Graph Theory
Assignment 1

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3. ~~DHRUV~~

Assignment 3

(1)

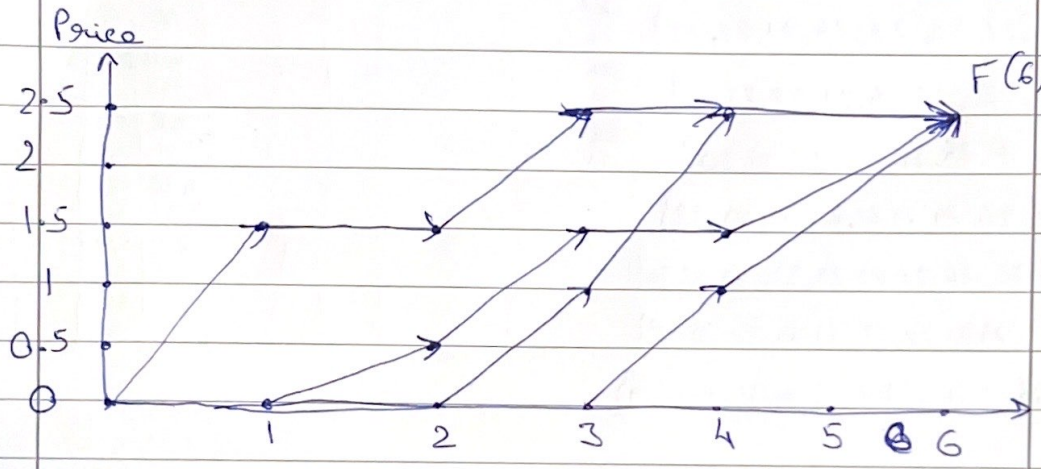
U	27	20	21	22	23	24	25	26	28	29	30	31	32
U = { }	0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
U = {27}		∞	∞	∞	∞	∞	41 27	∞	∞	∞	∞	∞	∞
U = {27, 25}		∞	∞	∞	∞	158 25	∞	132 25	∞	∞	∞	∞	∞
U = {27, 25, 26}		∞	∞	∞	249 26	158 25	∞	∞	∞	∞	∞	∞	∞
{27, 25, 26, 24}		~~188~~ ∞ ~~24~~	188 24	∞	249 26				∞	∞	∞	∞	∞
{27, 25, 26, 24, 21}		386 21		214 21	249 26				∞	∞	∞	∞	∞
{27, 25, 26, 24, 21, 23}		386 21			241 22				∞	∞	∞	284 22	∞
{27, 25, 26, 24, 21, 22, 23}		386 21							∞	∞	∞	284 22	365 23
{27, 25, 26, 24, 21, 22, 23, 31}		386 21							478 31	∞	345 31		365 23
{27, 25, 26, 24, 21, 22, 23, 31, 30}		386 21							478 31	558 30			365 23
{27, 25, 26, 24, 21, 22, 23, 31, 30, 32}									478 31	558 30			
{27, 25, 26, 24, 21, 22, 23, 31, 30, 32, 20}									478 21	558 30			
{27, 25, 26, 24, 21, 22, 23, 31, 30, 32, 20, 28}										558 30			
{27, 25, 26, 24, 21, 22, 23, 31, 30, 32, 20, 28, 29}													

⇒ Shortest path from 27 to 29 =

27 → 25 → 24 → 21 → 22 → 31 → 30 → 29

②

Item i	Price a_i	Benefit b_i
Bread	1.5€	4
Banana	0.5€	4
Cookies	1€	3
Rice	1.5€	5
Spinach	1€	4



Applying B-F algorithm,

A	E	C	G	H	D	F
0	-4	-4	-3	-5	-7	∞
0	-4	-4	-3	-5	-7	-11

Here, shortest path is 1→2, 2→D, D→F
∴ we should select items 2, 3 & 5.

(3) Theorem 3.5

Let $D=(V,A)$ be a connected graph $S \in V$ & $l : A \to Z$ a weight function on the edges, such that there is no directed circuit of negative length in D. The B-F algorithm computes the shortest path for all the vertices.

If $D=(V,A)$ is a directed graph without negative edges, then $d(v) = \delta(S,v)$ $\forall v \in V$

Let ve be any vertice from V. Consider shortest path k from S to v with min. no. of edges.

$$\delta(S, v_i) = \delta(S, v_{i-1}) + l(v_{i-1}, v_i)$$

We know, $\delta(s,s)=0$,

After first iteration,

$$d(v_1) = d(v_0) + l(v_0, v_1)$$
$$= \delta(S, v_0) + l(v_0, v_1)$$
$$= \delta(s, v_1)$$

After second pass,

$$d(v_2) = d(v_1) + l(v_1, v_2)$$
$$= \delta(S, v_2)$$

After n^{th} pass,

$$d(v_n) = \delta(s, v_n)$$

$\Rightarrow$ BF algo computes shortest path b/w s & all other vertices.

④ Running time of B-F algorithm

- It has two for loops.
- One iterates over vertices and other over set of edges.

⇒ Maximum time complexity = $O(|V| \cdot |E|)$