

A Beginning in the Reversible Logic Synthesis of Sequential Circuits

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Abstract

- This work presents Reversible Flip Flops useful in designing reversible sequential Circuits.
- Reversible circuits form the basic building blocks of quantum computers, as all quantum operations are reversible.
- Reversible gates used for reversible logic synthesis are New Gate, Feynman Gate, and Fredkin gate
- Novelty of the paper is the reversible logic synthesis of Flip Flops
- The Flip Flops that are synthesized using reversible logic are: RS Flip Flop, JK Flip Flop, D Flip Flop, T Flip Flop and Master Slave Flip Flop
- To the best of our knowledge and the survey of literature, this is the first work in this area.

Introduction

Reversible logic

- Reversible circuits are those circuits that do not lose information.
- Reversible circuits can generate a unique output vector from each input vector, and vice versa.
- In reversible circuits, there is a one-to-one mapping between input and output vectors.

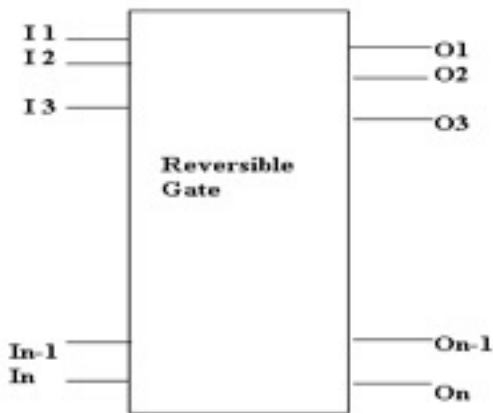


Figure 1. N X N reversible gate

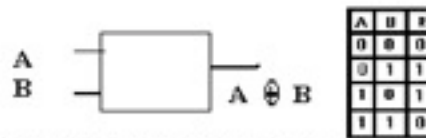


Figure2: XOR Gate(Classical) Irreversible

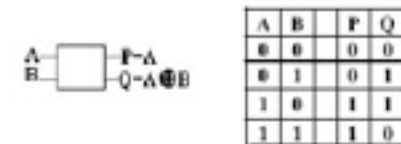


Figure 3: Reversible XOR Gate

Motivation behind reversible logic

- Landauer has shown that for irreversible logic computations, each bit of information lost, generates $kT\log 2$ joules of heat
- Bennett showed that $kT\ln 2$ energy dissipation would not occur, if a computation were carried out in a reversible way
- Whenever a logic operation is performed, the computer erases information. All these logic operations are irreversible dissipating a lot of heat.
- The current irreversible technologies will dissipate a lot of heat and can reduce the life of the circuit.
- As Moore's law continues to hold, processing power doubles every 18 months.
- Reversible logic operations do not erase (lose) information and dissipate much less heat.
- Reversible logic is likely to be in demand in high speed power aware circuits.
- Reversible circuits are of high interest in low-power CMOS design, optical computing, nanotechnology and quantum computing.

Application of Reversible Logic In Quantum Computing

- Prominent application of reversible logic lies in quantum computers.
- Quantum gates perform an elementary unitary operation on one, two or more two–state quantum systems called qubits.
- Any unitary operation is reversible and hence quantum networks also.
- Quantum networks effecting elementary arithmetic operations cannot be directly deduced from their classical Boolean counterparts (classical logic gates such as AND or OR are clearly irreversible).
- Thus, Quantum computers must be built from reversible logical components

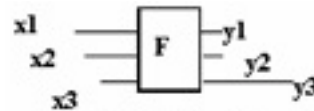
BASIC REVERSIBLE GATES

FREDKIN GATE:

- a (3*3) conservative reversible gate originally introduced by Petri
- The input triple (x1,x2,x3) associates with its output triple (y1, y2,y3) as follows

NEW

$$\begin{aligned} y_1 &= x_1 \\ y_2 &= (\neg x_1 \wedge x_2) \vee (x_1 \wedge x_3) \\ y_3 &= (x_1 \wedge x_2) \vee (\neg x_1 \wedge x_3) \end{aligned}$$



Fredkin Gate Symbol

Inputs			Outputs		
x1	x2	x3	y1	y2	y3
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	1	1	1

Table 1: Truth Table of Fredkin Gate

- **New Gate (NG)** is a 3*3 one-through reversible gate
- The input triple (A,B,C) associates with its output triple (P,Q,R) as follows.

$$P=A$$

$$Q=ABC$$

$$R=A'C' \oplus B'$$

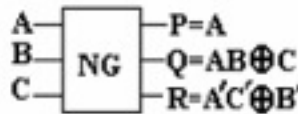


Figure : New Gate Symbol

A	B	C	P	Q	R
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	0	1
0	1	1	0	1	0
1	0	0	1	0	1
1	0	1	1	1	1
1	1	0	1	1	0
1	1	1	1	0	0

Table 4: Truth Table of New Gate(NG)

FEYNMAN GATE (FG):

- Feynman gate is a 2*2 one-through reversible gate
- The input double (x1,x2) associates with its output double (y1,y2) as follows.

$$y_1=x_1$$

$$y_2=x_1 \oplus x_2$$

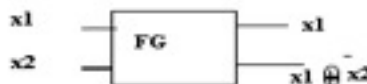


Figure 5: Feynman gate Symbol

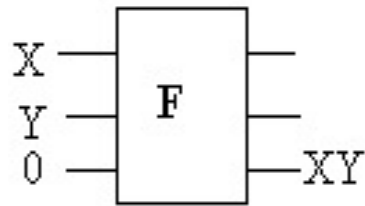
Inputs		Outputs	
x1	x2	y1	y2
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

Table 2: Truth Table of Feynman gate

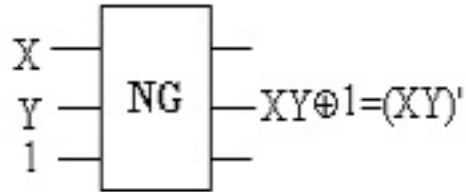
Novel Design of Reversible Flip Flops

- A novel attempt is made to design Sequential circuits using Reversible logic.
- According to the survey of literature and to the best of our knowledge, this is the first work in this area.
- In order to initiate the process of reversible logic synthesis of sequential circuits, Flip-Flops are designed using Reversible logic.
- The Flip Flops that are synthesized using reversible logic are RS Flip Flop, JK Flip Flop, D Flip Flop, T Flip Flop and Master Slave Flip Flop

Reversible Equivalent Gates Used for Designing



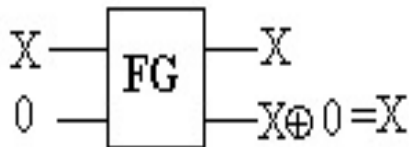
Fredkin Gate As Gate



AND New Gate As NAND Gate



New Gate As NOR Gate

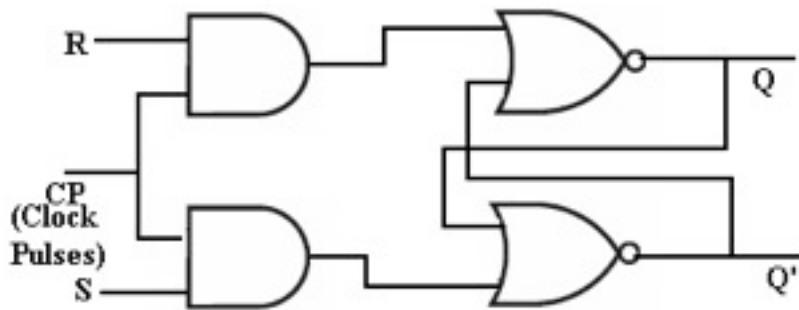


Feynman Gate As Copying Output

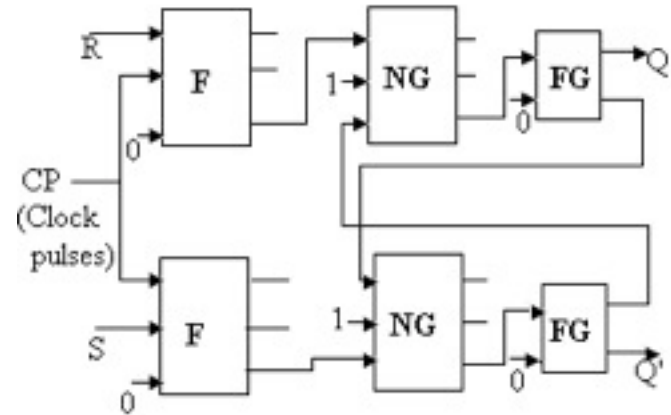


Feynman Gate As Not Gate

RS Flip Flop



Conventional RS Flip Flop

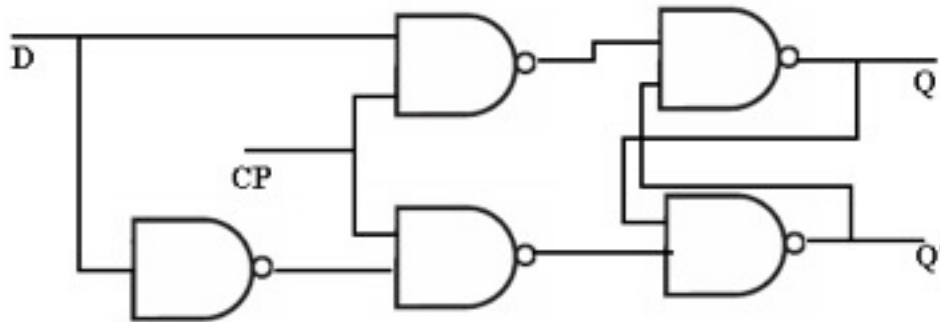


Proposed Reversible RS Flip Flop

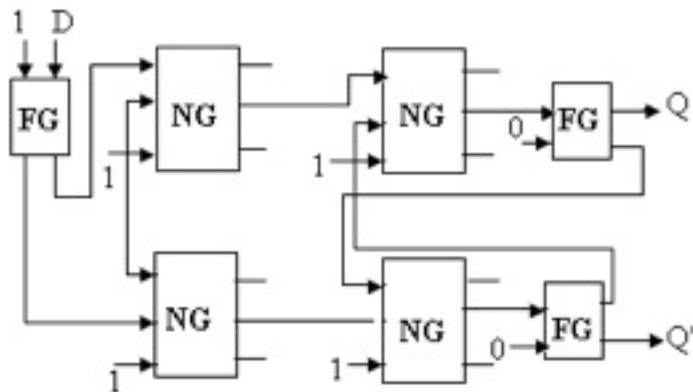
Evaluation of the Proposed RS Flip Flop

	No of gates	Garbage Outputs
Proposed Circuit	6	8
Existing One	None in literature	None in Literature

D Flip Flop



Conventional D Flip Flop

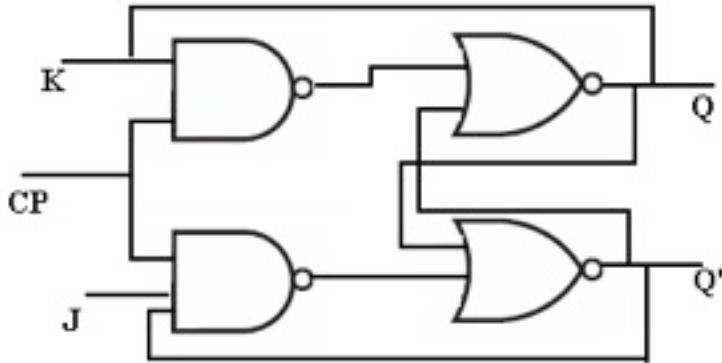


Proposed Reversible D Flip Flop

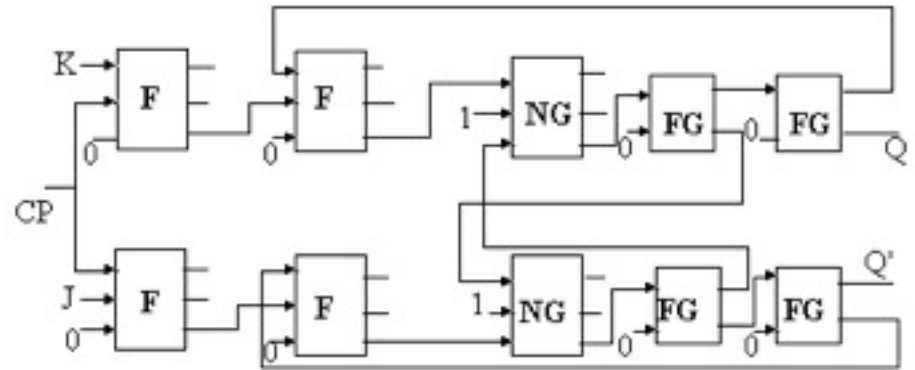
Evaluation of the Proposed D Flip Flop

	No of gates	Garbage Outputs
Proposed Circuit	7	8
Existing One	None in literature	None in Literature

JK Flip Flop



Conventional JK Flip Flop

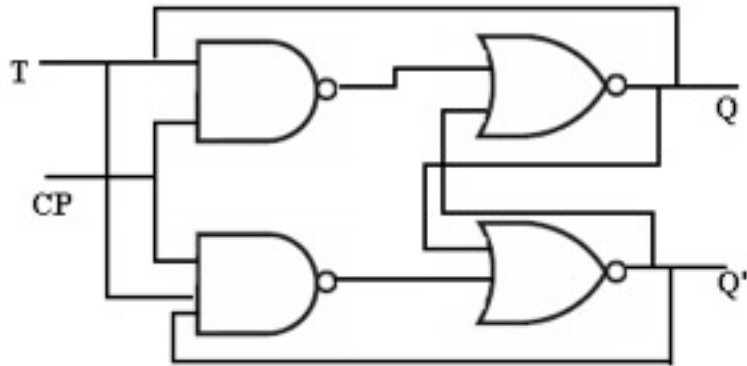


Proposed Reversible JK Flip Flop

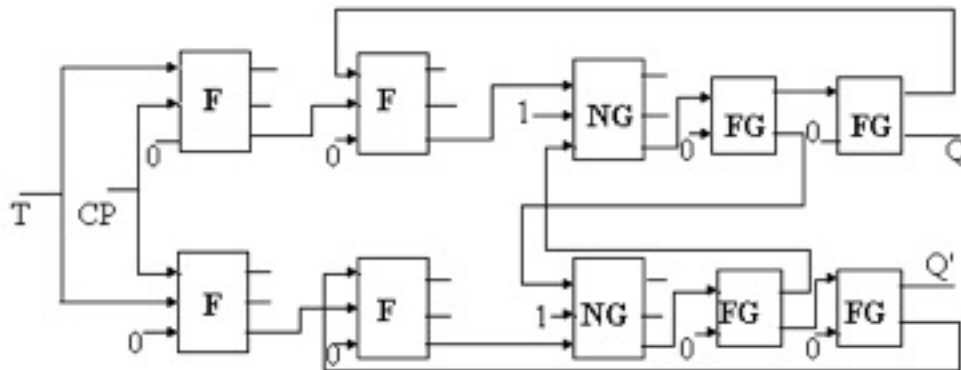
Evaluation of the Proposed JK Flip Flop

	No of gates	Garbage Outputs
Proposed Circuit	10	12
Existing One	None in literature	None in Literature

T Flip Flop



Conventional T Flip Flop

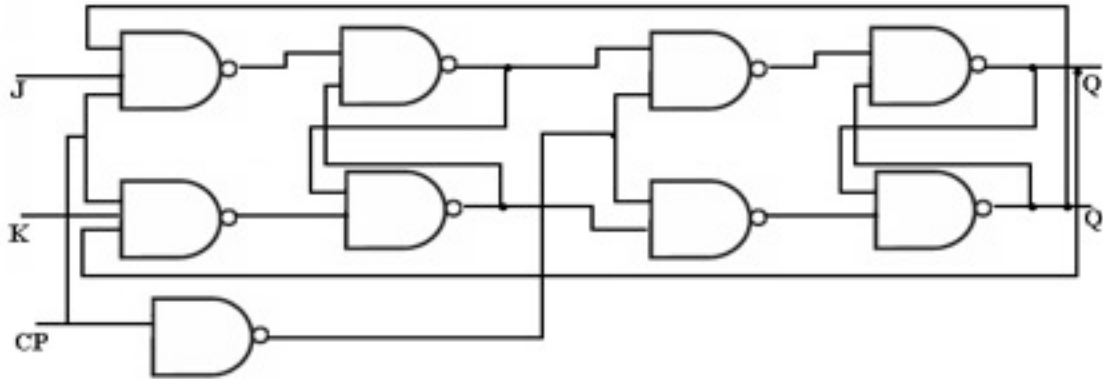


Proposed Reversible T Flip Flop

Evaluation of the Proposed
T Flip Flop

	No of gates	Garbage Outputs
Proposed Circuit	10	12
Existing One	None in literature	None in Literature

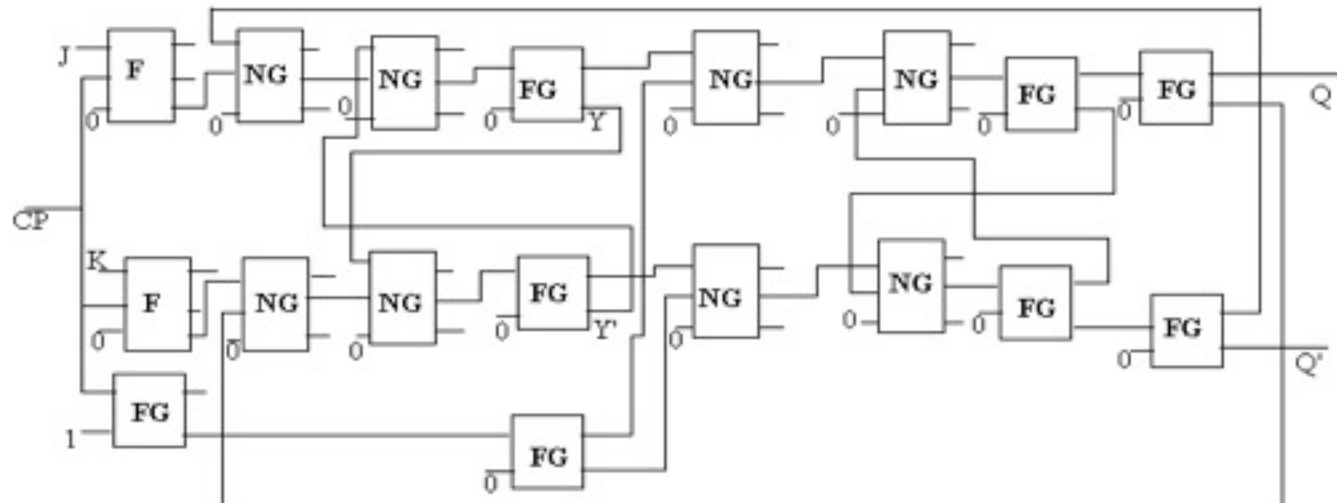
Master Slave JK Flip Flop



Conventional Master Slave JK Flip Flop

Evaluation of the Proposed Flip Flop

	No of gates	Garbage Outputs
Proposed Circuit	18	21
Existing One	None in literature	None in Literature



Proposed Reversible Master Slave JK Flip Flop

Results and Discussion

- Novel Reversible Flip Flops are designed Using Feynman Gate, New gate and Fredkin Gate.
- The designed FFs are highly optimized in terms of number of reversible gates and Garbage outputs.
- Modularization approach has been used to design the reversible Flip Flops.
- Fan out problem is avoided by using Feynman gate for copying the output .

Work In Progress

- Coming out with a new reversible gate specially designed for performing sequential operations.
- Designing of Complex Sequential circuits using the proposed designs.

Future Work

- Building of the proposed Reversible Flip Flops by using technologies such as
 - a. CMOS, in particular adiabatic CMOS
 - b. Optical, thermodynamic technology
 - c. Nanotechnology & DNA technology.
- Introducing online testability feature in Reversible designs of Sequential circuits

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