AI1110 Assignment 1

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ICSE class 12 paper 2019 Q15 (c)

Find the angle between the two lines 2x = 3y = -zand 6x = -y = -4z.

Solution:

The two lines can be expressed in vector form as

$$\mathbf{r} = \lambda \begin{pmatrix} 3\\2\\-6 \end{pmatrix} \tag{1}$$

$$\mathbf{r} = \mu \begin{pmatrix} 2 \\ -12 \\ -3 \end{pmatrix} \tag{2}$$

The angle between the two lines is given by

$$\theta = \cos^{-1} \frac{\mathbf{m}^{\top} \mathbf{n}}{\|\mathbf{m}\| \|\mathbf{n}\|})$$
 (5)

$$\theta = \cos^{-1} \frac{\mathbf{m} \mathbf{n}}{\|\mathbf{m}\| \|\mathbf{n}\|}$$
as
$$\Rightarrow \theta = \cos^{-1} \left(\frac{\left(3 \quad 2 \quad -6\right)^{\mathsf{T}} \begin{pmatrix} 2 \\ -12 \\ -3 \end{pmatrix}}{\| \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} \| \| \begin{pmatrix} 2 \\ -12 \\ -3 \end{pmatrix} \|} \right)$$
(6)
$$\Rightarrow \theta = \cos^{-1}(0)$$

$$\Rightarrow \theta = \frac{\pi}{2}$$
(8)

$$\implies \theta = \cos^{-1}(0) \tag{7}$$

$$\implies \theta = \frac{\pi}{2} \tag{8}$$

Let

$$\mathbf{m} = \begin{pmatrix} 3\\2\\-6 \end{pmatrix} \tag{3}$$

$$\mathbf{n} = \begin{pmatrix} 2 \\ -12 \\ -3 \end{pmatrix} \tag{4}$$

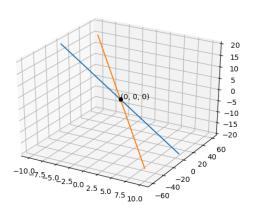


Fig. 1. 3D plot showing the two lines