Vidya Jyothi Institute of Technology
(AN AUTONOMOUS INSTITUTION)

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I B.Tech II Semester (24-25)
QUESTION BANK
MATHEMATICS-II
(ORDINARY DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS)

UNIT-I: First Order Ordinary Differential Equations and their Applications

	UNIT-I Very Short Answer Questions
1	Define solution, general solution and particular solution of a differential equation
2	Define exact differential equation
3	Define integrating factor
4	Define orthogonal trajectories of the family of curves
5	State Newton's law of cooling
6	State law of natural growth and decay
7	Solve $(x^2-ay)dx+(y^2-ax)dy=0$
8	Solve $(2x+e^y)dx+xe^y dy=0$
9	Solve $xdy-ydx+2x^3dx=0$
10	Solve $xdy-ydx+a(x^2+y^2)dy=0$
11	Find an integrating factor of $y(xy\sin xy+\cos xy)dx+x(xy\sin xy-\cos xy)dy=0$
12	Find an integrating factor of $(5x^3+12x^2+6y^2)dx+6xydy=0$
13	Find an integrating factor of $x\cos x\frac{dy}{dx}+(x\sin x+\cos x)y=1$
14	Reduce the differential equation $\frac{dy}{dx}+\left(\frac{y}{x}\right)\log y=\frac{y}{x}(\log y)^2$ into linear form
15	Find the orthogonal trajectories of the family of straight lines $y=ax$
16	Find the orthogonal trajectories of the family of curves $r=ae^{\theta}$
17	If $(\alpha xy^3+y\cos x)dx+(3x^2y^2+\beta\sin x)dy=0$ is exact then $\alpha,\beta=$ ___________ A) $\alpha=2,\beta=1$ B) $\alpha=1,\ \beta=2$ C) $\alpha=3,\beta=1$ D) $\alpha=1,\beta=3$
18	Which of the following is not an integrating factor of $ydx-xdy=0$? A) $\frac{1}{x^2}$ B) $\frac{1}{x^3}$ C) $\frac{1}{y^2}$ D) $\frac{1}{xy}$
19	The orthogonal trajectories of the family of parabolas $y^2=4a(x+a)$ is __________ A) $y^2=4b(y+b)$ B) $y^2=4b(x+b)$ C) $x^2=4b(x+b)$ D) $x^2=4b(y+b)$
20	The orthogonal trajectories of the family of curves $r^n=a^n\sin n\theta$ (a is parameter) is _______ A) $r^n=b^n\cos n\theta$ B) $r^n\sin n\theta=b^n$ C) $r^n=b^n\sin n\theta$ D) $r^n\cos n\theta=b^n$

UNIT-I Long Answer Questions	
1	Solve the following differential equations: (*i*) $\frac{dy}{dx}+\frac{y\cos x+\sin y+y}{\sin x+x\cos y+x}=0$ (*ii*) $(y^2e^{xy^2}+4x^3)dx+(2xye^{xy^2}-3y^2)dy=0$ (*iii*) $\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx+\left(x+\log x-x\sin y\right)dy=0$ (*iv*) $\left(2xy+y-\tan y\right)dx+\left(x^2-x\tan^2 y+\sec^2 y\right)dy=0$
2	Solve the following differential equations: (*i*) $(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$ (*iii*) $\left(x^2y^2+xy+1\right)ydx+\left(x^2y^2-xy+1\right)xdy=0$ (*iii*) $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$ (*iv*) $(4xy+3y^2-x)dx+x(x+2y)dy=0$
3	Solve the following differential equations: (*i*) $x\log x\frac{dy}{dx}+y=\log x^2$ (*ii*) $(1+y^2)dx=(\tan^{-1}y-x)dy$ (*iii*) $2y'\cos x+4y\sin x=\sin 2x, y(0)=\frac{\pi}{3}$ (*iv*) $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1$
4	Solve the following differential equations: (*i*) $2xy'=10x^3y^5+y$ (*ii*) $\frac{dy}{dx}=y\tan x-y^2\sec x$ (*iii*) $\frac{dy}{dx}-\frac{\tan y}{1+x}=(1+x)e^x\sec y$ (*iv*) $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$
5	Find the orthogonal trajectories of the following family of curves: (*i*) $\frac{x^2}{a^2}+\frac{y^2}{a^2+\lambda}=1$ (λ is the parameter) (*ii*) $x^2+y^2+2gx+c=0$ (g is the parameter) (*iii*) $r=a\left(1-\cos\theta\right)$ (*iv*) $r^n=a^n\cos n\theta$ (a is the parameter)
6	Show that the family of confocal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self orthogonal, λ being parameter
7	(*i*) A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original? (*ii*) If the temperature of air is 30°C and the substance cools from 100°C to 70°C in 15 minutes, find when the temperature will be 40°C.
8	(*i*) A cake is removed from an oven at 210°F and is placed in a room of constant temperature 70°F. After 30 minutes, the temperature of the cake is 150°F. When will the temperature of the cake be 120°F? (*ii*) If a substance cools from 370K to 330K in 10 minutes when the temperature of the surrounding air is 290K, find the temperature of the substance after 40 minutes. Also find when the temperature of the substance will become 310K.
9	(*i*) The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after $1\frac{1}{2}$ hours? (*ii*) The rate at which bacteria multiply is proportional to the instantaneous number present. If the original number doubles in 2 hours, in how many hours will it triple?
10	(*i*) Radium decomposes at a rate proportional to the amount present. If a fraction p of the original amount disappears in 1 year, how much will remain at the end of 21 years? (*ii*) If 30% of a radioactive substance disappears in 10 days, how long will it take for 90% of it to disappear?

UNIT-II: Higher Order Linear Differential Equations

UNIT-II Very Short Answer Questions	
1	Define linear differential equation
2	Solve $\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+11\frac{dy}{dx}-6y=0$
3	Solve $(D^2-4D+1)y=0$
4	Solve $\frac{d^4y}{dx^4}-4\frac{d^3y}{dx^3}+10\frac{d^2y}{dx^2}-12\frac{dy}{dx}+5y=0$
5	Find the complementary function of $(D^2+1)^2\,y=\cos x$
6	Find the complementary function of $(D^3+D^2+4D+4)y=e^x+\sin 2x$
7	Find the complementary function of $(D^4+4)y=e^x\cos x$
8	Find the particular integral of $(D-1)^3=6e^x$
9	Find the particular integral of $(D^3+4D)y=\sin 2x$
10	Find the particular integral of $(D^4-5D^2+4)y=10\cos x$
11	Find the particular integral of $(D-1)^2y=x^2$
12	Find the particular integral of $(D-1)^2\,y=6xe^x$
13	Find the particular integral of $(D+1)^4\,y=e^{-x}\cos x$
14	Find the Wronskian of the functions $e^x,\ e^{2x}$
15	Reduce $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+4y=\log x\cos(2\log x)$ into linear differential equation with constant coefficients
16	Reduce $x^3y'''-x^2y''-6xy'+18y=x^3$ into linear differential equation with constant coefficients
17	The set of linearly independent solutions of $(D^4-D^2)y=0$ is____________ A) $\{1,\ x,\ x^2,e^x\}$ B) $\{1,\ x,\ xe^{-x},\ e^x\}$ C) $\{1,\ x,\ e^x,\ e^{-x}\}$ D) $\{e^x,\ e^{-x},\ xe^x,\ xe^{-x}\}$
18	The particular integral of $(D^4-2D^3+2D^2-2D+1)y=\cos x$ is ____________ A) $x\cos x$ B) $\frac{x}{2}\cos x$ C) $\frac{x}{4}\cos x$ D) $\frac{x}{6}\cos x$
19	The particular integral of $(D^2-2D+2)y=\log 4$ is ____________ A) $\log 2$ B) $\log 4$ C) $2\log 4$ D) $4\log 4$
20	The particular integral of $(D^2-a^2)y=\cosh ax$ is ____________ A) $\frac{x}{2a}\cosh ax$ B) $-\frac{x}{2a}\sinh ax$ C) $\frac{x}{2a}\sinh ax$ D) $-\frac{x}{2a}\cosh ax$
UNIT-II Long Answer Questions	
1	Solve the following differential equations: (*i*) $(D^3-6D^2+11D-6)y=e^{-2x}+e^{-3x}$ (*ii*) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=(1-e^x)^2$ (*iii*) $y''-4y'+3y=4e^{3x},y(0)=-1,y'(0)=3$ (*iv*) $(D^3-5D^2+7D-3)y=e^{2x}\cosh x$
2	Solve the following differential equations: (*i*) $(D^3+1)y=\cos(2x-1)$ (*ii*) $y''+4y'+4y=3\sin x+4\cos x,y(0)=1,y'(0)=0$

3	Solve the following differential equations: (*ii*) $(D^2+D+1)y=x^3$ (*ii*) $(D^2+1)^2y=x^4+2\sin x\cos 3x$
4	Solve the following differential equations: (*i*) $(D-2)^2y=8(e^{2x}+\sin 2x+x^2)$ (*ii*) $(D^3-D)y=2x+1+4\cos x+2e^x$
5	Solve the following differential equations: (*i*) $(D^3-7D^2+14D-8)y=e^x\cos 2x$ (*ii*) $(D^2+4D+3)y=e^{-x}\sin x+xe^{3x}$ (*iii*) $\frac{d^4y}{dx^4}-y=\cos x\cosh x$
6	Solve the following differential equations: (*i*) $(D^2+2D+1)y=x\cos x$ (*ii*) $(D^2+16)y=x\sin 3x$ (*iii*) $(D^2-2D+1)y=xe^x\sin x$
7	Solve the following differential equations by the method of variation of parameters: (*i*) $\frac{d^2y}{dx^2}+a^2y=\operatorname{cosec} ax$ (*ii*) $(D^2+1)y=\sec x$ (*iii*) $\frac{d^2y}{dx^2}+4y=\tan 2x$
8	Solve the following differential equations by the method of variation of parameters: (*i*) $y''-6y'+9y=\frac{e^{3x}}{x^2}$ (*ii*) $y''-2y'+y=e^x\log x$ (*iii*) $y''-2y'+2y=e^x\tan x$
9	Solve the following differential equations: (*i*) $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^4$ (*ii*) $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$ (*iii*) $x^2y''-3xy'-5y=\sin(\log x)$
10	Find the current i in the $L-C-R$ circuit assuming zero initial current and charge. If $L=20$ henries, $C=0.01$ farads, $R=80$ ohms and $E=100$ volts.

UNIT-III: Laplace Transforms

UNIT-III Very Short Answer Questions	
1	Define Laplace transform
2	Write the sufficient conditions for the existence of Laplace transform of $f(t)$
3	State first shifting theorem
4	State change of scale property
5	Define unit step function and find its Laplace transform
6	State second shifting theorem
7	State Convolution theorem
8	If $L\{f(t)\}=\frac{1}{\sqrt{s^2+1}}$, evaluate $L\{f(2t)\}$
9	Find the Laplace transform of $4t^2+\sin 3t+e^{2t}$
10	Evaluate $L\{t\cos at\}$
11	Evaluate $L\left\{\frac{\sin t}{t}\right\}$
12	If $L\{\sin\sqrt{t}\}=\frac{\sqrt{\pi}}{2s^{3/2}}e^{-\frac{1}{4s}}$ then prove that $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\}=\sqrt{\frac{\pi}{s}}e^{-\frac{1}{4s}}$
13	Evaluate $L\left\{\int_0^t\int_0^t \cosh au\,du\,du\right\}$
14	Evaluate $L^{-1}\left\{\frac{s^2-s+2}{s^3}\right\}$

15	Evaluate $L^{-1}\left\{\frac{s^2}{(s-2)^3}\right\}$
16	Evaluate $L^{-1}\left\{\frac{s-3}{(s-3)^2+16}\right\}$
17	If $\delta(t-a)$ denotes Dirac's delta function then $L\{\delta(t-a)\} =$ ____________ A) $\frac{e^{as}}{s}$ B) $\frac{e^{-as}}{s}$ C) e^{-as} D) e^{as}
18	If $f(t)$ is a periodic function with period 2π then $L\{f(t)\} =$ ____________ A) $\frac{1}{1-e^{-s\pi}}\int_0^{2\pi} e^{-st}f(t)dt$ B) $\frac{1}{1-e^{-2s\pi}}\int_0^{\infty} e^{-st}f(t)dt$ C) $\frac{1}{1-e^{-2s\pi}}\int_0^{2\pi} e^{-st}f(t)dt$ D) $\frac{1}{1-e^{-s\pi}}\int_0^{\infty} e^{-st}f(t)dt$
19	If $L\{f(t)\} = \frac{1}{s^{3/2}}$ and $f(0)=0$ then $L\{f'(t)\} =$ ____________ A) $\frac{1}{\sqrt{s}}$ B) $\frac{1}{s}$ C) $\frac{1}{s^2}$ D) $\frac{1}{s^{5/2}}$
20	If $L\left\{\frac{\cos 3t-\cos 2t}{t}\right\} = \frac{1}{2}\log\left(\frac{s^2+4}{s^2+9}\right)$ then $\int_0^{\infty}\frac{\cos 3t-\cos 2t}{t}dt =$ ____________ A) $\log\left(\frac{4}{9}\right)$ B) $\log\left(\frac{9}{4}\right)$ C) $\log\left(\frac{3}{2}\right)$ D) $\log\left(\frac{2}{3}\right)$
UNIT-III Long Answer Questions	
1	Find the Laplace transforms of the following: (*i*) $e^{2t}+4t^3-2\sin 3t+3\cos 3t$ (*ii*) $\sin 2t\sin 3t$ (*iii*) $\cos^2 2t$ (*iv*) $\sin^3 2t$
2	(*i*) Find $L\left\{\frac{\sin at}{t}\right\}$, given that $L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left(\frac{1}{s}\right)$ (*ii*) If $L\{f(t)\} = \frac{1}{s}e^{-\frac{1}{s}}$, then prove that $L\{e^{-t}f(3t)\} = \frac{1}{s+1}e^{-\frac{3}{s+1}}$
3	Find the Laplace transforms of the following: (*i*) $e^{-3t}(2\cos 5t-3\sin 5t)$ (*ii*) $e^{-t}\sin^2 t$ (*iii*) $te^{2t}\cos t$ (*iv*) $t^2e^{-3t}\sin 2t$
4	Evaluate the following: (*i*) $L\left\{\frac{e^{-at}-e^{-bt}}{t}\right\}$ (*ii*) $L\left\{\frac{\cos at-\cos bt}{t}\right\}$ (*iii*) $L\left\{\frac{\sin 3t\cos t}{t}\right\}$
5	Using Laplace transforms evaluate the following: (*i*) $\int_0^{\infty}\frac{e^{-at}-e^{-bt}}{t}dt$ (*ii*) $\int_0^{\infty}\frac{\cos 6t-\cos 4t}{t}dt$ (*iii*) $\int_0^{\infty}\frac{e^{-t}\sin^2 t}{t}dt$
6	Find the Laplace transform of the triangular wave function of period $2a$ given by $f(t)=\begin{cases} t, & 0<t<a \\ 2a-t, & a<t<2a \end{cases}$
7	Find the inverse Laplace transforms of the following: (*i*) $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$ (*ii*) $\frac{4s+5}{(s-1)^2(s+2)}$ (*iii*) $\frac{5s+3}{(s-1)(s^2+2s+5)}$ (*iv*) $\frac{s}{(s-3)(s^2+4)}$
8	Evaluate the following: (*i*) $L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\}$ (*ii*) $L^{-1}\left\{\log\left(\frac{s^2+4}{s^2+9}\right)\right\}$ (*iii*) $L^{-1}\left\{\cot^{-1}\frac{s}{2}\right\}$

9	Evaluate the following using Convolution theorem $(i)\ L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$ $(ii)\ L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+9)}\right\}$ $(iii)\ L^{-1}\left\{\frac{s}{(s+2)(s^2+9)}\right\}$
10	Solve the following differential equation by using Laplace transforms: $(i)\ y''' + 2y'' - y' - 2y = 0, y(0) = y'(0) = 0$ and $y''(0) = 6$ $(ii)\ \frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$ with $x = 2$, $\frac{dx}{dt} = -1$ at $t = 0$ $(iii)\ \frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$ given that $y(0) = 0$ and $y'(0) = 1$ $(iv)\ \frac{d^2x}{dt^2} + 9x = \cos 2t$, if $x(0) = 1, x\left(\frac{\pi}{2}\right) = -1$

UNIT-IV: Multiple Integrals & Vector Differentiation

UNIT-IV Very Short Answer Questions	
1	Evaluate $\int_0^2 \int_0^3 xy\, dydx$
2	Evaluate $\int_0^1 \int_0^x e^x dydx$
3	Evaluate $\int_1^3 \int_0^{\log y} \frac{1}{\log y} dxdy$
4	Evaluate $\int_0^1 \int_0^1 \frac{1}{\sqrt{(1-x^2)(1-y^2)}} dxdy$
5	Evaluate $\int_0^{\pi/2} \int_0^2 \sin\theta\, drd\theta$
6	Evaluate $\int_0^1 \int_0^2 \int_0^4 xyz\, dzdydx$
7	Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\, dxdydz$
8	Define gradient of a scalar point function, divergence and curl of a vector point function
9	Define solenoidal and irrotational vector point functions
10	Find a unit vector normal to the surface $x^2y + 2xz = 4$ at the point $(2,-2,3)$
11	Find the maximum value of directional derivative of $\phi = x^2yz^3$ at $(2,1,-1)$
12	Find the greatest rate of increase of $u = xyz^2$ at $(1,0,3)$
13	If $\vec{A} = 3x^2\hat{i} + 5xy^2\hat{j} + xyz^3\hat{k}$ then find $\nabla \cdot \vec{F}$ at $(1,2,3)$
14	If $\vec{f} = (2x+3y)\hat{i} + (4y-z)\hat{j} + (x-pz)\hat{k}$ is solenoidal, then find p
15	Determine the constants a,b,c so that $\vec{F} = (4x+3y+az)\hat{i} + (bx-y+z)\hat{j} + (2x+cy+z)\hat{k}$ is irrotational.
16	Show that $r^n\vec{r}$ is irrotational
17	The iterated integral for $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dydx$ after changing the order of integration is _________ A) $\int_0^1 \int_0^{\sqrt{1-y^2}} y^2 dxdy$ B) $\int_0^1 \int_0^1 y^2 dxdy$ C) $\int_0^1 \int_0^{1-y^2} y^2 dxdy$ D) $\int_0^1 \int_0^y y^2 dxdy$

18	$\int_0^{\frac{\pi}{4}}\int_0^{\infty} e^{-r^2} r dr d\theta =$ __________ A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{8}$ D) $\frac{\pi}{16}$
19	If $\phi = x^3 + y^3 + z^3 - 3xyz$ then the value of $curl(grad\ \phi)$ at $(1,2,3) =$ __________ A) $\hat{i} - 2\hat{j}$ B) $2\hat{j} - 3\hat{k}$ C) $\hat{i} - 2\hat{j} + 3\hat{k}$ D) $\vec{0}$
20	If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = \lvert\vec{r}\rvert$ then $\nabla(r^n) =$ __________ A) $nr^{n-1}\vec{r}$ B) $nr^{n-2}\vec{r}$ C) $nr^{n}\vec{r}$ D) $nr^{n-3}\vec{r}$
UNIT-IV Long Answer Questions	
1	(i) Evaluate $\iint xy(x+y)\,dxdy$ over the area between $y = x^2$ and $y = x$. (ii) Evaluate $\iint_R xy\,dxdy$, where R is the domain bounded by the x-axis, the ordinate $x = 2a$ and the curve $x^2 = 4ay$.
2	(i) Evaluate $\iint r\sin\theta\,drd\theta$ over the cardioid $r = a(1-\cos\theta)$ above the initial line (ii) Evaluate $\iint r^3 drd\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$
3	(i) Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}dxdy$ by changing to polar coordinates. Hence show that $\int_0^{\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$ (ii) Evaluate $\int_0^2\int_0^{\sqrt{2x-x^2}} \frac{x}{x^2+y^2}\,dxdy$ by transforming into polar coordinates
4	Evaluate the following integrals by changing the order of the integration (i) $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$ (ii) $\int_0^1\int_{e^x}^{e} \frac{dydx}{\log_e y}$ (iii) $\int_0^3\int_0^{\sqrt{4-y}} (x+y)\,dxdy$ (iv) $\int_0^1\int_{x^2}^{2-x} xy\,dydx$
5	Evaluate the following (i) $\int_0^1\int_0^{1-x}\int_0^{1-x-y} dxdydz$ (ii) $\int_0^1\int_{y^2}^1\int_0^{1-x} x\,dz\,dx\,dy$ (iii) $\int_{-1}^1\int_0^z\int_{x-z}^{x+z} (x+y+z)\,dxdydz$ (iv) $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\,dxdydz$
6	(i) Find the directional derivative of $\phi(x,y,z) = x^2yz + 4xz^2$ at the point $(1,-2,-1)$ in the direction of the vector $2\hat{i} - \hat{j} - \hat{k}$ (ii) Find the directional derivative of $\phi = 5x^2y - 5y^2z + 2.5z^2x$ at the point $P(1,1,1)$ in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = z$ (iii) Find the directional derivative of $\phi(x,y,z) = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of the normal to the surface $x\log z - y^2 = -4$ at $(-1,2,1)$ (iv) Find the directional derivative of $f(x,y,z) = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ in the direction of the line PQ, where $Q(5,0,4)$

7	(i) Find the values of a and b such that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ cut orthogonally at $(1,-1,2)$. (ii) Calculate the angle between the normals to the surface $xy = z^2$ at the points $(4,1,2)$ and $(3,3,-3)$
8	(i) Find $div\,\vec{F}$ and $curl\,\vec{F}$ at the point $(1,-1,1)$, where $\vec{F} = xy^2\hat{i} + 2x^2yz\hat{j} - 3yz^2\hat{k}$ (ii) Find $div\,\vec{F}$ and $curl\,\vec{F}$, where $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$.
9	Show that the following vector point functions are irrotational and find the scalar potential in each case: (i) $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ (ii) $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$
10	(i) Prove that $div(r^n\vec{r}) = (n+3)r^n$. Hence show that $\frac{\vec{r}}{r^3}$ is solenoidal. (ii) Show that $div(grad\, r^n) = \nabla^2(r^n) = n(n+1)r^{n-2}$

UNIT-V: Vector Integration

UNIT-V Very Short Answer Questions	
1	Evaluate $\int_C ydx + xdy$, where C is the arc of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$
2	Find the circulation of $\vec{F} = y\hat{i} - x\hat{j} + z\hat{k}$ along the circle $x^2 + y^2 = 1$ in xy-plane
3	State Green's theorem
4	State Stoke's theorem
5	State Gauss's divergence theorem
6	Use Green's theorem to evaluate $\oint_C (2xy - x^2)dx + (x^2 + y^2)dy$, where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$.
7	Apply Green's theorem to evaluate $\oint_C xydx + x^2dy$, where C is the square bounded by $0 \le x \le 1$ and $0 \le y \le 1$
8	Apply Stoke's theorem to evaluate $\oint_C e^x dx + 2ydy - dz$, where C is the curve $x^2 + y^2 = 9$ and $z = 2$
9	Evaluate $\int_S \vec{r}\cdot\vec{n}\,ds$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 9$
10	Apply divergence theorem to evaluate $\iint_S xdydz + ydzdx + zdxdy$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$
11	If S is a closed surface enclosing a volume V and $\vec{F} = ax\hat{i} + by\hat{j} + cz\hat{k}$ then prove that $\int_S \vec{F}\cdot\vec{n}\,ds = (a+b+c)V$
12	If C is the curve $x = t^2, y = 2t$ $(0 \le t \le 1)$ then $\int_C (2x+3y)dx - xydy =$ __________ A) 2 B) 3 C) 4 D) 6

13	If C is the circle $x^2+y^2=4$, then $\int_C (3y+\cos x)dx+(7x-e^y)dy=$ __________ A) 16π B) 8π C) 4π D) 2π
14	If A is the area of a plane region bounded by a closed curve C in xy-plane then $A=$ _________ A) $\oint_C (xdy-ydx)$ B) $\frac{1}{2}\oint_C (xdy-ydx)$ C) $\frac{1}{2}\oint_C (ydx-xdy)$ D) $\oint_C (ydx-xdy)$
15	If S is the surface of the cuboid bounded by the planes $0\le x\le a,\ 0\le y\le b, 0\le z\le c$ then $\iint_S (x+z)dydz+(y+z)dzdx+(x+y)dxdy=$ _________ A) 0 B) abc C) $2abc$ D) $3abc$
UNIT-V Long Answer Questions	
1	(i) If $\vec{F}=3xy\hat{i}-y^2\hat{j}$, evaluate $\int_C \vec{F}\cdot d\vec{r}$, where C is the curve $y=2x^2$ in the xy-plane from $(0,0)$ to $(1,2)$. (ii) Compute the line integral $\int_C y^2dx-x^2dy$ about the triangle whose vertices are $(1,0),(0,1)$ and $(-1,0)$.
2	If $\vec{A}=(3x^2+6y)\hat{i}-14yz\hat{j}+20xz^2\hat{k}$, evaluate $\int \vec{A}\cdot d\vec{r}$ from $(0,0,0)$ to $(1,1,1)$ along the path $x=t, y=t^2, z=t^3$.
3	Find the work done in moving a particle in the force $\vec{F}=3x^2\hat{i}+(2xz-y)\hat{j}+z\hat{k}$ along (i) the straight line from $(0, 0, 0)$ to $(2, 1, 3)$ (ii) the curve defined by $x^2=4y, 3x^3=8z$ from $x=0$ to $x=2$.
4	Find the total work done in moving a particle in a force field given by $\vec{F}=3xy\hat{i}-5z\hat{j}+10x\hat{k}$ along the curve $x=t^2+1, y=2t^2, z=t^3$ from $t=0$ to $t=2$.
5	Evaluate $\int_S \vec{F}\cdot\vec{n}\,ds$, where $\vec{F}=18z\hat{i}-12\hat{j}+3y\hat{k}$ and S is the portion of the plane $2x+3y+6z=12$ in the first octant.
6	Evaluate $\int_S \vec{F}\cdot\vec{n}\,ds$, where $\vec{F}=z\hat{i}+x\hat{j}-3y^2z\hat{k}$ and S is the surface $x^2+y^2=16$ included in the first octant between $z=0$ and $z=5$.
7	(i) Verify Green's theorem for $\oint_C (xy+y^2)dx+x^2dy$, where C is bounded by $y=x$ and $y=x^2$ (ii) Verify Green's theorem for $\oint_C (3x-8y^2)dx+(4y-6xy)dy$, where C is the boundary of the region bounded by $x=0, y=0$ and $x+y=1$ (iii) Verify Green's theorem for $\oint_C (x^2-xy^3)dx+(y^2-2xy)dy$, where C is the square with vertices $(0,0),(2,0),(2,2)$ and $(0,2)$.
8	(i) Apply Green's theorem to evaluate $\oint_C (x^2-\cosh y)dx+(y+\sin x)dy$, where C is the rectangle formed by $0\le x\le\pi,\ 0\le y\le 1$

	(ii) Using Green's theorem, evaluate $\oint_C (y - \sin x)dx + \cos x dy$, where C is the plane triangle enclosed by the lines $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$
9	(i) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken around the rectangle bounded by the lines $x = \pm a, y = 0$ and $y = b.$ (ii) Apply Stoke's theorem to evaluate $\oint_C (x + y)dx + (2x - z)dy + (y + z)dz$, where C is the boundary of the triangle with vertices $(2,0,0)$, $(0,3,0)$ and $(0,0,6)$
10	(i) Verify divergence theorem for $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c.$ (ii) Use divergence theorem to evaluate $\int_S \vec{F} \cdot \vec{n}\, ds$, where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface bounding the region $x^2 + y^2 = 4,\ z = 0$ and $z = 3$

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