UNIT-I

INTRODUCTION TO ELECTRICAL ENGINEERING AND DC CIRCUITS:

Basic definitions, Ohm's law, types of elements, types of sources, Kirchhoff's laws, resistive networks-series, parallel circuits, delta- star and star- delta transformation, Network theorems- Superposition, Thevenin's - simple problems

1. 1. Basic definitions:

a) Voltage: The amount of energy required to move an electric charge from one point to another in an electric circuit is called voltage. Voltage is denoted by the symbol **V.**

Mathematically, voltage is defined as the ratio of the work done to the electric charge, i.e.,

$$\text{Voltage, V} = \frac{\text{Work done (W)}}{\text{Charge (Q)}}$$

Where, the work done is measured in Joules (J) and charge in Coulombs (C). Therefore, the unit of voltage is Joules per Coulomb (J/C). (Or)

The SI unit of voltage is Volt (V).

b) Current: It is defined as the rate of flow of electrons through a conductor.

The conventional direction of electric current is taken as opposed to the direction of flow of electrons. If a charge Q flows through the cross-section of a conductor in time t.

Then the current **I=Q/t.**

S.I unit of current: ampere (or) A

c) Power: The rate of doing work is defined as Power.

It is denoted by P.

S.I unit: watts (or) W

Mathematically, Power = work / time

P = (qV)/t where w= qV

P= (VIt)/t where q=It

Electrical power P= VI

d) Energy: The capacity to do work is known as Energy.

Electrical energy E= Power x Time

E=VIt

S.I unit: Watt-hr

e) Resistance:

Resistance is a measure of the opposition to current flow in an electrical circuit. It is denoted with R.

Resistance is measured in ohms, symbolized by the Greek letter ohm (Ω).

$$R = \frac{\rho L}{A} \qquad \begin{aligned} \rho &= \text{resistivity} \\ L &= \text{length} \\ A &= \text{cross sectional area} \end{aligned}$$

⇨ Factors effecting Resistance:

- **Length of the material:** The resistance 'R' of a conductor is directly proportional to the length, i.e., $R \propto l$
- The conductor's resistance increases with length.
- **Area of cross-section:** The resistance 'R' of a conductor is inversely proportional to the area of the cross-section, $R \propto (1/a)$
- The conductor's resistance decreases with increasing thickness. It indicates that a thin conductor has a high resistance.
- **Nature of Material:** The resistance 'R' of a conductor depends upon the nature of the material with which it is made.
- All materials have some level of resistance, varying from high to low. We categories materials as conductors or insulators based on how well they resist current flow.
- **Temperature:** The resistance 'R' of a conductor depends on its operating temperature.
- The relationship between resistivity and temperature is inversely proportional.

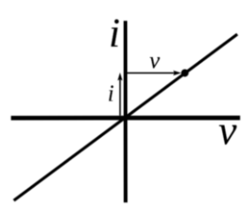
f) Specific Resistance or Resistivity: The specific resistance of a material is defined as the resistance offered by the 1 m length of a conducting material having a cross-sectional area of 1 m².

1.2 Ohm's law:

The electric current passing through the conductor is directly proportional to the potential difference across its ends provided temperature and other physical conditions remain constant.

If current **I** is flowing through the conductor and let **V** be the potential difference between the ends of the conductor,

$$V \propto I \text{ or } I \propto V \quad \Rightarrow V = IR.$$



R is the constant of proportionality, known as resistance.
Current is inversely proportional to the resistance.

Limitations:

1. Ohm's law is applicable only at constant temperature.

2. Ohm's law is also not applicable to non – linear elements. Non-linear elements are those which do not have current exactly proportional to the applied voltage that means the resistance value of those elements changes for different values of voltage and current. Examples of non – linear elements are diodes and thyristors.

1.3 Types of Elements:

A **network element** is the basic building block of an electrical network. The network element is sometimes also called a **circuit element** or **circuit component**.

Based on the behavior of a network element in the circuit, the network elements are classified into the following types:

a) Linear & Non-Linear Elements
b) Active & Passive Elements
c) Bilateral & Unilateral Elements
d) Lumped & Distributed Elements

(a). Linear Elements: Linear elements are the elements that show a linear relationship between voltage and current.

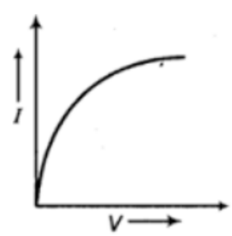
Examples: Resistors, Inductors, and capacitors.

The V-I characteristics of linear elements will be a straight line and always passes through the origin. Linear elements obey ohm's law.

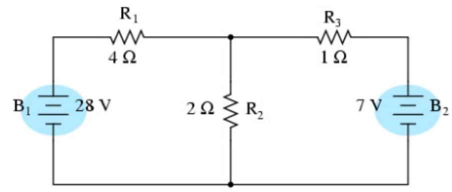
Ex: R, L, C

Non-Linear Elements: Non-Linear Elements are those that do not show a linear relation between voltage and current.

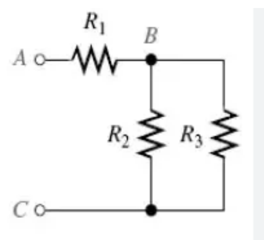
Ex: Diode, Transistors



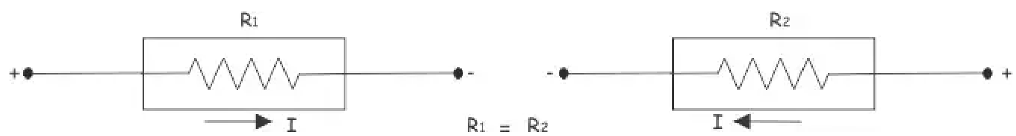
b) Active Elements: The elements which are capable of delivering energy are known as active elements.
Ex: Generators, Batteries



Passive elements: The elements which either absorb or dissipate energy are known as Passive elements.
Ex: R, L, C

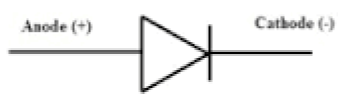


c) Bilateral Elements: are the elements that allow the current in both directions



Examples: Resistors, Inductors and capacitors.

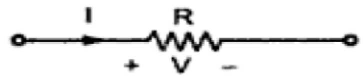
Unilateral Elements: are those that allow the current in only one direction.
Example: Diodes



d) Lumped Elements: The elements which are physically separable from the network are known as lumped elements.
Ex: All physical network elements

Distributed Elements: The elements which are physically not separable from the network are known as distributed elements.
Ex: transmission line parameters.

Basic circuit Elements: Basic circuit elements are
1. Resistor
2. Inductor
3. Capacitor
Resistor: Resistor is having a property called Resistance by which it opposes the flow of current through it.
Resistance is denoted by the symbol 'R'. And is measured in ohms.



The relation between voltage and current for a resistor is given by ohm's law.

$$V = Ri$$

$$\boxed{R = \frac{V}{i}\ \Omega}$$

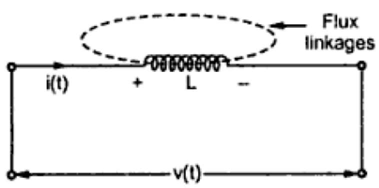
The Power absorbed by a resistor is given by,

$$\boxed{P = vi = \frac{v^2}{R} = i^2R \text{ Watt}}$$

The amount of energy converted into heat energy in time't' is given by

$$\boxed{w = VIt \text{ Joule}}$$

Inductor: An element in which energy is stored in the form of Electro Magnetic field. Inductor is having a property of inductance by which it opposes the flow of alternating current. Inductance is denoted by 'L' and is measured in Henries (H)



When time variying current is flowing through the coil an alternating flux will be produced. Let the the flux produced 'Ψ'

$$\Psi = N\phi \qquad (1)$$

Where N= No of turns and ϕ= flux per turn
As the total flux produced is proportional to current flowing through the coil,
i.e $\Psi \alpha I$

$$\Psi = LI \qquad (2)$$

Where L= Inductance of the coil
Equating (1) & (2), we get

$$N\phi = LI$$

$$\boxed{L = \frac{N\emptyset}{I}}$$

According to faraday's laws of electromagnetic induction, whenever the flux linking with a coil changes then an emf induced across the coil.

$$V = \frac{d\Psi}{dt}$$

$$V = \frac{dLI}{dt}$$

Voltage across inductor is given by

$$\boxed{V = \frac{LdI}{dt} \text{ Volt}}$$

$$V\,dt = LdI$$

$$dI = \frac{1}{L}Vdt$$

Integrate on both sides

$$\int dI = \int \frac{1}{L}Vdt$$

Current flowing through inductor is given by

$$\boxed{I = \frac{1}{L}\int Vdt \;\; Ampere}$$

Energy stored in inductor is given by

$$W = \int Pdt$$

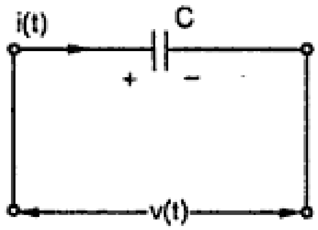
$$W = \int VI\,dt$$

$$W = \int \frac{LdI}{dt}\, I\,dt$$

$$W = \int LI\,dI$$

$$\boxed{W = \frac{1}{2}\,LI^2 \text{ Joules}}$$

Capacitor:



An element in which energy stored in the form of an electro static field is known as capacitor.

It is made up of two conducting plates separated by dielectric material. It is denoted as 'C' and measured in farads (F).

In a capacitor the current through it is proportional to the rate of change of voltage across it.

$$q \propto v$$

$$q = CV$$

Where 'C' is the constant of proportionality known as capacitance.

$$\boxed{C = \frac{q}{v}}$$

The current passing through the capacitor is given by

$$i = \frac{dq}{dt}$$

$$i = \frac{dCv}{dt}$$ Where q=cv

$$\boxed{i = C\frac{dv}{dt}}$$

$$i\,dt = C\,dv$$

Integrate on both sides

$$\int i\,dt = \int C\,dv$$

The voltage across capacitor is given by

$$\boxed{v = \frac{1}{C}\int i\,dt}$$

The power in the capacitor is given by

$$\boxed{P = vi = C\,v\,\frac{dv}{dt}}$$

The energy stored in the capacitor is given by

$$W = \int Pdt = \int C\, v\, \frac{dv}{dt}\, dt$$

$$W = \int C\, v\, dv$$

$$W = C\frac{V^2}{2}$$

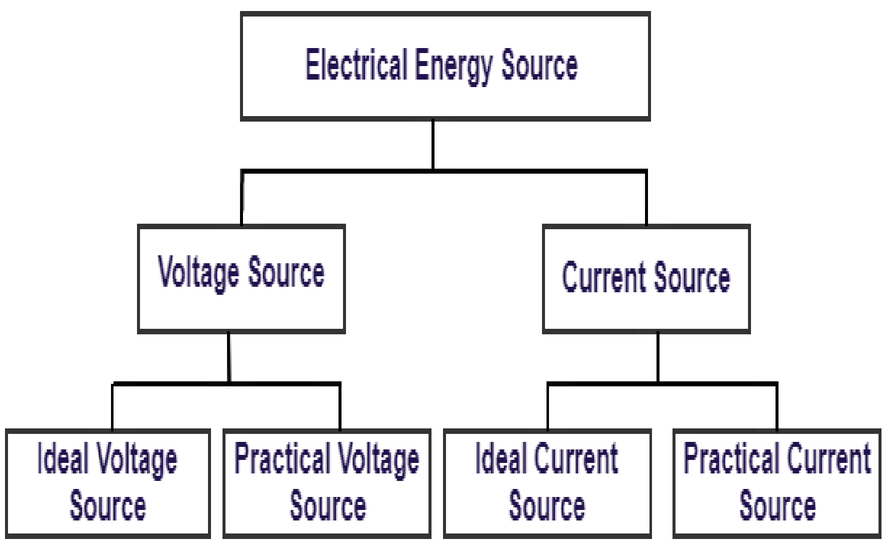
$$\boxed{W = \frac{1}{2}\, CV^2\, Joules}$$

1.4 Types of sources:

In electrical and electronic circuits, some components are used to supply the required electrical energy for the operation of the circuit. These components are known as **energy sources**.
They are classified into two categories

1. Independent Sources
2. Dependant Sources

1. Independent Sources:

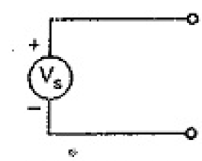


* The device or source which generates and maintains potential energy is called as 'Voltage Source'.

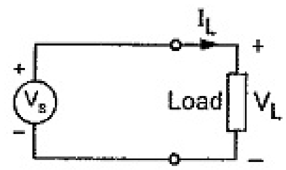
Voltage Sources:

a) Ideal Voltage Source

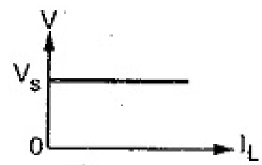
Ideal voltage source is defined as the energy source which gives constant voltage across its terminals irrespective of the current drawn through its terminals. The internal resistance of ideal voltage source is always zero.



a) Symbol



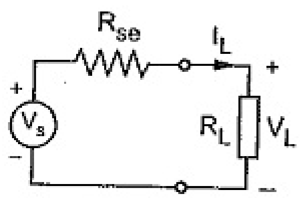
b) Circuit

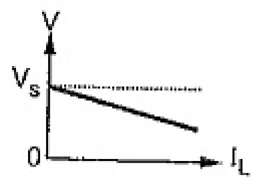


c) V-I Characteristics

b) Practical Voltage Source:

Practically every voltage source has small **internal resistance** shown in series with voltage source and is represented by R_{se} as shown in the Fig





Because of the R_{se} voltage across terminals decreases slightly with increase in current and it is given by expression,

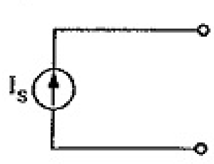
$$V_L = (-R_{se})\, I_L + V_S$$
$$= V_S - I_L R_{se}$$

* The device or source which generates electric current is called as 'Current Source'.

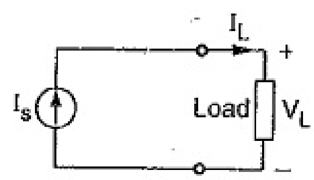
Current Sources:

a) Ideal current Source:

Ideal current source is the source which gives constant current at its terminals irrespective of the voltage appearing across its terminals. The internal resistance of ideal current source is always infinity.



a) Symbol

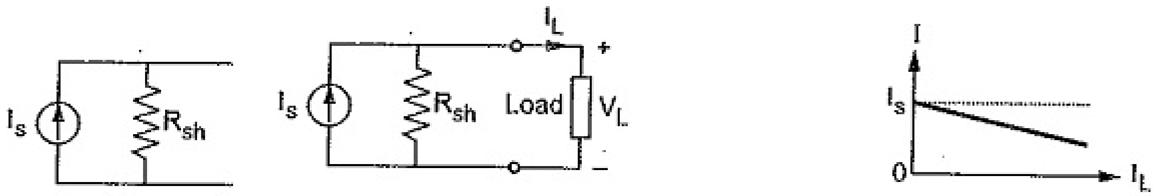


b) Circuit

c) V-I Characteristics

b) Practical Current Source:

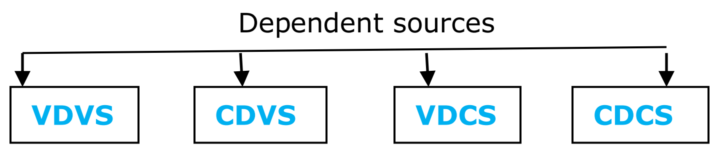
Practically every current source has high internal resistance, connected in parallel with current source and it is represented by R_{sh}.



Because of R_{sh}, current through its terminals decreases slightly with increase in voltage at its terminals.

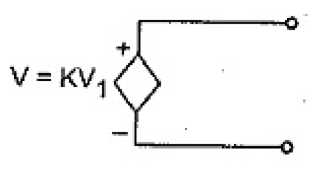
2. Dependant Sources

Dependent sources are those whose value of source depends on voltage or current in the circuit.



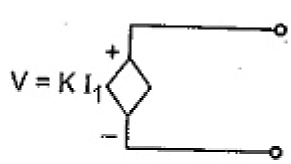
a) Voltage Dependent Voltage Source:

It produces a voltage as a function of voltages elsewhere in the given circuit. This is called VDVS.



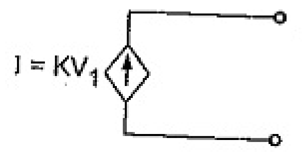
b) Current Dependent Voltage Source:

It produces a voltage as a function of current elsewhere in the given circuit. This is called CDVS.



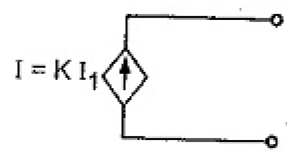
c) Voltage Dependent Current Source:

It produces a current as a function of voltage elsewhere in the given circuit. This is called VDCS.



d) Current Dependent Current Source:

It produces a current as a function of currents elsewhere in the given circuit. This is called CDCS.



1.5 Kirchhoff's Laws:

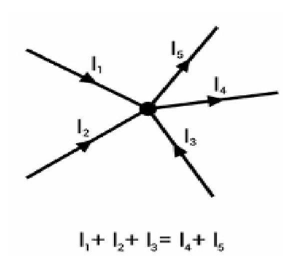
There are two types of Kirchhoff's Circuit Laws, which are,

1) Kirchhoff's Current Law and

2) Kirchhoff's Voltage Law.

1.5.1 Kirchhoff's Current Law (KCL):

Kirchhoff's Current Law states that **the algebraic sum of currents at any junction point of a circuit is equal to zero**. The total current entering a junction is exactly equal to the total current leaving the junction.
Example:



Here, the 3 currents entering the node, I1, I2, and I3 are all positive in value and the 2 currents leaving the node, I4 and I5 are negative in value.

This we can also write it as

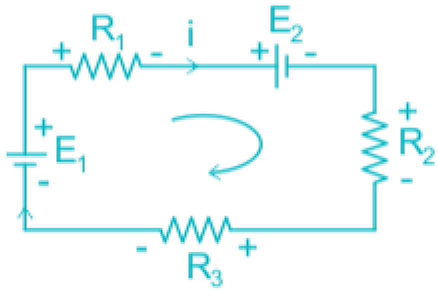
$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

1.5.2 Kirchhoff's Voltage Law (KVL):

Kirchhoff's Voltage Law states "**In any closed loop, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop**" (Or)

The algebraic sum of all voltages within the loop must be equal to zero.

Example: Consider the below circuit



Consider Voltage as a drop (-) ,when the current going from positive (+) to negative (-). (Or)
Consider Voltage as a Gain (+), when the current going from positive (+) to negative (-).

$$-iR1 - E2 - iR2 + E1 = 0$$

$$E1 - E2 = iR1 + iR2$$

1.6 Resistive Networks:

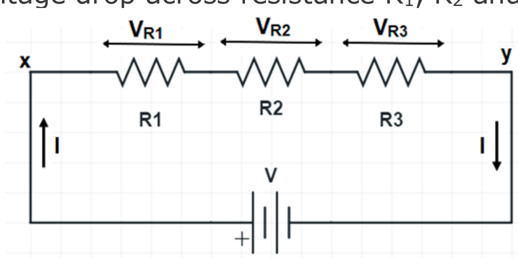
1.6.1 Series Resistive Networks:

When two resistors are connected in an end-to-end manner in a circuit then they are considered to be connected in the series. In a series combination, the flow of current is the same in all resistors

Determination of the Equivalent Resistance of Resistors When Connected in Series:

Let's consider that there are three resistances of values R_1, R_2 and R_3. Connected in a series combination.

Let V_{R1}, V_{R2} & V_{R3} be the voltage drop across resistance R_1, R_2 and R_3 respectively.



According to KVL, Total voltage V across terminal x-y must be equal to the sum of voltage drop across each of the resistances. Hence, we can write,

$$V = V_{R1} + V_{R2} + V_{R3} \qquad \text{-------------- } 1$$

According to Ohm's law **V=IR**

Therefore $V_{R1}=IR1$, $V_{R2}=IR2$, $V_{R3}=IR3$

Substitute the values in equation 1

$$V=IR1+IR2+IR3$$

$$IR_{eq}=I\ (R1+R2+R3)$$

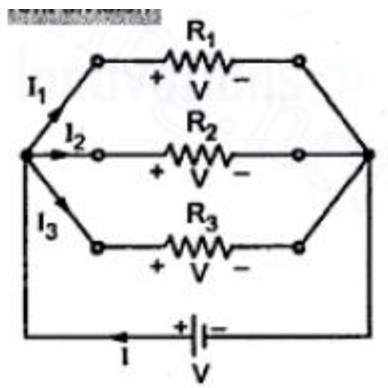
$$\boxed{R_{eq}= R1+R2+R3}$$

The equivalent resistance of three resistors connected in series is the sum of the individual resistances.

1.6.2 Parallel Resistive Networks:

When resistors are connected across each other in a circuit, then those resistors are considered to be connected in a parallel combination. In a parallel combination, the current flow is divided among the resistors, while the voltage drop across each resistor is the same.

Determination of the Equivalent Resistance of Resistors When Connected in Parallel:



In the parallel connection shown above, the three resistances R1, R2 & R3 are connected in parallel & combination is connected across a voltage source V. Voltage across each resistor is same i.e. V. Let I1, I2 & I3 are the currents passing through R1, R2,& R3.

$$V = I_1 R_1\ ,\ V = I_2R_2\ ,\ V = I_3R_3$$

$$I_1 = \frac{V}{R_1}\ ,\ I_2 = \frac{V}{R_2}\ ,\ I_3 = \frac{V}{R_3}$$

According to KCL, The sum of incoming currents is equal to the outgoing currents.

$$I = I_1 + I_2 + I_3$$

$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$= V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

For overall circuit if ohm's law is applied $V= IR_{eq}$

$$I = \frac{V}{R_{eq}}$$

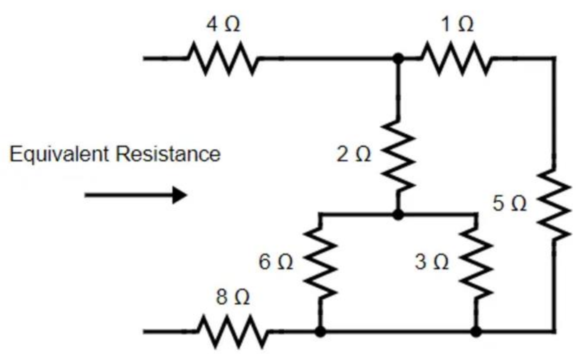
Where $\mathbf{R_{eq}}$ is the total resistance of the circuit

$$\frac{V}{R_{eq}} = V\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]$$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

⇨ Problems:

1. Find the equivalent resistance of the given circuit.



Solution:

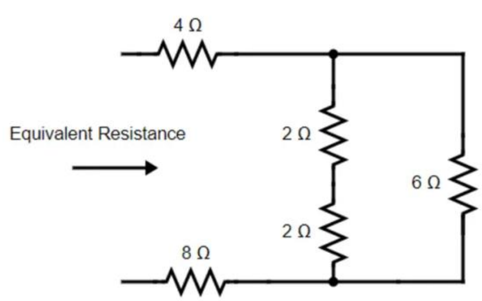
To get the Equivalent Resistance we combine resistors in series and in parallel. Here, **6Ω** and **3Ω** are in parallel. So, the equivalent resistance is given as

$$\frac{6 \times 3}{6+3} = 2\Omega$$

Also, the **1Ω** and **5Ω** resistors are in series. Hence the equivalent resistance will be given as,

$$1\Omega + 5\Omega = 6\Omega$$

The circuit will become



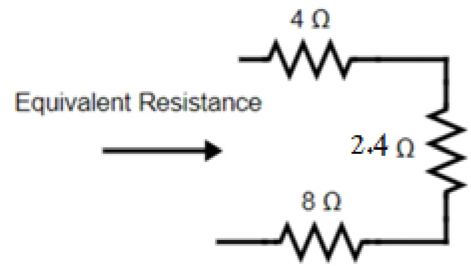
After reduction, we now notice, **2Ω** and **2Ω** are in series, so the equivalent resistance

$$2\Omega + 2\Omega = 4\Omega$$

This **4Ω** resistor is now in parallel with **6Ω** the resistor. So, their equivalent resistance will be given as

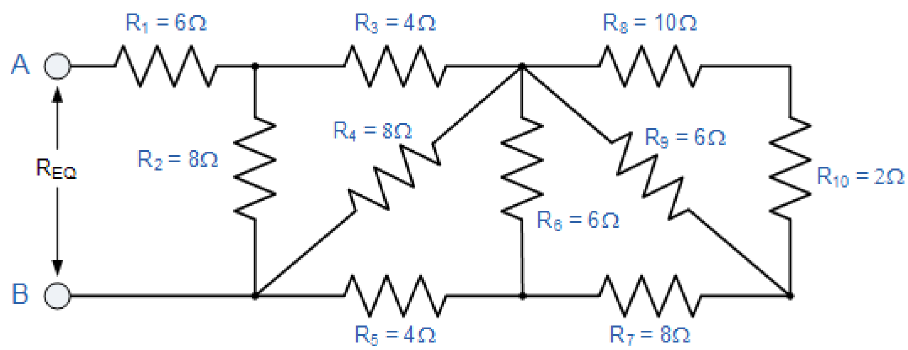
$$\frac{4 \times 6}{4+6} = 2.4\Omega$$

Now replacing the above circuit with appropriate values, the three resistors will be in series. So, the final equivalent resistance is given as



$$R_{eq} = 4\Omega + 2.4\Omega + 8\Omega = 14.4\Omega$$

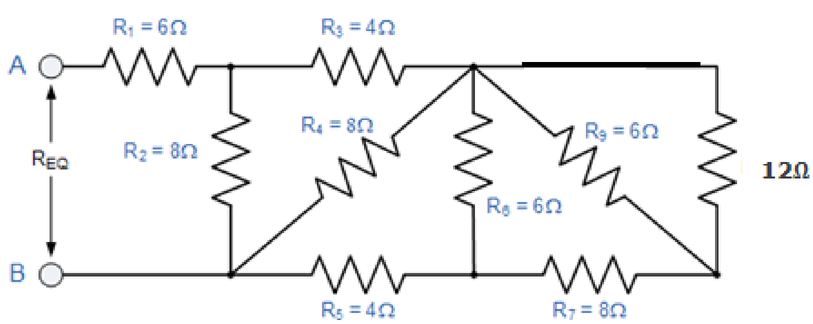
2. Find the equivalent resistance of the given circuit.



Solution:

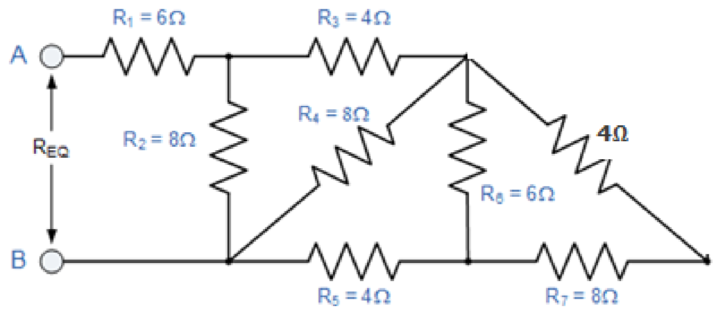
R8 & R10 are in series. Then equivalent of the series connected resistors are

$$\mathbf{R_{eq} = 10+2 = 12\Omega}$$



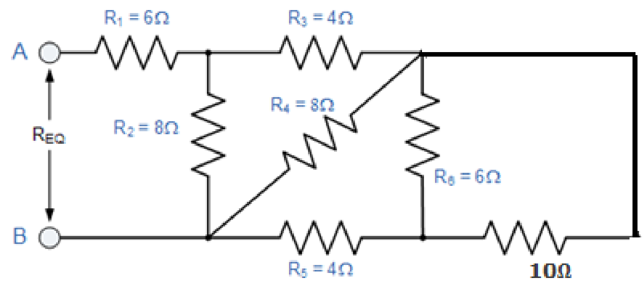
12Ω and 6Ω are in parallel. Then equivalent of the parallel combination is

$$\mathbf{R_{eq}} = \frac{(12*6)}{(12+6)} = \mathbf{4\Omega}$$



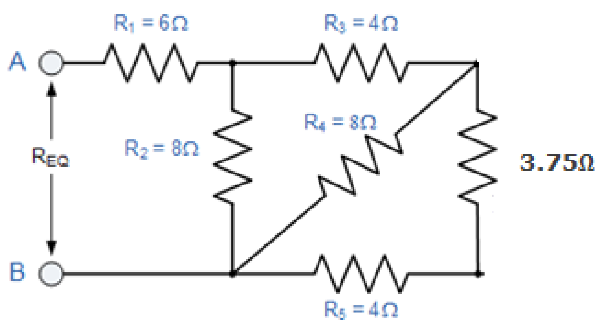
4Ω and 8Ω are in series Then equivalent of the series connected resistors are

R_{eq}= 4+8= 10Ω



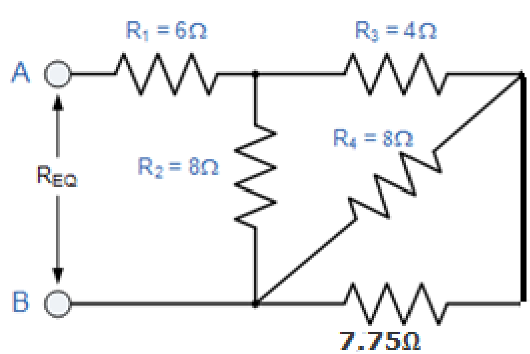
10Ω and 6Ω are in parallel. Then equivalent of the parallel combination is

$$R_{eq} = \frac{(10*6)}{(10+6)} = 3.75\Omega$$



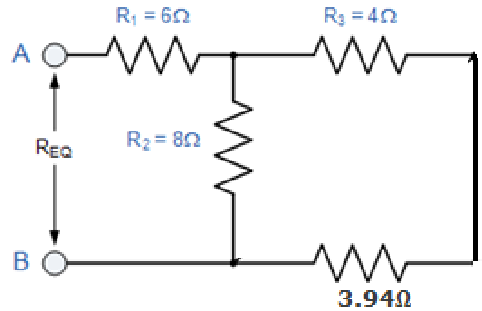
4Ω and 3.75Ω are in series Then equivalent of the series connected resistors are

R_{eq}= 4+3.75= 7.75Ω

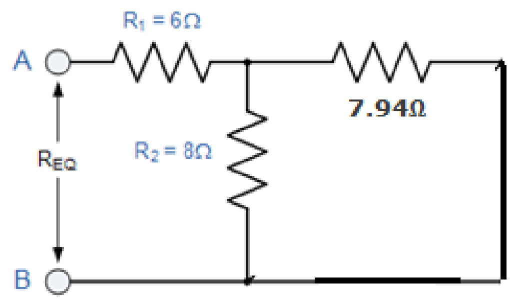


8Ω and 7.75Ω are in parallel. Then equivalent of the parallel combination is

$$\mathbf{R_{eq}} = \frac{(8*7.75)}{(8+7.75)} = \mathbf{3.94\Omega}$$

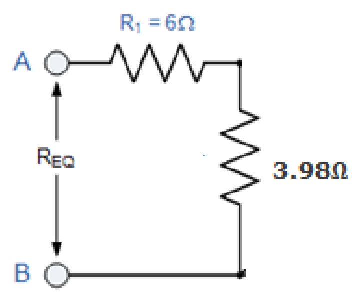


4Ω and 3.94Ω are in series Then equivalent of the series connected resistors are $\mathbf{R_{eq}}$= 4+3.94= 7.94Ω

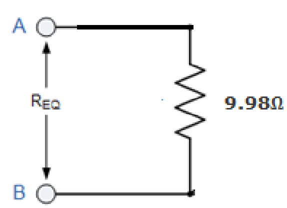


8Ω and 7.94Ω are in parallel. Then equivalent of the parallel combination is

$$\mathbf{R_{eq}} = \frac{(8*7.94)}{(8+7.94)} = \mathbf{3.98\Omega}$$



6Ω and 3.98Ω are in series Then equivalent of the series connected resistors are $\mathbf{R_{eq}}$= 6+3.98= 9.98Ω



Total equivalent resistance of the circuit $\mathbf{R_{EQ}}$= 9.98Ω

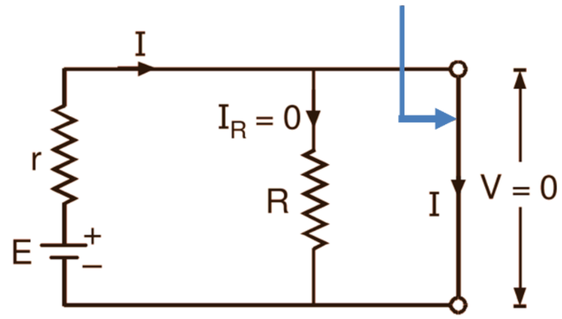
Note*

SHORT CIRCUIT:

When any two points in a circuit are joined directly to each other with thick metallic conducting wire, the two points are said to be short circuited. The resistance of such short circuit is zero.(R=0)

The current flowing through a short circuit is infinite. (As $I=\frac{V}{R}=\frac{V}{0}=\infty$)

And the voltage across short circuit is **V=0**

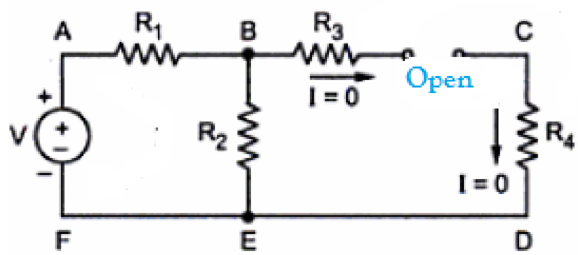


Since the current always prefers the path of low resistance, all the current will flow through the short circuited path and no current will flow through resistor R.

OPEN CIRCUIT:

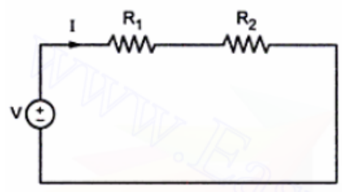
When there is no connection between the two points of a network, having some voltage across the two points then the two points are said to be open circuited. The resistance of the open circuit is **∞**.

The current flowing through open circuit = 0



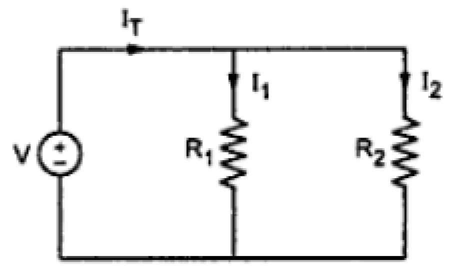
There is no current flowing through **R3** and **R4** elements.

Voltage division in series circuit of Resistors:



$V_{R1} = \left[\frac{R_1}{R_1+R_2}\right]V$, $V_{R2} = \left[\frac{R_2}{R_1+R_2}\right]V$

Current division in Parallel circuit of Resistors:

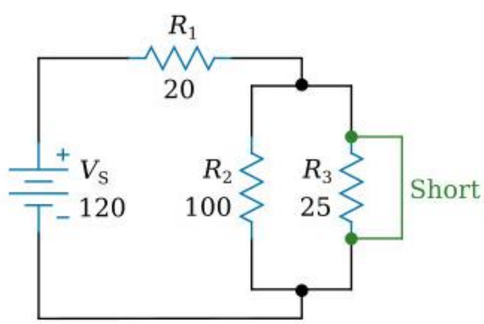


$$I_1 = \left[\frac{R_2}{R_1+R_2}\right] I_T \,, \quad I_2 = \left[\frac{R_1}{R_1+R_2}\right] I_T$$

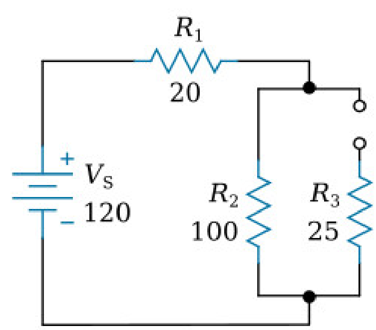
Problem:

1. Find the equivalent resistance of the below networks

i)



ii)



Solution:

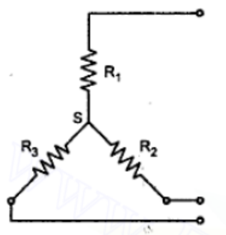
i) In the first network above, R_3 has shorted. With R_3 shorted there is a short circuit in parallel with R_2. The short circuit routes the current around R_2, effectively removing R_2 from the circuit. Total circuit resistance is now equal to the resistance of R_1, or **20 ohms.**

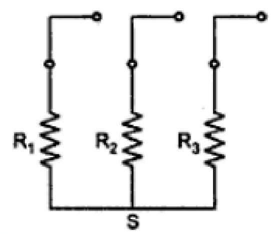
ii) In the second network given, an open is shown in the parallel branch of R_3. There is no path for current through R_3. In the circuit, current flows through R_1 and R_2 only. Since there is only one path for current flow, R_1 and R_2 are effectively in series. Total circuit resistance is now equal to (20+100) =**220Ω**

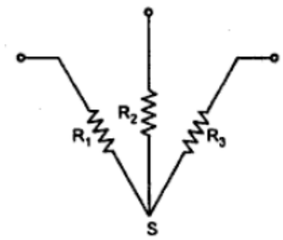
1.7 STAR & DELTA connection of resistances:

STAR Connection:

If the three resistors are connected in such a manner that one end of each is connected together to form a junction point called star point, then the resistances are said to be in star connection.

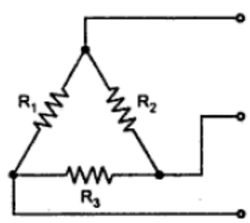


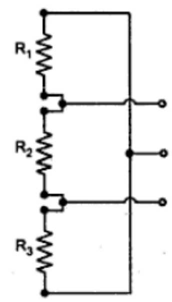


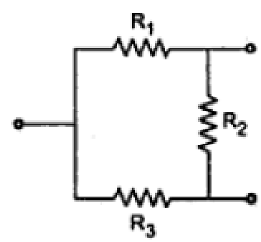


DELTA Connection:

If the three resistors are connected in such a manner that one end of the first is connected to first end of the second, and the second end of second to first end of third and so on to complete a loop then the resistors are said to be in Delta.

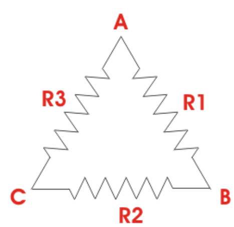


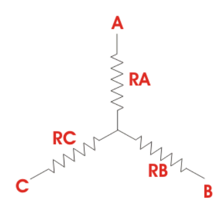




1.7.1 Delta – Star Transformation:

The replacement of delta by equivalent star connection is known as **delta – star transformation**.





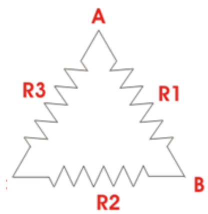
Consider three ressitances $\mathbf{R_1,R_2,R_3}$ connected in Delta connection. The terminals between which are conneted in Delta are named as A,B,C.

Now it is always possible to replace these Delta connected resistors by three equivalent star connected resistors $\mathbf{R_A, R_B, R_C}$ between the same terminals.

Now to call these two arangements as equivalent, the resistance between any two terminals must be same in both the type of connections.

⇨ **Consider the terminals A & B** then find the equivalent resistance between them in delta as well as star.

Delta connection

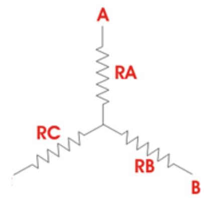


Resistances R_2,R_3 are in series and it is in parallel with R_1

The resistance between the points A and B will be

$$R_{AB} = \frac{R1(R2+R3)}{R1+R2+R3}$$

Star Connection



Resistances R_A,R_B are in series & R_C is negelected as it is not connected to any terminal

the resistance between the points A and B in star will be

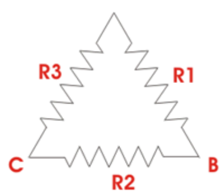
$$R_{AB} = R_A+R_B$$

Since the two systems are equivalent, resistance measured between terminals A and B in both systems must be equal.

$$\frac{R1(R2+R3)}{R1+R2+R3} = R_A+R_B \quad \text{-------} \ 1$$

⇨ **Consider the terminals B & C** then find the equivalent resistance between them in delta as well as star.

Delta connection

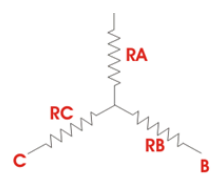


Resistances R_1,R_3 are in series and it is in parallel with R_2.

The resistance between the points B and C will be.

$$R_{BC} = \frac{R2(R1+R3)}{R1+R2+R3}$$

Star Connection



Resistances R_B,R_C are in series & R_A is negelected as it is not connected to any terminal.

the resistance between the points B and C in star will be.

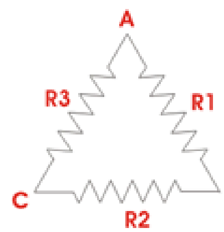
$$R_{BC} = R_B+R_C$$

Since the two systems are equivalent, resistance measured between terminals B and C in both systems must be equal.

$$\frac{R2(R1+R3)}{R1+R2+R3} = R_B+R_C \quad \text{-------} \ 2$$

⇨ **Consider the terminals A & C** then find the equivalent resistance between them in delta as well as star.

Delta connection

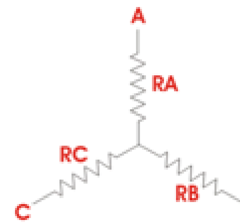


Resistances R_1,R_2 are in series and it is in parallel with R_3

The resistance between the points A and C will be

$$R_{AC} = \frac{R3(R1+R2)}{R1+R2+R3}$$

Star Connection



Resistances R_A,R_C are in series & R_B is negelected as it is not connected to any terminal

the resistance between the points A and C in star will be

$$R_{AC} = R_A+R_C$$

Since the two systems are equivalent, resistance measured between terminals A and B in both systems must be equal.

$$\frac{R3(R1+R2)}{R1+R2+R3} = R_A+R_C \quad \text{-------} \; 3$$

Adding equations 1, 2 and subtract 3 we get,

$$\frac{R1(R2+R3)}{R1+R2+R3} + \frac{R2(R1+R3)}{R1+R2+R3} - \frac{R3(R1+R2)}{R1+R2+R3} = R_A+R_B+R_B+R_C-R_A-R_C$$

$$2R_B = \frac{2R1R2}{R1+R2+R3}$$

$$\boxed{R_B = \frac{R1R2}{R1+R2+R3}}$$

Similarly

$$\boxed{R_C = \frac{R2R3}{R1+R2+R3}} \qquad \boxed{R_A = \frac{R1R3}{R1+R2+R3}}$$

1.7.2 Star - Delta Transformation:

From Delta – Star conversion derivation,

$$R_A = \frac{R1R3}{R1+R2+R3}, \quad R_B = \frac{R1R2}{R1+R2+R3}, \quad R_C = \frac{R2R3}{R1+R2+R3}$$

Now multiply R_A&R_B, R_B&R_C, R_A&R_C

$$\mathbf{R_A R_B} = \frac{R1R3}{R1+R2+R3}\;\frac{R1R2}{R1+R2+R3} \quad \text{--------------1}$$

$$\mathbf{R_B R_C} = \frac{R1R2}{R1+R2+R3}\;\frac{R2R3}{R1+R2+R3} \quad \text{--------------2}$$

$$\mathbf{R_A R_C} = \frac{R1R3}{R1+R2+R3}\;\frac{R2R3}{R1+R2+R3} \quad \text{--------------3}$$

Adding all the three equations,

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_1.{R_2}^2.R_3 + R_1.R_2.{R_3}^2 + {R_1}^2.R_2.R_3}{(R_1+R_2+R_3)^2}$$

$$= \frac{R_1.R_2.R_3(R_1+R_2+R_3)}{(R_1+R_2+R_3)^2}$$

$$= \frac{R_1.R_2.R_3}{R_1+R_2+R_3}. \quad \text{--------4}$$

From the equation of $\mathbf{R_C} = \frac{R2R3}{R1+R2+R3}$

Substitute it in equation 4

$\mathbf{R_A R_B + R_B R_C + R_A R_C = R1 * R_C}$

We will get

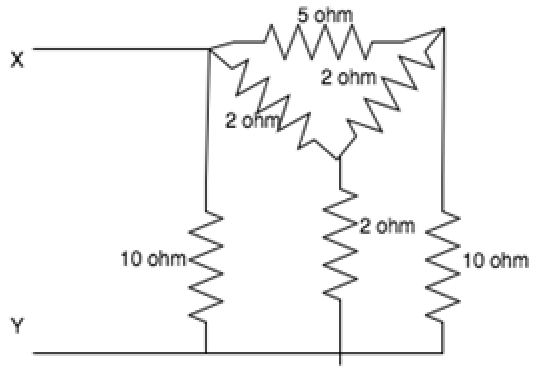
$$\boxed{R1 = \frac{RARB+RBRC+RARC}{RC}}$$

Similarly

$$\boxed{R2 = \frac{RARB+RBRC+RARC}{RA}} \qquad \boxed{R3 = \frac{RARB+RBRC+RARC}{RB}}$$

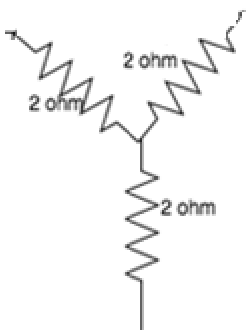
Problems:

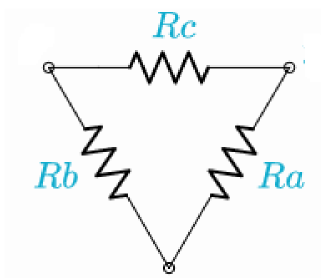
1. Find the equivalent resistance between X & Y



Solution:

In the given network the three 2Ω resistors are connected in the form of Star. Convert the star into Delta.

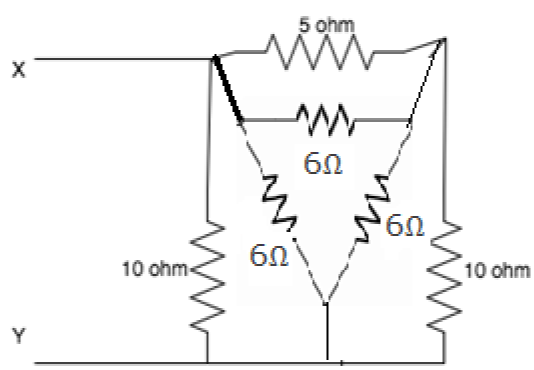




$$R_A = \frac{(2*2)+(2*2)+(2*2)}{2} = 6\Omega$$

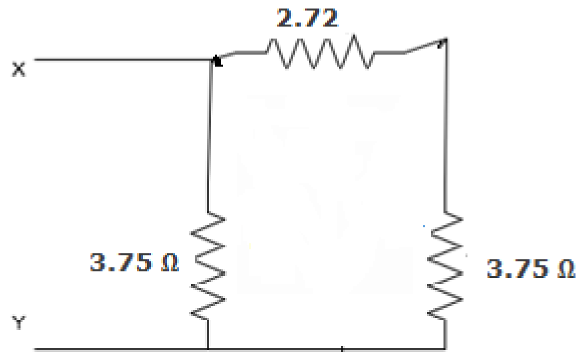
$$R_b = \frac{(2*2)+(2*2)+(2*2)}{2} = 6\Omega \quad R_c = \frac{(2*2)+(2*2)+(2*2)}{2} = 6\Omega$$

Now replace the star network with delta



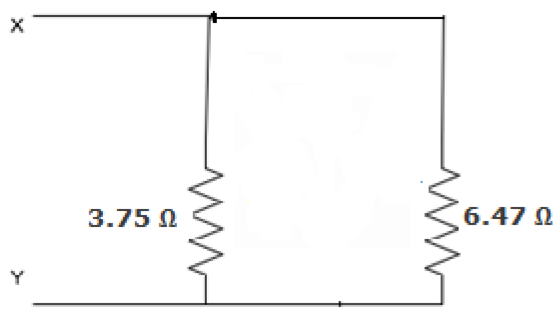
In the above network **5Ω, 6Ω** are in parallel, **10Ω, 6Ω** are in parallel.

Then equivalent is $\frac{(5*6)}{5+6}$ **= 2.72 Ω,** $\frac{(10*6)}{10+6}$ **= 3.75 Ω**



Now 2.72 & 3.75 Ω are in series

2.72+3.75 = 6.47 Ω



Now 3.75 Ω & 6.47 Ω are in Parallel

$$\frac{(3.75*6.47)}{3.75+6.47} = 2.37\ \Omega$$

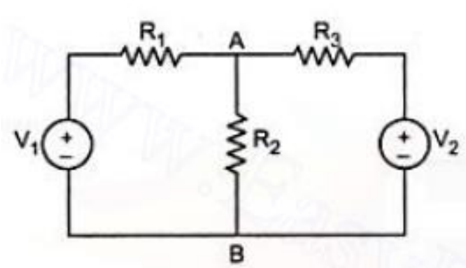
Therefore the equivalent resistance between X &Y is **2.37 Ω**

1.8 SUPERPOSITION THEOREM:

Superposition theorem states that in any linear, active, bilateral network having more than one source, the response across any element is the algebraic sum of the responses obtained from each source considered individually and all other sources are remaining inactive.

PROCEDURE:

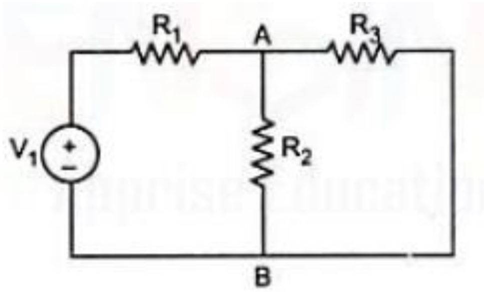
Consider a network shown below having two sources V1 & V2



Let us calculate the current in branch **A-B**

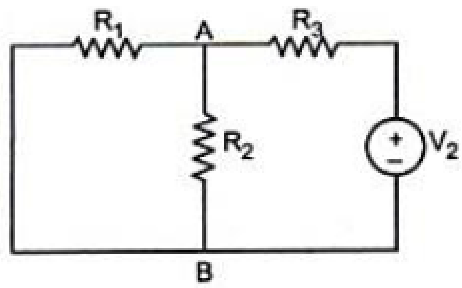
STEP 1: According to superposition theorem, consider each source independently.

Let source V1 acting independently and the source V2 must be replaced by short circuit. Hence circuit becomes, as shown in figure below



Using any network reduction method, obtain the value of current in branch A-B.
i.e $\mathbf{I_{AB}}$ due to source **V1** alone.

STEP 2: Now consider source V2 alone, and source V1 replaced by short circuit. Hence circuit becomes, as shown in figure below.



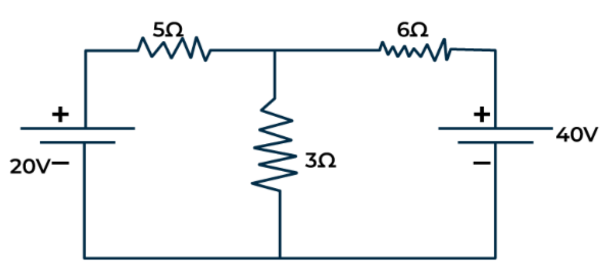
Using any network reduction method, obtain the value of current in branch A-B.
i.e $\mathbf{I_{AB}}$ due to source **V2** alone.

STEP 3: According to superposition theorem, the total current through branch A-B is the sum of the currents through branch A-B produced by each source acting independently.

$$I_{AB} = I_{AB} \text{ due to } V_1 + I_{AB} \text{ due to } V_2$$

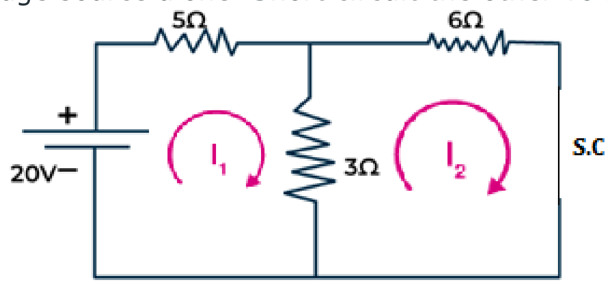
Problem:

Find the current through 3 Ω resistor using superposition theorem.



Solution:

Consider the 20 V voltage source alone. Short circuit the other voltage source.



Apply KVL to loop 1

$$+5I_1+3(I_1-I_2)-20=0$$

$$8I_1-3I_2=20 \quad \text{---------1}$$

Apply KVL to loop 2

$$6I_2+3(I_2-I_1)=0$$

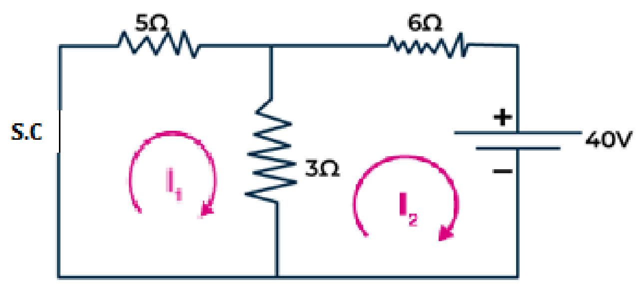
$$-3I_1+9I_2=0 \quad \text{---------2}$$

Solving equations 1 &2

$I_1 = 2.857$ **A** $\qquad$ $I_2=0.952$ **A**

The current passing through **3Ω** due to **20V** source= $I_1-I_2 = 2.857-0.952 = 1.905$ **A**

Consider the 40 V voltage source alone. Short circuit the other voltage source.



Apply KVL to loop 1

$$+5I_1+3(I_1-I_2)=0$$

$$8I_1-3I_2=0 \quad \text{---------1}$$

Apply KVL to loop 2

$$6I_2+40+3(I_2-I_1)=0$$

$$-3I_1+9I_2=-40 \quad \text{---------2}$$

Solving equations 1 &2

$I_1 = -1.904$ **A** $\qquad$ $I_2= -5.079$ **A**

The current passing through **3Ω** due to **40V** source= $I_1-I_2 = -1.904 - (-5.079) = 3.175$ **A**

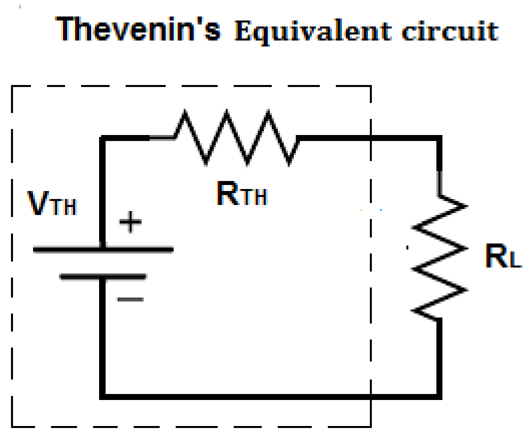
According to superposition theorem, the total current through **3Ω** is the sum of the currents through **3Ω** produced by each source acting independently.

Current passing through **3Ω,** $\quad$ **I3Ω = I3Ω due to 20V + I3Ω due to 40V**

= 1.905 + 3.175 = 5.08 A

1.9 Thevenin's Theorem:

Thevenin's theorem states that any linear circuit containing several voltage sources and resistors can be simplified to a Thevenin-equivalent circuit with a single voltage source called open circuit voltage and a resistance called Thevenin equivalent resistance connected in series with a load.



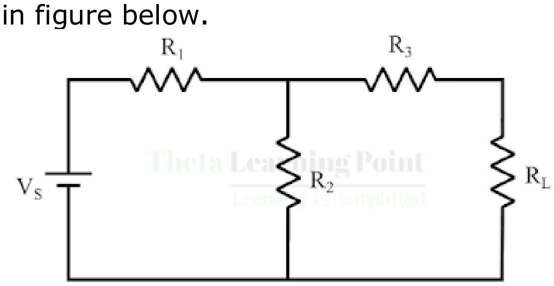
- Here**, Vth** represents the Thevenin voltage; **Rth** denotes the Thevenin resistance.
- The Thevenin voltage VTh is the open circuit voltage at the load terminals.
- Thevenin resistance RTh is the equivalent resistance looking into the load terminals.

The electric current through the load resistor will be,

$$I_L = \frac{V_L}{R_{Th} + R_L}$$

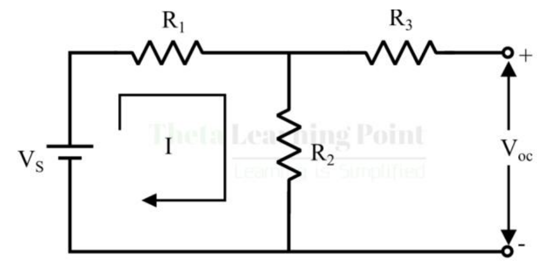
PROCEDURE:

We can explain Thevenin's theorem with the help of an electric circuit. For that let us consider a simple DC circuit as shown in figure below.



In this circuit, we are going to find the electric current $\mathbf{I_L}$ through the load resistor $\mathbf{R_L}$ by using Thevenin's theorem.

STEP 1: Remove the load resistor (R_L) and calculate the open circuit voltage (V_{oc}) across the open-circuited load terminals.

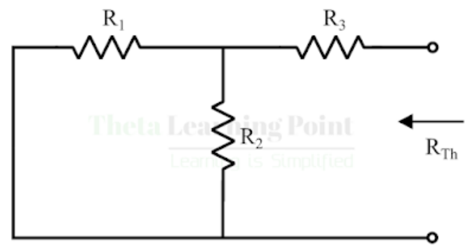


$$V_{oc} = IR_2$$

$$V_{oc} = \left(\frac{V_s}{R_1 + R_2}\right) R_2$$

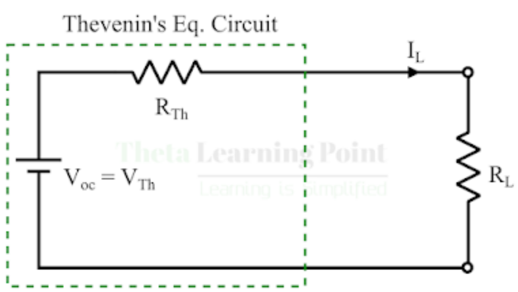
The open-circuited voltage $\mathbf{V_{oc}}$ is also called **Thevenin's voltage** and is denoted by $\mathbf{V_{Th}}$.

STEP 2: In order to find R_{Th}, remove all the sources by making voltage sources short circuit and current sources open circuit.



$$R_{Th} = \left(\frac{R_1 R_2}{R_1 + R_2}\right) + R_3$$

STEP 3: Obtain the Thevenin's equivalent circuit by connecting Thevenin's resistance R_{Th} in series with the open circuit voltage V_{oc}.

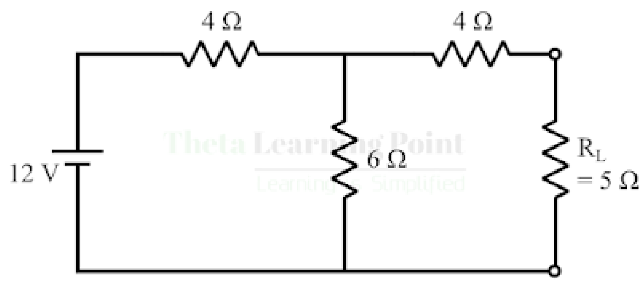


STEP 4: Calculate $\mathbf{I_L}$

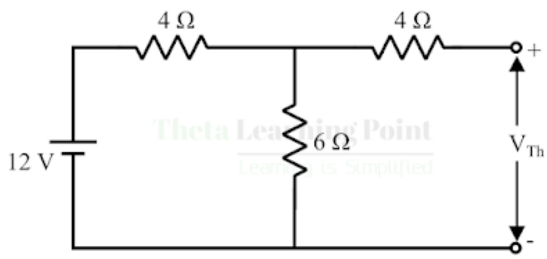
The electric current through the load resistor will be,

$$I_L = \frac{V_L}{R_{Th} + R_L}$$

⇨ **Problem: Find the current through the load resistor R_L = 5 Ω using Thevenin's Theorem.**

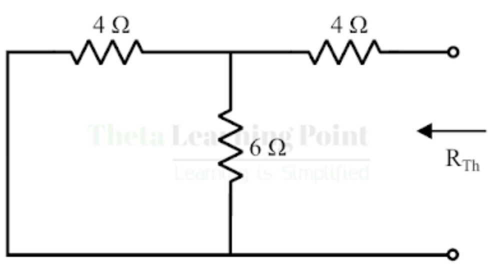


Solution: Remove the load resistance R_L, and calculate the voltage across the open-circuited load terminals.



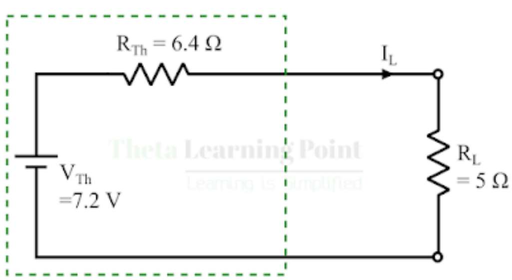
$$V_{Th} = 12 \times \frac{6}{4+6} = 7.2 \quad \text{V}$$

Deactivate the voltage source of 12 volts, and find Thevenin's resistance R_{Th}.



$$R_{Th} = \left(\frac{4 \times 6}{4+6}\right) + 4 = 6.4\Omega$$

Replace the network with its Thevenin equivalent circuit and connect the load resistance R_L, and calculate the load current.



$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$\Rightarrow I_L = \frac{7.2}{6.4+5} = 0.63 \quad \text{A}$$