AI1110 Assignment 1

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ICSE class 12 paper 2019 Q15 (c)
Find the angle between the two lines $2x = 3y = -z$ and $6x = -y = -4z$.

Solution:
The two lines can be expressed in vector form as

$$\mathbf{r} = \lambda \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} \tag{1}$$

$$\mathbf{r} = \mu \begin{pmatrix} 2 \\ -12 \\ -3 \end{pmatrix} \tag{2}$$

Let

$$\mathbf{m} = \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} \tag{3}$$

$$\mathbf{n} = \begin{pmatrix} 2 \\ -12 \\ -3 \end{pmatrix} \tag{4}$$

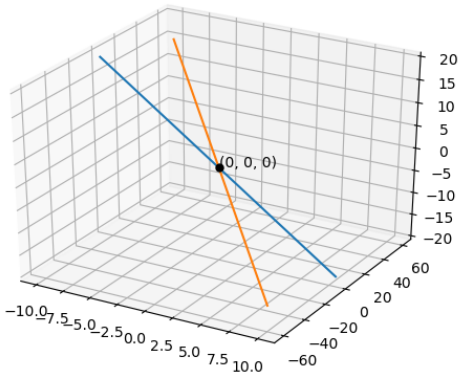

Fig. 1. 3D plot showing the two lines

The angle between the two lines is given by

$$\theta = \arccos\left(\frac{\mathbf{m}^\top \mathbf{n}}{\|\mathbf{m}\| \, \|\mathbf{n}\|}\right) \tag{5}$$

$$\implies \theta = \arccos\left(\frac{\begin{pmatrix} 3 & 2 & -6 \end{pmatrix}^\top \begin{pmatrix} 2 \\ -12 \\ -3 \end{pmatrix}}{\left\|\begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix}\right\| \left\|\begin{pmatrix} 2 \\ -12 \\ -3 \end{pmatrix}\right\|}\right) \tag{6}$$

$$\implies \theta = \arccos(0) \tag{7}$$

$$\implies \theta = \frac{\pi}{2} \tag{8}$$