# EN530.603 Applied Optimal Control
## Homework #4

October 23, 2013

Due: November 6, 2013 (before class)

Professor: Marin Kobilarov

1. Consider the minimization of

$$J = x(1)^2 + \int_0^1 \frac{1}{2}[x(t)u(t)]^2 dt,$$

subject to the nonlinear dynamics

$$\dot{x} = xu, \qquad x(0) = 1.$$

Derive an optimal feedback control using the HJB equation. In the process, show that the HJB partial differential equation has a solution that is a quadratic function in $x$.

2. Consider the LQR problem with known disturbance $w(t)$ which requires the minimization of

$$J = \frac{1}{2}x(t_f)^T P_f x(t_f) + \frac{1}{2}\int_{t_0}^{t_f} [x(t)^T Q(t)x(t) + u(t)^T R(t)u(t)]dt,$$

subject to

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + w(t).$$

a) Using the HJB equation, show that a possible value function has the form

$$V(x(t),t) = \frac{1}{2}x(t)^T P(t)x(t) + b(t)^T x(t) + c(t)$$

and show that the associate optimal control is

$$u(t) = -K(t)x(t) + k(t),$$

where $K(t)$ and $k(t)$ are feedback and feedforward terms, respectively. Derive the differential equations for $\dot{P}, \dot{b}, \dot{c}$ and their boundary conditions which satisfy the HJB equation.

b) Do the discrete-time equivalent of a)

3. Read the paper: J. Betts "Survey of Numerical Methods for Trajectory Optimization", Journal of Guidance, Control, and Dynamics, Vol. 21, No. 2, 1998

4. Nonlinear numerical optimal control implementation. You can select one of these two choices (or both for extra credit):

a) A general discrete optimal control code (ddp.zip) is provided along with two examples (2-dof robotic arm and a second-order wheeled vehicle model). You have two options: 1) *extend* one of the two provided models; or 2) implement a new model of your own choice in a similar manner as the two examples. If you choose option 1) you must include control bounds and add environmental obstacles (recall HW3#3) modeled as a barrier/penalty term.

b) Implement a nonlinear programming strategy (e.g. using parametrized trajectory or e.g. using direct transcription as explained in Betts, 1998) to one of the two provided models (car or arm), or to a model of your choice. This can be accomplished by defining the cost and constraints and finding a solution using Matlab `fmincon`.

   Note: email your code to marin@jhu.edu with a subject line starting with: **EN530.603.F2013.HW4**; in addition attach a printout of the code and plots to your homework solutions.