Graph Theory
Assignment 1

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1. Incidence Matrix

$G = (V, E)$

Undirected graph

$$b_{ij} = \begin{cases} 1 & \text{, node } i \text{ incident on } j \\ 2 & \text{, edge } j \text{ is loop in } i \\ 0 & \text{, otherwise} \end{cases}$$

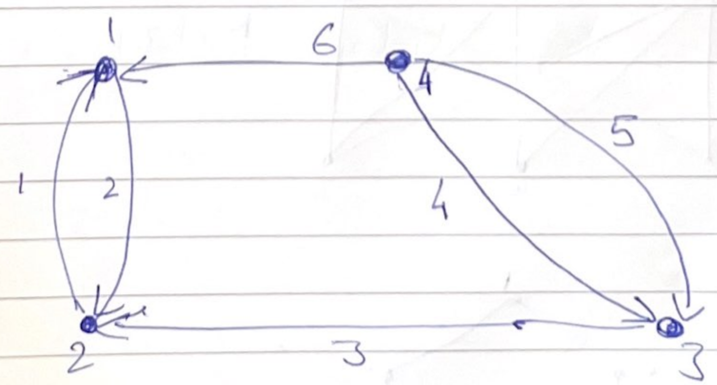
Directed loop free graph

$G = (V, A)$

$$b_{ij} = \begin{cases} 1 & , j = (i, k) \text{ for one } k \\ -1 & , j = (k, i) \text{ for one } k \\ 0 & \text{, otherwise} \end{cases}$$

2. Adjacency Matrix

$$a_{ij} = \begin{cases} k & \text{, if there are } k \text{ edges b/w } i \,\&\, j \\ k & \text{, if there are } k \text{ edges from } i \text{ to } j \end{cases}$$



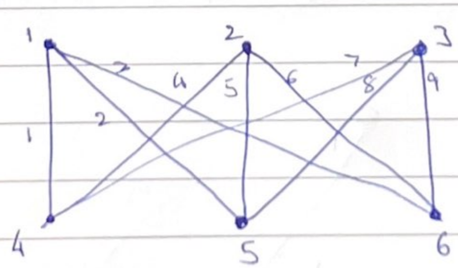
i) Adjacency Matrix

	V_1	V_2	V_3	V_4
V_1	0	1	0	0
V_2	1	0	0	0
V_3	0	1	0	0
V_4	1	0	2	0

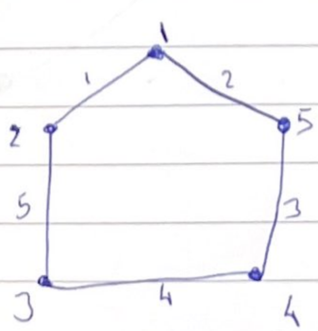
Incidence Matrix

	e_1	e_2	e_3	e_4	e_5	e_6
v_1	-1	1	0	0	0	-1
v_2	1	-1	-1	0	0	0
v_3	0	0	1	-1	-1	0
v_4	0	0	0	1	1	1

ii)



$K_{3,3}$



C_5

$K_{3,3}$: Adjacency Matrix

	V_1	V_2	V_3	V_4	V_5	V_6
V_1	0	0	0	1	1	1
V_2	0	0	0	1	1	1
V_3	0	0	0	1	1	1
V_4	1	1	1	0	0	0
V_5	1	1	1	0	0	0
V_6	1	1	1	0	0	0

Incidence Matrix

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
V_1	1	1	1	0	0	0	0	0	0
V_2	0	0	0	1	1	1	0	0	0
V_3	0	0	0	0	0	0	1	1	1
V_4	1	0	0	1	0	0	1	0	0
V_5	0	1	0	0	1	0	0	1	0
V_6	0	0	1	0	0	1	0	0	1

C_5: Adjacency Matrix

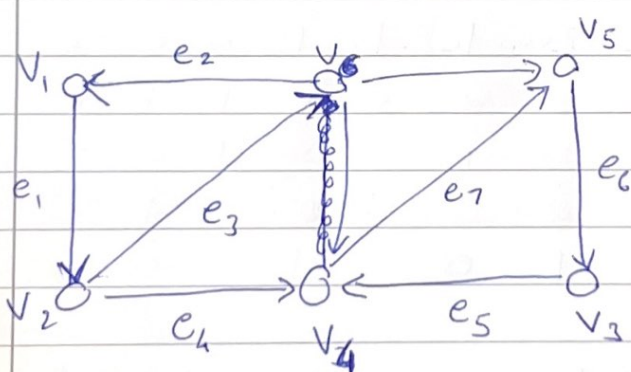
	V_1	V_2	V_3	V_4	V_5
V_1	0	1	0	0	1
V_2	1	0	1	0	0
V_3	0	1	0	1	0
V_4	0	0	1	0	1
V_5	1	0	0	1	0

Incidence Matrix

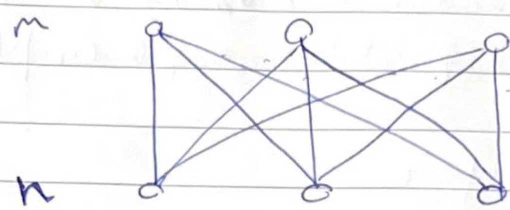
	e_1	e_2	e_3	e_4	e_5
V_1	1	1	0	0	0
V_2	1	0	0	0	1
V_3	0	0	0	1	1
V_4	0	0	1	1	0
V_5	0	1	1	0	0

(iii) Draw directed graph of given incidence matrix

$$
\begin{array}{c|ccccccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 \\
\hline
V_1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
V_2 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
V_3 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & -1 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 \\
V_6 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & 1
\end{array}
$$



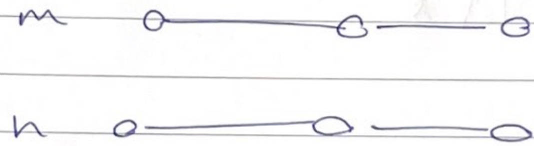
(2) i) $G =$



Bipartite Graph Properties:

1. Nodes of m are connected to nodes of n.
2. No nodes of m are neighbours
3. No nodes of n are neighbours

Complement of G



ii) Proof: $4m \leq n^2$ holds.

Bipartite Graph G with n nodes & m edges

Maximum edges $= x \cdot y = m$

Maximum no. of nodes $= x + y = n$

According to inequality,

$$\frac{x+y}{2} \geq \sqrt{xy}$$

$$\Rightarrow \frac{n}{2} \geq \sqrt{m}$$

$$\Rightarrow \frac{n^2}{4} \geq m$$

$$\therefore 4m \leq n^2$$

iii) Proof: for the nodes ~~$V = V_1 V_2 V_2$~~ $V = V_1 \cup V_2$ of a regular bipartite graph G. $|V_1| = |V_2|$ holds

Edges in $V_1 = |V_1| \cdot d$
Edges in $V_2 = |V_2| \cdot d$

We know that every edge in the bipartite graph connects a node in V_1 with node in V_2

$\therefore |V_1| \cdot d = |V_2| \cdot d$

$\Rightarrow |V_1| = |V_2|$

3) Proof: If all vertices have degree 3 in an undirected graph, the number of vertices is even.

By handshaking lemma,

$$\sum_{i=1}^{n} d(v_i) = 2e$$

$\Rightarrow$ sum of degree of vertices is even

$\Rightarrow$ sum of degrees of vertices with degree 3 is even

$\Rightarrow 3 \times n =$ even number

$\therefore n$ (no. of nodes) must be an even number.

④ Proof:

Given a graph $G = (V, E)$ with n nodes $|E| \leq \frac{1}{2}(n^2 - n)$ holds.

$\Rightarrow$ Let $V = \{V_1, V_2 \ldots V_n\}$

$E = \{e_1, e_2 \ldots e_n\}$

Max no. of edges for $V_1 = (n-1)$

$V_2 = (n-2)$

$\vdots \quad \vdots$

$V_n = 1$

$\Rightarrow$ total max. no. of edges in graph

$= (n-1) + (n-2) + \ldots + 3+2+1$

$= \frac{n \cdot (n-1)}{2}$