ASSIGNMENT-1

1 (a)

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 7 & 8 & 9 \end{bmatrix}$$

Sol.

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 7 & 8 & 9 \end{bmatrix}$$

$R_1 \longleftrightarrow R_4.$

$$\begin{bmatrix} 6 & 7 & 8 & 9 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 1 & 2 & 3 & 0 \end{bmatrix}$$

$R_3 \longrightarrow 2R_3 - R_1$

$R_2 \longrightarrow 3R_2 - R_1$

$R_4 \longrightarrow 6R_4 - R_1.$

$$\begin{bmatrix} 6 & 7 & 8 & 9 \\ 0 & 5 & 1 & -3 \\ 0 & -3 & -6 & -3 \\ 0 & 5 & 10 & -9 \end{bmatrix}$$

$R_4 \longrightarrow R_4 - R_2.$

$R_3 \longrightarrow 5R_3 + 3R_2.$

$$\begin{bmatrix} 6 & 7 & 8 & 9 \\ 0 & 5 & 1 & -3 \\ 0 & 0 & -27 & -24 \\ 0 & 0 & 9 & -6 \end{bmatrix}$$

$R_4 \longrightarrow 3R_4 + R_3$

$$\begin{bmatrix} 6 & 7 & 8 & 9 \\ 0 & 5 & 1 & -3 \\ 0 & 0 & -77 & -24 \\ 0 & 0 & 0 & -42 \end{bmatrix} \longrightarrow$$ Row echelon form of given matrix. & rank = 4.

(b)
Sol.

$$\begin{bmatrix} 2 & 4 & 6 \\ -1 & 3 & 2 \\ 1 & 4 & 6 \\ 2 & 8 & 7 \end{bmatrix}$$

$R_2 \rightarrow 2R_2 + R_1$

$R_3 \longrightarrow 2R_3 - R_1$

$R_4 \longrightarrow R_4 - R_1$

$$\begin{bmatrix} 2 & 4 & 6 \\ 0 & 10 & 10 \\ 0 & 4 & 6 \\ 0 & 4 & 1 \end{bmatrix}$$

$R_3 \longrightarrow 5R_3 - 2R_2$

$R_4 \rightarrow 5R_4 - 2R_2$

$$\begin{bmatrix} 2 & 4 & 6 \\ 0 & 10 & 10 \\ 0 & 0 & 10 \\ 0 & 0 & -15 \end{bmatrix}$$

$R_4 \longrightarrow 2R_4 + 3R_3$

$$\begin{bmatrix} 2 & 4 & 6 \\ 0 & 10 & 10 \\ 0 & 0 & 10 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow$$ **Ans** Rank = 3

(c) $\begin{bmatrix} 1 & -2 & 5 & -3 \\ 2 & 3 & 1 & -4 \\ 3 & 8 & 3 & -5 \end{bmatrix}$

Sol. $R_1 \rightarrow R_3$

$$\begin{bmatrix} 3 & 8 & 3 & -5 \\ 2 & 3 & 1 & 4 \\ 1 & -2 & 5 & -3 \end{bmatrix}$$

$R_2 \rightarrow 3R_2 - 2R_1$

$R_3 \rightarrow 3R_3 - R_1$

$$\begin{bmatrix} 3 & 8 & -3 & -5 \\ 0 & -7 & 9 & 2 \\ 0 & -14 & 18 & -4 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_2$

$$\begin{bmatrix} 3 & 8 & -3 & -5 \\ 0 & -7 & 9 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Ans $\boxed{\text{Rank} = 2.}$

(d) $\begin{bmatrix} 1 & 1 & 2 & 0 & -4 \\ 1 & 2 & 0 & -4 & 1 \\ 2 & 0 & -4 & 1 & 1 \\ 0 & -4 & 1 & 1 & 2 \\ -4 & 1 & 1 & 2 & 0 \end{bmatrix}$

Sol. $R_1 \leftrightarrow R_5$

$$\begin{bmatrix} -4 & 1 & 1 & 2 & 0 \\ 1 & 2 & 0 & -4 & 1 \\ 2 & 0 & -4 & 1 & 1 \\ 0 & -4 & 1 & 1 & 2 \\ 1 & 1 & 2 & 0 & -4 \end{bmatrix}$$

$R_2 \to 4R_2 + R_1$

$R_3 \to 2R_3 + R_1$

$R_4 \to 4R_4 + R_1$

$$\begin{bmatrix} -4 & 1 & 1 & 2 & 0 \\ 0 & 9 & 1 & -14 & 4 \\ 0 & 1 & -7 & 4 & 2 \\ 0 & -4 & 1 & 1 & 2 \\ 0 & 5 & 9 & 2 & -16 \end{bmatrix}$$

$R_3 \to 9R_3 - R_2$. $R_4 \to 9R_4 + 4R_2$. $R_5 \to 9R_5 - 5R_2$

$$\begin{bmatrix} -4 & 1 & 1 & 2 & 0 \\ 0 & 9 & 1 & -14 & 4 \\ 0 & 0 & -64 & 50 & 14 \\ 0 & 0 & 13 & -47 & 34 \\ 0 & 0 & 76 & 88 & -164 \end{bmatrix}$$

$R_3 \to \frac{1}{2}R_3$ $R_5 \to \frac{1}{4}R_5$

$$\begin{bmatrix} -4 & 1 & 1 & 2 & 0 \\ 0 & 9 & 1 & -14 & 4 \\ 0 & 0 & -32 & 25 & 7 \\ 0 & 0 & 13 & -47 & 34 \\ 0 & 0 & 19 & 22 & -41 \end{bmatrix}$$

$R_4 \to 32R_4 + 13R_3$ $R_5 \to 32R_5 + 19R_3$

$$\begin{bmatrix} -4 & 1 & 1 & 2 & 0 \\ 0 & 9 & 1 & -14 & 4 \\ 0 & 0 & -32 & 25 & 7 \\ 0 & 0 & 0 & -1179 & 1179 \\ 0 & 0 & 0 & 1179 & -1179 \end{bmatrix}$$

$R_5 \to R_5 + R_4$

$$\begin{bmatrix} -4 & 1 & 1 & 2 & 0 \\ 0 & 9 & 1 & -14 & 4 \\ 0 & 0 & -32 & 25 & 7 \\ 0 & 0 & 0 & -1179 & 1179 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Rank} = 4}$

Ans

1 2 a

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 5 & 3 & 4 \end{pmatrix}$$

$R_2 \rightarrow R_2 - R_1$. $\quad R_3 \rightarrow R_3 - 5R_1$

$$\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & -2 & -6 \end{pmatrix}$$

$R_3 \rightarrow R_3 + 2R_2$.

$$\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}.$$

Rank of matrix = 2 which is less than no. of vectors hence it is linearly dependent.

b

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 8 & 1 & 5 \end{pmatrix}$$

$R_2 \rightarrow R_2 - 2R_1 \quad R_3 \rightarrow R_3 - 8R_1$.

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 3 & -1 \\ 0 & 9 & -3 \end{pmatrix}.$$

$R_3 \rightarrow R_3 - 3R_2$.

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

Rank = 2 which is less than 'n' Hence linearly dependent.

(c)

$$\begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{pmatrix}$$

$R_2 \to R_2 - R_1$ $R_3 \to R_3 - 3R_1$

$$\begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 0 & -2 & 3 & -2 \end{pmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Rank = 2 Hence linearly dependent.

(d)

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & 2 \end{pmatrix}$$

$R_2 \to R_2 - 2R_1$ $R_3 \to R_3 - R_1$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -3 & -2 \\ 0 & -3 & 1 \end{pmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -3 & -2 \\ 0 & 0 & 3 \end{pmatrix}.$$

Rank = 3 which is equal to the number of vectors

Hence, it is linearly ~~dependent~~ independent.

(3) (a) $\begin{pmatrix} \alpha & 1 & 2 \\ 0 & 2 & \beta \\ 1 & 3 & 6 \end{pmatrix}$

Sol. $R_1 \longleftrightarrow R_3$

$$\begin{pmatrix} 1 & 3 & 6 \\ 0 & 2 & \beta \\ \alpha & 1 & 2 \end{pmatrix}$$

$R_3 \longrightarrow R_3 - \alpha R_1$.

$$\begin{pmatrix} 1 & 3 & 6 \\ 0 & 2 & \beta \\ 0 & 1-3\alpha & 2-6\alpha \end{pmatrix}$$

$R_3 \longrightarrow 2R_3 - (1-3\alpha)R_2$.

$$\begin{pmatrix} 1 & 3 & 6 \\ 0 & 2 & \beta \\ 0 & 0 & (\beta-4)(1-3\alpha) \end{pmatrix}$$

(i) Rank = 1 Not possible as atleast 2 rows are non-zero

(ii) Rank = 2. $\beta = 4$ or $\alpha = \frac{1}{3}$

(iii) Rank = 3 $\beta \neq 4$ & $\alpha \neq \frac{1}{3}$.

(b) For a system to be consistent $Rank(A) = Rank(A/b)$

So,

$$\left(\begin{array}{ccc|c} 2 & 4 & \alpha+3 & 2 \\ 1 & 3 & 1 & 2 \\ \alpha-2 & 2 & 3 & \beta \end{array}\right)$$

$R_2 \longrightarrow 2R_2 - R_1$ $\quad R_3 \longrightarrow 2R_3 - (\alpha-2)R_1$

$$\left(\begin{array}{ccc|c} 2 & 4 & \alpha+3 & 2 \\ 0 & 2 & -1-\alpha & 2 \\ 0 & 4(3-\alpha) & 6-(\alpha-2)(\alpha+3) & 2\beta-2(\alpha-2) \end{array}\right)$$

$R_3 \longrightarrow 2R_3 - (12-4\alpha)R_2$

$$\left(\begin{array}{ccc|c} 2 & 4 & \alpha+3 & 2 \\ 0 & 2 & -\alpha-1 & 2 \\ 0 & 0 & -6(\alpha^2-\alpha-6) & 4(\beta+\alpha-4) \end{array}\right)$$

For, $Rank(A/b) = Rank(A)$

Either $\quad \alpha^2-\alpha-6=0$ & $\beta+\alpha-4=0$

or

$\alpha^2-\alpha-6 \neq 0$

Case ① $\quad \alpha=3$ or $\alpha=-2$
$\beta=1 \qquad \beta=6$

Case ② $\quad \alpha \neq 3$ & -2

(4) (a)

$$\left(\begin{array}{ccc|c} 1 & 4 & -1 & 4 \\ 1 & 1 & -6 & -4 \\ 3 & -1 & -1 & 1 \end{array}\right)$$

$R_2 \longrightarrow R_2 - R_1$

$R_3 \longrightarrow R_3 - 3R_1$

$$\left(\begin{array}{ccc|c} 1 & 4 & -1 & 4 \\ 0 & -3 & -5 & -8 \\ 0 & -13 & 2 & -11 \end{array}\right)$$

$R_3 \longrightarrow R_3 - \frac{13}{3} R_2$

$$\left(\begin{array}{ccc|c} 1 & 4 & -1 & 4 \\ 0 & -3 & -5 & -8 \\ 0 & 0 & \frac{71}{3} & \frac{71}{3} \end{array}\right)$$

$x \quad y \quad z$

$\boxed{z=1}$

$-3y - 5z = -8$

$3y + 5z = 8$

$\boxed{y=1}$

$x + 4y - z = 4$

$x + 4 - 1 = 4$

$\boxed{x=1}$

(b)

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & 1 & 5 & 2 \\ 3 & 3 & 4 & 1 \end{array}\right)$$

$R_2 \longrightarrow R_2 - 2R_1$

$R_3 \longrightarrow R_3 - 3R_1$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 7 & 0 \\ 0 & -3 & 7 & -2 \end{array}\right)$$

$R_3 \longrightarrow R_3 - R_2$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 7 & 0 \\ 0 & 0 & 0 & -2 \end{array}\right)$$

Rank $(A/b) \neq$ Rank(A)

$\therefore$ No solution.

(c)

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 3 & 1 & -2 & 1 \\ 2 & 4 & 2 & 4 \end{array}\right)$$

$R_2 \longrightarrow R_2 - 3R_1$ $R_3 \longrightarrow R_3 - 2R_1$

$$\begin{array}{c} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -5 & -5 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right) \\ \begin{array}{ccc} x & y & z \end{array} \end{array}$$

Rank (A/b) = Rank(A) = 2 but no. of ~~unknow~~ unknown variables = 3

Hence, infinite solutions

$-5y - 5z = -5$

$\boxed{y + z = 1}$

$\boxed{x + 2y + z = 2}$

$\boxed{x + y = 1}$

let $y = k$ (an arbitrary constant)

$\boxed{x = 1 - k. \quad y = k. \quad z = 1 - k.}$ Ans

5 (a)

$$\left(\begin{array}{ccc|c} -2 & 1 & 1 & a \\ 1 & -2 & 1 & b \\ 1 & 1 & -2 & c \end{array}\right)$$

$R_2 \rightarrow R_1 + 2R_2.$

$R_3 \rightarrow R_1 + 2R_3$

$$\left(\begin{array}{ccc|c} -2 & 1 & -1 & a \\ 0 & -3 & 3 & a+2b \\ 0 & 3 & -3 & a+2c \end{array}\right)$$

$R_3 \rightarrow R_3 + R_2$

$$\left(\begin{array}{ccc|c} -2 & 1 & 1 & a \\ 0 & -3 & 3 & a+2b \\ 0 & 0 & 0 & 2a+2b+2c \end{array}\right)$$

(i) No solution $\quad Rank(A) = 2$, ~~Rank(A|b) ≠~~ so

so, $Rank(A|b) \neq 2.$

$a + b + c \neq 0$

(ii) Not possible as there are 3 variables and

Rank of $A = 2$

(iii) $\boxed{a+b+c=0}$

(5)
$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 2 & 3 & \lambda & 3 \\ 1 & \lambda & 3 & 2 \end{array}\right)$$

$R_2 \longleftrightarrow R_3$
$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & \lambda & 3 & 2 \\ 2 & 3 & \lambda & 3 \end{array}\right).$$

$R_2 \longrightarrow R_2 - R_1$, $R_3 \longrightarrow R_3 - 2R_1$
$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & \lambda-1 & 4 & 1 \\ 0 & 1 & \lambda+2 & 1 \end{array}\right)$$

$R_2 \longleftrightarrow R_3$
$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & \lambda+2 & 1 \\ 0 & \lambda-1 & 4 & 1 \end{array}\right)$$

$R_3 \longrightarrow R_3 - (\lambda-1)R_2$.
$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & \lambda+2 & 1 \\ 0 & 0 & 4-(\lambda+2)(\lambda-1) & 2-\lambda \end{array}\right).$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & \lambda+2 & 1 \\ 0 & 0 & -\lambda^2-\lambda+6 & 2-\lambda \end{array}\right)$$

$\quad\quad\hookrightarrow (\lambda+3)(\lambda-2)$

(i) Ran $(A|b) \neq$ Rank(A). $\boxed{\lambda = -3}$

(ii) $\lambda \neq -3, 2$

(iii) $\lambda = 2$ so that ~~Rank A~~ Rank $(A|b)$ = Rank (A).

(2)

(c)

$$\left(\begin{array}{ccc|c} \lambda & 1 & 1 & p \\ 1 & \lambda & 1 & q \\ 1 & 1 & \lambda & r \end{array}\right)$$

$R_2 \rightarrow \lambda R_2 - R_1 \quad R_3 \rightarrow \lambda R_3 - R_1$

λ

0

0

$R_1 \longleftrightarrow R_2$

$$\left(\begin{array}{ccc|c} 1 & \lambda & 1 & q \\ \lambda & 1 & 1 & p \\ 1 & 1 & \lambda & r \end{array}\right)$$

$R_2 \rightarrow R_2 - \lambda R_1 \quad R_3 \rightarrow R_3 - R_1$

q

$$\left(\begin{array}{ccc|c} 1 & \lambda & 1 & q \\ 0 & 1-\lambda^2 & 1-\lambda & p-\lambda q \\ 0 & 1-\lambda & \lambda-1 & r-q \end{array}\right)$$

$R_3 \longrightarrow (\lambda+1)R_3 - R_2$

$$\left(\begin{array}{ccc|c} 1 & \lambda & 1 & q \\ 0 & 1-\lambda^2 & 1-\lambda & p-\lambda q \\ 0 & 0 & \lambda^2+\lambda-2 & (\lambda+1)(r-q)-p \end{array}\right)$$

(i) For no solution. $\lambda^2+\lambda-2=0$ & $\lambda r-\lambda q+r-q-p+\lambda q \neq 0$

$\boxed{\lambda=1 \quad p+q-2r\neq 0}$ or $\boxed{\lambda=-2 \quad p+q+r\neq 0}$

$\boxed{\text{or } p-q\neq 0}$

(ii) For unique solution ~~Rank A~~ Rank(A|b) = Rank(A) = 3

$\lambda \neq 1, -2.$

(iii) For infinite solution.

$\lambda=1$ & $p=q=r$ or $\lambda=-2$ & $p+q+r=0$

NOTE: For $\lambda=1$ system reduces to

$x+y+z=p$

$x+y+z=q$

$x+y+z=r$

$p=q=r$

6 a) let $\sin x = a$, $\cos y = b$ & $\tan z = c$

then,

$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 3 \\ 4 & 2 & -2 & 10 \\ 6 & -3 & 1 & 9 \end{array}\right)$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 3 \\ 0 & 4 & -8 & 4 \\ 0 & 0 & -8 & 0 \end{array}\right)$$

Rank of $(A|b) \neq$ Rank(A) Hence no solution

b)

$$\left(\begin{array}{cccc|c} 0 & 2 & 2 & 3 & b_1 \\ 2 & 4 & 6 & 7 & b_2 \\ 1 & 1 & 2 & 2 & b_3 \end{array}\right)$$

$R_1 \leftrightarrow R_3$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 2 & b_3 \\ 2 & 4 & 6 & 7 & b_2 \\ 0 & 2 & 2 & 3 & b_1 \end{array}\right)$$

$R_2 \to R_2 - 2R_1$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & b_3 \\ 0 & 2 & 2 & 3 & b_2 - 2b_3 \\ 0 & 2 & 2 & 3 & b_1 \end{array}\right)$$

$R_3 \longrightarrow R_3 - R_2$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 2 & b_3 \\ 0 & 2 & 2 & 3 & b_2 - 2b_3 \\ 0 & 0 & 0 & 0 & b_1 - b_2 + 2b_3 \end{array}\right)$$

Hence, if for some $b_1, b_2, b_3 \in R$ $b_1 - b_2 + 2b_3 \neq 0$ then system has no solution.

7 P_2: ax^2+bx+c.

$f(1)=2$ $a+b+c=2$

$f(-1)=6$. $a-b+c=6$.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & -1 & 1 & \cancel{8}\,6 \end{array}\right)$$

$R_2 \longrightarrow R_2 - R_1$

$$\begin{array}{c} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & 0 & 4 \end{array}\right) \\ \begin{array}{ccc} a & b & c \end{array} \end{array}$$

$-2b=4$

$\boxed{b=-2}$

$a+b+c=2$

$\boxed{a+c=4}$

let $c=k$ (arb. const)

$a=4-k$.

Hence P_2:
$(4-k)x^2-2x+k$ Ans

8 (a)

$$\begin{pmatrix} 3u-8 & 3 & 3 \\ 3 & 3u-8 & 3 \\ 3 & 3 & 3u-8 \end{pmatrix}.$$

For the equation to have non-trivial solution. $D=0$.

$$(3u-8)\left[(3u-8)^2-9\right]+3\left[-3(3u-8)+9\right]+3\left(9-3(3u-8)\right)=$$

Let $3u-8=t$

$$t^3-27t+54=0.$$

$$(t-3)^2(t+6)=0$$

$3u-8=3$. & $3u-8=-6$

$u=\frac{2}{3}$ or $u=\frac{11}{3}$

(i) $u\neq\frac{2}{3},\frac{11}{3}$.

(ii) $u=\frac{2}{3}$ or $u=\frac{11}{3}$

(b)

$$\begin{pmatrix} u-1 & 3u+1 & 2u \\ u-1 & 4u-2 & u+3 \\ 2 & 3u+1 & 3u-3 \end{pmatrix}.$$

Determinant:

$$\begin{vmatrix} u-1 & 3u+1 & 2u \\ u-1 & 4u-2 & u+3 \\ 2 & 3u+1 & 3u-3 \end{vmatrix}$$

$R_2 \rightarrow R_2-R_1$.

$$\begin{vmatrix} k-1 & 3k+1 & 2k \\ 0 & k-3 & 3-k \\ 2 & 3k+1 & 3k-3 \end{vmatrix} = 0$$

$$(k-3)\begin{vmatrix} k-1 & 3k+1 & 2k \\ 0 & 1 & -1 \\ 2 & 3k+1 & 3k-3 \end{vmatrix} = 0$$

$$(k-3)\left[k-1\left(3k-3+\cancel{3k+1}\right) - \cancel{(3k+1)}\right.$$

$R_1 \longrightarrow R_1 - R_3$

$$(k-3)\begin{vmatrix} k-3 & 0 & 3-k \\ 0 & 1 & -1 \\ 2 & 3k+1 & 3k-3 \end{vmatrix} = 0$$

$$(k-3)^2\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 2 & 3k+1 & 3k-3 \end{vmatrix} = 0$$

$$(k-3)^2\left[3k-\cancel{3}+\cancel{3k+1}+\cancel{2}-\cancel{6}\right] = 0$$

$k = 3, 0$.

(i) $k \neq 3, 0$ (ii) $k = 0$ or $k = 3$

9 (a)

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 3 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$

$R_2 \longrightarrow R_2 - 2R_1$. $\quad R_3 \longrightarrow R_3 - 3R_1$.

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 3 & -4 & -2 & 1 & 0 \\ 0 & 7 & -5 & -3 & 0 & 1 \end{array}\right)$$

$R_2 \longrightarrow \frac{1}{3} R_2$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{-4}{3} & \frac{-2}{3} & \frac{1}{3} & 0 \\ 0 & 7 & -5 & -3 & 0 & 1 \end{array}\right)$$

$R_1 \longrightarrow R_1 + R_2$. $\quad R_3 \longrightarrow R_3 - 7R_2$.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & \frac{-4}{3} & \frac{-2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{13}{3} & \frac{5}{3} & \frac{-7}{3} & 1 \end{array}\right)$$

$R_3 \longrightarrow \frac{3}{13} \times R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & \frac{-4}{3} & \frac{-2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & \frac{5}{13} & \frac{-7}{13} & \frac{3}{13} \end{array}\right)$$

$R_2 \longrightarrow R_2 + \frac{4}{3} R_3$

$R_1 \longrightarrow R_1 - \frac{2}{3} R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{13} & \frac{9}{13} & \frac{-2}{13} \\ 0 & 1 & 0 & \frac{-2}{13} & \frac{-5}{13} & \frac{4}{13} \\ 0 & 0 & 1 & \frac{5}{13} & \frac{-7}{13} & \frac{3}{13} \end{array}\right)$$

Inverse of matrix Ans

(b)

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 3 & 0 \\ 3 & 0 & 2 & 5 \\ 2 & 1 & 1 & 3 \end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 3 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 2 & 5 & 0 & 0 & 1 & 0 \\ 2 & 1 & 1 & 3 & 0 & 0 & 0 & 1 \end{array}\right)$$

$R_2 \longrightarrow R_2 - 2R_1$ $\quad R_3 \longrightarrow R_3 - 3R_1$. $\quad R_4 \longrightarrow R_4 - 2R_1$

$$\left(\begin{array}{c|ccc} 1 & \cancel{0} & 0 & 3 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -2 & -2 & 1 & 0 & 0 \\ 0 & -3 & -1 & 2 & -3 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & -2 & 0 & 0 & 1 \end{array}\right)$$

$R_1 \to R_1 + R_2$. $\quad R_3 \to R_3 - 3R_2$ $\quad R_4 \to R_4 - R_2$.

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 2 & -1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -2 & -2 & 1 & 0 & 0 \\ 0 & 0 & -4 & 8 & 3 & -3 & 1 & 0 \\ 0 & 0 & -2 & 3 & 0 & -1 & 0 & 1 \end{array}\right)$$

$R_2 \to (-1) R_2$. $\quad R_3 \to -\frac{1}{4} R_3$.

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 2 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 2 & 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & -2 & -\frac{3}{4} & \frac{3}{4} & -\frac{1}{4} & 0 \\ 0 & 0 & -2 & 3 & 0 & -1 & 0 & 1 \end{array}\right)$$

$R_1 \to R_1 - 2R_3$ $\quad R_2 \to R_2 + R_3$ $\quad R_4 \to R_4 + 2R_3$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 3 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & \frac{5}{4} & -\frac{1}{4} & -\frac{1}{4} & 0 \\ 0 & 0 & 1 & -2 & -\frac{3}{4} & \frac{3}{4} & -\frac{1}{4} & 0 \\ 0 & 0 & 0 & -1 & -\frac{3}{2} & \frac{1}{2} & -\frac{1}{2} & 1 \end{array}\right)$$

$R_4 \to (-1) R_4$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 3 & \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & \frac{5}{4} & \frac{-1}{4} & \frac{-1}{4} & 0 \\ 0 & 0 & 1 & -2 & \frac{-3}{4} & \frac{3}{4} & \frac{-1}{4} & 0 \\ 0 & 0 & 0 & 1 & \frac{3}{2} & \frac{-1}{2} & \frac{1}{2} & -1 \end{array}\right)$$

$R_1 \longrightarrow R_1 - 3R_4 \quad R_3 \longrightarrow R_3 + 2R_4$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -4 & 1 & -1 & 3 \\ 0 & 1 & 0 & 0 & \frac{5}{4} & \frac{-1}{4} & \frac{-1}{4} & 0 \\ 0 & 0 & 1 & 0 & \frac{9}{4} & \frac{-1}{4} & \frac{3}{4} & -2 \\ 0 & 0 & 0 & 1 & \frac{3}{2} & \frac{-1}{2} & \frac{1}{2} & -1 \end{array}\right)$$

Inverse of matrix

Ans

(10) Soln.: ~~Case~~ v is a solution of $AX=0$

$\Rightarrow Av = 0$ & as p is solution of $AX=Y$

$Ap = Y$.

Then, $A(v+p) = Av + Ap$

$= 0 + Y$

$= Y$.

$\therefore$ $v+p$ is a solution of $AX=Y$.

11) a) i) If A is an invertible matrix, Hence A^{-1} exists.

As $AB = 0$

Taking inverse on both sides

~~$A^{-1}(AB) = 0$~~

$$A^{-1}AB = 0$$

$$IB = 0$$

$$\boxed{B = 0}$$

ii) If A is not invertible i.e. $|A| = 0$,

So for $b \neq 0$ such that $Ab = 0$

Now, ~~a mat of~~ consider a matrix

$$B = [b \ 0 \ 0 \ \ldots \ 0]_{n \times n}$$

such that $b \neq 0$

Then B is a non-zero matrix

$$AB = A[b \ 0 \ \ldots \ 0]$$

$$= [Ab \ A0 \ \ldots \ A0]$$

$$= [0 \ 0 \ 0 \ \ldots \ 0] \quad (\because Ab = 0)$$

$$\Rightarrow \boxed{AB = 0}$$

(b) Consider the system. $Ax = 0$

$$\begin{pmatrix} 1 & 2 & -1 & 3 \\ -2 & 1 & 0 & 1 \\ 0 & 5 & -2 & 7 \\ -1 & 3 & -1 & 9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = 0$$

$$x_1 + 2x_2 - x_3 + 3x_4 = 0$$

$$-2x_1 + x_2 + x_4 = 0$$

$$5x_2 - 2x_3 + 7x_4 = 0$$

$$-x_1 + 3x_2 - x_3 + 4x_4 = 0$$

Performing gaussian elimination.

Let $x_4 = 1$ & $x_3 = 1$

Then, $x_2 = -1$ & $x_1 = 0$

Let $b = (0, -1, 1, 1)^T$

Then $\cancel{B = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}}$ $B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ Ans

12 (a)

$$\begin{pmatrix} 1 & 7 & -1 & -2 & -1 \\ 3 & 21 & 0 & 9 & 0 \\ 2 & 14 & 0 & 6 & 1 \\ 6 & 42 & -1 & 13 & 0 \end{pmatrix}$$

$R_2 \rightarrow R_2 - 3R_1 \quad R_3 \rightarrow R_3 - 2R_1 \quad R_4 \rightarrow R_4 - 6R_1$

$$\begin{pmatrix} 1 & 7 & -1 & -2 & -1 \\ 0 & 0 & 3 & 15 & 3 \\ 0 & 0 & 2 & 10 & 3 \\ 0 & 0 & 5 & 25 & 6 \end{pmatrix}$$

$R_2 \rightarrow \frac{1}{3} \times R_2$

$$\begin{pmatrix} 1 & 7 & -1 & -2 & -1 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & 0 & 2 & 10 & 3 \\ 0 & 0 & 5 & 25 & 6 \end{pmatrix}$$

$R_1 \rightarrow R_1 + R_2 \quad R_3 \rightarrow R_3 - 2R_2 \quad R_4 \rightarrow R_4 - 5R_2$

$$\begin{pmatrix} 1 & 7 & 0 & 3 & 0 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$R_2 \rightarrow R_2 - R_3 \quad R_4 \rightarrow R_4 - R_3$

$$\begin{pmatrix} 1 & 7 & 0 & 3 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ Ans

(b) Consider matrix A as

$A = IA$

$$\begin{pmatrix} 1 & 7 & -1 & -2 & -1 \\ 3 & 21 & 0 & 9 & 0 \\ 2 & 14 & 0 & 6 & 1 \\ 6 & 42 & -1 & 13 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} A$$

$R_2 \to R_2 - 3R_1 \quad R_3 \to R_3 - 2R_1$

$R_4 \to R_4 - 6R_1$

$$\begin{pmatrix} 1 & 7 & -1 & -2 & -1 \\ 0 & 0 & 3 & 15 & 3 \\ 0 & 0 & 2 & 10 & 3 \\ 0 & 0 & 5 & 25 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ -6 & 0 & 0 & 1 \end{pmatrix} A.$$

$\downarrow$

PA.

Hence. $P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ -6 & 0 & 0 & 1 \end{pmatrix}$. Ans

Consider the matrix

(c)

$$\left(\begin{array}{ccccc|c} 1 & 7 & -1 & -2 & -1 & x \\ 3 & 21 & 0 & 9 & 0 & y \\ 2 & 14 & 0 & 6 & 1 & z \\ 6 & 42 & -1 & 13 & 0 & 5 \end{array}\right)$$

$R_2 \longrightarrow R_2 - 3R_1 \quad R_3 \longrightarrow R_3 - 2R_1 \quad R_4 \longrightarrow R_4 - 6R_1$

$$\left(\begin{array}{ccccc|c} 1 & 7 & -1 & -2 & -1 & x \\ 0 & 0 & 3 & 15 & 3 & y-3x \\ 0 & 0 & 0 & 0 & 3 & 3z-2y \\ 0 & 0 & 0 & 0 & 3 & 15-5y-3x \end{array}\right)$$

$R_3 \longrightarrow 3R_3 - 2R_2 \quad R_4 \longrightarrow 3R_4 - 5R_2$

$$\left(\begin{array}{ccccc|c} 1 & 7 & -1 & -2 & -1 & x \\ 0 & 0 & 3 & 15 & 3 & y-3x \\ 0 & 0 & 2 & 10 & 3 & z-2x \\ 0 & 0 & 5 & 25 & 6 & 5-6x \end{array}\right)$$

$R_3 \longrightarrow 3R_3 - 2R_2. \quad R_4 \longrightarrow 3R_4 - 5R_2.$

$$\left(\begin{array}{ccccc|c} 1 & 7 & -1 & -2 & -1 & x \\ 0 & 0 & 3 & 15 & 3 & y-3x \\ 0 & 0 & 0 & 0 & 3 & 3z-2y \\ 0 & 0 & 0 & 0 & 3 & 15-5y-3x \end{array}\right)$$

R_4

$R_4 \longrightarrow R_4 - R_3$

$$\left(\begin{array}{ccccc|c} 1 & 7 & -1 & -2 & -1 & x \\ 0 & 0 & 3 & 15 & 3 & y-3x \\ 0 & 0 & 0 & 0 & 3 & 3z-2y \\ 0 & 0 & 0 & 0 & 0 & 15-3x-3y-3z \end{array}\right)$$

Since $AX=Y$ has a solution, we must have.

$$15-3x-3y-3z=0$$

$$\boxed{x+y+z=5}$$

(d) By row-reduced echelon form of the matrix A, the system $AX=0$ can be reduced to the following system

$$\begin{pmatrix} 1 & 7 & 0 & 3 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = 0$$

$$\boxed{\begin{aligned} x_1+7x_2+3x_4&=0 \\ x_3+5x_4&=0 \\ x_5&=0 \end{aligned}}$$