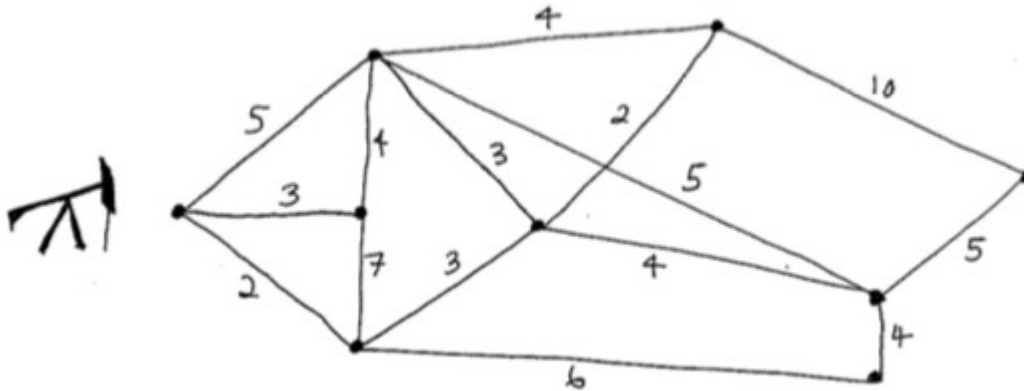


### Extra Exercises 3

1. Use induction to prove that any tree with at least 2 vertices is 2-colorable. *Do not use in your proof the theorem that states any tree is a bipartite graph.*
2. Let Düing & Brothers be a company where the employees are required to perform a large set of tasks  $T$ . Associated with each task in  $T$  is a set of employees who together can perform this task in 2-hour slots. The scheduling problem of assigning 2-hour slots to the tasks in  $T$  can be viewed as a coloring problem of a graph  $G_T$ . Define the vertices and edges in graph. Also, define the colors that can be used in coloring graph  $G_T$ .
3. An oil well is located on the left side of the graph below; each other vertex is a storage facility. The edges represent pipes between the well and storage facilities or between two storage facilities. The weights on the edges represent the time it takes for oil to travel from one vertex to another. Using Dijkstra's algorithm find a shortest path and the total time it takes oil to get from the well to the facility on the right side. Use a table.



4. Give a careful proof by induction on the number of vertices, that every tree is bipartite.
5. Consider a subset of the German ICE-network (right). Draw the graph  $G_{ICE}$  and add weights to edges. You may either search for time distances or provide an educated guess, e.g. for distances. Use Dijkstra's algorithm to find the shortest path between Hamburg and Augsburg, and Passau and Köln. Use a table.

