AI1110 - Probability and Random Variables
Assignment 7

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CBSE class 12 Exercise 13.4

Question 6

From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Solution

Let us define **success** as **Ball drawn is not defective**
Then probability of success is $p = \frac{6}{30} = \frac{4}{5}$
and probability of failure is $q = \frac{1}{5}$
Let the random variable $X \in \{0, 1, 2, 3, 4\}$ denote the number of defective bulbs
This is Bernauli trial. The probability of $i \in \{0, 1, 2, 3, 4\}$ balls being defective is given by

$$\Pr(X = i) = \binom{4}{i} p^i q^{4-i} \tag{1}$$

Therefore, the probability distribution is

No. of defective balls	Probability
0	$\binom{4}{0} \times \left(\frac{4}{5}\right)^4 = \frac{256}{625}$
1	$\binom{4}{1} \times \frac{1}{5} \times \left(\frac{4}{5}\right)^3 = \frac{256}{625}$
2	$\binom{4}{2} \times \left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right)^2 = \frac{96}{625}$
3	$\binom{4}{3} \times \left(\frac{1}{5}\right)^3 \times \left(\frac{4}{5}\right)^1 = \frac{16}{625}$
4	$\binom{4}{4} \times \left(\frac{1}{5}\right)^4 \times \left(\frac{4}{5}\right)^0 = \frac{1}{625}$

Code

```python
import numpy as np

num = 30

t = 100000
balls = np.array([0]*6 + [1]*(num-6))

choices = [4-np.count_nonzero(np.random.choice(balls, 4)) for i in range(t)]
print("pr(X=0) = ", choices.count(0)/t)
print("pr(X=1) = ", choices.count(1)/t)
print("pr(X=2) = ", choices.count(2)/t)
print("pr(X=3) = ", choices.count(3)/t)
print("pr(X=4) = ", choices.count(4)/t)
```

Figure: 1