\theta)} + \frac{d \ln P(\theta)}{d\theta} = 0$$

$$\frac{d \ln P(\theta)}{d\theta} =$$

$$\left\{ \begin{array}{c} \dfrac{d \ln (\theta_1 . \theta_2 \dots \theta_n)\, e^{-(\theta_1 + \theta_2 \dots \theta_n)}}{d\theta_1} \\ \vdots \\ \dfrac{d \ln (\theta_1 . \theta_2 \dots \theta_n) . e^{(-\theta_1 - \theta_2 - \theta \dots \theta_n)}}{d\theta_n} \end{array} \right\}$$

$$= \begin{bmatrix} \frac{1}{\theta_1} - 1 \\ \frac{1}{\theta_2} - 1 \\ \vdots \\ \frac{1}{\theta_n} - 1 \end{bmatrix}$$

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$$\frac{d \ln f(\theta)}{d\theta} = \frac{1}{\theta} - 1 \quad — \textcircled{2}$$

from 1 & 2

$$\sum_{i=1}^{N} \frac{x_i - \theta}{\theta(1-\theta)} = -\frac{1}{\theta} + 1$$

$$\sum_{i=1}^{N} x_i - \sum_{i=1}^{N} \theta = \left(1 - \frac{1}{\theta}\right)\theta * (1-\theta).$$

$$\sum_{i=1}^{N} x_i \quad - N\theta$$

$$= (\theta - 1) * (1 - \theta)$$

$$\sum_{i=1}^{N} x_i = -(1 + \theta^2 - 2\theta) + N\theta)$$

$$\theta^2 - (N+2)\theta + 1 + \sum_{i=1}^{N} x_i = 0 \quad - \textcircled{3}$$

6 $$X = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{2*4}$$

from ③

$$\theta_1^2 - (4+2)\theta + 4 = 0$$

$$\theta_1 = 5.23 \quad \text{or} \quad \theta_1 = 0.76$$

θ_1 is prob. $\Rightarrow$ $\theta_1 = 0.76$

for θ_2

$$\theta_2^2 - 6\theta_2 + 2 = 0$$

$\theta_2 = 5.64$

$\theta_2 = 0.35$

θ_2 is 0.35

$\theta_2 = 0.35$.

③ $x = \begin{bmatrix} 4 & 7 \\ 9 & 6 \end{bmatrix}$

$y = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$

$x - y = \begin{bmatrix} -2 & +1 \\ 6 & 3 \end{bmatrix}$

$\text{Cov}(x, x) = \frac{1}{3}\left[(4-6)^2 + (7-6)^2\right]$

$= \frac{1}{3}\left[(-2)^2 + (1)^2\right]$

$= \frac{1}{3}\left[4 + 1\right] = \frac{5}{3}$

$\text{Cov}(x, y) = \frac{1}{3}\left((4-6)(9-3) + (7-6)(6-3)\right)$

$= \frac{1}{3}\left((-2)(6) + (1)(3)\right)$

$= \frac{1}{3}\left(-12 + 3\right]$

$= \frac{-9}{3} = \underline{\underline{-3}}$

$$\operatorname{cov}(y,y) = \frac{1}{3}\left[(9-3)^2 + (6-3)^2\right]$$

$$= \frac{1}{3}\left[(6)^2 + (3)^2\right]$$

$$= \frac{45}{3} = \underline{\underline{15}}$$

$$S = \begin{bmatrix} xx & xy \\ yx & yy \end{bmatrix}$$

$$= \begin{bmatrix} 5/3 & -3 \\ -3 & 15 \end{bmatrix}$$

(i) <u>Eigen value calculation</u>

$$\det(S - \lambda I) = 0$$

$$\det \begin{bmatrix} 5/3 - \lambda & -3 \\ -3 & 15 - \lambda \end{bmatrix} = 0.$$

$$\left(\frac{5}{3} - \lambda\right)(15 - \lambda) - 9 = 0.$$

$$\frac{5}{\cancel{3}} * \overset{5}{\cancel{15}} - \frac{5}{3}\lambda - 15\lambda + \lambda^2 - 9 = 0.$$

$$25 - \frac{5}{3}\lambda - 15\lambda + \lambda^2 - 9 = 0.$$

$$75 - \cancel{5\lambda} - \cancel{45}\lambda + 3\lambda^2 - 27 = 0.$$

$$3\lambda^2 - 50\lambda + 48 = 0$$

$$\left\{ \begin{array}{c} \lambda_1 = 15.6 \\ \text{or} \\ \lambda_2 = 1.022 \end{array} \right\}$$

Eigen values of λ_1.

$$(S - \lambda I)\,\mu_1 = 0$$

$$\begin{pmatrix} 5/3 - \lambda & -3 \\ -3 & 15 - \lambda \end{pmatrix} \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\frac{5}{3} - \lambda\right)\mu_1 - 3\mu_2 = 0$$

$$-3\mu_1 + (15 - \lambda)\,\mu_2 = 0$$

(i) $-3\mu_1 + (15-\lambda)\mu_2 = 0$

$(15-\lambda)\mu_2 = 3\mu_1$

$$\frac{\mu_2}{3} = \frac{\mu_1}{(15-\lambda)} = t$$

Let $t = 1$

$\frac{\mu_2}{3} = 1$ $\qquad$ $\frac{\mu_1}{(15-\lambda)} = 1$

$\mu_2 = 3$ $\qquad$ $\mu_1 = (15-\lambda)$

Eigen vector μ_1 of λ_1 $= \begin{bmatrix} 3 \\ 15-\lambda \end{bmatrix}$

when $\lambda = 15.6$

$\lambda = 1$

λ (i)

u $x_1 = \begin{bmatrix} 3 \\ -6/10 \end{bmatrix}$ $\qquad$ $x_2^{\mu} = \begin{bmatrix} 3 \\ 14 \end{bmatrix}$

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(iii) $\lambda_1 =$

$$e_1 = \begin{bmatrix} 3 / \sqrt{3^2 + (-6/10)^2} \\ -6/10 / \sqrt{3^2 + (-6/10)^2} \end{bmatrix}$$

$$e_2 = \begin{bmatrix} 3 / \sqrt{3^2 + (14)^2} \\ 14 / \sqrt{3^2 + (14)^2} \end{bmatrix}$$

$$e_1 = \begin{bmatrix} 1 \\ -.6/3 \end{bmatrix} = \begin{bmatrix} 1 \\ -.2 \end{bmatrix}$$

$$e_2 = \begin{bmatrix} 3/14.3 \\ 14/14.3 \end{bmatrix} = \begin{bmatrix} .2 \\ .97 \end{bmatrix}$$

$$P_{12} = \begin{bmatrix} \frac{2}{10} & \frac{97}{100} \end{bmatrix} \begin{bmatrix} -8 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{-160}{100} + \frac{291}{100} \end{bmatrix}$$

$$= \frac{+131}{100} = 1.31$$

$$P_{11} = \begin{bmatrix} 1 & \frac{-2}{10} \end{bmatrix} \begin{bmatrix} -8 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -8 - \frac{6}{10} \end{bmatrix}$$

$$= -8.6$$

$$\begin{bmatrix} -8.6 & 1.31 \end{bmatrix}$$

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(b) $\mu = \begin{bmatrix} 3 \\ -6/10 \end{bmatrix}$

$y = \begin{bmatrix} -8.6 & 1.31 \end{bmatrix}$

$uy = \begin{bmatrix} 3 \\ -6/10 \end{bmatrix} \begin{bmatrix} -8.6 & 1.31 \end{bmatrix}$

$= \begin{bmatrix} 3 * -8.6 & 3.93 \\ \frac{-6}{10} * \frac{-8.6}{10} & \frac{-6}{10} * 1.31 \end{bmatrix}$

$= \begin{bmatrix} -25.8 & 3.93 \\ .5.16 & .7.86. \end{bmatrix}$

$y = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$

$uy + y = \begin{bmatrix} -19.8 & 9.93 \\ 3.516 & 3.786 \end{bmatrix}$

$$x = \begin{bmatrix} 4 & 7 \\ 9 & 1 \end{bmatrix}$$

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - y)^2$$

$$= \frac{1}{4} \Big((4-(-19.8))^2 + (7 - 9.93)^2$$

$$(3.516 - 9)^3$$

$$+ (3.786 - 6)^2 \Big)$$

$$= \frac{1}{4} \left[\cancel{(4 - (392.04))^2 + 7} \right]$$

$$= \frac{1}{4} \Big(+1507575. + 8.52$$

$$\cancel{72.45} + 30.07$$

$$+ 4.901 \Big]$$

$$= \underline{37659.622}$$