Graph Theory
Assignment 2

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(1) Determine the asymptotic running time of Algorithm ("Recursive DFS") in the lecture notes.

Edges & nodes to be traversed
$= V_1 + V_1$ incidente edges $+ V_2 + V_2$ incident edges
$+ \ldots + V_n + V_n$ incident edges

$= (V_1 + V_2 + V_3 + \ldots + V_n) + (E_1 + E_2 + \ldots E_n)$

$\therefore$ Time Complexity $= O(V) + O(E)$
$= O(|V| + |E|)$

(2) Determine the asymptotic running time of the given algorithm

Time complexity $= O(E) + O(E) + O(V) \cdot O(E)$

$= O(E) + O(V \cdot E)$
$= O(V \cdot E)$
$= O(|V| \cdot |E|)$

(3) Proof: If two different circles in a graph G contain the same edge e, then a circle exists, which does not contain e

Consider a graph G, which has two subgraphs, $A'(V(A), E(A)$ and $B'(V(B), E(B))$ both of them are circles connected by a common edge "e". Assume e connects V_x and V_y. As a result, there must be two additional path/edge between x & y, given both graphs are circles.

$\therefore$ even if we remove edge e, it won't make a difference as $A-e$ & $B-e$ will form a circle.

$\Rightarrow$ if two separate circles in G have same edge e, then a circle without edge e exists.

(4) Let T be a connected graph with no cycles. Then deleting any edge from T disconnects the graph

Because T is a tree, there exist a unique simple path between any 2 vertices. If an edge (u, v) is deleted, then there will be no path between them, so the graph will be disconnected.

~~Assumption~~

Let (u, v) be an edge in G, since G is a graph there are atleast 2 paths from u to v, one directly and one indirectly. Assume p is a path between u and v, which does not go through (u, v). If the edge b/w (u, v) is deleted, there will still be one path connecting vertices through p.

(5) Every tree T of order $n \geq 2$ has at least two leafs

Let $T = (V, E)$ be a tree with $|V| \geq 2$.
There cannot be a vertex with degree 0, otherwise it will be disconnected from the graph.
Let L be the set of leaves. All vertices in $|L|$ have degree 1. All vertices in $|V - L|$ have degree at least 2.

$$\sum_{v \in V} \deg(V) = \sum_{v \in L} \deg(V) + \sum_{v \in (V-L)} \deg(V)$$

$$\geq |L| + 2|(V-L)|$$

$$\geq 2 \cdot |V| - |L|$$

For any Tree,
$|E| = |V| - 1$

$$\therefore 2|V| - 2 = \sum_{v \in V} \deg(v) \geq 2|V| - |L|$$

$$\Rightarrow |L| \geq 2$$