Graph Theory

Assignment: 4

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* **Task I** : Let G be a plane drawing of a connected planar graph. Let n and m denote the number of vertices and edges and let $m \geq 3$. Show: $m \leq 3n - 6$

Considering,

n - vertices

m - edges

$n \geq 3$.

For, $m \leq 3n - 6$

P_3  where,

$n = 3$,

$m = 2$

$\therefore 2 \leq 3(3) - 6 = \underline{3}$

Let G be a planar graph.

Then the Regions are:

$R_1, R_2, R_3, \ldots R_r$

Where,

Number of edges in boundary of $R_i = m_1, m_2, m_3, \ldots, m_r$

Given,

$m_i \geq 3$

$$M = \sum_{i=1}^{r} m_i \geq 3$$

$$3r \leq M \leq 2m$$ (If we count atleast two cycle of the edges.)

From,

$n - m + r = 2$

$6 = 3n - 3m + 3r$ (multiplied by 3)

$\therefore 6 = 3n - 3m + 3r \le 3n - 3m + 2m = 3n - m$

$\therefore 6 \le 3n - m$

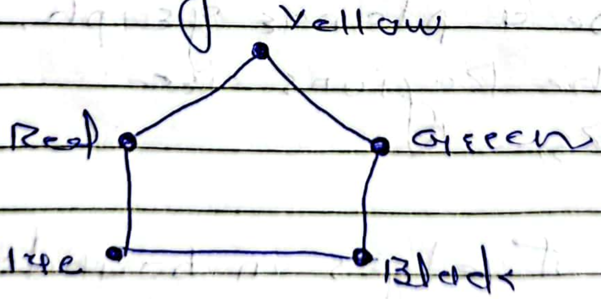
$\therefore \boxed{m \le 3n - 6}$ //

+ Task 2 : Each planar graph can be coloured with five colors.

Given,

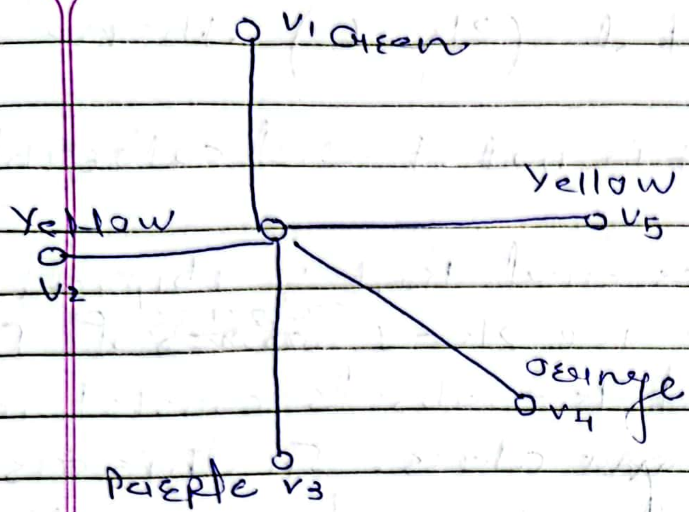
→ $P(n \le 5)$ i.e. Graph can be coloured using 5 different colors.

- Lemma: Every planar graph contains a vertex with degree $deg(v) \le 5$.



- induction : Let $k = 4$, $deg(v) \le 4 \Rightarrow$ True

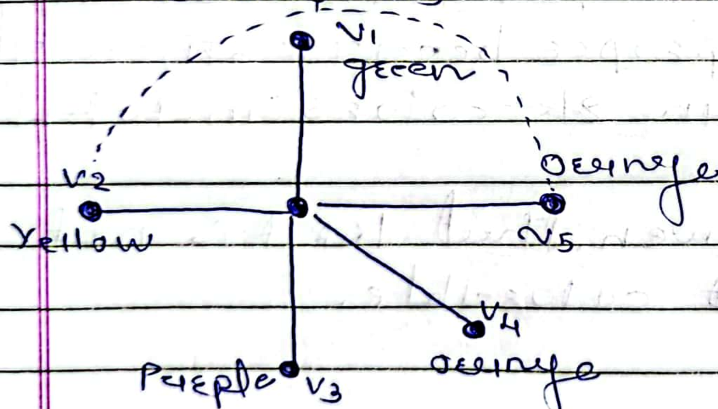
- verify for $k+1$
  $deg(v) \le 5$



Case 1: No direct edges.

Case 2:

Let there be a direct edge between V_2 and V_5



[color change from Yellow]

→ Here, we are changing the color from Yellow to any other color since it is a direct edge from V_2, other that is adjacent of Yellow. Therefore as it is a planar graph none of the other edges should cross each others.

→ Hence, it is proved that every planar graph can be colored with five colors.

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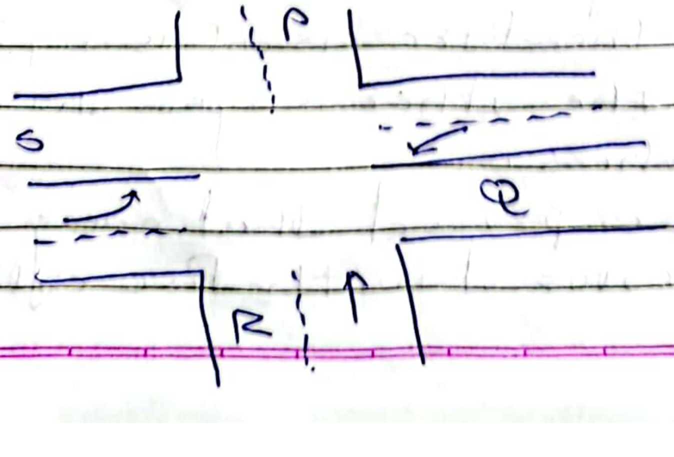
Task 3: Let $t = \binom{2k-1}{k}$ then $K_{t,t}$ is not t-list colorable.

Here, T vertices and $K_{t,t}$ is a bipartite graph. There are $2k-1$ colours of T different and k-subsets consist in each partite class. In this case there is a vertex in each class that contains none of the colors in its list for any $k-1$ of the colors. As a result any coloring from these lists, whether proper or not, must use at least k-color on each class, and the coloring can not be proper because there are fewer than $2k$ colors in total.

Hence, proven that $K_{t,t}$ is not t-list colorable.

Task 4: Construct an ideal traffic light circuit for the following intersection with node coloring.



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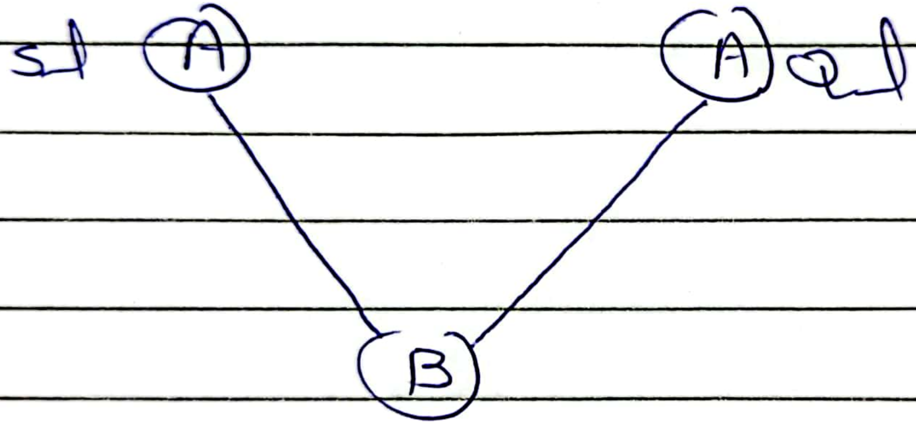

→ We have used two minimal colors (A & B) to construct the traffic light circuit.