CLASS 3 Date ....

Inverse of a Matrix

Let $AX = B$

$$A_{n\times n} X_{n\times 1} = I_{n\times n} b_{n\times 1}$$

If A^{-1} exists. $A^{-1}AX = A^{-1}b$

$I_{n\times n} X = A^{-1} b$

$$(A / I) \rightarrow (I_{n\times n} \mid A^{-1})$$

- We start from A and try to row reduce it to $I_{n\times n}$ (Identity Matrix)
- If the same row reduction process is applied to I_n, then we end up with A^{-1}.

Eg: Let $A = \begin{bmatrix} -1 & 1 & 1 \\ 3 & 1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$ • Find Inverse using row reduction.

Th: Let $A_{n\times n}$ be a square matrix.
→ Then A^{-1} exists if rank $(A) = n$
→ A^{-1} doesn't exist if rank$(A) < n$ and $\det(A) = 0$

Soln:
$$\left[\begin{array}{ccc|ccc} -1 & 1 & -1 & 1 & 0 & 0 \\ 3 & 1 & -1 & 0 & 1 & 0 \\ 2 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \leftrightarrow -R_1$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & -1 & -1 & 0 & 0 \\ 3 & 1 & -1 & 0 & 1 & 0 \\ 2 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

Date ....

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 2R_1$

then, $R_2 \leftrightarrow \frac{1}{4}R_2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0.5 & 0.75 & 0.25 & 0 \\ 0 & 4 & 3 & 2 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + R_2 \\ R_3 \to R_3 - 4R_2 \end{array}$$

$\approx$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -0.5 & -0.25 & 0.25 & 0 \\ 0 & 1 & 0.5 & 0.75 & 0.25 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + \frac{R_3}{2} \\ R_2 \to R_2 - \frac{R_3}{2} \end{array}$$

$\approx$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3/4 & -1/4 & 1/2 \\ 0 & 1 & 0 & 5/4 & 3/4 & -1/2 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right]$$

A^{-1} exists and A^{-1}

Example: $A = \begin{bmatrix} 1 & -1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, Find A^{-1}, if it exists

$$(A/I) = \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 3 & -1 & 2 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 3R_1 \\ \text{then } R_2 \to \frac{1}{2}R_2 \end{array}$$

$\approx$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1/2 & -3/2 & 1/2 & 0 \\ 0 & 4 & -2 & -3 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 + R_2$

$R_3 \to R_3 - 4R_2$

Date ....

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1/2 & -1/2 & 1/2 & 0 \\ 0 & 1 & -1/2 & -3/2 & 1/2 & 0 \\ 0 & 0 & 0 & 3 & -2 & 1 \end{array}\right]$$

Rank$(A) = 2 \neq n$, cannot be reduced to $I_{3\times3}$

* Row Echelon form (R.E.F) Already defined.
* Row Reduced Echelon form (R.R.E.F)
  1. A should be in REF
  2. The leading element (pivot) in each non zero row is 1.
  3. Each column containg a pivot '1' must have 0 in all its other entries

$$\begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & 5 \\ 6 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(RREF)

* Identity Matrix is always in RREF form.

Ex:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} \rightarrow \text{RREF}$$

Spiral

Date ....

Homogenous Linear System

$AX = 0$ $\quad (A_{m\times n} X_{n\times 1} \quad 0_{m\times 1}$

It always non trivial solⁿ if rank $(A) < n$ $\quad A_{m\times n}$

If $m=n$, then | or if rank $(A) = n$

① If $|A| \neq 0$, then $AX = 0$ has only trivial solⁿ.

② If $|A| = 0$, or if rank $(A) < n$, then $AX = 0$ has ~~trivial~~ non trivial solⁿ.

Ex:
$$(k-1)x + (3k-1)y + 2kz = 0$$
$$(k-1)x + (4k-2)y + (k+3)z = 0$$
$$2x + (3k+1)y + 3(k-1)z = 0$$

Find values of k for which the system has →

(i) trivial solⁿ.

(ii) non trivial solⁿ.

(i) $k \neq 0, 3$

(ii) $k = 0$ or 3.

Spiral