Regression

(*) We want to predict x_1 on the basis of the information provided by other variables $x_2, x_3, \ldots, x_p$.

The MSE $f(\underset{\sim}{x}_{(2)})$ in predicting x_1 is,

$E(e^2) = E(x_1 - f(\underset{\sim}{x}_{(2)}))^2$ is minimised

when $f(\underset{\sim}{x}_{(2)}) = M(\underset{\sim}{x}_{(2)}) = E(x_1 | \underset{\sim}{x}_{(2)})$ with prob. 1.

(*) $corr(x_1, M(\underset{\sim}{x}_{(2)})) = \rho(x_1, M(\underset{\sim}{x}_{(2)})) \geq 0$

and $|\rho(x_1, f(\underset{\sim}{x}_{(2)}))| \leq \rho(x_1, M(\underset{\sim}{x}_{(2)}))$.

equality holds iff $f(\underset{\sim}{x}_{(2)}) = M(\underset{\sim}{x}_{(2)})$.

(*)

Least Square Linear Regression

Suppose the model is,

$x_1 = \alpha + \beta_2 x_2 + \cdots + \beta_p x_p + e$

Then the L.S. estimates of α, β_i's are

(*) $\hat{\underset{\sim}{\beta}} = \Sigma_2^{-1} \underset{\sim}{\sigma}_{(1)}$

and $\hat{\alpha} = \mu_1 - \hat{\underset{\sim}{\beta}}^T \underset{\sim}{\mu}_2$

where, $\Sigma_2 = \begin{pmatrix} \sigma_{22} & \cdots & \sigma_{2p} \\ \vdots & & \vdots \\ \sigma_{p2} & \cdots & \sigma_{pp} \end{pmatrix}$, $\underset{\sim}{\sigma}_{(1)} = \begin{pmatrix} \sigma_{21} \\ \sigma_{31} \\ \vdots \\ \sigma_{p1} \end{pmatrix}$

$\underset{\sim}{\mu}_2 = \begin{pmatrix} \mu_2 \\ \mu_3 \\ \vdots \\ \mu_p \end{pmatrix}$

$[x_{1.23\ldots p} + e_{1.23\ldots p} = x_1]$

(*) $E(e_{1.23\cdots p}) = 0$

$cov(e_{1.23\cdots p}, x_j) = 0, \quad j = 2(1)p$

$cov(e_{1.23\cdots p}, x_{1.23\cdots p}) = 0$

$V(e_{1.23\cdots p}) = \sigma_{11} - \underset{\sim}{\sigma}_{(1)}^T \Sigma_2^{-1} \underset{\sim}{\sigma}_{(1)}$

$= \dfrac{|\Sigma|}{|\Sigma_2|}, \quad \Sigma = \begin{bmatrix} \sigma_{11} & \underset{\sim}{\sigma}_{(1)}^T \\ \underset{\sim}{\sigma}_{(1)} & \Sigma_2 \end{bmatrix}$

* The correlation coefficient between x_1 and the LS multiple linear regression $X_{1\cdot 23\cdots p}$ is the max^m correlation between x_1 and any linear function of $\underset{\sim}{x}_{(2)}$.

* $0 \leq \rho(x_1, X_{1\cdot 23\cdots p}) \leq 1$
As it is a measure of degree of linear relationship of x_1 ~~and~~ with $\underset{\sim}{x}_{(2)}$ and there is no ~~rato~~ direction in this relationship.

* $$\rho(x_1, X_{1\cdot 23\cdots p}) = \max \rho(x_1, l_0 + \underset{\sim}{l}^T \underset{\sim}{x}_{(2)})$$
$$= \sqrt{\frac{\hat{\underset{\sim}{\beta}}^T \Sigma_2 \hat{\underset{\sim}{\beta}}}{\sigma_{11}}} = \sqrt{\frac{var(X_{1\cdot 23\cdots p})}{var(x_1)}}$$

$$\rho^2_{1\cdot 23\cdots p} = 1 - \frac{\sigma^2_{1\cdot 23\cdots p}}{\sigma_{11}} = \left(\sqrt{1 - \frac{|\Sigma|}{\sigma_{11}|\Sigma_2|}}\right)^2$$

[Due to L.S. principle]

* $$\rho^2_{1,23\cdots p} \geq \rho^2_{1\cdot 34\cdots p}$$

* $$\sigma^2_{1\cdot 23\cdots p} = \sigma_1^2 (1 - \rho^2_{1\cdot 23\cdots p})$$

* Partial Correlation Coefficient

$$\rho_{12\cdot 34\cdots p} = \frac{cov(e_{1\cdot 34\cdots p}, e_{2\cdot 34\cdots p})}{\sqrt{V(e_{1\cdot 34\cdots p}) V(e_{2\cdot 34\cdots p})}}$$

$$cov(e_{1\cdot 34\cdots p}, e_{2\cdot 34\cdots p}) = \frac{(-1)^{1+2} \Sigma_{12}}{|\Sigma_3|}$$

Cov(e_1).

Rank Correlation

\# $R = 1 - \dfrac{6 \Sigma d_i^2}{n(n^2-1)}$ where n is the number of individuals who are ranked, d_i is the difference between ranks given by two judges to ith individual

\# $-1 \leq R \leq 1$

\# For perfect agreement $R = 1$ $[\text{when } \Sigma d_i^2 = 0]$

" " dis- " $R = -1$ $[\text{when } x_i + x_i' = n+1 \ \forall i]$

\# $\sum_{i=1}^{n} d_i^2 \leq \dfrac{n(n^2-1)}{3}$