1ST CLASS

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Mathematics - I : MAN001

CWS - 50 %

ETE - 50 %

Credits - 4

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Contents → ① Matrix Algebra
② Differential Calculus
③ Integral Calculus
④ Vector Calculus

Books → ① E. Kreyszig, Adv. Engg. Maths (John Willey & Sons)
② R.K Jain & Iyengar, Adv. Engg. Maths
③ Thomas and Finney (Calculus), Pearson Education.

Matrix Algebra

1) Elementary Row Operations

* $R_i \leftrightarrow R_j$
* kR_i or $\frac{1}{k}R_i$
* $R_i \to R_i + cR_j$

Matrix A is said to be in row reduced Echelon form if B satisfies the following →

① If a column, contains the first non-zero element of any row, then every subsequent element in this column is 0.

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(2) the zero rows, if any occur below all non-zero rows.

(3) If the first non zero element of the i^{th} non zero row occurs in the column k_i,

then, $k_1 < k_2 < k_3 \ldots\ldots$

<u>Ex:</u>

$$\begin{array}{c c} \begin{bmatrix} 2 & 3 & 5 & 7 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & 9 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array}$$

Thus the matrix A is in row reduced echelon form.

<u>Example:</u>

$$B = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 0 & 7 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{l} (1) \checkmark \\ (2) \checkmark \\ (3) \checkmark \end{array}$$

B is in row reduced echelon form.

<u>Example:</u>

$$X = \begin{bmatrix} 2 & 3 & 5 & 7 \\ 0 & 0 & 0 & 9 \\ 0 & 5 & 4 & 0 \end{bmatrix} \qquad \begin{array}{l} (1) \checkmark \\ (2) \sim \\ (3) \times \end{array}$$

$(k_1 = 1, k_2 = 4, k_3 = 2)$

$k_1 \nless k_2$

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Q Reduce the matrix A to its row reduced Echelon form.

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 8 & 2 \end{bmatrix}$$

Solⁿ ~

$R_2 \leftrightarrow R_2 - 2R_1$

$R_3 \leftrightarrow R_3 + 2R_1$

$$= \begin{bmatrix} 1 & 3 & 5 \\ 0 & -7 & -6 \\ 0 & 14 & 12 \end{bmatrix}$$

$R_3 \leftrightarrow R_3 + 2R_2$

$$B = \begin{bmatrix} 1 & 3 & 5 \\ 0 & -7 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

$k_1 = 1$

$k_2 = 2$

$k_1 < k_2$

Here B is the row reduced echelon form of A.

- Row reduced echelon form is not unique.

Ex: Let A be same as above

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 2 & -1 & 4 \\ 1 & 3 & 5 \\ -2 & 8 & 2 \end{bmatrix}$$

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$R_2 \to R_2 - 0.5R_1$

$R_3 \to R_3 + R_1$

$$\begin{bmatrix} 2 & -1 & 4 \\ 0 & 7/2 & 3 \\ 0 & 7 & 6 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 2 & -1 & 4 \\ 0 & 7/2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

↓

Row Reduced Echelon form

It's different from the previous form, hence R.R Echelon form of same matrix is not unique.

Example: $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{bmatrix}$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{bmatrix} //$$

Rank(A) = 3

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Rank(A) = number of non-zero rows in its row reduced Echelon form of A.

Q What is the rank of

$$P = \begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ 2 & 8 & 2 \end{bmatrix}$$

From previous question

Rank(P) = 2.

Rank of a Matrice is unique.

Result : ① the elementary row operations do not change the rank of a matrix

② $A \approx A$

row equivalent

③ if $A \approx B$, then $B \approx A$

④ if $A \approx B$, $B \approx C$, then $A \approx C$