

AI1110 Assignment 1

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Find the angle between the two lines $2x = 3y = -z$ and $6x = -y = -4z$.

Solution:

The two lines can be expressed in vector form as

$$\mathbf{r} = \lambda \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} \quad (1)$$

$$\mathbf{r} = \mu \begin{pmatrix} 2 \\ -12 \\ -3 \end{pmatrix} \quad (2)$$

Let

$$\mathbf{m} = \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} \quad (3)$$

$$\mathbf{n} = \begin{pmatrix} 2 \\ -12 \\ -3 \end{pmatrix} \quad (4)$$

The angle between the two lines is given by

$$\theta = \cos^{-1} \left(\frac{\mathbf{m}^T \mathbf{n}}{\|\mathbf{m}\| \|\mathbf{n}\|} \right) \quad (5)$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{(3 \ 2 \ -6)^T \begin{pmatrix} 2 \\ -12 \\ -3 \end{pmatrix}}{\left\| \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} \right\| \left\| \begin{pmatrix} 2 \\ -12 \\ -3 \end{pmatrix} \right\|} \right) \quad (6)$$

$$\Rightarrow \theta = \cos^{-1}(0) \quad (7)$$

$$\Rightarrow \theta = \frac{\pi}{2} \quad (8)$$

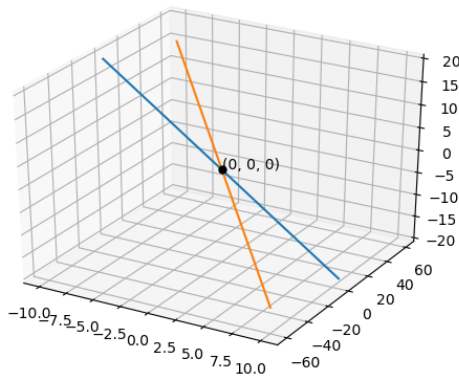


Fig. 1. 3D plot showing the two lines