EN530.603 Applied Optimal Control
Homework #3

October 8, 2014

Due: October 22, 2014 (before class)

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1. (Kirk, 5-34.) Consider the system

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -ax_2(t) + u(t),$$

for $a > 0$ and $|u(t)| \leq 1$. The system must be transferred to the origin $x(t_f) = 0$ while minimizing the performance measure

$$J = \int_{t_0}^{t_f} [\gamma + |u(t)|]dt$$

The final time is free and $\gamma > 0$ is a constant.

   a) Determine the adjoint equations and the control that minimizes H
   b) What are the possible optimal control sequences?
   c) Show that a singular interval cannot exist.
   d) Determine the optimal control law.

2. (Bryson, p. 115) Consider the problem of minimizing

$$J = \|x(t_f)\|^2$$

for the system

$$\dot{x} = Ax + Bu, \qquad x(0) = x_0, \qquad t_f \text{ given}$$

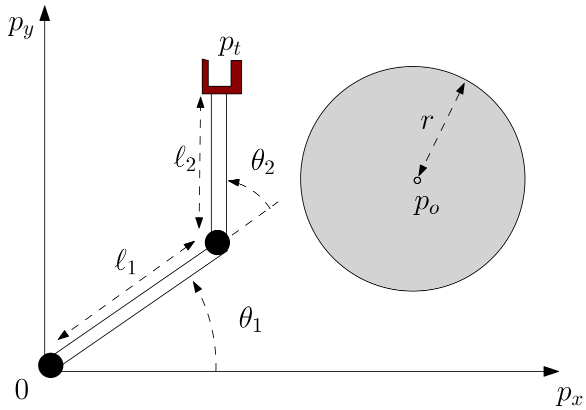
where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}$ with constraints

$$\|u(t)\| \leq 1.$$

Show that the optimal control for $J_{\min} > 0$ is bang-bang.

What is the analog to the figure of the time-optimal trajectories for the double-integrator problem (that we drew in class)? Either draw it by hand or simulate using Matlab by setting A and B to match the dynamics of a double integrator in 2-D.

3. Consider a two degree of freedom robotic arm operating in a workspace with a spherical obstacle. The arm base is at the origin $(0, 0)$ while the obstacle center is at position $p_o \in \mathbb{R}^2$ and its radius is r meters.



The arm must move so that its tip does not penetrate the obstacle. The arm configuration consists of its joint angles θ_1, θ_2 and thus the state of the arm is defined by

$$x = (\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2).$$

Ignoring gravity, assume that the arm is controlled using torque inputs $u = (u_1, u_2)$ so that

$$\ddot{\theta}_1 = u_1, \qquad \ddot{\theta}_2 = u_2.$$

The coordinates of the arm tip are given by

$$p_t = \begin{pmatrix} \cos(\theta_1)\ell_1 + \cos(\theta_1 + \theta_2)\ell_2 \\ \sin(\theta_1)\ell_1 + \sin(\theta_1 + \theta_2)\ell_2 \end{pmatrix}.$$

Give the expression for obstacle avoidance *state* inequality constraint $c(x(t), t) \leq 0$. Then derive the q-th order *state-control* inequality constraint that must be satisfied on the surface of the obstacle.

4. (Kirk, 5-37) The equations of motion of a rocket in horizontal flight are given by

$$\dot{x}_1(t) = \frac{cu(t)}{x_2(t)} - \frac{D}{x_2(t)},$$
$$\dot{x}_2(t) = -u(t),$$

where $x_1(t)$ is the horizontal velocity, $x_2(t)$ is the mass of the rocket, c is the exhaust gas speed and D is the aerodynamic drag force given by

$$D = \alpha x_1^2(t) + \frac{\beta x_2^2(t)}{x_1^2(t)} \geq 0, \tag{1}$$

where α and β are positive constants. The control input $u(t)$ can be regarded as the fuel burn rate and is limited by $0 \leq u(t) \leq u_{\max}$. It is desired to *maximize* the range of the rocket. The initial and final values of the mass and the velocity are specified, and the terminal time is free.

a) Determine the adjoint equations of the boundary condition relationships

b) Investigate the possibility of singular control intervals

Note: in the original homework D was given as constant. If you have already solved the problem with constant D then it is OK. Otherwise you will get extra credit for using the actual definition of D given in (1).