Bracketing the Beetle: How Wittgenstein's Understanding of Language Can Guide Our Practice in AGI and Cognitive Science

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Abstract. We advocate for a novel connectionist modeling framework as an answer to a set of challenges to AGI and cognitive science put forth by classical formal systems approaches. We show how this framework, which we call Vector Symbolic Architectures, or VSAs, is also the kind of model of mental activity that we arrive at by taking Ludwig Wittgenstein's critiques of the philosophy of mind and language seriously. We conclude by describing how VSA and related architectures provide a compelling solution to three central problems raised by Wittgenstein in the *Philosophical Investigations* regarding rule-following, aspect-seeing, and the development of a "private" language.

Keywords: Wittgenstein, language-games, connectionism, Vector Symbolic Architectures, Sparse Distributed Memory, Raven's Progressive Matrices, Necker Cube.

1 Connectionism and the Classical Approach

The traditional and perhaps still-dominant view of mental activity describes it in terms of symbols and rules of the sort used in writing predicate calculi or formal grammars. For example, the concept of romantic jealousy might be described by a rule

$$\{(X \text{ loves } Y) \text{ and } (Y \text{ loves } Z)\} \rightarrow (X \text{ is jealous of } Z)$$

The precise details of the symbols and rules do not much matter; what is important here is the hypothesis that any physical system that instantiates the symbols and rules in an explicit, consistent way is a reasonable candidate for being a model of mind [1]. The brittleness of such rule-based systems and the

difficulty of scaling them up to real-world problems[1] led to the connectionist (neural network, Parallel Distributed Processing) renaissance of the 1980's and 90's, centered around the back-propagation algorithm for training networks with hidden layers of nodes [3]. PDP advocates cited the "graceful degradation" displayed by such systems, in contrast to the all-or-nothing brittleness of rule-based systems, as evidence in favor of the PDP / connectionist approach. With network nodes roughly corresponding to nodes and connections to synapses, PDP networks also looked to be a promising avenue to showing how cognitive capacities, such as thought and language, could be based in hidden neural activity.

Researchers favoring connectionist models typically cite the past-tense model of English verbs [4], which indicated that a single neural network exposed to representations of the present and past tenses of English verbs could learn both rule-like patterns (*walks/walked*) and exceptions (*goes/went*),without anything corresponding to an explicit syntactic rule. The word or other form being represented at any given time was encoded not as an explicit symbol, but in the "sub-symbolic" state of activations distributed across the hidden (internal) neurons. Such *distributed representations* [5] offered a number of desirable features, such as content-addressability and robustness to noise, that were not intuitively available to classicists.

Advocates of the symbols-and-rules approach were quick to point out the limitations: although connectionist models showed an impressive ability to learn both rule-like and exception-based patterns, there was little evidence that they were capable of modeling the systematic, compositional nature of language and thought [6]. Without the ability to compose and decompose propositions and other structures in systematic ways – relating e.g., *John loves Mary* to *Mary loves John* – there was little reason to expect connectionist models to work for more abstract reasoning as in the jealousy example above. Further, the back-propagation algorithm used to train the network required explicit supervision (repeated error-correction by a teacher) in a way that was not consistent with actual language acquisition, which consists mainly of the experience of positive exemplars [7]. In addition to concerns about the ability of connectionist networks to scale up to bigger problem domains, these observations made connectionist models seem implausible.

2   VSA Representation and Operation

Partly in response to such criticisms, we have spent the past decade or so developing connectionist models that support the acquisition of systematic, compositional behavior from a small number of positive examples and provide plausible, scalable models for language and thought.

The general term we use for these models – Vector Symbolic Architecture, or VSA [8] – describes a class of connectionist networks that use high-dimensional

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[1] An oft-cited model was SHRDLU [2], which could converse in English about a virtual world containing children's toy blocks using a vocabulary of around 50 words, but was never successfully extended to a more realistic, complicated domain.

vectors of low-precision numbers to encode systematic, compositional information as distributed representations. VSAs can represent complex entities such as multiple role/filler relations or attribute/values pairs in such a way that every entity – no matter how simple or complex – corresponds to a pattern of activation distributed over all the elements of the vector.

For our purposes in this paper, we make use of three operations on vectors: an element-wise multiplication operation $\otimes$ that associates or binds vectors of the same dimensionality; an element-wise vector-addition operation $+$ that superposes such vectors or adds them to a set; and a permutation operator $P()$ that can be used to encode precedence relations.

For example, given a two-place predicate like *kisses* and the representations of two individuals John and Mary, one possible way of representing the proposition that Mary kisses John is

$$\langle kisses \rangle \otimes \langle \text{SUBJECT} \rangle \otimes \langle Mary \rangle + \langle kisses \rangle \otimes \langle \text{OBJECT} \rangle \otimes \langle John \rangle$$

where the angle brackets $\langle \rangle$ around an item stand for the vector representation of that item. If the vector elements are taken from the set $\{-1, +1\}$, then each vector is its own binding inverse.[2] Binding and unbinding can therefore both be performed by the same operator, thanks to its associativity:

$$X \otimes (X \otimes Y) = (X \otimes X) \otimes Y = Y$$

Because these vector operations are also commutative and distribute over addition, another interesting property holds: the unbinding operation can be applied to a set of associations just as easily as it can to a single association:

$$Y \otimes (X \otimes Y + W \otimes Z) =$$
$$Y \otimes X \otimes Y + Y \otimes W \otimes Z = X \otimes Y \otimes Y + Y \otimes W \otimes Z =$$
$$X + Y \otimes W \otimes Z$$

If the vector elements are chosen randomly (e.g., either +1 or -1 chosen by coin flip), then we can rewrite this equation as

$$Y \otimes (X \otimes Y + W \otimes Z) = X + noise$$

where *noise* is a vector completely dissimilar (having a dot-product or vector cosine of zero) to any of our original vectors W, X, Y, and Z. If we like, the noise can be removed through a "cleanup memory," such as a Hopfield network [9], that stores the original vectors in a neurally plausible way. In a multiple-choice setting, the cleanup isn't even necessary, because we can use the vector dot-product to find the item having the highest similarity to $X + noise$.

Going beyond simple associative binding, we can use the permutation operator to encode directionality or precedence. For example, to encode the simple directed graph in Figure 1:

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[2] It may seem dubious to represent predicates or individuals by simple vectors taken (randomly) from the set $\{-1, +1\}$. Our goal here is not to provide a theory of representation, but rather to illustrate the basic functioning of VSA, which can in turn serve as a foundation for a fully fleshed-out representational system of actual vectors derived through the interaction of sensory-motor processes, linguistics experience, etc.
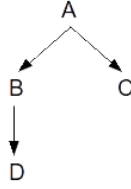
Fig. 1. A simple directed graph

$$\langle G \rangle = A \otimes P(B) + A \otimes P(C) + B \otimes P(D)$$

Querying the child(ren) of B then corresponds to applying the inverse permutation $P^{-1}()$ to the result of the same product operation:

$$P^{-1}(B \otimes \langle G \rangle) =$$
$$P^{-1}(B \otimes (A \otimes P(B) + A \otimes P(C) + B \otimes P(D))) =$$
$$P^{-1}(B \otimes A \otimes P(B) + B \otimes A \otimes P(C) + B \otimes B \otimes P(D)) =$$
$$P^{-1}(B \otimes A \otimes P(B) + B \otimes A \otimes P(C) + P(D)) =$$
$$D + noise$$

In sum, VSA provides a principled connectionist alternative to classical symbolic systems (e.g., predicate calculus, graph theory) for encoding and manipulating a variety of useful structures. The biggest advantage of VSA representations over other connectionist approaches is that a single association (or set of associations) can be quickly recovered from a set (or larger set) of associations using the same simple operator that creates the associations, in a time that is independent of the number of associations. VSA thus answers the scalability problem, and also shows how to build compositionally without using the grammatical rules and atomic symbols that classical approaches require. VSA also no longer needs to rely on back-propagation for learning. We can thus get to the same phenomenal results we see in language use efficiently, without positing a deep grammar or logic.

3   VSA and Wittgenstein

Having outlined the details of Vector Symbolic Architectures, we will now argue for VSA as the kind of AGI framework that accords well with the critiques presented by Wittgenstein in his *Philosophical Investigations* [10] and other later works. As pointed out by several researchers ([11]; [12]; [13]), connectionist networks are to the classical approaches of Fodor and Pylyshyn, what Wittgenstein's later philosophy of language was to the formal approaches of Gottlob Frege and Bertrand Russell. We have shown how VSAs can fulfill the promise of connectionism by responding to classicist concerns. We will now show how VSAs coordinate well with several of Wittgenstein's important observations concerning meaningful language.

First, we note that the sub-symbolic content of VSA representations (which are arbitrary or literally random) accords nicely with the "Beetle in the Box" metaphor from section 293 of the *Investigations*:

> Suppose everyone had a box with something in it: we call it a "beetle". No one can look into anyone else's box, and everyone says he knows what a beetle is only by looking at his beetle. – Here it would be quite possible for everyone to have something different in his box. One might even imagine such a thing constantly changing. – But suppose the word "beetle" had a use in these people's language? – If so it would not be used as the name of a thing. The thing in the box has no place in the language-game at all; not even as a something: for the box might even be empty. – No, one can 'divide through' by the thing in the box; it cancels out, whatever it is. That is to say: if we construe the grammar of the expression of sensation on the model of 'object and designation' the object drops out of consideration as irrelevant.

Wittgenstein highlights the role of the symbol in linguistic practices. Symbols commonly do not derive their meaning by directly representing a thing, i.e., by ostensive definition. The language-game shows us how to use the word meaningfully. In other words, the atomic thing can cancel out without loss of meaning. Indeed, VSAs use of random vectors guarantees that my "beetle," i.e., any atomic sensation particular to me and my use of the word, will be different from yours from the very start. Further, as the concept of *beetle* evolves in the experience of an individual or its use by that individual changes in different contexts, the random vector itself may be "constantly changing."

Second, we observe that the distributed nature of VSA representations accords well with Wittgenstein's notion of "family resemblance" terms. This connection was noted by Smolensky [14], who described how a "family of distributed patterns" informed the meaning of a word in connectionist models. Mills expanded on this point [11] and explicitly linked it to Wittgenstein's critique of essentialist concepts and formalisms. Mills notes that connectionist systems reflect the reliance on overlapping and criss-crossing resemblances and contextual cues for meaningful word use ([10] #66; [11],139-141).

Mills also notes that Wittgenstein rejects a psycho-physical parallelism ([15], 608-611), and this also accords well with the distributed nature of the symbols ([11], 151, 152). A distributed view of symbols stands in contrast to recent efforts to localize thought in a particular organ or brain region – the most extreme version being the putative "grandmother cell" neuron whose sole job is to recognize your grandmother [16].

Third, we note that the relation between the symbols does not require the classical linguistic or propositional form in order to be meaningful. The non-linguistic nature of the distributed representations employed in VSA carries over into natural language expressions. As Wittgenstein indicates in the building crew example at the beginning of the *Investigations*, it is a mistake to claim that the utterance "Slab!" really means "Bring me a slab!" ([10] #19). I.e., in order

for an utterance to be meaningful there is no need for it to fit an underlying grammatical form that looks like a proposition in predicate calculus. In contrast to the sentence-like representations employed in traditional symbol systems, in VSA there is no sense in which any item is located in any grammatical "position."

Last, we note that the uniform nature of representation in VSA eliminates the sort of problems that Wittgenstein criticized in Russell's theory of types and related formalisms. VSA dispenses with grammatical categories and types. As early as 1914, Wittgenstein was suspicious of any artificial hierarchical structures that might be used to designate, from the top-down, when a particular combination of signs were symbols with sense [17]. The idea that all symbols are of the same type, coordinates well with VSA representation, which, in turn, coordinates well with Wittgenstein's later view: meaningful utterances emerge from linguistic practices and not from the artificial characterizations we impose by designating symbols and manipulating them with formal rules.

VSA fits desiderata Wittgenstein established in the *Investigations* by (1) not relying on ostension or an atomic identification for symbol meaning, (2) recognizing the distributed and "family resemblance" nature of symbol construction, (3) not relying on predicate logic or grammatical form for the composition of meaningful thoughts or sentences, and (4) repudiating different orders or types for symbols in favor of a more organic approach. Moreover, we will now show how VSA can solve three interrelated puzzles that Wittgenstein raised in a manner consistent with his observations.

4   Learning Patterns without Explicit Rule-Following

Wittgenstein had serious reservations about attempts to characterize language or mental processes using symbols-and-rules methodology ([10] #81; [12] 138). For Wittgenstein, the productivity of human thought and linguistic behavior is underdetermined by the rules of logic and grammar. A rule on its own cannot be properly applied without some sort of training.

Having illustrated the way in which VSA supports compositionality and systematicity, we now illustrate how it can generalize from exemplars without recourse to explicit rules. This example is due to Rasmussen and Eliasmith [18], who show how a VSA-based neural architecture can solve the Raven's Progressive Matrices task for intelligence testing. In this task, subjects are given a puzzle like the one in the left side of Figure 2 (simplified for our purposes here) and are asked to select the missing piece from a set of possibilities like the ones on the right.

Asked how they arrived at this solution, people might report that they followed these two rules:

1. *Put one item in the first column, two in the second, and three in the third.*
2. *Put ● in the first row, ♦ in the second, and ▲ in the third.*

A compelling feature of VSA is that it can solve this problem using the representation of the matrix itself; that is, VSA can (so to speak) *learn the rule(s)*
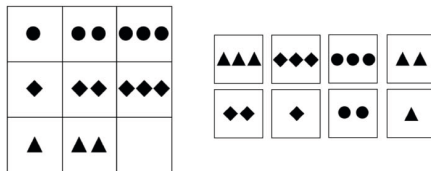
Fig. 2. Raven's Progressive matrix example (left) and candidate solutions (right)

through exposure to the problem. Each element of the matrix can be represented as a set of attribute/value pairs; for example, the center element would be

$$\langle shape \rangle \otimes \langle diamond \rangle + \langle number \rangle \otimes \langle two \rangle$$

Solving the matrix then corresponds to deriving a mapping from one item to the next. As Rasmussen and Eliasmith show, such a mapping can be obtained by computing the vector transformation from each item to the item in the row or column next to it. The overall transformation for the entire matrix is then the vector sum of such transformations.

Details of our VSA solution to the Raven's Matrices (a simplified version of [18]), along with Matlab code for this task and others mentioned in this paper, are available from `tinyurl.com/wittvsa`. A similar solution, using a different kind of VSA encoding, is presented in [19].

5 Seeing-as: Determining Perceptual Experience in Ambiguous Contexts *without* Interpretation

The paradox of visually ambiguous figures, or "seeing-that vs. seeing-as," occupied Wittgenstein all the way from the Necker cube example in *Tractatus* [20] (5.5423) through the duck-rabbit example in the *Investigations* (II.xi; see Figure 3), about which he says

> I may, then, have seen the duck-rabbit simply as a picture-rabbit from the first. That is to say, if asked "What's that?" or "What do you see here?" I should have replied: "A picture-rabbit". I should not have answered the question "What do you see here?" by saying: "Now I am seeing it as a picture-rabbit". I should have described my perception: just as if I had said "I see a red circle over there."

There is no sense in which we simultaneously perceive one alternative and the possibility of the other: either the duck or the rabbit must win. Wittgenstein's point is that the possibility of interpretation, when the perceptual information is ambiguous, does not mean that we are interpreting in normal circumstances, e.g., in the case of seeing just the rabbit. Connectionist approaches show how the same perceptual process that shows us the red circle can show us the duck or the rabbit, but not both at the same time.
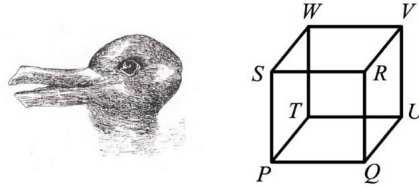
Fig. 3. The Duck-Rabbit(*courtesy of Wikimedia Commons*) and the Necker Cube

Modeling the perception of visually ambiguous images like this was one of the first accomplishments of the connectionist renaissance of the 1980s. As Rumelhart *et al.* [21] showed, such images could be represented as a network of constraints that excited or inhibited each other in a way that drove the network quickly into one of the possible solutions. For example, in the Necker cube in Figure 3, the two solutions are (1) $PQRS$ front, $TUVW$ back and (2)) $PQRS$ back, $TUVW$ front. Rumelhart et al. modeled these constraints as a localist ("grandmother cell") neural network each of whose units represented a possible position of each vertex. They showed that inhibitory or excitatory synaptic connections between pairs of constraints (P_f excites W_b; R_f inhibits W_f), combined with a simple update rule, are sufficient to drive the entire network quickly into one of the two consistent yet incompatible solutions.

This excitation / inhibition model provides a nice explanation for Wittgenstein's observation about seeing-as: presented with a set of vertices, the model, like human observers, cannot help but "see" one global pattern of organization or another. The network is, however, localist, and as a general model of constraint satisfaction it therefore raises the philosophical and practical concerns expressed earlier. Is it possible to design an excitation / inhibition network that uses distributed (VSA) representations?

In [22] we provide an example of such a network, and show how it can be used to solve a Necker-cube-like problem called "graph isomorphism" (optimally matching up the vertices of two similar shapes). Our solution works by a Bayesian process that repeatedly updates a candidate solution state x using evidence w, until x converges to a stable solution. Inhibition of inconsistent solutions by consistent solutions is implemented by normalizing the values in x to a fixed interval at the end of every update. This same approach can be used to solve the Necker Cube. In our Necker Cube program, the candidate solution state x is initially just the vector sum of the representations of all possible solution components:

$$x_0 = P_f + Q_f + R_f + S_f + Tb + Ub + Vb + Wb + Pb + Qb + Rb + Sb + T_f + U_f + V_f + W_f$$

where the subscripts stand for $f$orward and $b$ackward. As usual for VSA, each term of the sum is a vector of high dimensionality with elements chosen randomly

from the set $\{-1, +1\}$. The constraints (evidence) w can then be represented as the sum of the pairwise products of mutually-consistent components:

$$w = P_f \otimes Q_f + P_f \otimes R_f + P_f \otimes S_f + P_f \otimes T_b + P_f \otimes U_b + ... + W_b \otimes U_b + W_b \otimes V_b$$

The update of x from w can likewise be implemented by using the binding (elementwise product) operator $\otimes$. If any vertex/position vector (e.g. P_f) has greater representation in x than others do, multiplying this consistency vector w by the state vector x has the effect of "unlocking" (unbinding) the components of w consistent with this evidence. As an example, consider the extreme case in which x contains only the component P_f:

$$x_t \otimes w =$$

$$P_f \otimes (P_f \otimes Qf + P_f \otimes Rf + P_f \otimes Sf + P_f \otimes Tb + P_f \otimes Ub + ... + Wb \otimes Ub + Wb \otimes V_b) =$$

$$Q_f + R_f + S_f + T_b + U_b + V_b + W_b + noise$$

6   Boxing the Beetle: The Emergence of Schemata from Repeated Exposure

VSA and related connectionist technologies support the view of mental processes that we get by taking Wittgenstein's critiques seriously. These technologies all involve (1) representations distributed over high-dimensional vectors of numerical elements and (2) psychologically plausible learning mechanisms.

Sparse Distributed Memory or SDM [23] is a technology for content-based storage and retrieval of high-dimensional vector representations like the ones used in VSA. An SDM consists of some (arbitrary) number of address vectors, each with a corresponding data vector. Addresses and data can be binary 0/1 values, or +1,-1 values as in VSA. The address values are initially random, and the data values are initially zero. To enter a new address/data pair into the SDM, the Hamming distance (count of the elementwise differences) of the new address vector with each of the existing address vectors is first computed. If the new address is less than some fixed distance from an existing address, the new data is added to the existing data at that address. To retrieve the item at a novel address, a similar comparison is made between the novel address and the existing addresses, resulting in a set of addresses less than a fixed distance from the probe. The data vectors at these addresses are summed, and the resulting vector sum is converted to a vector of 0's and 1's (or -1s and +1s) by converting each non-negative value to 1 and each negative value to 0 (or -1).

As illustrated in [24], the distribution of each pattern across several locations produces a curious property: given a set of degraded exemplars of a pattern (such as the pixels for an image with some noise added), an SDM can often reconstruct the "ideal" form of the pattern through retrieval, even though no exemplar of this ideal was presented to it. Because of these powerful properties of SDM, our research group and others (e.g., [19]) have begun to construct models combining

VSA representations with SDM. For example, the VSA might be used to encode sequence information (through the binding and permutation operators described above), and the SDM would then be used as a memory of previously-encountered sequences. We are currently investigating the use of this architecture as a model of the encapsulation and chunk extraction that are necessary for the acquisition of skilled behaviors like language and planning.[3]

In allowing each address to portray a slight variant of the same concept, SDM reminds us of the family-resemblance approach to categories in the *Investigations.* Moreover, this property of SDM also proposes a solution to the problem of how we might come to use a particular leaf pattern as a schema for leaves in general ([10] #47) or how we might come to develop a sign for a particular yet elusive sensation using experience and memory.

> Let us imagine the following case. I want to keep a diary about the recurrence of a certain sensation. To this end I associate it with the sign "S" and write this sign in a calendar for every day on which I have the sensation.—-I will remark first of all that a definition of the sign cannot be formulated.–But still I can give myself a kind of ostensive definition. – How? ... in the present case I have no criterion of correctness. ([10]#258)

This remark is meant to support the idea that there can be no language that ties words directly to private sensations. It emphasizes the need for a background of reliable cues and uses to be in place before a sign can be meaningful. The connectionist approach helps explicate the manner in which private experiences can develop into meaningful uses of language even here when the information is sparse.

If Wittgenstein is not denying a kind of private language (e.g., the sort that a Robinson Crusoe could still speak) as insightful readers of the so-called "private language" argument believe ([26], ch. 10), then we have a riddle in this passage that distributed representations can solve: just as we saw how a rule could emerge from linguistic practices, and how a symbol could be determined from possibly ambiguous perceptual information, we now see how meaning might emerge for a sign that has a sparse and ambiguous heritage.

This ties directly back into the beetle in the box example. Just as we do not get the meaning of "beetle" by pointing to an object, we do not get the meaning of a sensation word by an internal "pointing" to a sensation. There need be no unique private sensation that the sign "S" captures in order for a meaningful language containing the sign "S" to get off the ground. Wittgenstein suggests that one cannot tell, even privately, if one is using the word correctly if there is nothing but the sensation to rely on; one does not have a grip on the right use of the term without a context and other behavioral cues that give sense to the sign [27]. Although SDM works with degraded versions of an original concept or ideal that it can reconstruct, it can also construct a meaningful representation from a set of experiences in want of criteria of correctness. What VSA, SDM,

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[3] Contrast this approach with the state-of-the-art model for such tasks, which relies on back-propagation for the memory component. [25]

and the like show here is how an ideal or proper use for the sign can be built up from the uses of "S" even if the iterations of the "S" tying it to the sensation were not "correctly" used (from some imagined God's eye point of view that we do not have).

Without any external checks to help provide a use for the sign we could not reliably establish a meaning, but the iterations and their associations, e.g., my stomach growling and it being around 12 o'clock when I say "S", can begin to build a box around the beetle – and then we can bracket the beetle: the private sensation no longer functions to provide the rule by which I will use the word: for the purposes of meaningful language, the sensation itself "is not a *something*, but not a *nothing* either!" ([10] #304) And the rule by which I use the word is established in a network of sub-symbolic connections; it is not fixed permanently, nor is it a rule of ordinary English grammar, or a rule of Mentalese that such a grammar purports to approximate. Wittgenstein shows us that we do not always have a special internal or intentional grip on what the sign means. We cannot grasp the inner sensation or the outer beetle, nor need we.

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