Semantic Web – Tutorial #3

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Correction Regarding Assignment 2A, Task 2.1

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Consider the interpretation $I = (\Delta^I, \cdot^I)$, where

$$
\begin{aligned}
\Delta^I &= \{n_1, n_2, p_1, j_1, i_1\} \\
\text{Nurse}^I &= \{n_1, n_2\} \\
\text{Patient}^I &= \{p_1\} \\
\text{Janitor}^I &= \{j_1, n_2, p_1, i_1\} \\
\text{Illness}^I &= \{i_1\} \\
\text{Physician}^I &= \{n_1, n_2, p_1, i_1\} \\
\text{takesCareOf}^I &= \{(n_1, p_1), (j_1, p_1)\} \\
\text{canTreat}^I &= \{(j_1, i_1), (j_1, p_1)\}
\end{aligned}
$$

I satisfies $\mathcal{T}_1$ but does not satisfy neither of the statements (4)–(6). Both nurses n_1 and n_2 only take care of patients (n_2 satisfies it vacuously, since it takes care of no patient), but it violates (4) since n_2 takes care of no patient. Moreover, (5) is violated, because j_1, a janitor, takes care of the patient p_1. Statement (6) is violated, because j_1 treats the illness i_1.

1: Interpretations

Consider the following knowledge base $\mathcal{K}_1 = (\mathcal{T}_1, \mathcal{A}_1)$ in $\mathcal{ALC}$:

$$
\begin{aligned}
\mathcal{T}_1 = \{\exists R.A \sqsubseteq B,\\
A \sqcap D \equiv E,\\
A \sqsubseteq \neg\exists Q.D\}\\
\mathcal{A}_1 = \{y : A, (x, y) : R, (y, z) : Q\}
\end{aligned}
$$

Task: Formally define an interpretation I that satisfies $\mathcal{K}_1$.

$I = (\Delta^I, \cdot^I)$ with

$$
\begin{aligned}
\Delta^I &= \{x_1, y_1, z_1\}\\
x^I &= \{x_1\}\\
y^I &= \{y_1\}\\
z^I &= \{z_1\}\\
A^I &= \{x_1, y_1, z_1\}\\
B^I &= \{x_1\}\\
D^I &= \emptyset\\
E^I &= \emptyset\\
R^I &= \{(x_1, y_1)\}\\
Q^I &= \{(y_1, z_1)\}
\end{aligned}
$$

Task: Demonstrate that I satisfies $\mathcal{K}_1$.

- We have to show that I is a model of both $\mathcal{A}_1$ and $\mathcal{T}_1$.
- For $\mathcal{A}_1$, show that:
  - for each $a : C, a^I \in C^I$ and
  - for each $(a, b) : r, (a^I, b^I) \in r^I)$ (where C is a concept name, and r a role name)

1) Only one concept assertion: $y : A$. From $\cdot^I$, we have $y^I = y_1$, and $y_1 \in A^I$. Thus, $y^I \in A^I$.

2.1) $(x, y) : R$. We have $x^I = x_1$ and $y^I = y_1$, and $(x_1, y_1) \in R^I$. Thus, $(x^I, y^I) \in R^I$.

2.2) $(y, z) : Q$. We have $y^I = y_1$ and $z^I = z_1$, and $(y_1, z_1) \in Q^I$. Thus, $(y^I, z^I) \in Q^I$.

For $\mathcal{T}_1$, we have to show that $C^I \subseteq D^I$, for each $C \sqsubseteq D$. (Note that $C \equiv D$ is equivalent to $C \sqsubseteq D$ and $D \sqsubseteq C$.)

- $\exists R.A \sqsubseteq B$. We have $(\exists R.A)^I = \{x_1\}$. Also, $B^I = \{x_1\}$. It follows that $\{x_1\} \subseteq \{x_1\}$. Thus, $(\exists R.A)^I \subseteq B^I$.
- $A \sqcap D \equiv E$. In this case, it suffices to show that $A^I \cap D^I = E^I$. We have $A^I = \{x_1, y_1, z_1\}$ and $D^I = \emptyset$. Thus, $A^I \cap D^I = \emptyset = E^I$.
- $A \sqsubseteq \neg\exists Q.D\}$. We have $(\exists Q.D)^I = \emptyset$. Thus, $(\neg\exists Q.D)^I = \Delta^I$. Also, $A^I = \{x_1, y_1, z_1\} \subseteq \Delta^I$. It follows that $A^I \subseteq (\neg\exists Q.D)^I$.

2: Tableau Algorithm

Consider the following Knowledge base $\mathcal{K}_2 = (\mathcal{T}_2, \mathcal{A}_2)$ with

$$
\begin{aligned}
\mathcal{T}_2 = \{&A \sqsubseteq D, \\
&\neg(B \sqcup \neg D) \sqsubseteq E, \\
&D \sqsubseteq \neg\exists R.E, \\
&B \sqcap E \sqsubseteq \bot, \\
&\neg\neg D \sqsubseteq E\} \\
\mathcal{A}_2 = \{&x : \neg(\neg D \sqcap \neg E), y : A \sqcap B, (y, x) : R\}
\end{aligned}
$$

Task: Transfer both $\mathcal{T}_2$ and $\mathcal{A}_2$ to NNF.

$\neg(B \sqcup \neg D) \sqsubseteq E$
- $\neg B \sqcap D \sqsubseteq E$

$D \sqsubseteq \neg\exists R.E$
- $D \sqsubseteq \forall R.\neg E$

$\neg\neg D \sqsubseteq E$
- $D \sqsubseteq E$

$x : \neg(\neg D \sqcap \neg E)$
- $x : D \sqcup E$

Is $\mathcal{K}_2$ consistent or not? In either case, apply the tableau algorithm for your answer. Document and explain each step you take when applying the tableau algorithm.

Solution 1

TBox

$$\mathcal{T}_2 = \{A \sqsubseteq D,\ \neg B \sqcap D \sqsubseteq E,\ D \sqsubseteq \forall R.\neg E,\ B \sqcap E \sqsubseteq \bot,\ D \sqsubseteq E\}$$

ABox

$$\mathcal{A}_2 = \{x : D \sqcup E,\ y : A \sqcap B,\ (y,x) : R\}$$

1.	$x : D \sqcup E$	ABox
2.	$y : A \sqcap B$	ABox
3.	$(y,x) : R$	ABox
4.	$y : A$	($\sqcap$, 2)
5.	$y : B$	($\sqcap$, 2)
6.	$y : \neg A \sqcup D$	($\sqsubseteq$, $A \sqsubseteq D$)
7.	$y : \neg A$ $y : D$	($\sqcup$, 6)
8.	$\otimes$ 4, 7 $y : \neg D \sqcup (\forall R.\neg E)$	($\sqsubseteq$, $D \sqsubseteq \forall R.\neg E$)
9.	$y : \neg D$ $y : \forall R.\neg E$	($\sqcup$, 8)
10.	$\otimes$ 7, 9 $x : \neg E$	($\forall$, 9)
11.	$x : \neg(\neg B \sqcap D) \sqcup E$	($\sqsubseteq$, $\neg B \sqcap D \sqsubseteq E$)
12.	$x : B \sqcup \neg D$ $x : E$ $\otimes$ 10, 12	($\sqcup$, 11)
13.	$x : B$ $x : \neg D$	($\sqcup$, 12)
14.	$x : D$ $\otimes$ 13, 14 $x : E$ $\otimes$ 10, 14	
15.	$x : (\neg B \sqcup \neg E) \sqcup \bot$	($\sqsubseteq$, $B \sqcap E \sqsubseteq \bot$)
16.	$x : \neg B \sqcup \neg E$ $x : \bot$ $\otimes$	($\sqcup$, 15)
17.	$x : \neg B$ $\otimes$ 13, 17 $x : \neg E$	($\sqcup$, 16)
18.	$x : \neg D \sqcup E$	$\sqsubseteq$, $D \sqsubseteq E$
19.	$x : \neg D$ $x : E$ $\otimes$ 17, 19	($\sqcup$, 18)
20.	$x : D$ $\otimes$ 19, 20 $x : E$ $\otimes$ 17, 20	

Solution 2

TBox

$$\mathcal{T}_2 = \{A \sqsubseteq D,\ \neg B \sqcap D \sqsubseteq E,\ D \sqsubseteq \forall R.\neg E,\ B \sqcap E \sqsubseteq \bot,\ D \sqsubseteq E\}$$

ABox

$$\mathcal{A}_2 = \{x : D \sqcup E,\ y : A \sqcap B,\ (y, x) : R\}$$

1.	$x : D \sqcup E$	ABox
2.	$y : A \sqcap B$	ABox
3.	$(y, x) : R$	ABox
4.	$y : A$	($\sqcap$, 2)
5.	$y : B$	($\sqcap$, 2)
6.	$y : \neg A \sqcup D$	($\sqsubseteq$, $A \sqsubseteq D$)
7.	$y : \neg A$	$y : D$	($\sqcup$, 6)
8.	$\otimes$ (4, 7)	$y : \neg D \sqcup E$	($\sqsubseteq$, $D \sqsubseteq E$)
9.	$y : \neg D$	$y : E$	($\sqcup$, 8)
10.	$\otimes$ (7, 9)	$y : \neg(B \sqcap E) \sqcup \bot$	($\sqsubseteq$, $B \sqcap E \sqsubseteq \bot$)
11.	$y : \neg B \sqcup \neg E$	$y : \bot$ $\otimes$	($\sqcup$, 10)
12.	$y : \neg B$ $\otimes$ (5, 12)	$y : \neg E$ $\otimes$ (9, 12)	

3: Consistency and Inference

Let $\mathcal{S}_1 = (N_C, N_R, N_O)$ be a signature where

$$N_C = \{A, B, D, E, F, G\}$$

$$N_R = \{P, Q\}$$

$$N_O = \{x, y\}$$

Consider the following *Tbox* and *Aboxes*:

$$\begin{array}{lllll} \mathcal{T}_3 & = \{ & A & \equiv & \neg(E \sqcup F), \\ & & B \sqcup D & \sqsubseteq & A, \\ & & E & \sqsubseteq & \exists P.G, \\ & & A \sqcap \exists P.G & \sqsubseteq & \bot, \\ & & E \sqcup F & \sqsubseteq & \neg\exists Q.G \,\} \end{array}$$

$$\begin{array}{lllll} \mathcal{A}_3 & = \{ & x & : & G, \\ & & (x, y) & : & Q \,\} \end{array}$$

$$\begin{array}{lllll} \mathcal{A}_4 & = \{ & x & : & G, \\ & & y & : & B, \\ & & (x, y) & : & Q \,\} \end{array}$$

Task: Is the knowledge base $\mathcal{K}_3 = (\mathcal{T}_3, \mathcal{A}_3)$ consistent? If your answer is positive, then define an interpretation I that satisfies $\mathcal{K}_3$. Otherwise, prove that $\mathcal{K}_3$ is inconsistent.

$\mathcal{K}_3 = (\mathcal{T}_3, \mathcal{A}_3)$

$$
\begin{array}{llll}
\mathcal{T}_3 & = \{ & A & \equiv & \neg(E \sqcup F), \\
 & & B \sqcup D & \sqsubseteq & \text{A}, \\
 & & E & \sqsubseteq & \exists P.G, \\
 & & A \sqcap \exists P.G & \sqsubseteq & \bot, \\
 & & E \sqcup F & \sqsubseteq & \neg\exists Q.G \,\}
\end{array}
\qquad
\begin{array}{llll}
\mathcal{A}_3 & = \{ & x & : & G, \\
 & & (y, x) & : & Q \,\}
\end{array}
$$

Yes, $\mathcal{K}_3$ is consistent. We define $I = (\Delta^I, \cdot^I)$ with

$$\Delta^I = \{x_1, y_1\}$$

$$
\begin{aligned}
A^I &= \{x_1, y_1\} \\
B^I &= \emptyset \\
D^I &= \{y_1\} \\
E^I &= \emptyset \\
F^I &= \emptyset \\
G^I &= \{x_1\}
\end{aligned}
$$

$$
\begin{aligned}
P^I &= \emptyset \\
Q^I &= \{(y_1, x_1)\}
\end{aligned}
$$

$$
\begin{aligned}
x^I &= \{x_1\} \\
y^I &= \{y_1\}
\end{aligned}
$$

Task: Is the knowledge base $\mathcal{K}_4 = (\mathcal{T}_3, \mathcal{A}_4)$ consistent? If your answer is positive, then define an interpretation I that satisfies $\mathcal{K}_4$. Otherwise, prove that $\mathcal{K}_4$ is inconsistent.

$\mathcal{K}_4 = (\mathcal{T}_3, \mathcal{A}_4)$

$$\mathcal{T}_3 = \{\begin{array}{lll} A & \equiv & \neg(E \sqcup F), \\ B \sqcup D & \sqsubseteq & A, \\ E & \sqsubseteq & \exists P.G, \\ A \sqcap \exists P.G & \sqsubseteq & \bot, \\ E \sqcup F & \sqsubseteq & \neg\exists Q.G \,\} \end{array} \qquad \mathcal{A}_4 = \{\begin{array}{lll} x & : & G, \\ y & : & B, \\ (y, x) & : & Q \,\} \end{array}$$

- The knowledge base $\mathcal{K}_4$ is inconsistent. Let us assume, for contradiction, that $\mathcal{K}_4$ is consistent. Then, there is an interpretation I that satisfies $\mathcal{K}_4$.

1. $\xrightarrow{(\mathcal{A}_4)}$ $x : G$ and $y : B$ and $(y, x) : P$
2. $\xrightarrow{(1.)}$ $x^{I} \in G^{I}$ and $y^{I} \in B^{I}$ and $(y^{I}, x^{I}) \in P^{I}$
3. $\xrightarrow{(\mathcal{T}_3)}$ $B \sqcup D \sqsubseteq A$
4. $\xrightarrow{(3.)}$ $B^{I} \cup D^{I} \subseteq A^{I}$
5. $\xrightarrow{(2.,4.)}$ $y^{I} \in A^{I}$
6. $\xrightarrow{(\mathcal{T}_3)}$ $A \sqcap \exists P.G \sqsubseteq \bot$
7. $\xrightarrow{(6.)}$ $A^{I} \cap (\exists P.G)^{I} \subseteq \emptyset$
8. $\xrightarrow{(2.,5.)}$ $y^{I} \in A^{I}$ and $x^{I} \in G^{I}$ and $(y^{I}, x^{I}) \in P^{I}$
9. $\xrightarrow{(7.,8.)}$ $\{y^{I}\} \cap \{y^{I}\} \subseteq \emptyset$ **contradiction**

Task: Prove that $(\mathcal{T}_3, \mathcal{A}_3) \models y : A$.

$(\mathcal{T}_3, \mathcal{A}_3)$

$$\mathcal{T}_3 = \{ \begin{array}{lcl} A & \equiv & \neg(E \sqcup F), \\ B \sqcup D & \sqsubseteq & \text{A}, \\ E & \sqsubseteq & \exists P.G, \\ A \sqcap \exists P.G & \sqsubseteq & \bot, \\ E \sqcup F & \sqsubseteq & \neg\exists Q.G \,\} \end{array}$$

$$\mathcal{A}_3 = \{ \begin{array}{lcl} x & : & G, \\ (y, x) & : & Q \,\} \end{array}$$

- We need to show that for every interpretation I, if $I \models (\mathcal{T}_3, \mathcal{A}_3)$, then $I \models y : A$. Let us assume $I \models (\mathcal{T}_3, \mathcal{A}_3)$. We will show that $y^I \in A^I$. As $I \models (\mathcal{T}_3, \mathcal{A}_3)$, we get that $I \models \mathcal{T}_3$ and $I \models \mathcal{A}_3$.

1. $\xRightarrow{(\mathcal{A}_3)}$ $x : G$ and $(y, x) : Q$
2. $\xRightarrow{(1.)}$ $x^I \in G^I$ and $(y^I, x^I) \in Q^I$
3. $\xRightarrow{(\mathcal{T}_3)}$ $E \sqcup F \sqsubseteq \neg\exists Q.G$
5. $\xRightarrow{(4.)}$ $E^I \cup F^I \subseteq \Delta^I \setminus (\exists Q.G)^I$
6. $\xRightarrow{(5.)}$ $E^I \cup F^I \subseteq \Delta^I \setminus \{y^I\}$
7. $\xRightarrow{(\mathcal{T}_3)}$ $A \equiv \neg(E \sqcup F)$
8. $\xRightarrow{(7.)}$ $A^I = \neg(E^I \cup F^I)$
9. $\xRightarrow{(6.)}$ $y^I \notin (E^I \cup F^I)$
10. $\xRightarrow{(6.,9.)}$ $y^I \in \neg(E^I \cup F^I)$
11. $\xRightarrow{(7.,10.)}$ $y^I \in A^I$