EN530.603 Applied Optimal Control
Homework #2

September 25, 2013

Due: October 9, 2013 (before class)

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1. Find the curve x^* that minimizes

$$J(x) = \int_0^1 [\frac{1}{2}\dot{x}^2(t) + 3x(t)\dot{x}(t) + 2x^2(t) + 3x(t)]dt$$

and passes through the points $x(0) = 0$ and $x(1) = 4$.

2. Find the extremals for

$$J(x) = \int_0^{t_f} [\dot{x}_1^2(t) + \dot{x}_2^2(t) + 3x_1(t)x_2(t)]dt,$$

with

a) $x_1(0) = 0$, $x_2(0) = 0$, $t_f = 1$, $x_1(t_f)$ free, $x_2(t_f) = 1$.

b) $x_1(0) = 0$, $x_2(0) = 0$, t_f free, $x(t_f)$ must lie on the surface

$$x_1(t) + 3x_2(t) + 5t = 15$$

Note: the last constraint is of the general form $\psi(x(t_f), t_f) = 0$. See last section in Lecture#4 posted on the website for details.

3. (Kirk 4-5.) Let η be a continuously differentiable function of time t that is arbitrary on the interval $[t_0, t_f]$ except at the end-points where $\eta(t_0) = \eta(t_f) = 0$. If ϵ is an arbitrary real parameter, then $x^* + \epsilon\eta$ represents a family of curves. Evaluating the functional

$$J(x) = \int_{t_0}^{t_f} g(x(t), \dot{x}(t), t)dt$$

on $x = x^* + \epsilon\eta$ makes J a function of ϵ, and if x^* is an extremal this function must have a relative extremum at $\epsilon = 0$.

Show that the Euler-Lagrange equations can be equivalently obtained from the necessary condition

$$\frac{dJ(x^* + \epsilon\eta)}{d\epsilon}\Big|_{\epsilon=0} = 0.$$

4. Given the first-order system with quadratic criterion

$$\dot{x} = -ax + bu, \quad x(t_0) \quad \text{given}, \tag{1}$$

$$J = \frac{1}{2}c[x(t_f)]^2 + \frac{1}{2}\int_{t_0}^{t_f}[u(t)]^2 dt, \tag{2}$$

where x, u and scalar variables and a, b, c are constant. Compute analytically the optimal control $u(t)$ minimizing J.

5. Consider the second-order system

$$\dot{x}_1 = x_2, \tag{3}$$

$$\dot{x}_2 = 3x_1 - x_2 + u, \tag{4}$$

$$\tag{5}$$

with cost function

$$J = \int_0^T [x_1^2 + \frac{1}{2}x_2^2 + \frac{1}{2}u^2]dt, \tag{6}$$

Find the optimal control law by finding the Riccati ODE. Implement the control law from initial condition $x(0) = [-5,\ 5]^T$ until final time $T = 20$ using Matlab (you can either integrate $P(t)$ analytically or numerically backwards in time using e.g. `ode45`, whichever is applicable). Plot the resulting elements of the matrix $P(t)$, the control input $u(t)$ and state histories $x(t)$.

Note: email your code to marin@jhu.edu with subject line starting with: **EN530.603.F2013.HW2**; in addition attach a printout of the code and plots to your homework solutions.

6. Why was the ODE resulting from the LQR necessary conditions named after Riccati, and who was he?