

ASSIGNMENT - I

① @

$$\left[\begin{array}{cccc} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 7 & 8 & 9 \end{array} \right] \quad \left[\begin{array}{cccc} 1 & 2 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

*(row 10 most suitable for
Gauss-Jordan method)*

Sol.

$$\left[\begin{array}{cccc} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 7 & 8 & 9 \end{array} \right] \quad \left[\begin{array}{cccc} 1 & 2 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

②

Ans

$$R_1 \longleftrightarrow R_4$$

$$\left[\begin{array}{cccc} 6 & 7 & 8 & 9 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 1 & 2 & 3 & 0 \end{array} \right] \quad \begin{array}{l} \text{Subtract } 2R_1 \text{ from } R_2 \\ \text{Subtract } 3R_1 \text{ from } R_3 \\ \text{Subtract } R_1 \text{ from } R_4 \end{array}$$

$$3R_2 \rightarrow R_2 \rightarrow R_2 - R_1$$

$$\left[\begin{array}{cccc} 6 & 7 & 8 & 9 \\ 0 & 1 & 0 & 0 \\ 3 & 2 & 1 & 3 \\ 1 & 2 & 3 & 0 \end{array} \right]$$

$$R_2 \rightarrow 3R_2 - R_1$$

$$R_2 \rightarrow 3R_2 - R_1 \quad \text{Ans - 2nd row}$$

$$\left[\begin{array}{cccc} 6 & 7 & 8 & 9 \\ 0 & 5 & 1 & -3 \\ 0 & -3 & -6 & -3 \\ 0 & 5 & 10 & -9 \end{array} \right] \quad \begin{array}{l} \text{Subtract } 3R_2 \text{ from } R_3 \\ R_3 \rightarrow R_3 + 3R_2 \\ R_3 \rightarrow R_3 + 3R_2 \end{array}$$

$$R_3 \rightarrow R_3 - 3R_2$$

$$R_3 \rightarrow 5R_3 + 3R_2$$

$$\left[\begin{array}{cccc} 6 & 7 & 8 & 9 \\ 0 & 5 & 1 & -3 \\ 0 & 0 & -22 & -24 \\ 0 & 0 & 9 & -6 \end{array} \right] \quad \left[\begin{array}{cccc} 6 & 7 & 8 & 9 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$R_1 \rightarrow 3R_1 + R_2$

$$\left[\begin{array}{cccc} 6 & 2 & 8 & 9 \\ 0 & 5 & 1 & -3 \\ 0 & 0 & -n & 24 \\ 0 & 0 & 0 & -42 \end{array} \right]$$

Row echelon form of given matrix. $\text{rank } 4$.

(b)

Sol.

$$\left[\begin{array}{ccc} 2 & 4 & 6 \\ -1 & 3 & 2 \\ 1 & 4 & 6 \\ 2 & 8 & 7 \end{array} \right]$$

$$\left[\begin{array}{cccc} 0 & \epsilon & \epsilon & 1 \\ \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & 1 & \epsilon & \epsilon \\ \epsilon & 8 & f & d \end{array} \right]$$

$\epsilon_1 \leftrightarrow \epsilon_2$

$R_2 \rightarrow 2R_2 + R_1$

$R_3 \rightarrow 2R_3 - R_1$

$R_4 \rightarrow R_4 - R_1$

$$\left[\begin{array}{ccc} 2 & 4 & 6 \\ 0 & 10 & 10 \\ 0 & 4 & 6 \\ 0 & 4 & 1 \end{array} \right]$$

$$\left[\begin{array}{cccc} p & \epsilon & f & d \\ \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & 1 & \epsilon & \epsilon \\ 0 & \epsilon & \epsilon & \epsilon \end{array} \right]$$

$\cancel{\epsilon_2 - \epsilon_1} \leftarrow \cancel{\epsilon_3 - \epsilon_1} \leftarrow \cancel{\epsilon_4 - \epsilon_1}$

④ $R_3 \rightarrow 5R_3 - 2R_2$ $\leftarrow \cancel{\epsilon_3 - \epsilon_2} \leftarrow \cancel{\epsilon_4 - \epsilon_2}$

$R_4 \rightarrow 5R_4 - 2R_2$

$$\left[\begin{array}{ccc} 2 & 4 & 6 \\ 0 & 10 & 10 \\ 0 & 0 & 10 \\ 0 & 0 & -15 \end{array} \right]$$

$$\left[\begin{array}{cccc} p & \epsilon & f & d \\ \epsilon & 1 & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & 0 & \epsilon & \epsilon \end{array} \right]$$

$R_4 \rightarrow 2R_4 + 3R_3$

$$\left[\begin{array}{ccc} 2 & 4 & 6 \\ 0 & 10 & 10 \\ 0 & 0 & 10 \\ 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc} p & \epsilon & f & d \\ \epsilon & 1 & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Ans Rank 3

$$\textcircled{c} \quad \begin{bmatrix} 1 & -2 & 5 & -3 \\ 2 & 3 & 1 & -4 \\ 3 & 8 & 3 & -5 \end{bmatrix}$$

Step 1: $R_1 \leftrightarrow R_3$
 Step 2: $R_2 \rightarrow R_2 + R_1$
 Step 3: $R_3 \rightarrow R_3 + R_1$

Sol.

$$R_1 \rightarrow R_3$$

$$\begin{bmatrix} 3 & 8 & 3 & -5 \\ 2 & 3 & 1 & -4 \\ 1 & -2 & 5 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 8 & 1 & -2 \\ 2 & 3 & 1 & -4 \\ 1 & -2 & 5 & -3 \end{bmatrix}$$

Step 4: $R_2 \rightarrow R_2 - 2R_1$

$$R_2 \rightarrow 3R_2 - 2R_1$$

$$R_3 \rightarrow 3R_3 - R_1$$

$$\begin{bmatrix} 3 & 8 & -3 & -5 \\ 0 & -7 & 9 & 2 \\ 0 & -14 & 18 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 8 & 1 & -2 \\ 0 & -7 & 9 & 2 \\ 1 & -2 & 5 & -3 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 2R_2$$

$$\begin{bmatrix} 3 & 8 & -3 & -5 \\ 0 & -7 & 9 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 8 & 1 & -2 \\ 0 & -7 & 9 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Ans Rank 2

\textcircled{d}

$$\begin{bmatrix} 1 & 1 & 2 & 0 & -3 \\ 1 & 2 & 0 & -1 & 2 \\ 2 & 0 & -1 & 1 & 2 \\ 0 & -1 & 1 & 1 & 0 \\ -1 & 1 & 1 & 2 & 0 \end{bmatrix}$$

Step 1: $R_1 \leftrightarrow R_3$

Sol. $R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 2 & 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & -1 & 2 \\ 1 & 1 & 1 & 1 & 2 \\ 2 & 0 & -1 & 1 & 2 \\ 0 & -1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 & -1 \end{bmatrix}$$

Step 2: $R_1 \leftrightarrow R_4$

Step 3: $R_1 \leftrightarrow R_5$

$$R_2 \rightarrow R_2 + R_1$$

$$R_1 \rightarrow 2R_1$$

$$R_3 \rightarrow 4R_3$$

$$\left[\begin{array}{ccccc} -4 & 1 & 1 & 2 & 0 \\ 0 & 9 & 1 & -14 & 4 \\ 0 & 0 & -2 & 4 & 2 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & -4 & 1 & 1 & 2 \\ 0 & 8 & 9 & 2 & -16 \end{array} \right]$$

$$\left[\begin{array}{ccccc} 2 & 8 & 8 & 8 & 0 \\ 0 & 1 & 8 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$R_3 \rightarrow 9R_3 - R_2$$

$$R_1 \rightarrow 4R_1$$

$R_2 \rightarrow \text{only } R_2$

$$\left[\begin{array}{ccccc} -4 & 1 & 1 & 2 & 0 \\ 0 & 9 & 1 & -14 & 4 \\ 0 & 0 & -64 & 50 & 14 \\ 0 & 0 & 13 & -47 & 33 \\ 0 & 0 & 76 & 88 & -164 \end{array} \right]$$

$\leftarrow R_2 - R_1$

$\leftarrow R_3 - R_2$

$\leftarrow R_4 - R_3$

$$R_3 \rightarrow \frac{1}{2}R_3$$

$$R_3 \rightarrow \frac{1}{4}R_3$$

$$\left[\begin{array}{ccccc} -4 & 1 & 1 & 2 & 0 \\ 0 & 9 & 1 & -14 & 4 \\ 0 & 0 & -32 & 25 & 7 \\ 0 & 0 & 13 & -47 & 33 \\ 0 & 0 & 19 & 22 & -41 \end{array} \right]$$

$\leftarrow R_2 - R_1$

$\leftarrow R_3 - R_2$

$$R_4 \rightarrow 3R_4 + 13R_3$$

$$R_5 \rightarrow 3R_5 + 9R_3$$

$$\left[\begin{array}{ccccc} -4 & 1 & 1 & 2 & 0 \\ 0 & 9 & 1 & -14 & 4 \\ 0 & 0 & -32 & 25 & 7 \\ 0 & 0 & 0 & -1179 & 1179 \\ 0 & 0 & 0 & 1179 & -1179 \end{array} \right]$$

$$R_5 \rightarrow R_5 + R_4$$

$$\left[\begin{array}{ccccc} -4 & 1 & 1 & 2 & 0 \\ 0 & 9 & 1 & -14 & 4 \\ 0 & 0 & -32 & 25 & 7 \\ 0 & 0 & 0 & -1179 & 1179 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Rank = 4

Ans

① ② ③

$$\left(\begin{array}{ccc|cc} 1 & 1 & 2 & 1 & 1 \\ 1 & 2 & 5 & -1 & 1 \\ 5 & 3 & 4 & 0 & -1 \end{array} \right)$$

$R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - 5R_1$

$$\left(\begin{array}{ccc|cc} 1 & 1 & 2 & 1 & 1 \\ 0 & 1 & 3 & -1 & 1 \\ 0 & -2 & -6 & 0 & -6 \end{array} \right)$$

$R_3 \rightarrow R_3 + 2R_2$

$$\left(\begin{array}{ccc|cc} 1 & 1 & 2 & 1 & 1 \\ 0 & 1 & 3 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Rank of matrix = 2

which is less than.

no. of vectors hence.

It is linearly dependent.

④

$$\left(\begin{array}{ccc|cc} 1 & -1 & 1 & 1 & 1 \\ 2 & 1 & 10 & 1 & 1 \\ 8 & 1 & 5 & -1 & 1 \end{array} \right)$$

$R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 8R_1$

$$\left(\begin{array}{ccc|cc} 1 & -1 & 1 & 1 & 1 \\ 0 & 3 & -5 & -1 & -1 \\ 0 & 9 & -3 & 1 & 1 \end{array} \right)$$

$R_3 \rightarrow R_3 - 3R_2$

$$\left(\begin{array}{ccc|cc} 1 & -1 & 1 & 1 & 1 \\ 0 & 3 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Rank = 2 which is less
than 'n' Hence. Linearly
dependent.

(c)

$$\left(\begin{array}{cccc} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{array} \right) \xrightarrow{\text{R}_2 - R_1} \left(\begin{array}{cccc} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 3 & 1 & 0 & 1 \end{array} \right) \xrightarrow{\text{R}_3 - 3R_1} \left(\begin{array}{cccc} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 0 & -2 & 3 & -2 \end{array} \right)$$

$$R_2 \rightarrow R_2 - R_3, R_3 \rightarrow R_3 - 3R_1$$

$$\left(\begin{array}{cccc} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 0 & -2 & 3 & -2 \end{array} \right) \xrightarrow{\text{R}_3 \rightarrow R_3 - R_2} \left(\begin{array}{cccc} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$R_3 \rightarrow R_3 - R_2$$

$$\left(\begin{array}{cccc} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\text{Row echelon form}} \left(\begin{array}{cccc} 1 & 1 & -1 & 1 \\ 0 & 1 & -\frac{3}{2} & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\text{Row echelon form}} \left(\begin{array}{cccc} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{3}{2} & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

Rank = 2 Hence linearly dependent.

(d)

$$\left(\begin{array}{ccc} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & 2 \end{array} \right) \xrightarrow{\text{R}_2 \rightarrow R_2 - 2R_1} \left(\begin{array}{ccc} 1 & 2 & 1 \\ 0 & -3 & -2 \\ 1 & -1 & 2 \end{array} \right) \xrightarrow{\text{R}_3 \rightarrow R_3 - R_1} \left(\begin{array}{ccc} 1 & 2 & 1 \\ 0 & -3 & -2 \\ 0 & -3 & 1 \end{array} \right)$$

$$R_2 \rightarrow R_2 - R_3, R_3 \rightarrow R_3 - R_2$$

$$\left(\begin{array}{ccc} 1 & 2 & 1 \\ 0 & -3 & -2 \\ 0 & -3 & 1 \end{array} \right) \xrightarrow{\text{Row echelon form}} \left(\begin{array}{ccc} 1 & 2 & 1 \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{array} \right)$$

$$R_3 \rightarrow R_3 - R_2$$

possible with '0' next to diagonal

$$\left(\begin{array}{ccc} 1 & 2 & 1 \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{array} \right)$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -3 & -2 \\ 0 & 0 & 3 \end{pmatrix}$$

Rank=3 which is equal to the number of vectors
 Hence, it is ~~linearly dependent~~ independent.

$$\textcircled{3} \quad \textcircled{6} \quad \left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & \beta & \\ \hline 1 & 3 & 6 & \end{array} \right) \quad \text{condition is ab integers so not } \textcircled{7}$$

Sol.

$$R_1 \longleftrightarrow R_3$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & \\ 0 & 2 & \beta & \\ \hline 2 & 1 & 2 & \end{array} \right)$$

$$R_3 \rightarrow R_3 - 2R_1$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & \\ 0 & 2 & \beta & \\ \hline 0 & 1-3 & 2-2\cdot 2 & \end{array} \right) \quad \text{so } (n-5) = 0 \Leftrightarrow n=5$$

$$R_3 \rightarrow 2R_3 - (1-3\alpha)R_2$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & \\ 0 & 2 & \beta & \\ \hline 0 & 0 & (2-3\alpha)(1-3\alpha) & \end{array} \right)$$

$\textcircled{1}$ Rank=1 Not possible as at least 2 rows are non-zero
 $\textcircled{2}$ $\alpha=n-b+\beta$ $\beta=d-a-\beta$

\textcircled{ii} Rank=2. $\beta=n$ or $\alpha=\frac{1}{3}$

\textcircled{iii} Rank=3 $\beta \neq n$ & $\alpha \neq \frac{1}{3}$.

$$n-b = d-a \quad \textcircled{1} \text{ and}$$

$$\beta = d - 1 = a$$

$$n-b \neq d-a \quad \textcircled{2} \text{ and}$$

(b) For a system to be consistent $\text{Rank}(A) = \text{Rank}(A/b)$

So,

$$\left(\begin{array}{ccc|c} 2 & 4 & \alpha+3 & 2 \\ 0 & 3 & 1 & 2 \\ \alpha-2 & 2 & 3 & \beta \end{array} \right)$$

$$R_2 \rightarrow 2R_2 - R_1$$

$$R_3 \rightarrow 2R_3 - (\alpha-2)R_1$$

$$\left(\begin{array}{ccc|c} 2 & 4 & \alpha+3 & 2 \\ 0 & 2 & 1-\alpha & 2 \\ 0 & 4(3-\alpha) & 6-(\alpha-2)(\alpha+3) & 2\beta-2(\alpha-2) \end{array} \right)$$

$$R_3 \rightarrow 2R_3 - (12-4\alpha)R_2$$

$$\left(\begin{array}{ccc|c} 2 & 4 & \alpha+3 & 2 \\ 0 & 2 & 1-\alpha & 2 \\ 0 & 0 & -6(\alpha^2-1-\alpha) & 4(\beta+\alpha-4) \end{array} \right)$$

For, $\text{Rank}(A/b) = \text{Rank}(A)$

Case I $\alpha^2-\alpha-6=0 \Rightarrow \alpha=3 \text{ or } \alpha=-2$ (i) $\beta+\alpha-4=0 \Rightarrow \beta=1$ (ii)

or

$\alpha^2-\alpha-6 \neq 0 \Rightarrow \alpha \neq 3 \text{ and } \alpha \neq -2$ (iii)

Case (i) $\alpha=3 \text{ or } \alpha=-2$

$$\beta=1 \quad \beta=6$$

Case (ii) $\alpha \neq 3 \text{ and } \alpha \neq -2$

$$\textcircled{4} \textcircled{a} \quad \left(\begin{array}{ccc|cc} 1 & 4 & -1 & 1 & 1 \\ 1 & 1 & -6 & 5 & -5 \\ 3 & -1 & -1 & 1 & 1 \end{array} \right) \xrightarrow{\substack{R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - 3R_1}} \left(\begin{array}{ccc|cc} 1 & 4 & -1 & 1 & 1 \\ 0 & -3 & -5 & 0 & -10 \\ 0 & -13 & 2 & -8 & 1 \end{array} \right)$$

$$R_2 \rightarrow R_2 - R_1 \quad \xrightarrow{\substack{R_2 - 5R_1 \\ R_3 - 2R_1}}$$

$$R_3 \rightarrow R_3 - 3R_1 \quad \xrightarrow{\substack{R_3 - 2R_1 \\ R_3 - 13R_1}}$$

$$\left(\begin{array}{ccc|cc} 1 & 4 & -1 & 1 & 1 \\ 0 & -3 & -5 & 0 & -10 \\ 0 & -13 & 2 & -8 & 1 \end{array} \right) \xrightarrow{\substack{R_3 - 13R_2 \\ R_3 - 2R_1}}$$

$$R_3 \rightarrow R_3 - \frac{13}{3}R_2$$

$$\left(\begin{array}{ccc|cc} 1 & 4 & -1 & 1 & 1 \\ 0 & -3 & -5 & 0 & -10 \\ 0 & 0 & \frac{21}{3} & \frac{21}{3} & 0 \end{array} \right)$$

$$x \quad y \quad \text{(2)nd} \neq (\text{1st}) \text{ and}$$

$$\boxed{2=1}$$

not true obs.

$$-3y - 5z = -8$$

$$3y + 5z = 8$$

$$\boxed{y=1}$$

$$\left(\begin{array}{ccc|cc} 5 & 1 & 1 & 1 & 1 \\ x+4y-2z & 4 & & & \\ x+y-z & 4 & & & \end{array} \right)$$

$$\boxed{x=1}$$

$$\xrightarrow{\substack{R_2 - R_1 \\ R_3 - R_1}}$$

(1st)nd = (2nd)nd

or true

then $S =$

original condition to

$\epsilon = \text{calculated value}$

$$\left(\begin{array}{ccc|cc} 1 & 4 & -1 & 1 & 1 \\ 0 & -3 & -5 & 0 & -10 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

(b)

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & 1 & 5 & 2 \\ 3 & 3 & 4 & 1 \end{array} \right) \xrightarrow{\begin{matrix} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{matrix}} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 7 & 0 \\ 0 & -3 & 7 & -2 \end{array} \right)$$

$$R_3 \rightarrow R_3 - R_2$$

$$R_3 \rightarrow R_3 - R_2$$

1

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 7 & 0 \\ 0 & -3 & 7 & -2 \end{array} \right) \xrightarrow{\begin{matrix} R_3 \rightarrow R_3 - R_2 \\ R_3 \rightarrow R_3 - R_2 \end{matrix}} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 7 & 0 \\ 0 & 0 & 0 & -2 \end{array} \right)$$

$$R_3 \rightarrow R_3 - R_2$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 7 & 0 \\ 0 & 0 & 0 & -2 \end{array} \right) \xrightarrow{\begin{matrix} R_3 \rightarrow -\frac{1}{2}R_3 \\ R_3 \rightarrow R_3 + R_2 \end{matrix}} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 7 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\text{Rank}(A/b) \neq \text{Rank}(A)$$

\therefore No solution.

(c)

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 3 & 1 & -2 & 1 \\ 2 & 4 & 2 & 5 \end{array} \right) \xrightarrow{\begin{matrix} R_2 \rightarrow R_2 - 3R_1 \\ R_3 \rightarrow R_3 - 2R_1 \end{matrix}} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -5 & -5 & -8 \\ 0 & 0 & 0 & 1 \end{array} \right)$$

$$\boxed{1=5}$$

$$8 = 5x - 5x$$

$$8 = x + 5x$$

$$\boxed{1=4}$$

$$R_2 \rightarrow R_2 - 5R_1 \quad R_3 \rightarrow R_3 - 2R_1$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -5 & -5 & -8 \\ 0 & 0 & 0 & 1 \end{array} \right)$$

$$\text{Rank}(A/b) = \text{Rank}(A)$$

$= 2$ last no.

of unknown variables = 3

Hence, infinite solutions

$S = A + \lambda B$

$$-sy -sz = -s$$

$$\boxed{y+2z=1}$$

$$\boxed{x+2y+2z=2}$$

$$\boxed{x+y=1}$$

$$\boxed{0 = 0 + 0 + 0}$$

let $y = k$ (an arbitrary constant.)

$$\boxed{x=1-k, y=k, z=1-k} \quad \underline{\text{Ans}}$$

⑤ a

$$\left(\begin{array}{ccc|c} -2 & 1 & 1 & a \\ 1 & -2 & 1 & b \\ 1 & 1 & -2 & c \end{array} \right) \xrightarrow{\text{R}_1 \rightarrow -R_1, \text{R}_2 \rightarrow R_2 + 2R_1, \text{R}_3 \rightarrow R_3 + R_1} \left(\begin{array}{ccc|c} 1 & -1 & -1 & a \\ 0 & -1 & 1 & a+2b \\ 0 & 2 & -1 & a+c \end{array} \right)$$

$$R_2 \rightarrow R_2 + 2R_1$$

$$R_3 \rightarrow R_3 + 2R_1$$

$$\left(\begin{array}{ccc|c} -2 & 1 & -1 & a \\ 0 & -3 & 3 & a+2b \\ 0 & 3 & -3 & a+2c \end{array} \right) \xrightarrow{\text{R}_3 \rightarrow R_3 + R_2} \left(\begin{array}{ccc|c} -2 & 1 & -1 & a \\ 0 & -3 & 3 & a+2b \\ 0 & 0 & 0 & a+2b+2c \end{array} \right)$$

$$R_3 \rightarrow R_3 + R_2$$

$$\left(\begin{array}{ccc|c} -2 & 1 & -1 & a \\ 0 & -3 & 3 & a+2b \\ 0 & 0 & 0 & a+2b+2c \end{array} \right)$$

① no solution $\text{Rank}(A) = 2, \text{Rank}(A+b) \neq 2$
so, $\text{Rank}(A+b) \neq 2$
 $a+2b+2c \neq 0$

(ii) Not possible as there are 3 variables and
only 2 equations

Rank of A = 2

(iii) $a+b+c=0$

(5)

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 2 & 3 & 1 & 3 \\ 1 & 1 & 3 & 2 \end{array} \right)$$

(not in row echelon form) $n=p=3$

$$x_1 + x_2 + x_3 = 0$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & 1 & 3 & 2 \\ 2 & 3 & 1 & 3 \end{array} \right)$$

$$N-1=5 \quad N=6 \quad N-1=5$$

$$R_2 \rightarrow R_2 - R_1 \quad R_3 \rightarrow R_3 - 2R_1$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & \lambda-1 & 4 & 2 \\ 0 & 1 & \lambda+2 & 3 \end{array} \right)$$

$$R_2 \leftrightarrow R_3 \quad \text{and } R_2 \leftarrow \text{row } 2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & \lambda+2 & 3 \\ 0 & \lambda-1 & 4 & 2 \end{array} \right)$$

$$R_3 \rightarrow R_3 - (\lambda-1)R_2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & \lambda+2 & 3 \\ 0 & 0 & 4-(\lambda+2)(\lambda-1) & 2-\lambda \end{array} \right)$$

multiple of ①

or 2 + 2 + 2

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & \lambda+2 & 1 \\ 0 & 0 & -\lambda^2 + \lambda + 6 & 2-\lambda \end{array} \right) \xrightarrow{\text{L3} - \text{L2}} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & \lambda+2 & 1 \\ 0 & 0 & (\lambda+3)(\lambda-2) & 2-\lambda \end{array} \right)$$

i) $\text{Rank}(A/b) \neq \text{Rank}(A)$. $\boxed{\lambda = -3}$ and

ii) $\lambda \neq -3, 2$

iii) $\lambda = 2$, so that $\text{Rank}(A/b) = \text{Rank}(A)$.

④ $\lambda = 2$ $\Rightarrow \lambda = 2$ is a root of $\lambda^2 - 5\lambda + 6 = 0$ $\Rightarrow \lambda = 2$ or $\lambda = 3$

⑤

$$\left(\begin{array}{ccc|c} \lambda & 1 & 1 & \lambda-p \\ 1 & \lambda & 1 & p+\lambda+1 \\ 1 & 1 & \lambda & \lambda \end{array} \right) \xrightarrow{\text{R}_2 - \text{R}_1, \text{R}_3 - \text{R}_1} \left(\begin{array}{ccc|c} \lambda & 1 & 1 & \lambda-p \\ 0 & \lambda-1 & 0 & p+\lambda+1 \\ 0 & 0 & \lambda-1 & \lambda \end{array} \right)$$

$$\begin{cases} \lambda = 2 \\ \lambda = 3 \end{cases}$$

$$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$$

$\mathcal{E}(A/b) = \mathcal{E}(A/b)$ has 2 pivot elements \Rightarrow ⑥

$$R_1 \leftrightarrow R_2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & \lambda-p \\ 0 & \lambda-1 & 0 & p+\lambda+1 \\ 0 & 0 & \lambda-1 & \lambda \end{array} \right) \xrightarrow{\text{divide by } \lambda-1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & \lambda-p \\ 0 & 1 & 0 & p+\lambda+1 \\ 0 & 0 & 1 & \lambda \end{array} \right)$$

iii)

$$R_2 \rightarrow R_2 - \lambda R_1, R_3 \rightarrow R_3 - \lambda R_1$$

or

$$\begin{cases} \lambda = 2 \\ \lambda = 3 \end{cases}$$

$$\lambda = 2$$

$$\left(\begin{array}{ccc|cc} 1 & \lambda & 1 & 1 & a \\ 0 & 1-\lambda^2 & 1-\lambda & -\lambda & p-\lambda q \\ 0 & 1-\lambda & \lambda(1-\lambda) & \lambda & r-q \end{array} \right)$$

$$R_3 \rightarrow (1-\lambda)R_3 - R_2 \quad (\text{After } + (A/A) \text{ ad})$$

$$\left(\begin{array}{ccc|cc} 1 & \lambda & 1 & 1 & a \\ 0 & 1-\lambda^2 & 1-\lambda & -\lambda & p-\lambda q \\ 0 & 0 & \lambda^2 - \lambda + 1 & \lambda^2 - \lambda - 2 & (1-\lambda)(r-q) \end{array} \right)$$

$(A/\text{ad}) = ((1-\lambda)/A) \text{ ad}$

i) For no solution. $\lambda^2 - \lambda - 2 \neq 0 \Leftrightarrow \lambda \neq -1, 2$

$\lambda \neq 1$	or	$\lambda \neq -2$
$p+q+r \neq 0$		$p+q+r \neq 0$
$a+b+c \neq 0$		
$p+q+r \neq 0$		

ii) For unique solution ~~Rank(A) = Rank(A|b) = 3~~

$$\lambda \neq 1, -2$$

iii) For infinite solution.

$$\lambda = 1 \text{ in proper or } \lambda = -2 \text{ in proper} \neq 0$$

NOTE: For $\lambda = 1$ system reduces to

$$ay+bx=0$$

$$ay+bx=a$$

$$ay+bx=0$$

$$p+q+r=0$$

⑥ (a) Let there be a system of linear equations

then,

$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 3 \\ 4 & 2 & -2 & 10 \\ 6 & -3 & 1 & 9 \end{array} \right)$$

now if we consider the second row, we find that $2R_1 - R_2$, which

$$R_2 \rightarrow R_2 - 2R_1$$

$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 3 \\ 0 & 4 & -8 & 4 \\ 0 & 0 & -8 & 0 \end{array} \right)$$

Rank of $(A|b) \neq \text{Rank}(A)$ Hence no solution

(b)

$$\left(\begin{array}{ccc|c} 0 & 2 & 2 & 2 \\ 2 & 4 & 6 & 7 \\ 1 & 1 & 2 & 2 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 2 & 4 & 6 & 7 \\ 0 & 2 & 2 & 3 \end{array} \right)$$

$$R_1 \leftarrow R_1 - R_3$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & 2 & 2 & 3 \\ 2 & 4 & 6 & 7 \end{array} \right)$$

(row swap) $R_2 \leftrightarrow R_3$

Now $R_2 \rightarrow R_2 - 2R_1$

$$R_3 \rightarrow R_3 - R_2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & 2 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\text{Row operations}} \left(\begin{array}{ccc|c} 1 & 1 & 2 & b_3 \\ 0 & 2 & 2 & b_2 - 2b_3 \\ 0 & 0 & 0 & b_1 - b_2 + 2b_3 \end{array} \right)$$

Hence, if for some b_1, b_2, b_3 or $b_1 - b_2 + 2b_3 \neq 0$ then system has no solution.

⑦ P_2 : $a+b+c$

$$f(A)=2 \quad a+b+c=2$$

$$f(A)=6 \quad a+b+c=6$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 6 \end{array} \right)$$

$$R_2 \rightarrow R_2 - R_1$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & 0 & 4 \end{array} \right) \xrightarrow{\text{Row operations}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & -2 \end{array} \right) \xrightarrow{\text{Row operations}} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & -2 \end{array} \right)$$

$$\boxed{b = -2}$$

$$a+b+c=2$$

$$\boxed{a+c=4}$$

let $c=4$ (const. const.)

$$a=4-b$$

Hence. P_2

$$(4-b)^2 - 2ab + b^2 \text{ Ans}$$

8 a

$$\begin{array}{ccccccc}
 3k-8 & 3 & 3 & N & 1+MC & 1-N \\
 3 & 3k-8 & 3 & NE & S-N & 0 \\
 3 & 3 & 3k-8 & ENC & 1+MS & S
 \end{array}$$

For the equation to have non-trivial solution, $D = 0$.

$$(3u-8) \left[(3u-8)^2 - 9 \right] + 3 \left[-3(3u-8) + 9 \right] + 3(9 - 3(3u-8)) =$$

Let $3w - 8 = t$

$$t^3 - 27t + 54 = 0.$$

$(t-3)^2(t+6) = 0$

$$3u - 8 = 3 \quad |+8 \quad 3u = 11$$

$$K = \frac{2}{3} \quad \text{or} \quad K = \frac{11}{3}$$

$$\textcircled{1} \quad u \neq \frac{2}{3}, -\frac{2}{3}$$

ii) $k^2 \geq \frac{2}{3}$ or $k^2 \leq \frac{11}{3}$

b

$$\left(\begin{array}{ccc|c} u_1 & 3u+1 & 1-u & 8 \\ u_1 & u_1-2 & u+3 & 0 \\ 2 & 3u+1 & 3u-3 & 5 \end{array} \right) \xrightarrow{\text{E-N8}} \left(\begin{array}{ccc|c} u_1 & 3u+1 & 1-u & 8 \\ 0 & -1 & 1 & 0 \\ 2 & 3u+1 & 3u-3 & 5 \end{array} \right)$$

$$v = \left[\frac{D_{\text{max}} + D_{\text{min}}}{2} \right]^{1/(E-N)}$$

$w_1 \quad w_2 \quad w_3$

$$2^{3n+1} - 3^{n-3}$$

o, e+n Ⓛ

$$R_2 \rightarrow R_2 - R_1$$

$$\left| \begin{array}{ccc|c} u_1 & 3u_1 & 2u_1 & \\ 0 & u-3 & 3-u & 20 \\ 2 & 3u+1 & 3u-3 & \end{array} \right|$$

$\therefore 0 = 0$ satisfies the system of equations

$$D = ((u-3)(u+1)) \times \left| \begin{array}{ccc|c} u_1 & 3u_1 & 2u_1 & 0 \\ 0 & u-3 & 3-u & 20 \\ 2 & 3u+1 & 3u-3 & \end{array} \right| = 0$$

Therefore $D=0$

$$(u-3) \times u_1 (3u-3 + 3u+1) - (3u+1) = 0$$

$\therefore D = (u+1)(u-3)$

$$R_1 \rightarrow R_1 - R_3$$

$$(u-3) \left| \begin{array}{ccc|c} u-3 & 0 & 3-u & \\ 0 & 1 & -1 & 20 \\ 2 & \frac{11-u}{2} & \frac{3u+1}{2} & \frac{3u-3}{2} \end{array} \right|$$

$$(u-3)^2 \left| \begin{array}{ccc|c} 1 & 0 & -1 & \\ 0 & 1 & -1 & 20 \\ 2 & 3u+1 & 3u-3 & \end{array} \right|$$

$$(u-3)^2 [3u-1 + 3u+1 + 2 - 20] = 0$$

$$u=3, 0 \quad \text{and } u \neq 1$$

① $u \neq 3, 0$

② $u=0$ or $u=3$

∴ $\boxed{\text{No solution}}$

Q@Q

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 0 \\ 3 & 5 & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 3 & 5 & 1 & 0 & 0 & 1 \end{array} \right)$$

$$R_2 \rightarrow R_2 - 2R_1, \quad R_3 \rightarrow R_3 - 3R_1.$$

to second
interation

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 3 & -4 & -2 & 1 & 0 \\ 0 & 7 & -5 & -3 & 0 & 1 \end{array} \right)$$

$$R_2 \rightarrow \frac{1}{3}R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{4}{3} & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 7 & -5 & -3 & 0 & 1 \end{array} \right)$$

$$R_1 \rightarrow R_1 + R_2, \quad R_3 \rightarrow R_3 - 7R_2.$$

standard
form

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & -\frac{4}{3} & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{13}{3} & \frac{5}{3} & -\frac{7}{3} & 1 \end{array} \right)$$

$$R_3 \rightarrow \frac{3}{13} \times R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & -\frac{4}{3} & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & \frac{5}{13} & -\frac{7}{13} & \frac{3}{13} \end{array} \right)$$

$$R_2 \rightarrow R_2 + \frac{4}{3} R_3$$

$$R_1 \longrightarrow R_1 - \frac{2}{3} R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{13} & \frac{9}{13} & \frac{2}{13} \\ 0 & 1 & 0 & -\frac{2}{13} & -\frac{5}{13} & \frac{4}{13} \\ 0 & 0 & 1 & \frac{5}{13} & -\frac{7}{13} & \frac{3}{13} \end{array} \right)$$

the ~~other~~ ~~the~~ ~~the~~

Inverse of
matrix And

1

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 3 & 0 \\ 3 & 0 & 2 & 5 \\ 2 & 0 & 1 & 3 \end{pmatrix}$$

$$\left(\begin{array}{cccc|ccccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 3 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 2 & 5 & 0 & 0 & 1 & 0 \\ 2 & 1 & 1 & 3 & 0 & 0 & 0 & 1 \end{array} \right)$$

$$R_1 \rightarrow R_2 - 2R_1 \quad R_3 \rightarrow R_3 - 3R_1. \quad R_4 \rightarrow R_4 - 2R_1$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & -3 & -1 & 2 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 & 0 & 1 \end{array} \right)$$

$R_1 \rightarrow R_1 + R_2$, $R_3 \rightarrow R_3 - 3R_2$, $R_4 \rightarrow R_4 - R_2$.

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 2 & -1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -2 & -2 & 1 & 0 & 0 \\ 0 & 0 & -1 & 8 & 3 & -3 & 1 & 0 \\ 0 & 0 & -2 & 3 & 0 & -1 & 0 & 1 \end{array} \right)$$

System to solve
 $R_2 \rightarrow -1R_2$, $R_3 \rightarrow -\frac{1}{3}R_3$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 2 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 2 & 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & -2 & -\frac{3}{3} & \frac{3}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & -2 & 3 & 0 & 1 & 0 & 1 \end{array} \right)$$

$R_1 \rightarrow R_1 - 2R_2$, $R_2 \rightarrow R_2 + R_3$, $R_4 \rightarrow R_4 + 2R_3$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 3 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & \frac{5}{3} & -\frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & -2 & -\frac{3}{3} & \frac{3}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & -1 & -\frac{3}{2} & \frac{1}{2} & \frac{1}{2} & 1 \end{array} \right)$$

$R_4 \rightarrow (-1)R_4$ without writing and get ...

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 3 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & \frac{5}{2} & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -2 & -\frac{3}{2} & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{3}{2} & -\frac{1}{2} & \frac{1}{2} & -1 \end{array} \right)$$

$$R_1 \rightarrow R_1 - 3R_4, \quad R_3 \rightarrow R_3 + 2R_4$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -4 & 1 & -1 & 3 \\ 0 & 1 & 0 & 0 & \frac{5}{2} & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & \frac{9}{2} & -\frac{1}{2} & \frac{3}{2} & -2 \\ 0 & 0 & 0 & 1 & \frac{3}{2} & -\frac{1}{2} & \frac{1}{2} & -1 \end{array} \right)$$

Inverse of matrix

Ans

⑩ Soln.: ~~Given~~ v is a solution of $AX=0$

$\Rightarrow Av=0$ $\&$ v is a solution of $Av=y$

$$Av=y$$

Then, $A(v+y) = Av+Ay$

$$= 0 + y$$

$\therefore v+y$ is a solution of $AX=y$

ii) (a) If A is an invertible matrix, hence A^{-1} exists.

$$A_b \quad Ab = 0$$

(i) Taking inverse on both sides

$$\cancel{A^{-1}Ab = A^{-1}0}$$

$$A^{-1}Ab = 0 \quad \text{or } I_b = 0$$

$$B = 0$$

$$\boxed{B = 0}$$

$$0 = 0 + 0 + 0 + \dots + 0$$

ii) If A is not invertible, i.e., $|A| = 0$

so for $b \neq 0$ such that $Ab = 0$

Now, consider a matrix B such that

$$B = [b \ 0 \ 0 \ \dots \ 0]_{n \times n+1} \quad \text{and} \quad b \neq 0$$

such that $b \neq 0$

Then B is a non-zero matrix

$$AB = A[b \ 0 \ \dots \ 0]$$

$$= [Ab \ Ab \ \dots \ Ab]$$

$$= [0 \ 0 \ 0 \ \dots \ 0] \quad (\because Ab = 0)$$

$$\Rightarrow \boxed{AB = 0}$$

b) Consider the system $Ax = 0$ and a TI (A)

$$\left\{ \begin{array}{l} \left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 \\ -2 & 1 & 0 & 1 \\ 0 & 5 & -2 & 7 \\ -1 & 3 & -1 & 9 \end{array} \right) \left(\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right) = 0 \\ \text{this is not a pivot} \\ \text{not a pivot} \end{array} \right.$$

$$x_1 + 2x_2 - x_3 + 3x_4 = 0$$

$$-2x_1 + x_2 + x_3 = 0$$

$$5x_2 - 2x_3 + 7x_4 = 0$$

$$-x_1 + 3x_2 - x_3 + 4x_4 = 0$$

Performing gaussian elimination. Want to get A TI

let $x_1 = 1$, $x_2 = 1$, $x_3 = 1$, $x_4 = 0$ test since 0 is not a pivot

Then, $x_1 = 1$ denoted reduces to forms with

let $b = (0, -1, 1, 1)^T$ $0 = [0 \ 0 \ 0 \ 0]$ $= 0$

$$\text{Then } Bx = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \end{bmatrix} \quad B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Want to get a TI

$$[0 \ 0 \ 0 \ 0] A = 0A = 0$$

$$(0 = 0A :) [0 \ 0 \ 0 \ 0] :$$

$$0 = 0A \Leftrightarrow$$

12@

$$\left(\begin{array}{cccccc} 1 & 7 & -1 & -2 & -1 \\ 3 & 21 & 0 & 9 & 0 \\ 2 & 14 & 0 & 6 & 1 \\ 6 & 42 & -1 & 13 & 0 \end{array} \right)$$

A drawn below

$$AI = A$$

$$A \left(\begin{array}{cccc|c} 0 & 0 & 0 & 1 & \\ 0 & 0 & 1 & 0 & \\ \hline 1 & 7 & -1 & -2 & -1 \\ 0 & 0 & 3 & 15 & 3 \\ 0 & 0 & 2 & 10 & 3 \\ 0 & 0 & 5 & 25 & 6 \end{array} \right) = \left(\begin{array}{cccc|c} 1 & 7 & -1 & -2 & -1 \\ 0 & 0 & 0 & 15 & 3 \\ 0 & 0 & 0 & 10 & 3 \\ 0 & 0 & 0 & 25 & 6 \end{array} \right)$$

$R_2 \rightarrow R_2 - 3R_1$, $R_3 \rightarrow R_3 - 2R_1$, $R_4 \rightarrow R_4 - 6R_1$

$$R_2 \rightarrow \frac{1}{3} \times R_2$$

$$A \left(\begin{array}{cccc|c} 1 & 7 & -1 & -2 & -1 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & 0 & 2 & 10 & 3 \\ 0 & 0 & 5 & 25 & 6 \end{array} \right) = \left(\begin{array}{cccc|c} 1 & 7 & -1 & -2 & -1 \\ 0 & 0 & 0 & 10 & 3 \\ 0 & 0 & 0 & 25 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$R_1 \rightarrow R_1 + R_2$, $R_3 \rightarrow R_3 - 2R_2$, $R_4 \rightarrow R_4 - 5R_2$.

$$\left(\begin{array}{ccccc} 1 & 7 & 0 & 3 & 0 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right)$$

↓

A9

$$R_2 \rightarrow R_2 - R_3$$
, $R_4 \rightarrow R_4 - R_3 = 9$ such

$$\left(\begin{array}{ccccc} 1 & 7 & 0 & 3 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Ans

(b) Consider matrix A as

$$A = IA$$

$$\left(\begin{array}{cccccc} 1 & 7 & -1 & -2 & -1 \\ 3 & 21 & 0 & 9 & 0 \\ 2 & 14 & 0 & 6 & 1 \\ 6 & 42 & -1 & 13 & 0 \end{array} \right) = \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) A$$

$R_2 \rightarrow R_2 - 3R_1$ $R_3 \rightarrow R_3 - 2R_1$

$$R_4 \rightarrow R_4 - 6R_1$$

$$\left(\begin{array}{cccccc} 1 & 7 & -1 & -2 & -1 \\ 0 & 0 & 3 & 15 & 3 \\ 0 & 0 & 2 & 10 & 3 \\ 0 & 0 & 0 & 25 & 6 \end{array} \right) = \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ -6 & 0 & 0 & 1 \end{array} \right) A.$$

Q

PA.

Hence. $P = \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ -6 & 0 & 0 & 1 \end{array} \right)$

row ~~row~~

Consider the matrix

$$\left(\begin{array}{cccccc|c} 1 & 7 & -1 & -2 & -1 & & x \\ 3 & 21 & 0 & 9 & 0 & & y \\ 2 & 14 & 0 & 6 & 1 & & z \\ 6 & 42 & -1 & 13 & 0 & & s \end{array} \right)$$

$$R_2 \rightarrow R_2 - 3R_1, \quad R_3 \rightarrow R_3 - 2R_1, \quad R_4 \rightarrow R_4 - 6R_1$$

Since there are 3 non-zero entries in each row, we can use the row echelon form.

$$\left(\begin{array}{cccccc|c} 1 & 7 & -1 & -2 & -1 & & x \\ 0 & 0 & 3 & 15 & 3 & & y-3x \\ 0 & 0 & 0 & 0 & 3 & & 3z-2y \\ 0 & 0 & 0 & 0 & 3 & & 15-5y-3x \end{array} \right)$$

Set to meet condition between L2 and L3
 $R_3 \rightarrow 3R_3 - 2R_2$ $R_4 \rightarrow 3R_4 - 5R_2$

(b)

$$\left(\begin{array}{cccccc|c} 1 & 7 & -1 & -2 & -1 & & x \\ 0 & 0 & 3 & 15 & 3 & & y-3x \\ 0 & 0 & 0 & 21 & 10 & & 3z-2y \\ 0 & 0 & 0 & 5 & 6 & & 15-5y-3x \end{array} \right)$$

$$R_3 \rightarrow 3R_3 - 2R_2 \quad R_4 \rightarrow 3R_4 - 5R_2$$

$$\left(\begin{array}{cccccc|c} 1 & 7 & -1 & -2 & -1 & & x \\ 0 & 0 & 3 & 15 & 3 & & y-3x \\ 0 & 0 & 0 & 0 & 0 & & 3z-2y \\ 0 & 0 & 0 & 0 & 3 & & 15-5y-3x \end{array} \right)$$

R_4

$$R_4 \rightarrow R_4 - R_3$$

$$\left(\begin{array}{cccc|c} 1 & 7 & -1 & -2 & 1 \\ 0 & 0 & 3 & 15 & 3 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\begin{matrix} R_1 \rightarrow R_1 - 7R_2 \\ R_3 \rightarrow R_3 - R_2 \\ R_4 \rightarrow R_4 - R_2 \end{matrix}} \left(\begin{array}{cccc|c} 1 & 7 & -1 & -2 & 1 \\ 0 & 0 & 3 & 15 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\begin{matrix} R_1 \rightarrow R_1 - 7R_3 \\ R_2 \rightarrow R_2 - 5R_3 \\ R_3 \rightarrow R_3 - R_4 \end{matrix}} \left(\begin{array}{cccc|c} 1 & 7 & -1 & -2 & 1 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

~~Since Ax = y has a solution, we must have.~~

$$\left(\begin{array}{cccc|c} 1 & 7 & -1 & -2 & 1 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\begin{matrix} R_1 \rightarrow R_1 - 7R_2 \\ R_2 \rightarrow R_2 - 3R_3 \\ R_3 \rightarrow R_3 - R_4 \end{matrix}} \left(\begin{array}{cccc|c} 1 & 7 & -1 & -2 & 1 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\boxed{3x_1 + 7x_2 + x_3 + 2x_4 = 1}}$$

④

By row-reduced echelon form of the matrix A, the system $Ax = 0$ can be reduced to the following system:

$$\left(\begin{array}{cccc|c} 1 & 7 & -1 & -2 & 1 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\begin{matrix} R_1 \rightarrow R_1 - 7R_2 \\ R_2 \rightarrow R_2 - 3R_3 \\ R_3 \rightarrow R_3 - R_4 \end{matrix}} \left(\begin{array}{cccc|c} 1 & 0 & 7 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \right] = 0$$

$$\boxed{\begin{array}{l} x_1 + 7x_2 + 3x_3 + 2x_4 = 0 \\ x_3 + 5x_4 = 0 \\ x_5 = 0 \end{array}}$$