MA5710: Assignment-5

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Instructions :

- Please do not copy. Any sign of copy will lead to zero marks only.
- Submit all your findings as report in a single .pdf file to **ma19d201@smail.iitm.ac.in** within **15th November**

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1. Using Matlab's **ode45 solver** or equivalent function in python, evaluate trajectories of $N(t)$ and $P(t)$ for Lotka-Volterra Model(**with and without fishing**) taking $(N(0), P(0)) = (10, 5)$ as initial condition and setting the parameters as $a = 4, b = 2, c = 1.5, d = 3$ and for **with fishing model** set δ as 0.2
2. Plot the **Phase portrait** for both cases of **Q1**
3. Linearize the Lotka Volterra Model(with and without fishing) and solve it by applying ode solver as in Q1. Evaluate the trajectories and draw respective phase portraits.
4. This question requires use of **symbolic computation**. Look for **sympy** module in python or equivalent computational toolbox in Matlab. Consider the **Four species Model**.It has already been derived in the slides and also in the reference. Using symbolic compuation, derive the equilibrium points of the model and classify those points by figuring out respective eigen values and eigen vectors of the linearized system. **Please note that,** deriving equilibrium points, computing eigen values and eigen vectors of the linearized system, the whole computation process has to be done either in matlab or in python, using the module for symbolic compuation.
5. Consider the **Infectious disease model**. The model is derived and the equilibrium points are evaluated in the slide provided. **Verify** the equilibrium points using symbolic computation. Derive the conditions for equilibrium points to be **asymptotically stable**. Verify your calculation through symbolic computation.
6. In **Q5** system model, initial conditions are $V(0) = V^0 > 0, F(0) = F^*, C(0) = C^*, m(0) = 0$. The immunological barrier is defined as $V^* = \frac{a(pF^* - \alpha)}{\alpha \gamma p}$.
Choose the parameter values and initial condition such that $\alpha < pF^*$. This is **subclinical form** of the disease. Solve the system of ode's for the **chosen** values and plot $V(t)$ when, $V^0 < V*$ and $V^0 > V*$. **Conclude** how the plot differs for these two subcases
Next,Choose the parameter values and initial condition such that $\alpha > pF^*$. This is called **Acute** form of the disease. For this case immunological barrier does not exist. Plot $V(t)$ when $k\beta > \mu\gamma p$ and $k\beta < \mu\gamma p$. The first subcase denotes normal immune response to acute disease thus leading to recovery and the second subcase denotes immunodeficiency

response,thus leading to more severe form of the disease. Also plot $V(t)$ when value of σ is **gradually increased**. **See** whether with increased value of sigma, you are reaching the **chronic** state or not.
For subclinical, acute or chronic disease the plots for $V(t)$ are mentioned in the slide. The plots you generate for the above parameter values and initial condition should resemble the given plots.