Exercise Sheet 01

Deadline for submission is Friday May-06-22, 23.59h at your Olat course.

Task 1 *Data Structures*

8 p.

Graphs can be represented in multiple ways. These different representations can be relevant for analysis, various mathematical calculations but also, from a computer science perspective, for storage, and also calculations for runtimes and memory requirements of algorithms.

1 *Incidence matrix*: Let B be an n x m - Matrix.

- For an undirected graph $G = (V, E)$

$$b_{ij} = \begin{cases} 1 & \text{node } i \text{ is incident to } j \\ 2 & \text{edge } j \text{ is loop in } ii \\ 0 & \text{otherwise} \end{cases}$$

- and for a directed, loop-free graph $G = (V, A)$

$$b_{ij} = \begin{cases} 1 & \text{if } j = (i, k) \text{ for one } k \\ -1 & \text{if } j = (k, i) \text{ for one } k \\ 0 & \text{otherwise} \end{cases}$$

2 *Adjacency matrix*: Let A be an n x n-Matrix

$$a_{ij} = \begin{cases} k & \text{if there are } k \text{ edges between i and j (undirected)} \\ k & \text{if there are} k \text{ edges from } i \text{ to } j \text{ (directed)} \end{cases}$$

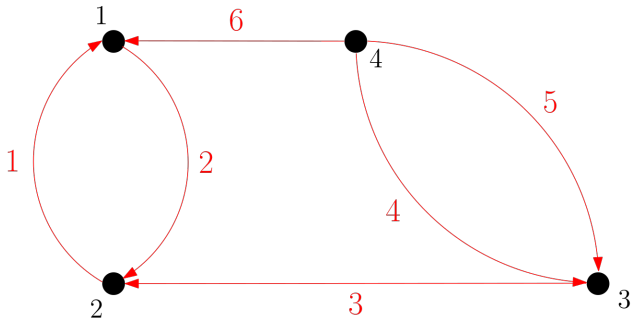

Figure 1: The red numbers on the edges represent labels, and not weights.

- Note the adjacency matrix and the incidence matrix of the graph.
- Note the adjacency matrix and the incidence matrix of the $K_{3,3}$ and C_5.
- Draw the directed graph to the following incidence matrix:

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Task 2 *Bipartite graphs*

4 p.

- What are the complements of the graphs $K_{n,m}$?
- Proof: For an undirected bipartite graph G with n nodes and m edges, $4m \leq n^2$ holds.
- Proof: For the nodes $V = V_1 \cup V_2$ of an regular bipartite graph, $|V_1| = |V_2|$ holds.

Task 3

2 p.

Proof: If all vertices have degree 3 in an undirected graph, the number of vertices is even.

Task 4

2 p.

Proof: Given a graph $G = (V, E)$ with n nodes $|E| \leq \frac{1}{2}(n^2 - n)$ holds.

Task	1	2	3	4	total
Points	8	4	2	2	16
reached					