MA5710 - Assignment 4

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1. Implement Gudanov Method to numerically solve the first order hyperbolic partial differential equation, that is the scalar conservation law. Assume different speed density relations and initial values as mentioned in the slide provided, on case by case basis(when the signal is red, then red to green and then red again, with and without speed breaker before and after the signal). Generate the tables provided in the slide for each case and subcases.

Code: traffic_flow_model.m

```
% Main file
   clear;
3
   clc;
4
   rhoMax = 1;
                   % Maximum rho
   rhoMin = 0;
6
                   % Minimum rho
7
   h = 0.1;
                  % space step size
8
   k = 0.1;
                  % time step size
   x = -1:h:1;
                   % Road - (-1 to 1)
                                % No. of space steps
10
   x_divs = length(x);
11
12
                                % Total time under consideration
   total_time= 3;
13
   t_steps = total_time / k;
                               % No. of time steps
14
15
   % Initial conditions
16
   rho0 = 0.65 * ones(1, x_divs);
17
   u = zeros(t_steps, x_divs);
18
                                % Array to store u
19
20
  % Initial and boundary conditions for u
21
   u(1,:) = 1 - 2 .* rho0/rhoMax;
                                     % Burger's equation
   u(:,1) = u(1,1)*ones(t_steps,1);
22
23
24
   % Signal
25
   x_signal = 0.2;
                       % Position of signal
26
   x_signal_idx = floor((x_signal - x(1))/h);
                                               % Index of signal position in x array
27
28
   signal_times = [0.5, 1.5, 2.5];
                                                % Signal timings (in min.)
29
                                                % Index of signal time in time array
   signal_time_idx = signal_times/k;
30
31
32
   f = Q(u) (u.^2)/2; % Flux function
33
   df = @(u) u;
                        % Derivative of Flux
34
   % Apply boundary condition (1)
36
   for i = 1:2:length(signal_time_idx)-1
      u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx) = -1;
38
      u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx + 1) = 1;
39
   end
40
```

```
41 | % Numerical calculation using Godunov method
   for t = 1:t_steps-1
42
43
       % Apply boundary condition (2)
44
       for i = 1:2:length(signal_time_idx)-1
45
          u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx) = -1;
46
           u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx + 1) = 1;
47
       end
48
49
       for i = 2:x_divs - 1
50
            u(t+1,i) = u(t,i) - (k/h)*(f(u_star(t,i, u,f, df))-f(u_star(t,i-1, u,f, df))
               );
51
       end
52
   end
   u(:,1) = u(:,2);
54 | u(:,end) = u(:,end-1);
56 % Get rho value from u
57 | \text{rho\_tab} = (\text{rhoMax} / 2).*(1 - u);
58
59
60 % Plotting at different cases
61 figure;
62 | First subplot - when green light (initial cond.)
63 | subplot(2, 2, 1);
64 | plot(x, rho_tab(1,:), 'LineWidth', 1.5);
65 | % Add labels and title
66 | xlabel('Road');
67 | ylabel('Traffic density');
68 | title('t = 0, Uniform traffic flow (Green signal)');
69 % Display grid
70 grid on;
71 | hold on;
72 | % Add a vertical line at signal position
73 | xline(x_signal, 'g--', 'LineWidth', 2); % Signal position
74 hold off;
75
76 | % Second subplot - when red light
77 | subplot(2, 2, 2);
78 | plot(x, rho_tab(10,:), 'LineWidth', 1.5);
79 \frac{1}{\%} Add labels and title
80 | xlabel('Road');
81 | ylabel('Traffic density');
82 | title('During Red signal');
83 grid on;
84 hold on;
85 | % Add a vertical line at signal position
86 | xline(x_signal, 'r--', 'LineWidth', 2);
87 hold off;
88
89 | % Third subplot - when red light (just before green light)
90 | subplot(2, 2, 3);
91 | plot(x, rho_tab(15,:), 'LineWidth', 1.5);
92 | % Add labels and title
93 | xlabel('Road');
94 | ylabel('Traffic density');
95 | title('Just before Green signal');
96 grid on;
97 hold on;
98 | % Add a vertical line at signal position
```

```
xline(x_signal, 'r--', 'LineWidth', 2);
100 hold off;
102 % forth subplot
103 | subplot(2, 2, 4);
    plot(x, rho_tab(25,:), 'LineWidth', 1.5);
104
105 | % Add labels and title
106 | xlabel('Road');
107
    ylabel('Traffic density');
108
    title('Traffic flow 1 min after signal turned green');
109 grid on;
110 | hold on;
    % Add a vertical line at signal position
111
112
    xline(x_signal, 'g--', 'LineWidth', 2);
113
    hold off;
114
115
116
    \% Function for u_star calculation used in Godunov scheme
117
118
    function u_star_out = u_star(t, i, u, f, df)
119
        gradF = df(u(t, i));
                                   % Gradient of flux for u(t,i)
120
        gradF1 = df(u(t, i + 1)); % Gradient of flux for u(t, i+1)
121
        % Apply different conditions...
122
        if gradF >= 0 && gradF1 >= 0
123
            u_star_out = u(t, i);
124
        elseif gradF < 0 && gradF1 < 0
125
            u_star_out = u(t, i + 1);
126
        elseif gradF >= 0 && gradF1 < 0
127
            s = (f(u(t, i + 1))-f(u(t, i)))/(u(t, i + 1)-u(t, i));
128
            if s >= 0
129
                u_star_out = u(t, i);
130
            else
131
                u_star_out = u(t, i + 1);
132
            end
133
        else
134
            u_star_out = 0;
        end
135
136
    end
```

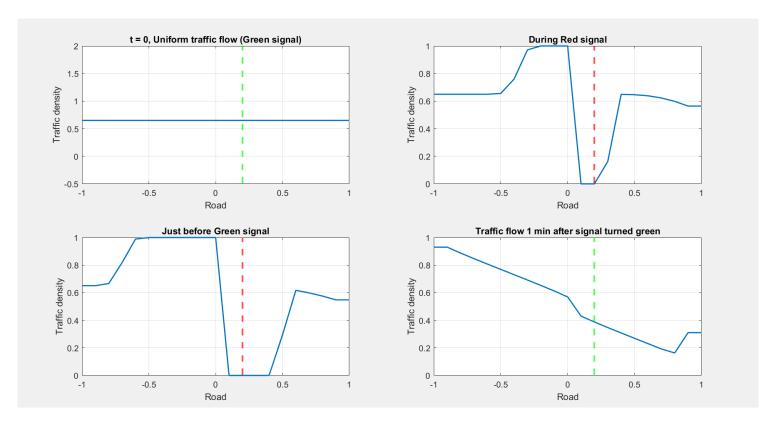
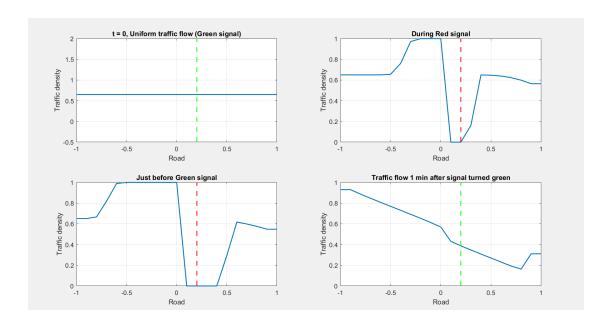


Figure 1: Density vs Road length for different traffic signal conditions

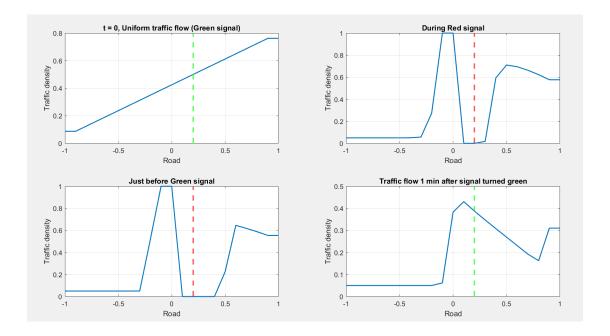
												Traffic sign	al position									
	t/x	-1	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	0	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65
	0.1	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65
	0.2	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.6275	0.6275
	0.3	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.643756	0.611244	0.611244
Signal turned red	0.4	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	1	0	0.65	0.65	0.65	0.65	0.65	0.648166	0.635466	0.598869	0.598869
	0.5		0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.8775	1	0	0.4225	0.65	0.65	0.65	0.649453	0.644564	0.62689	0.589094	0.589094
	0.6	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.770006	0.984994	1	0	0.195	0.65	0.65	0.649836	0.648016	0.639766	0.618726	0.581156	0.581156
	0.7	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.700403	0.932322	0.999775	1	0	0.038025	0.579475	0.649951	0.649294	0.645641	0.634327	0.611217	0.57457	0.57457
	0.8	0.65	0.65	0.65	0.65	0.65	0.65	0.667662		0.995195	1	1	0	0.001446	0.388539	0.649754	0.648217	0.642474		0.604408	0.569009	0.569009
	0.9	0.65	0.65	0.65	0.65	0.65	0.65561	0.76006	0.971853	0.999977	1	1	0	2.09E-06	0.16241	0.649296	0.646547	0.638727	0.623002	0.598269	0.564247	0.564247
	1	0.65	0.65	0.65	0.65			0.915074	0.999185	1	1	1	0	4.37E-12	0.026379		0.644316		0.61753		0.560119	0.560119
	1.1	0.65	0.65	0.65	0.650517	0.668309			0.999999	1	1	1	0	0	0.000696	0.353315	0.641609	0.630304	0.612317	0.587754	0.556505	0.556505
	1.2	0.65	0.65		0.65619	0.750007	0.963713	0.999935	1	1	1	1	0	0	4.84E-07	0.125527	0.637072	0.62594			0.553312	0.553312
	1.3	0.650047	0.650047	0.652004	0.694298	0.902533	0.998618	1	1	1	1	1	0	0	2.34E-13	0.015758	0.512703	0.621615	0.602797	0.579158	0.55047	0.55047
	1.4	0.650638	0.650638	0.666651	0.818579		0.999998	1	1	1	1	1	0	0	0	0.000248	0.293002	0.617392			0.547923	0.547923
Signal turned green	1.5	0.655719			0.956325	0.99988	1	1	1	1	1	0.75	0.25	0	0	0.2.2	0.086098	0.584245	0.594486	0.572045	0.545626	0.545626
	1.6	0.689248			0.997972	1	1	1	1	1	0.8125	0.6875	0.3125	0.1875	-		0.007413		0.590748	0.568936		0.543544
	1.7	0.806178			0.999996	1		1		0.847656			0.347656		0.152344		5.50E-05	0.187451		0.56608	0.541648	0.541648
	1.8	0.948685		0.999801	1	1	1			0.789291		0.629135	0.370865		0.210709	0.129135		0.035193	0.493945	0.563448	0.539914	0.539914
	1.9	0.997168		1	1	1		0.887541		0.75	0.682986	0.612459		0.317014	0.25	0.182986	0.112459	0.001239	0.281925		0.538321	0.538321
	2	0.999992	0.999992	1	1	_	0.900188			0.720984	0.662149	0.599812	0.400188		0.279016	0.220984	0.162149	0.099814	0.080719	0.514927	0.536852	0.536852
	2.1		1	1	1		0.854181				0.645819	0.58985	0.41015				0.198443	0.14582	0.096366	0.340489	0.535494	0.535494
	2.2		1	_	0.918223			0.773121		0.680326	0.632629	0.581777	0.418223					0.180326	0.133843		0.51005	0.51005
	2.3			0.924911		0.834601		0.75	0.707923	0.665399	0.621726	0.575089		0.378274		0.292077	0.25	0.207923			0.421849	0.421849
	2.4	0.930549			0.84714		0.769268		0.692048	0.65286	0.612547	0.569451	0.430549		0.34714		0.269268		0.192155			0.310406
	2.5			0.85784	0.821468	0.785623	0.75		0.678532	0.64216		0.564627	0.435373		0.35784	0.321468	0.285623	0.25	0.214418		0.232751	0.232751
	2.6	0.867086			0.799707			0.700293	0.666868	0.632914	0.597918	0.560451	0.439549		0.367086		0.299707	0.266542	0.233475	0.201489	0.202912	0.202912
	2.7	0.843311				0.75			0.656689	0.624836	0.591984	0.556796	0.443204				0.311979	0.280928	0.250008		0.202064	0.202064
	2.8	0.82278	0.82278		0.764507	0.735493	0.706431	0.67722		0.617713	0.586749	0.553571	0.446429		0.382287	0.352279	0.32278	0.293569	0.264511	0.235712		0.212185
	2.9	0.804776	0.804776	0.77735	0.75	0.72265	0.695224	0.667635	0.639756	0.611382	0.582093	0.550701	0.449299	0.417907	0.388618	0.360244	0.332365	0.304776	0.277352	0.250105	0.225174	0.225174

Figure 2: Output Density data

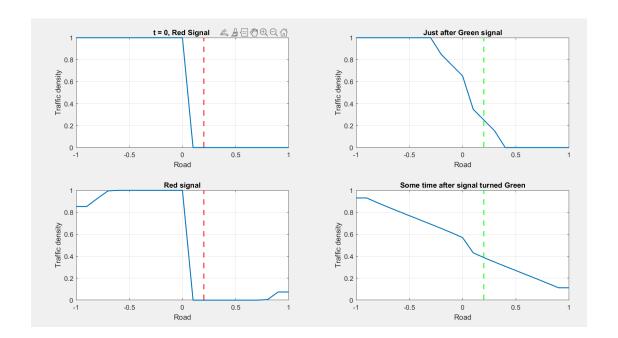
Different initial conditions (Comparisons): 1. $rho0 = 0.65 * ones(1, x_divs)$



2. $rho0 = linspace(0.05, 0.8, x_divs)$



3. $\mathbf{rho0}(x \le 0) = 1$; $\mathbf{rho0}(x > 0) = 0$;



Traffic flow with speed breaker before signal: Code: traffic_flow_model_spdbrk_left.m

```
| % Traffic flow model with speed breaker before signal
1
2
   clear;
3 clc:
4
   rhoMax = 1;
                   % Maximum rho
5
6 \mid \text{rhoMin} = 0;
                   % Minimum rho
7 \mid h = 0.1;
                  % space step size
8 k = 0.1;
                   % time step size
9 \mid x = -1:h:1;
                   % Road - (-1 to 1)
10 \mid x_{divs} = length(x);
                                % No. of space steps
11
12 | total_time= 3;
                                 % Total time under consideration
13 | t_steps = total_time / k; % No. of time steps
14
15 | % Initial conditions
16 | \text{rho0} = 0.65 * \text{ones}(1, x_divs);
17
18 \mid \% \ rho0 = np.linspace(0.1, 0.5, num_divs)
19 \ \% \ rho0 = np.zeros(num_divs)
20 \ \ \% \ \ rho0[x <= 0] = 1
21 \quad | \% \quad rho0[x > 0] = 0
22
23 | u = zeros(t_steps, x_divs);
                                 % Array to store u
24
25 | % Initial and boundary conditions for u
26 | u(1,:) = 1 - 2 .* rho0/rhoMax;
27 | u(:,1) = u(1,1)*ones(t_steps,1);
28
29 % Signal
30 | x_signal = 0.2;  % Position of signal
   x_signal_idx = floor((x_signal - x(1))/h); % Index of signal position in x array
32
33 | signal_times = [0.5, 1.5, 2.5];
                                                  % Signal timings (in min.)
34 | signal_time_idx = signal_times/k;
                                                  % Index of signal time in time array
35
36 % Speed breaker
37 | x_spd_brk = -0.2;
                                                      % Position of speed breaker
   x_spd_brk_idx = floor((x_spd_brk - x(1))/h);  % Index of speed breaker position in
       x array
39
   v_spd_brk = 0.15;
                                                      % Velocity at speed breaker
40
41 | f = Q(u) (u.^2)/2; % Flux function
42
   df = 0(u) u;
                       % Derivative of Flux
43
44 | % Apply boundary condition (1)
45
   for i = 1:2:length(signal_time_idx)-1
46
      u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx) = -1;
47
      u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx + 1) = 1;
48 end
49
   u(:, x_spd_brk_idx) = -1;
50
51 | % Numerical calculation using Godunov method
52 for t = 1:t_steps-1
       % Apply boundary condition (2)
```

```
54
        for i = 1:2:length(signal_time_idx)-1
           u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx) = -1;
56
           u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx + 1) = 1;
57
58
        u(:, x_spd_brk_idx) = -1;
59
60
        for i = 2:x_{divs} - 1
61
            u(t+1,i) = u(t,i) - (k/h)*(f(u_star(t,i, u,f, df))-f(u_star(t,i-1, u,f, df))
                );
62
             if i == x_spd_brk_idx
63
                 u(t + 1, i) = v_spd_brk * u(t + 1, i);
64
             end
65
        end
66
    end
67
    u(:,1) = u(:,2);
68 \ u(:,end) = u(:,end-1);
69
70 % Get rho value from u
71 | \text{rho\_tab} = (\text{rhoMax} / 2).*(1 - u);
72
73
74 | % Plotting at different cases
75 | figure;
76 | % First subplot - when green light (initial cond.)
    subplot(2, 2, 1);
78 | plot(x, rho_tab(1,:), 'LineWidth', 1.5);
79 | % Add labels and title
80 | xlabel('Road');
81 | ylabel('Traffic density');
82 | title('t = 0, Uniform traffic flow (Green signal)');
83 | % Display grid
84
    grid on;
85 hold on;
86 | % Add a vertical line at signal position
87 | xline(x_signal, 'g--', 'LineWidth', 2); % Signal position
88 | xline(x_spd_brk, 'm--', 'LineWidth', 1); % Speed breaker position
89 hold off;
90
91 | % Second subplot - when red light
92 | subplot(2, 2, 2);
93 | plot(x, rho_tab(10,:), 'LineWidth', 1.5);
94 | % Add labels and title
95 | xlabel('Road');
96 | ylabel('Traffic density');
97 | title('During Red signal');
98 grid on;
99 hold on;
    xline(x_signal, 'r--', 'LineWidth', 2); % Add a vertical line at signal position
100
    xline(x_spd_brk, 'm--', 'LineWidth', 1); % Speed breaker position
102 hold off;
103
104 | % Third subplot - when red light (just before green light)
105 | subplot(2, 2, 3);
106 | plot(x, rho_tab(15,:), 'LineWidth', 1.5);
107 | % Add labels and title
108 | xlabel('Road');
109 | ylabel('Traffic density');
110 | title('Just before Green signal');
111 grid on;
```

```
112 | hold on;
    113
114
   xline(x_spd_brk, 'm--', 'LineWidth', 1); % Speed breaker position
115 hold off;
116
117 | % forth subplot
118
   subplot(2, 2, 4);
119 | plot(x, rho_tab(25,:), 'LineWidth', 1.5);
120 | % Add labels and title
   xlabel('Road');
122 | ylabel('Traffic density');
123 | title('Traffic flow 1 min after signal turned green');
124
   grid on;
125 hold on;
126 | xline(x_signal, 'g--', 'LineWidth', 2); % Add a vertical line at signal position
    xline(x_spd_brk, 'm--', 'LineWidth', 1); % Speed breaker position
127
128 hold off;
129
130
131 |% Function for u_star calculation used in Godunov scheme
132
    function u_star_out = u_star(t, i, u, f, df)
133
       gradF = df(u(t, i)); % Gradient of flux for given u(n, i)
134
       gradF1 = df(u(t, i + 1)); % Gradient of flux for given u(n, i+1)
       % Apply different conditions...
136
       if gradF >= 0 && gradF1 >= 0
137
           u_star_out = u(t, i);
138
       elseif gradF < 0 && gradF1 < 0
139
           u_star_out = u(t, i + 1);
140
       elseif gradF >= 0 && gradF1 < 0
141
           s = (f(u(t, i + 1))-f(u(t, i)))/(u(t, i + 1)-u(t, i));
142
           if s >= 0
143
               u_star_out = u(t, i);
144
           else
145
               u_star_out = u(t, i + 1);
146
           end
147
       else
148
           u_star_out = 0;
149
        end
150
   end
```

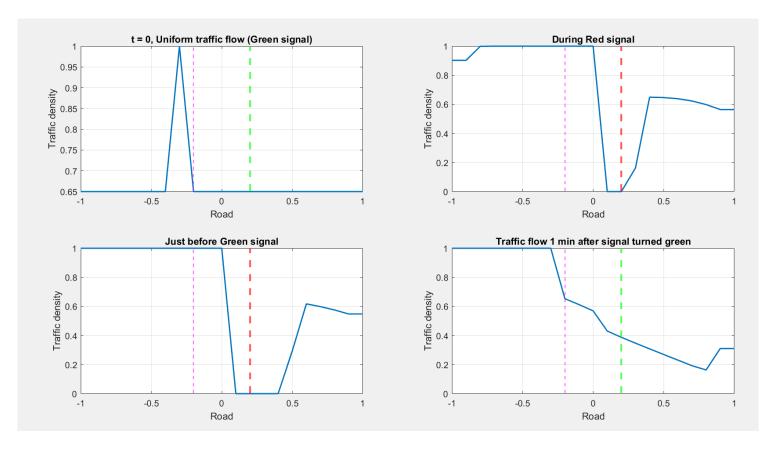


Figure 3: Density plot for different traffic signal conditions with Speed breaker before signal

								5	Spd brk po	s.		Traffic sign	al position									
	t/x	-1	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
	0	0.65	0.65	0.65	0.65	0.65	0.65	0.65	1	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.6
	0.1	0.65	0.65	0.65	0.65	0.65	0.65	0.8775	1	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.6
	0.2	0.65	0.65	0.65	0.65	0.65	0.770006	0.984994	1	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.6275	0.627
	0.3	0.65	0.65	0.65	0.65	0.700403	0.932322	0.999775	1	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.643756	0.611244	0.61124
Signal turned red	0.4	0.65	0.65	0.65	0.667662	0.847144	0.995195	1	1	0.65	0.65	1	0	0.65	0.65	0.65	0.65	0.65	0.648166	0.635466	0.598869	0.598869
	0.5	0.65	0.65	0.65561	0.76006	0.971853	0.999977	1	1	0.65	0.8775	1	0	0.4225	0.65	0.65	0.65	0.649453	0.644564	6.27E-01	0.589094	0.589094
	0.6	0.651715	0.651715	0.699027	0.915074	0.999185	1	1	1	0.770006	0.984994	1	0	0.195	0.65	0.65	0.649836	0.648016	0.639766	6.19E-01	0.581156	0.58115
	0.7	0.668309	0.668309	0.831702	0.991973	0.999999	1	1	1	0.932322	0.999775	1	0	0.038025	0.579475	0.649951	0.649294	0.645641	0.634327	0.611217	0.57457	0.5745
	0.8	0.750007	0.750007	0.963713	0.999935	1	1	1	1	0.995195	1	1	0	0.001446	0.388539	0.649754	0.648217	0.642474	0.628653	0.604408	5.69E-01	5.69E-0
	0.9	0.902533	0.902533	0.998618	1	1	1	1	1	0.999977	1	1	0	2.09E-06	0.16241	0.649296	0.646547	0.638727	0.623002	0.598269	5.64E-01	5.64E-0
	1	0.98912	0.98912	0.999998	1	1	1	1	1	1	1	1	0	4.37E-12	0.026379	0.556805	0.644316	0.634611	0.61753	0.59274	0.560119	0.56011
	1.1	0.99988	0.99988	1	1	1	1	1	1	1	1	1	0	0	0.000696	0.353315	0.641609	0.630304	0.612317	0.587754	0.556505	0.55650
	1.2	1	1	1	1	1	1	1	1	1	1	1	0	0	4.84E-07	0.125527	0.637072	0.62594	0.607403	0.583246	0.553312	0.55331
	1.3	1	1	1	1	1	1	1	1	1	1	1	0	0	2.34E-13	0.015758	0.512703	0.621615	0.602797	0.579158	0.55047	0.5504
	1.4	1	1	1	1	1	1	1	1	1	1	1	0	0	0	2.48E-04	0.293002	0.617392	0.598496	0.575439	0.547923	0.54792
Signal turned green	1.5	1	1	1	1	1	1	1	1	1	1	0.75	0.25	0	0	6.17E-08	0.086098	0.584245	0.594486	0.572045	0.545626	0.54562
	1.6	1	1	1	1	1	1	1	1	1	0.8125	0.6875	0.3125	0.1875	0	3.77E-15	7.41E-03	0.421858	0.590748	0.568936	0.543544	0.54354
	1.7	1	1	1	1	1	1	1	1	0.847656	0.75	0.652344	0.347656	0.25	0.152344	0	5.50E-05	0.187451	0.587265	0.56608	0.541648	0.54164
	1.8	1	1	1	1	1	1	1	1	0.789291	0.710709	0.629135	0.370865	0.289291	0.210709	0.129135	3.02E-09	0.035193	0.493945	0.563448	0.539914	0.53991
	1.9	1	1	1	1	1	1	1	1	0.75	0.682986	0.612459	0.387541	0.317014	0.25	0.182986	0.112459	0.001239	0.281925	0.561015	0.538321	0.53832
	2	1	1	1	1	1	1	1	1	0.720984	0.662149	0.599812	0.400188	0.337851	0.279016	0.220984	0.162149	0.099814	0.080719	0.514927	0.536852	0.53685
	2.1	1	1	1	1	1	1	1	1	0.698443	0.645819	0.58985	0.41015	0.354181	0.301557	0.25	0.198443	0.14582	0.096366	0.340489	0.535494	0.53549
	2.2	1	1	1	1	1	1	1	1	0.680326	0.632629	0.581777	0.418223	0.367371	0.319674	0.273121	0.226879	0.180326	0.133843	0.203013	0.51005	0.5100
	2.3	1	1	1	1	1	1	1	1	0.665399	0.621726	0.575089	0.424911	0.378274	0.334601	0.292077	0.25	0.207923	0.165723	0.157143	0.421849	0.42184
	2.4	1	1	1	1	1	1	1	1	0.65286	0.612547	0.569451	0.430549	0.387453	0.34714	0.307952	0.269268	0.230732	0.192155	0.162953	0.310406	0.31040
	2.5	1	1	1	1	1	1	1	1	0.64216	0.604704	0.564627	0.435373	0.395296	0.35784	0.321468	0.285623	0.25	0.214418	0.181785	0.232751	0.23275
	2.6	1	1	1	1	1	1	1	1	0.632914	0.597918	0.560451	0.439549	0.402082	0.367086	0.333132	0.299707	0.266542	0.233475	0.201489	0.202912	0.20291
	2.7	1	1	1	1	1	1	1	1	0.624836	0.591984	0.556796	0.443204	0.408016	0.375164	0.343311	0.311979	0.280928	0.250008	0.219562	0.202064	0.20206
	2.8	1	1	1	1	1	1	1	1	0.617713	0.586749	0.553571	0.446429	0.413251	0.382287	0.352279	0.32278	0.293569	0.264511	0.235712	0.212185	0.21218
	2.9	1	1	1	1	1	1	1	0.539578	0.611382	0.582093	0.550701	0.449299	0.417907	0.388618	0.360244	0.332365	0.304776	0.277352	0.250105	0.225174	0.22517

Figure 4: Density data with speed breaker and signal

Traffic flow with speed breaker after signal:

Code: traffic_flow_model_spdbrk_right.m

```
1 % Traffic flow model with speed breaker after signal
2 | clear;
3 clc:
4
5
   rhoMax = 1;
               % Maximum rho
                  % Minimum rho
6 \mid \text{rhoMin} = 0;
7 \mid h = 0.1;
                 % space step size
                 % time step size
8 | k = 0.1;
9 | x = -1:h:1; % Road - (-1 to 1)
10 \mid x_{divs} = length(x);
                        % No. of space steps
11
                                % Total time under consideration
12
   total_time= 3;
13 | t_steps = total_time / k; % No. of time steps
14
15 | % Initial conditions
16 | \text{rho0} = 0.65 * \text{ones}(1, x_divs);
17
18 \mid \% \ rho0 = np.linspace(0.1, 0.5, num_divs)
19 \mid \% \quad rho0 = np.zeros(num_divs)
20 \ \ \% \ \ rho0[x <= 0] = 1
21 \quad \% \quad rho0[x > 0] = 0
22
23 | u = zeros(t_steps, x_divs);   % Array to store u
24
25 | % Initial and boundary conditions for u
26 | u(1,:) = 1 - 2 .* rho0/rhoMax ;
27 | u(:,1) = u(1,1)*ones(t_steps,1);
28
29 % Signal
31 | x_signal_idx = floor((x_signal - x(1))/h); % Index of signal position in x array
32
33 | signal_times = [0.5, 1.5, 2.5];
                                                % Signal timings (in min.)
34 | signal_time_idx = signal_times/k;
                                               % Index of signal time in time array
35
36 % Speed breaker
37 \mid x_spd_brk = 0.7;
                                                   % Position of speed breaker
|x_{spd_brk_idx}| = floor((x_{spd_brk} - x(1))/h);
                                                    % Index of speed breaker position in
       x array
39 | v_spd_brk = 0.15;
                                                    % Velocity at speed breaker
40
41 \mid f = Q(u) (u.^2)/2; % Flux function
42 | df = Q(u) u;
                      % Derivative of Flux
43
44 | % Apply boundary condition (1)
45 | for i = 1:2:length(signal_time_idx)-1
     u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx) = -1;
46
47
      u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx + 1) = 1;
48
   end
49 | u(:, x_spd_brk_idx+1) = 1;
50
51 | % Numerical calculation using Godunov method
52 | for t = 1:t_steps-1
```

```
53
        % Apply boundary condition (2)
54
        for i = 1:2:length(signal_time_idx)-1
           u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx) = -1;
56
           u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx + 1) = 1;
57
        end
58
        u(:, x_spd_brk_idx+1) = 1;
59
60
        for i = 2:x_{divs} - 1
61
            u(t+1,i) = u(t,i) - (k/h)*(f(u_star(t,i, u,f, df))-f(u_star(t,i-1, u,f, df))
62
             if i == x_spd_brk_idx
63
                 u(t + 1, i) = v_spd_brk * u(t + 1, i);
64
             end
65
        end
66
    end
67
    u(:,1) = u(:,2);
    u(:,end) = u(:,end-1);
68
69
70 | % Get rho value from u
71 | rho_tab = (rhoMax / 2).*(1 - u);
72
73
74 | % Plotting at different cases
75 | figure;
76 | % First subplot - when green light (initial cond.)
77 | subplot(2, 2, 1);
78 | plot(x, rho_tab(1,:), 'LineWidth', 1.5);
79 | % Add labels and title
80 | xlabel('Road');
81 | ylabel('Traffic density');
82 | title('t = 0, Uniform traffic flow (Green signal)');
83 | % Display grid
84 grid on;
85 hold on;
86 | % Add a vertical line at signal position
87 | xline(x_signal, 'g--', 'LineWidth', 2); % Signal position
88 | xline(x_spd_brk, 'm--', 'LineWidth', 1); % Speed breaker position
89 hold off;
90
91 % Second subplot - when red light
92 | subplot(2, 2, 2);
93 | plot(x, rho_tab(10,:), 'LineWidth', 1.5);
94 | % Add labels and title
95 | xlabel('Road');
96 | ylabel('Traffic density');
97 | title('During Red signal');
98
    grid on;
99 hold on;
100 \mid \text{xline}(\text{x\_signal}, \text{'r--'}, \text{'LineWidth'}, 2); \% Add a vertical line at signal position
101 | xline(x_spd_brk, 'm--', 'LineWidth', 1); % Speed breaker position
102 hold off;
103
104 | % Third subplot - when red light (just before green light)
105 | subplot(2, 2, 3);
    plot(x, rho_tab(15,:), 'LineWidth', 1.5);
106
107 % Add labels and title
108 | xlabel('Road');
109 | ylabel('Traffic density');
110 | title('Just before Green signal');
```

```
111 grid on;
112 hold on;
113 | xline(x_signal, 'r--', 'LineWidth', 2); % Add a vertical line at signal position
    xline(x_spd_brk, 'm--', 'LineWidth', 1); % Speed breaker position
115 hold off;
116
117 % forth subplot
118 | subplot(2, 2, 4);
119
    plot(x, rho_tab(25,:), 'LineWidth', 1.5);
120 % Add labels and title
121 | xlabel('Road');
122 | ylabel('Traffic density');
123 | title('Traffic flow 1 min after signal turned green');
124 grid on;
125 hold on;
    xline(x_signal, 'g--', 'LineWidth', 2); % Add a vertical line at signal position
126
    xline(x_spd_brk, 'm--', 'LineWidth', 1); % Speed breaker position
127
128
    hold off;
129
130
    \% Function for u_star calculation used in Godunov scheme
132
    function u_star_out = u_star(t, i, u, f, df)
133
        gradF = df(u(t, i));
                                    % Gradient of flux for given u(n,i)
134
        gradF1 = df(u(t, i + 1));
                                    % Gradient of flux for given u(n, i+1)
        % Apply different conditions...
136
        if gradF >= 0 && gradF1 >= 0
137
            u_star_out = u(t, i);
        elseif gradF < 0 && gradF1 < 0
138
            u_star_out = u(t, i + 1);
139
        elseif gradF >= 0 && gradF1 < 0
141
            s = (f(u(t, i + 1))-f(u(t, i)))/(u(t, i + 1)-u(t, i));
142
            if s >= 0
143
                u_star_out = u(t, i);
144
            else
145
                u_star_out = u(t, i + 1);
146
            end
147
        else
148
            u_star_out = 0;
149
        end
150
    end
```

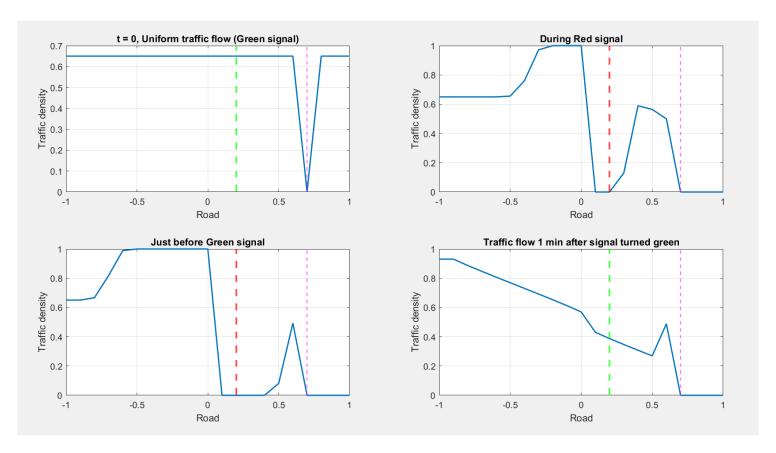


Figure 5: Density plot for different traffic signal conditions with Speed breaker before signal

1												Traffic sign	al position						Speed bre	aker pos		
1	t/x	-1	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	0	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0	0.65	0.65	0.65
	0.1	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.519125	0	0.4225	0.65	0.65
	0.2	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.627866	0.502814	0	0.195	0.6275	0.6275
	0.3	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.64385	0.611524	0.500421	0	0.038025	0.534475	0.534475
gnal turned red	0.4	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	1	0	0.65	0.648193	0.635595	0.599087	0.500063	0	0.001446	0.321054	0.321054
	0.5	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.8775	1	0	0.421961	0.644618	0.627027	0.589268	0.500009	0	2.09E-06	0.10452	0.10452
	0.6	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.770006	0.984994	1	0	0.192875	0.639839	0.61886	0.5813	0.500001	0	4.37E-12	0.010926	0.010926
	0.7	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.700403	0.932322	0.999775	1	0	0.037201	0.559641	0.611342	0.57469	0.5	0	0	0.000119	0.000119
	0.8	0.65	0.65	0.65	0.65	0.65	0.65	0.667662	0.847144	0.995195	1	1	0	0.001384	0.357855	0.604523	0.569111	0.5	0	0	1.43E-08	1.43E-08
	0.9	0.65	0.65	0.65	0.65	0.65	0.65561	0.76006	0.971853	0.999977	1	1	0	1.92E-06	0.129442	0.589095	0.564335	0.5	0	0	2.22E-16	2.22E-16
	1	0.65	0.65	0.65	0.65	0.651715	0.699027	0.915074	0.999185	1	1	1	0	3.67E-12	0.016757	0.455921	0.560196	0.5	0	0	0	(
	1.1	0.65	0.65	0.65	0.650517	0.668309	0.831702	0.991973	0.999999	1	1	1	0	0	0.000281	0.226021	0.556572	0.5	0	0	0	(
	1.2	0.65	0.65	0.650155	0.65619	0.750007	0.963713	0.999935	1	1	1	1	0	0	7.89E-08	0.051366	0.481508	0.5	0	0	0	(
	1.3	0.650047	0.650047	0.652004	0.694298	0.902533	0.998618	1	1	1	1	1	0	0	6.22E-15	0.002639	0.280577	0.499949	0	0	0	(
	1.4	0.650638	0.650638	0.666651	0.818579	0.98912	0.999998	1	1	1	1	1	0	0	0	6.96E-06	0.081355	0.49277	0	0	0	(
ignal turned green	1.5	0.655719	0.655719	0.740371	0.956325	0.99988	1	1	1	1	1	0.75	0.25	0	0	4.85E-11	0.006626	0.472634	0	0	0	(
	1.6	0.689248	0.689248	0.890825	0.997972	1	1	1	1	1	0.8125	0.6875	0.3125	0.1875	0	0.00E+00	4.39E-05	0.459495	0	0	0	(
	1.7	0.806178	0.806178	0.986057	0.999996	1	1	1	1	0.847656	0.75	0.652344	0.347656	0.25	0.152344	0	1.93E-09	0.456677	0	0	0	(
	1.8	0.948685	0.948685	0.999801	1	1	1	1	0.870865	0.789291	0.710709	0.629135	0.370865	0.289291	0.210709	0.129135	0.00E+00	0.456283	0	0	0	(
	1.9	0.997168	0.997168	1	1	1	1	0.887541	0.817014	0.75	0.682986	0.612459	0.387541	0.317014	0.25	0.182986	0.112459	0.456229	0	0	0	(
	2	0.999992	0.999992	1	1	1	0.900188	0.837851	0.779016	0.720984	0.662149	0.599812	0.400188	0.337851	0.279016	0.220984	0.162149	0.471194	0	0	0	(
	2.1	1	1	1	1	0.91015	0.854181	0.801557	0.75	0.698443	0.645819	0.58985	0.41015	0.354181	0.301557	0.25	0.198443	0.478682	0	0	0	(
	2.2	1	1	1	0.918223	0.867371	0.819674	0.773121	0.726879	0.680326	0.632629	0.581777	0.418223	0.367371	0.319674	0.273121	0.226879	0.48323	0	0	0	(
	2.3	1	1	0.924911	0.878274	0.834601	0.792077	0.75	0.707923	0.665399	0.621726	0.575089	0.424911	0.378274	0.334601	0.292077	0.25	0.486337	0	0	0	(
	2.4	0.930549	0.930549	0.887453	0.84714	0.807952	0.769268	0.730732	0.692048	0.65286	0.612547	0.569451	0.430549	0.387453	0.34714	0.307952	0.269268	0.488604	0	0	0	(
	2.5	0.895296	0.895296	0.85784	0.821468	0.785623	0.75	0.714377	0.678532	0.64216	0.604704	0.564627	0.435373	0.395296	0.35784	0.321468	0.285623	0.490324	0	0	0	(
	2.6	0.867086	0.867086	0.833132	0.799707	0.766542	0.733458	0.700293	0.666868	0.632914	0.597918	0.560451	0.439549	0.402082	0.367086	0.333132	0.299707	0.491669	0	0	0	(
	2.7	0.843311	0.843311	0.811979	0.780928	0.75	0.719072	0.688021	0.656689	0.624836	0.591984	0.556796	0.443204	0.408016	0.375164	0.343311	0.311979	0.492743	0	0	0	(
	2.8	0.82278	0.82278	0.793569	0.764507	0.735493	0.706431	0.67722	0.647721	0.617713	0.586749	0.553571	0.446429	0.413251	0.382287	0.352279	0.32278	0.493617	0	0	0	(
	2.9	0.804776	0.804776	0.77735	0.75	0.72265	0.695224	0.667635	0.639756	0.611382	0.582093	0.550701	0.449299	0.417907	0.388618	0.360244	0.332365	0.494338	0.249959	0	0	(

Figure 6: Enter Caption

2. Implement Gudanov Method to numerically solve the same PDE equation when the speed density relation follows the car following model, that is the traffic microscopic model. Generate the same tables as in Question no 1 for each case. This creates a bridge between traffic macroscopic and microscopic model.

```
% Traffic flow model for speed-density relation follows the car following model.
2
   clear;
3
   clc;
4
5
   rhoMax = 1;
                   % Maximum rho
6
   rhoMin = 0;
                    % Minimum rho
7
8
  h = 0.1;
                   % space step size
9
   k = 0.1:
                   % time step size
                    % Road - (-1 to 1)
10
   x = -1:h:1;
11
   x_divs = length(x);
                                % No. of space steps
12
                                 % Total time under consideration
13
   total_time= 3;
14
   t_steps = total_time / k;
                                 % No. of time steps
16
   % Initial conditions
17
   rho0 = linspace(0.1, 0.5, x_divs);
18
19
  u = zeros(t_steps, x_divs);
                                    % Array to store u
20
21
   % Initial and boundary conditions for u
22
   u(1,:) = rho0; % 1 - 2 .* rho0/rhoMax;
23
   u(:,1) = u(1,1)*ones(t_steps,1);
24
25
   % Numerical calculation using Godunov method
26
   for t = 1:t_steps-1
27
       % Apply boundary condition (2)
28
       % for i = 1:2: length(signal\_time\_idx)-1
29
            u(signal\_time\_idx(i):signal\_time\_idx(i+1), x\_signal\_idx) = -1;
30
             u(signal\_time\_idx(i):signal\_time\_idx(i+1), x\_signal\_idx + 1) = 1;
31
       % end
32
33
       for i = 2:x_divs - 1
34
           u(t+1,i) = u(t,i) - (k/h)*(f(u_star(t,i, u,0f, 0df))-f(u_star(t,i-1, u,0f, 0df))
               df)));
       end
36
   end
37
   u(:,1) = u(:,2);
38
   u(:,end) = u(:,end-1);
40
  % Get rho value from u
41
   rho_tab =
              u;
42
43
44 | % Plotting at different cases
45 figure;
46
   % First subplot
   subplot(2, 2, 1);
47
48 | plot(x, rho_tab(1,:), 'LineWidth', 1.5);
49 | % Add labels and title
50
   xlabel('Road');
51 | ylabel('Traffic density');
52 | title('t = 0');
```

```
53 % Display grid
54
    grid on;
56
57 % Second subplot
58 | subplot(2, 2, 2);
59 | plot(x, rho_tab(10,:), 'LineWidth', 1.5);
60 | % Add labels and title
61 | xlabel('Road');
62 | ylabel('Traffic density');
63 | title('t1');
64 grid on;
65
66 | % Third subplot
67 | subplot(2, 2, 3);
68 | plot(x, rho_tab(15,:), 'LineWidth', 1.5);
69 % Add labels and title
70 | xlabel('Road');
71 | ylabel('Traffic density');
72 | title('t2');
73 grid on;
74
75 % forth subplot
76 | subplot(2, 2, 4);
77
    plot(x, rho_tab(25,:), 'LineWidth', 1.5);
78 | % Add labels and title
79 | xlabel('Road');
80 | ylabel('Traffic density');
81 | title('t3');
82 grid on;
83
84
    \% Function for u_star calculation used in Godunov scheme
    function u_star_out = u_star(n, i, u, f, df)
85
86
                              % Gradient of flux for given u(n,i)
        gradF = df(u(n, i));
        gradF1 = df(u(n, i + 1)); % Gradient of flux for given u(n, i+1)
87
88
        % Apply different conditions...
        if gradF >= 0 && gradF1 >= 0
89
90
            u_star_out = u(n, i);
        elseif gradF < 0 && gradF1 < 0
91
92
            u_star_out = u(n, i + 1);
93
        elseif gradF >= 0 && gradF1 < 0
94
            s = (f(u(n, i + 1))-f(u(n, i)))/(u(n, i + 1)-u(n, i));
95
            if s >= 0
96
                u_star_out = u(n, i);
97
            else
98
                u_star_out = u(n, i + 1);
99
            end
100
        else
            u_star_out = 0;
102
        end
103
    end
104
105 | % Flux function for car following model
106
    function result = f(rho)
107
        rhoCrit = 0.45; % Critical rho
108
        rhoJam = 1;
                       % Jam rho
109
        vMax = 0.95;
                      % Max velocity
110
111
        if rho <= rhoCrit
```

```
112
             result = rho * vMax;
113
        elseif rhoCrit < rho && rho < rhoJam
114
             result = vMax * (1 - rho / rhoJam);
115
        elseif rho >= rhoJam
116
             result = 0;
117
        end
118
    end
119
120
    \% Derivative of flux
121
    function result = df(rho)
122
        rhoCrit = 0.45; % Critical rho
123
        rhoJam = 1;
                         % Jam rho
124
        vMax = 0.95;
                         % Max velocity
126
        if rho <= rhoCrit
127
             result = vMax;
128
        elseif rhoCrit < rho && rho < rhoJam
129
             result = -vMax / rhoJam;
130
        elseif rho >= rhoJam
131
             result = 0;
132
        end
133
    end
```

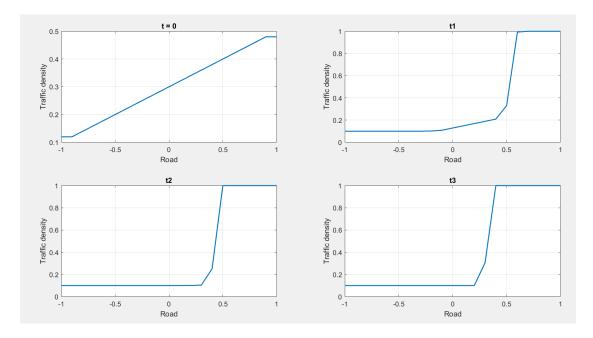


Figure 7: Density plots for different t for car following model

```
% Traffic flow model for speed-density relation follows the car following model with
       signal
   clear:
3
   clc;
4
              % Maximum rho
5 \mid \text{rhoMax} = 1;
                 % Minimum rho
6 \mid \text{rhoMin} = 0;
7
8 \mid h = 0.1;
               % space step size
9 | k = 0.1;
                 % time step size
               % Road - (-1 to 1)
10 | x = -1:h:1;
11 \mid x_{divs} = length(x);
                           % No. of space steps
12
13 | total_time= 3;
                             % Total time under consideration
14 | t_steps = total_time / k; % No. of time steps
15
16 | % Initial conditions
17 | rho0 = linspace(0.1, 0.5, x_{divs});
18
20
21 | % Initial and boundary conditions for u
22 | u(1,:) = rho0;
23 u(:,1) = u(1,1)*ones(t_steps,1);
24
25 | % Signal
26 | x_signal = 0.2;  % Position of signal
27 | x_signal_idx = floor((x_signal - x(1))/h); % Index of signal position in x array
28
29 | signal_times = [0.5, 1.5, 2.5];
                                             % Signal timings (in min.)
31
32 | % Apply boundary condition (1)
33 | for i = 1:2:length(signal_time_idx)-1
34
     u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx) = -1;
35
     u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx + 1) = 1;
36 end
37
38 | % Numerical calculation using Godunov method
39 \mid for t = 1:t_steps-1
40
       % Apply boundary condition (2)
41
      for i = 1:2:length(signal_time_idx)-1
42
         u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx) = -1;
43
         u(signal_time_idx(i):signal_time_idx(i+1), x_signal_idx + 1) = 1;
44
      end
45
46
      for i = 2:x_divs - 1
47
          u(t+1,i) = u(t,i) - (k/h)*(f(u_star(t,i, u,@f, @df))-f(u_star(t,i-1, u,@f, @df))
              df)));
48
       end
49 end
50 | u(:,1) = u(:,2);
51 | u(:,end) = u(:,end-1);
52
53 | % Get rho value from u
54 | rho_tab = u;
```

```
55
56
57 % Plotting at different cases
58 figure;
59 | % First subplot - when green light (initial cond.)
60 | subplot(2, 2, 1);
61 | plot(x, rho_tab(1,:), 'LineWidth', 1.5);
62 | % Add labels and title
63 | xlabel('Road');
64 | ylabel('Traffic density');
65 | title('t = 0, Uniform traffic flow (Green signal)');
66 | % Display grid
67 grid on;
68 hold on;
69 | % Add a vertical line at signal position
70 | xline(x_signal, 'g--', 'LineWidth', 2); % Signal position
71 hold off;
72
73 | % Second subplot - when red light
74 | subplot(2, 2, 2);
75 | plot(x, rho_tab(10,:), 'LineWidth', 1.5);
76 % Add labels and title
77 | xlabel('Road');
78 | ylabel('Traffic density');
79 | title('During Red signal');
80 grid on;
81 hold on;
82 | % Add a vertical line at signal position
83 | xline(x_signal, 'r--', 'LineWidth', 2);
84 hold off;
85
86 | % Third subplot - when red light (just before green light)
87 | subplot(2, 2, 3);
88 | plot(x, rho_tab(15,:), 'LineWidth', 1.5);
89 | % Add labels and title
90 | xlabel('Road');
91 | ylabel('Traffic density');
92 | title('Just before Green signal');
93 grid on;
94 hold on;
95 | % Add a vertical line at signal position
96 | xline(x_signal, 'r--', 'LineWidth', 2);
97 hold off;
98
99 % forth subplot
100 | subplot(2, 2, 4);
    plot(x, rho_tab(25,:), 'LineWidth', 1.5);
102 | % Add labels and title
103 | xlabel('Road');
104 | ylabel('Traffic density');
105 title('Traffic flow 1 min after signal turned green');
106 grid on;
107 | hold on;
108 | % Add a vertical line at signal position
109
    xline(x_signal, 'g--', 'LineWidth', 2);
110 hold off;
111
112
113
```

```
114 | % Function for u_star calculation used in Godunov scheme
115
    function u_star_out = u_star(n, i, u, f, df)
        gradF = df(u(n, i));
116
                                   % Gradient of flux for given u(n,i)
        gradF1 = df(u(n, i + 1)); % Gradient of flux for given u(n, i+1)
117
118
        % Apply different conditions...
119
        if gradF >= 0 && gradF1 >= 0
            u_star_out = u(n, i);
120
121
        elseif gradF < 0 && gradF1 < 0
122
            u_star_out = u(n, i + 1);
123
        elseif gradF >= 0 && gradF1 < 0
124
            s = (f(u(n, i + 1))-f(u(n, i)))/(u(n, i + 1)-u(n, i));
125
            if s >= 0
126
                u_star_out = u(n, i);
127
            else
128
                u_star_out = u(n, i + 1);
129
            end
130
        else
131
           u_star_out = 0;
132
        end
    end
134
135
    % Flux function for car following model
136
    function result = f(rho)
137
        rhoCrit = 0.45; % Critical rho
138
        139
                     % Max velocity
        vMax = 0.95;
140
141
        if rho <= rhoCrit
            result = rho * vMax;
142
143
        elseif rhoCrit < rho && rho < rhoJam
144
            result = vMax * (1 - rho / rhoJam);
145
        elseif rho >= rhoJam
146
            result = 0;
147
        end
148
    end
149
150 % Derivative of flux
    function result = df(rho)
152
        rhoCrit = 0.45; % Critical rho
        153
154
        vMax = 0.95;
                     % Max velocity
156
        if rho <= rhoCrit
157
            result = vMax;
        elseif rhoCrit < rho && rho < rhoJam
158
159
            result = -vMax / rhoJam;
        elseif rho >= rhoJam
            result = 0;
162
        end
    end
```

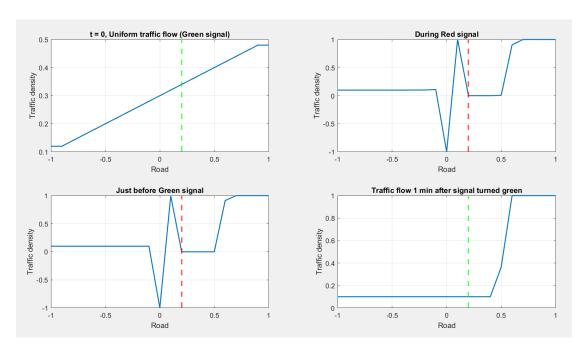


Figure 8: Density plots for different t for car following model with traffic signal

3. How will you formulate the traffic flow modelling problem as a black box model?

To formulate the traffic flow model to a black box model, we focus on observing the inputs and collecting the outputs of the system instead of looking at the complex formulations. Some of the input variables for the black box model are:

- (a) Traffic Density vehicles per unit length
- (b) Traffic Speed avg. speed of vehicle on the road
- (c) Traffic Flow no. of vehicles passing through a point per unit time.
- (d) Traffic Composition types of vehicles on the road.
- (e) External variables: Road type, Time of day, Weather, etc.

We collect data for these inputs and train the black-box model to predict the traffic flow of the road and conditions like traffic jams etc.
