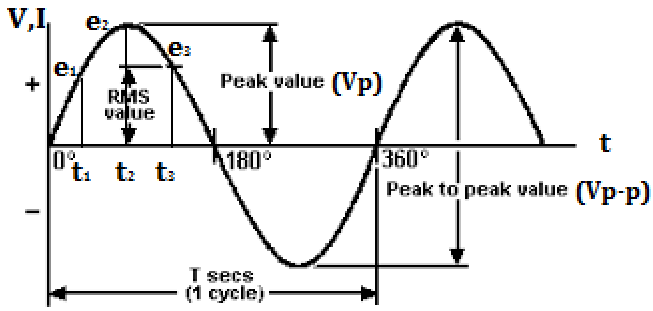
UNIT- II

AC CIRCUITS: Representation of sinusoidal waveforms, peak, RMS and average values. Phase representation of alternating quantities, analysis of AC circuits with single basic network element (R, L, C), single phase series circuits, concept of resonance, three-phase balanced circuits-voltage and current relations in star and delta connections.

2.1 Representation of sinusoidal waveform:



Waveform: A waveform is a graphical representation of instantaneous values of an alternating quantity plotted against time.

Instantaneous Value: The value of an alternating quantity (ac voltage, ac current or ac power) at a particular instant of time in a cycle.

Ex: e_1 (t1), e_2 (t2), e3 (t3)

Cycle: Set of one positive and negative half cycle of alternate quantity is called cycle.

Time Period (T): The time taken by an alternating quantity to complete one cycle is called time period. It is denoted by **T**.

Frequency (f): The number of cycles completed by an alternate quantity in one second is called Frequency. It is denoted by '**f**' and measured in Hz,

$$\mathbf{f} = \frac{1}{T}\, Hz$$

Peak Value or Amplitude (V_m or I_m or V_p): The maximum value attained by an alternating quantity during positive or negative half cycle is called its Peak value.

Angular Frequency (ω): It is the frequency expressed in electrical radians per second.

$$\omega = 2\pi f \text{ radians /sec} \quad \text{(or)} \quad \omega = 2\pi \left(\frac{1}{T}\right)$$

Alternating Current (A.C): An alternating current is the current which changes periodically both in magnitude and direction.

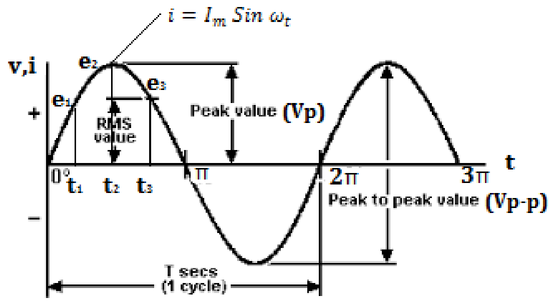
2.2 R.M.S VALUE: (Root mean square value):

RMS or root mean square value of the alternating quantity (voltage/current) represents the d.c. quantity (volgage/current) that dissipates the same amount of power as the average power dissipated by the alternating quantity (voltage/current).

or

RMS value is defined as the value of AC current which when flowing through a resistance produces the same amount of heat as DC current flowing through that resistance.

R.M.S VALUE of Alternating quantity:



$$RMS\,Value = \sqrt{\frac{1}{T}\int_0^T V^2(t)dt}$$

The voltage $v = V_m \sin \omega t$, Time period T=2 π

Square of the voltage $v^2 = {V_m}^2 sin^2\omega t$

$$RMS\,Value = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} {V_m}^2 sin^2\omega t\,dt}$$

$$= \sqrt{\frac{{V_m}^2}{2\pi}\int_0^{2\pi} sin^2\omega t\,dt}$$

$$= \sqrt{\frac{{V_m}^2}{2\pi}\int_0^{2\pi}\left[\frac{1-\cos 2\omega t}{2}\right]dt}$$

$$= \sqrt{\frac{{V_m}^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{{V_m}^2}{4\pi}[2\pi]}$$

$$\boxed{\boldsymbol{V_{r.m.s} = \frac{V_m}{\sqrt{2}} = 0.707\,V_m}}$$

The r.m.s value of the sinusoidal alternating current is 0.707 times the maximum or peak value or amplitude of that alternating current.

2.3 Average Value:

The average value of an alternating current is the average of all the instantaneous values during one alternation.

Average Value of Alternating Quantity:

The voltage $v = V_m \sin \omega t$

Average value of a sine wave over a half cycle= $\frac{1}{\pi}\int_0^{\pi} v\, dt$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t \;\, dt$$

$$= \frac{V_m}{\pi}\int_0^{\pi} \sin \omega t \;\, dt$$

$$= \frac{V_m}{\pi}[-\cos \omega t]_0^{\pi}$$

$$= \frac{V_m}{\pi}[-\cos \pi + \cos 0]$$

$$= \frac{V_m}{\pi}[2]$$

$$\boxed{\boldsymbol{V_{avg} = \frac{2V_m}{\pi} = 0.637\,V_m}}$$

2.4 Form Factor (K_f):

The form factor of an alternating quantity is defined as the ratio of r.m.s value to the average value.

$$Form\,factor, K_f = \frac{r.m.s\ value}{Average\ value}$$

$$\boxed{\boldsymbol{K_f = \frac{0.707\,V_m}{0.637\,V_m} = 1.11}}$$

2.5 Peak Factor (K_p):

The peak factor for alternating quantity is defined as ratio of maximum value to the r.m.s value.

$$Peak\ factor, K_p = \frac{Maximum\ value}{r.m.s\ value}$$

$$\boxed{K_p = \frac{V_m}{0.707V_m} = 1.414}$$

2.6 Phase representation of alternating quantities:

Phase: The phase of an alternating quantity specifies the position of a sine wave with respect to a reference waveform. Generally, the phase of the alternating quantity varies from **0** to **2π** in **radians** or **0º** to **360º**.

When the phase of the alternating quantity is 0 then the instantaneous value of the sinusoidal quantity is at t = 0 which is considered as reference. The figure given below indicates φ = 0º.

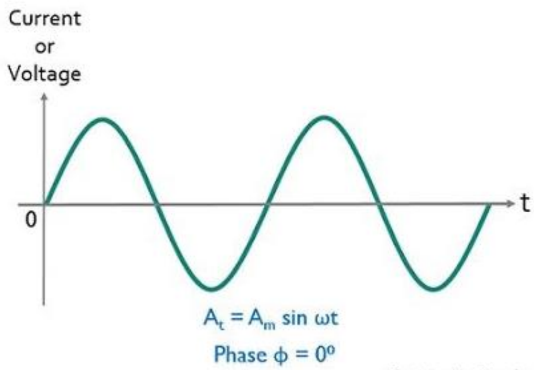


Fig: Reference wave form (Zero Phase)

Lagging Phase: In the figure given below, we have current and voltage wave forms and it is clearly shown that the voltage waveform is started after the reference and the current waveform is started exactly at the reference. Therefore Voltage is said to be lagging w.r.t to current (Reference).

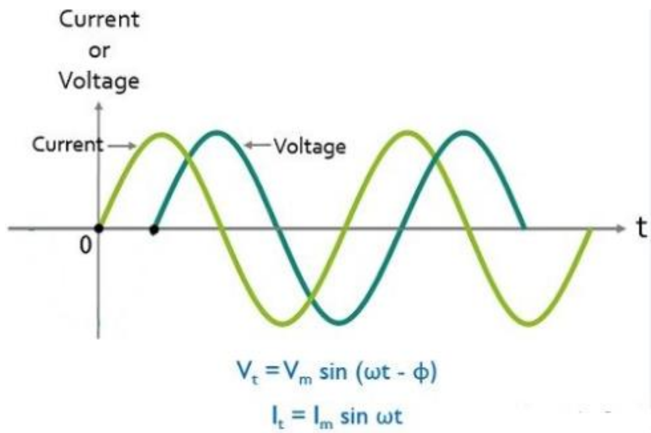


Fig: Lagging phase diagram

Leading Phase: In the below-given figure, one is the voltage waveform which is started before the reference point and the other is the current waveform which exactly starts at t=0.Therefore Voltage is said to be leading w.r.t to current (Reference).

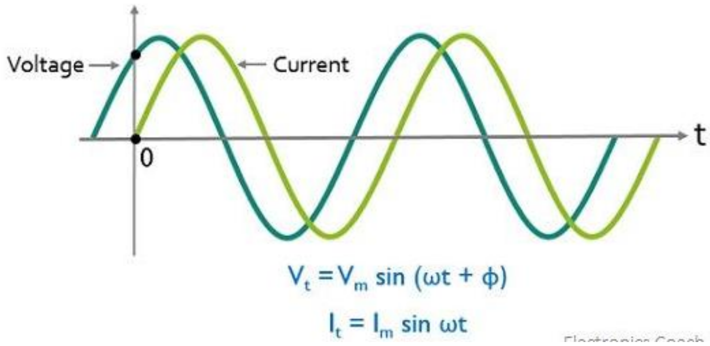


Fig: Leading phase diagram

Phase Difference: It is the difference in phase angles of two waveforms.

Equation for Phase Difference:

The general equation of the alternating quantities is given as:

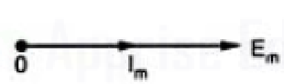
$$v(t) = V_m \sin(\omega t \pm \phi)$$

"+" sign for Leading Phase.

"-" sign for Lagging Phase.

Phasor: The sinusoidally varying alternating quantity is represented graphically by a straight line with an arrow in the phasor representation method. The length of the line represents the magnitude of the quantity and arrow indicates its direction. This is similar to vector representation, such a line is known as Phasor.

The phasors are assumed to be rotated in anti clockwise direction with an angular velocity **ω**.



Phasor Diagram: The diagram in which different alternating quantities of the same frequency are represented by individual phasors indicating exact phase interrelationship is known as phasor diagram.

* Generally the reference phasor chosen is shown along the positive X-axis direction.
* Length of the phasor is equal to maximum value of an alternating quantity.
Let's analyze phasor diagrams for different cases.

1. Zero Phase Difference: Consider two alternating quantities Voltage & Current having same frequency 'f' Hz and different maximum values.

Let maximum value of Voltage is greater than current. (i.e $V_m > I_m$)

$$V\,(t) = V_m \sin \omega t, \quad I(t) = I_m \sin \omega t$$

There is no phase difference between two alternating quantities. Then the two quantities are said to be **in phase.**

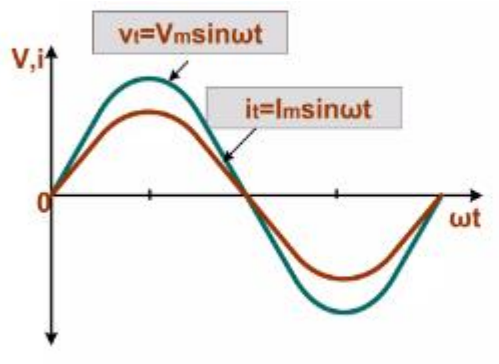


fig: Waveform

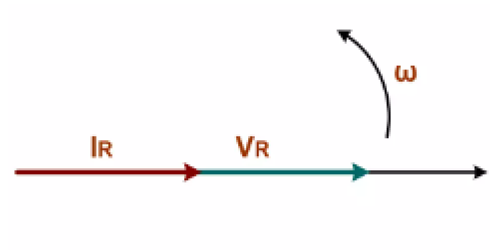


fig: Phasor diagram

2. Lagging Phase Difference:

Consider **V (t)= V_m sin ωt,** **I(t)= I_m sin(ωt-90^0)**

As shown in the above voltage and current equations, the current i is lagging with respect to the voltage by an angle of 90^0.

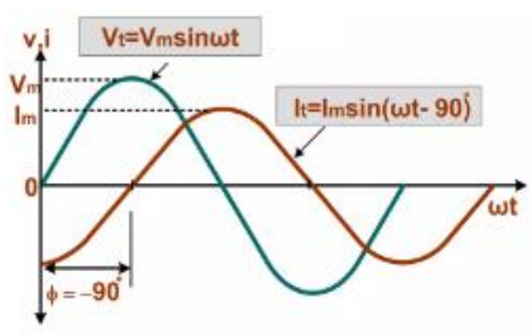


fig: Waveform

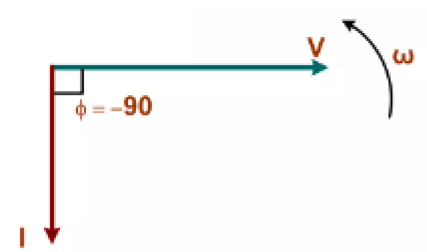


fig: Phasor diagram

3. Leading Phase Difference:

$$consider, \quad V\,(t) = V_m \sin \omega t, \quad I(t) = I_m \sin\,(\omega t + 90°)$$

As shown in the above voltage and current equations, the current i is leading with respect to the voltage by an angle of 90^0.

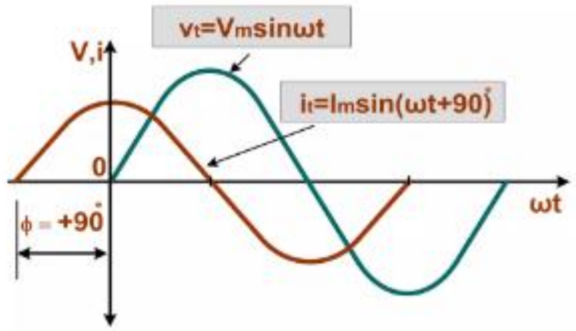


fig: Waveform

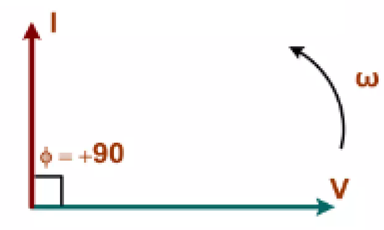


fig: Phasor diagram

Mathematical representation of Phasors :

The algebraic operations such as addition, subtraction etc with waveforms is more complicated and time consuming. Hence it is necessary to represent the phasors mathematically.

Any phasor can be represented mathematically in 2 ways.

1. Polar co-ordinate system
2. Rectangular co-ordinate system.

Polar co-ordinate system: Consider an alternating quantity $I(t) = I_m \sin(\omega t + \emptyset)$

The polar form of an alternating quantity can be expressed as $r\angle\pm\emptyset$

Where **r**= R.M.S value of an alternating quantity.

Φ= Phase angle w.r.t positive X-axis.

Polar form for given alternating quantity is $\frac{I_m}{\sqrt{2}} \angle\emptyset$

Example: Represent V= 40 sin (ωt+60^0) in polar co-ordinate system

Compare the equation with $\boldsymbol{V(t) = V_m \sin \omega t}$

Vm =40, $\emptyset = \mathbf{60°}$

r= r.m.s value = $\frac{\boldsymbol{Max\ value}}{\sqrt{\mathbf{2}}} = \frac{\mathbf{40}}{\sqrt{\mathbf{2}}}$

Polar form = $\frac{\mathbf{40}}{\sqrt{\mathbf{2}}} \angle\mathbf{60°}$

Rectangular co-ordinate system: Consider an alternating quantity $I(t) = I_m \sin(\omega t + \emptyset)$

Mathematically any alternating quantity can be divided into two components, X- component, Y-component.

For the above alternating quantity X- Component **X** = $\mathbf{I_m \cos}\ \emptyset$

Y- Component **Y** = $\mathbf{I_m \sin}\ \emptyset$

The rectangular form of an alternating quantity can be expressed as $\pm$**X**$\pm$**jY**

Mathematically **j=** $\sqrt{-1}$

In polar representation **j= 1** $\angle 90°$

Note: 'j' operator is used to indicate X and Y components are perpendicular to each other.

Represent i=20 sin (ωt+30^0) in rectangular co-ordinate system.

Compare the equation with i=I_m sin (ωt$\pm\emptyset$)

I_m =20, $\emptyset = 30°$

X- Component X = $I_m \cos \emptyset$ = 20 cos $30°$ **=**$10\sqrt{3}$

Y- Component Y = $I_m \sin \emptyset$ **=**20 sin $30°$ = 10

Rectangular form is **10$\sqrt{3}$+10**

Impedance: It is defined as the ratio of alternating voltage to the alternating current. It is denoted by 'Z', and is measured in terms of ohms.

$$\mathbf{Z = \frac{V}{I}\ \text{ohms}}$$

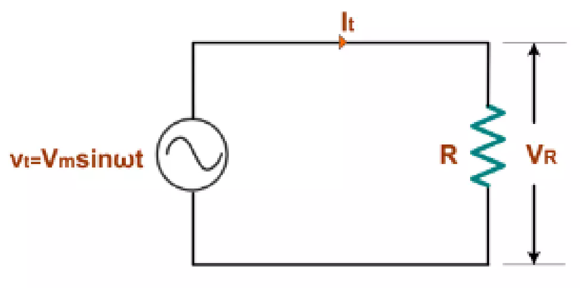
(Or) It is the opposition to the flow of alternating current.

2.7 Analysis of AC circuits with single basic network element (R, L, C):

2.7.1 A.C through pure Resistance:

Consider a simple circuit consisting of pure 'R' ohms connected across a voltage

$\boldsymbol{V\,(t) = V_m \sin \omega t}$**.** The circuit is shown below



Where V_t= instantaneous voltage

V_m= maximum voltage

V_R =voltage across resistor

According to ohm's law,

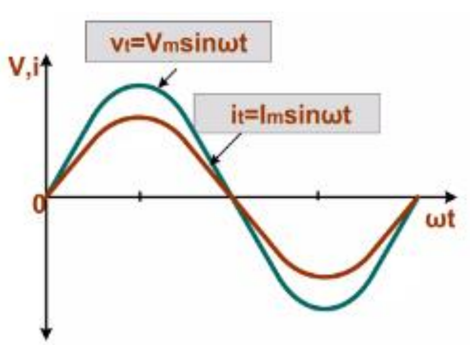
$$i= \frac{V}{R} = \frac{V_m \sin\omega t}{R}$$

$$i= \left(\frac{V_m}{R}\right) \sin\omega t$$

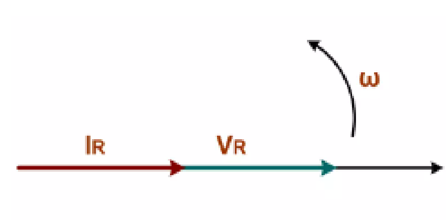
Compare this with $i = Im\ sin\ (\omega t \pm \emptyset)$

$$\mathbf{I_m} = \left(\frac{V_m}{R}\right) \quad , \emptyset = \mathbf{0}°$$

As $\emptyset$ = ° , it indicates that current is in phase with the voltage applied.



Waveform



Phasor diagram

Impedance of pure resistive circuit:

Impedance $\quad \mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\mathbf{V_m sin\omega t}}{(\frac{\mathbf{V_m}}{\mathbf{R}})\ \mathbf{sin\omega t}}$

To perform division on alternating quantities, polar representation can be used.

Polar form for $V_m \sin\omega t = \quad \frac{\mathbf{V_m}}{\sqrt{2}} \angle \mathbf{0}°$

Polar form for $(\frac{\mathbf{V_m}}{\mathbf{R}})\ \mathbf{sin\omega t} = \quad \frac{\mathbf{V_m}}{\sqrt{2R}} \angle \mathbf{0}°$

$$Z = \frac{\frac{\mathbf{V_m}}{\sqrt{2}} \angle \mathbf{0}°}{\frac{\mathbf{V_m}}{\sqrt{2R}} \angle \mathbf{0}°}$$

Therefore

$$\boxed{\mathbf{Z = R}}$$

Power:

The instantaneous power in a.c circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$\boldsymbol{P = v \times i = V_m \sin \omega t \times I_m \sin \omega t = V_m I_m \, sin^2(\omega t)}$$

$$\boldsymbol{= \frac{V_m I_m}{2}\,(1 - \cos 2\omega t)}$$

$$\boldsymbol{P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}(\cos 2\omega t)}$$

From the above equation, it is clear that the instantaneous power consist of two components.

1) Constant power component $(\frac{V_m I_m}{2})$

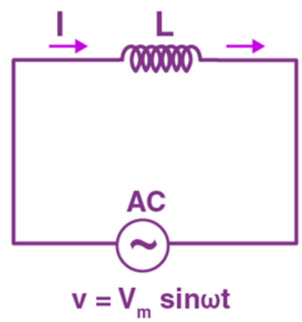
2) Fluctuating component $-\frac{V_m I_m}{2}(\cos 2\omega t)$ having frequency, double the frequency of the applied voltage.

2.7.2 A.C through pure Inductance:

Consider a simple circuit consisting of pure 'L' Henry connected across a voltage

$\mathbf{v = Vm \sin \omega t}$.   Pure inductance has zero ohmic resistance and its internal resistance is zero.

The circuit is shown below



Where v= instantaneous voltage

V_m= maximum voltage

V_L =voltage across Inductor

The current flowing through the inductor is given by

$$\mathbf{I} = \frac{1}{L}\int V\,dt$$

$$= \frac{1}{L}\int V_m \sin\omega t\,dt$$

$$= \frac{V_m}{L}\left[\frac{-\cos\omega t}{\omega}\right]$$

$$= \frac{V_m}{\omega L}\left[-\sin(90-\omega t)\right]$$

$$= -\frac{V_m}{\omega L}\left[\sin(-(\omega t-90))\right]$$

$$= \frac{V_m}{\omega L}\left[\sin(\omega t-90)\right]$$

$$= \frac{V_m}{X_L}\left[\sin(\omega t-90)\right]$$

$$where \quad X_L = \omega L = 2\pi f L\,\Omega = \textbf{Inductive reactance}$$

Compare this with $i = Im\,sin\,(\omega t \pm \emptyset)$

$$\mathbf{I_m} = \left(\frac{V_m}{X_L}\right) \quad , \emptyset = -90^{\circ}$$

As $\emptyset = -90°$ **,** it indicates that current is lagging with the voltage by 90^0.

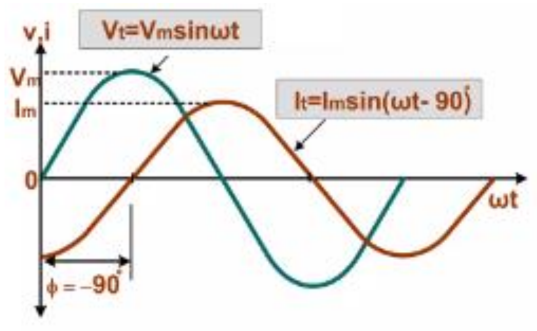


fig: Waveform

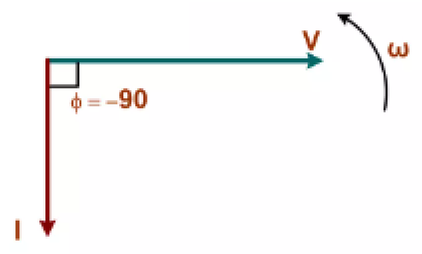


fig: Phasor diagram

Inductive reactance (X_L): Inductive reactance is the property of an inductive coil that resists the change in alternating current (AC) through it

$$X_L = \omega L = 2\pi f L\ \Omega$$

Impedance (Z) of pure inductive circuit:

$$Z = \frac{V}{I} = \frac{V_m \sin\omega t}{\left(\frac{V_m}{X_L}\right)\sin\omega t}\ \Omega$$

$$Z = \frac{\frac{V_m}{\sqrt{2}}\angle 0°}{\frac{V_m}{\sqrt{2}X_L}\angle -90°}$$

$$Z = X_L\angle 90°\ \Omega$$

$$Z = jX_L\ \Omega\ \ (Where\ j=1\angle 90°)$$

Power:

The instantaneous power in a.c circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$P = v \times i = V_m \sin\omega t \times I_m\left(\sin\omega t - \frac{\pi}{2}\right)$$

$$\left(\sin\omega t - \frac{\pi}{2} = -\cos\omega t\right)$$

$$= -V_m I_m\, sin(\omega t) cos(\omega t)$$

$$= -\frac{2\, V_m I_m\, sin(\omega t) cos(\omega t)}{2}$$

$$as\ 2\, sin(\omega t) cos(\omega t) = \sin 2\omega t$$

$$P = -\frac{V_m I_m}{2}\sin(2\omega t)$$

Power consumed by inductor,

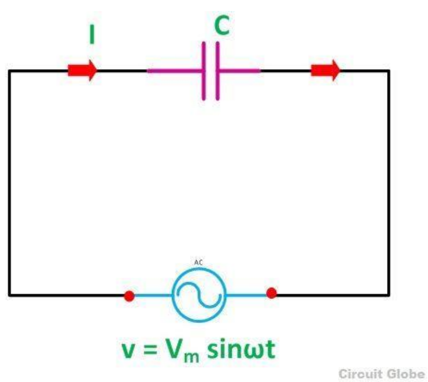
$$P = \frac{V_m I_m}{2} = 0$$

2.7.3 A.C through pure Capacitace:

Consider a simple circuit consisting of pure 'C' farad connected across a voltage

$\boldsymbol{v = V_m \sin \omega t}$. Pure capacitor has zero ohmic resistance and its internal resistance is zero.

The circuit is shown below



Where v= instantaneous voltage

V_m= maximum voltage

V_c =voltage across Capacitor

The current flowing through the capacitor is given by

$$\mathbf{I} = C\frac{dv}{dt}$$

$$= C\frac{d}{dt}V_m \sin\omega t$$

$$= C\,V_m \cos\omega t\ \omega$$

$$= C\,V_m\omega\cos\omega t$$

$$= \frac{V_m}{(\frac{1}{\omega C})}\cos\omega t$$

$$= \frac{V_m}{X_C}[\sin(\omega t + 90)]$$

$$where \quad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \textbf{Capacitive reactance}$$

Compare the current equation with $i = I_m \sin(\omega t \pm \emptyset)$

$$I_m = \left(\frac{V_m}{X_C}\right) \quad , \emptyset = +90°$$

As $\emptyset = +90°$, it indicates that current is leading with the voltage by 90^0.

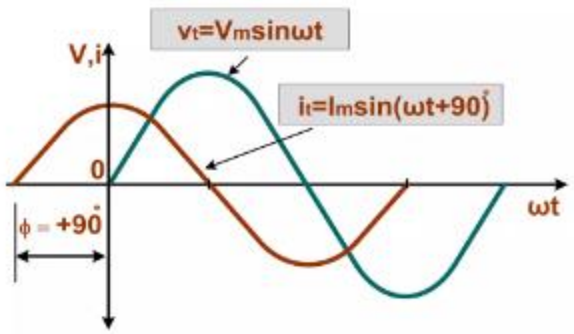


fig: Waveform

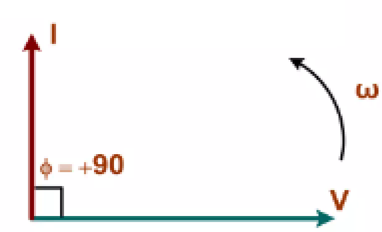


fig: Phasor diagram

Capacitive reactance (X_C): Capacitive reactance is the property of capacitor that which opposes the changes in voltage.

Impedance (Z) of pure capacitive circuit:

$$Z = \frac{V}{I} = \frac{V_m \sin \omega t}{\left(\frac{V_m}{X_C}\right)\sin\omega t} \ \Omega$$

$$Z = \frac{\frac{V_m}{\sqrt{2}} \angle 0°}{\frac{V_m}{\sqrt{2}X_C} \angle 90°}$$

$$Z = X_C \angle -90° \ \Omega$$

$$Z = -jX_C \ \Omega \quad (\text{Where } j=1\angle 90°)$$

Power:

The instantaneous power in a.c circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$P = v \times i = V_m \sin \omega t \times I_m \left(\sin \omega t + \frac{\pi}{2}\right)$$

$$(\sin(\omega t + \tfrac{\pi}{2}) = \cos \omega t)$$

$$= V_m I_m \, sin(\omega t) cos(\omega t)$$

$$= \frac{2\, V_m I_m \, sin(\omega t) cos(\omega t)}{2}$$

$$as\ 2\, sin(\omega t) cos(\omega t) = \sin 2\omega t$$

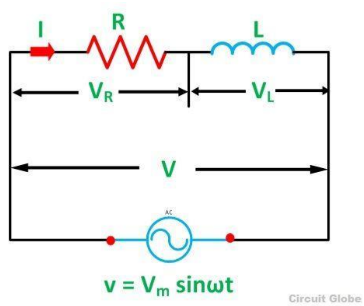
$$P = \frac{V_m I_m}{2} \sin(2\omega t)$$

Power consumed by inductor,

$$P = \frac{V_m I_m}{2} = 0$$

2.8 Single Phase Series Circuits:

2.8.1 Series R-L Circuit:



Consider a circuit consisting of pure resistance R ohms connected in series with a pure inductance of L henries. The series combination is connected across a.c supply given by

$$\boldsymbol{v = V_m \sin(\omega t)}$$

Circuit draws a current 'I' then there are two voltage drops,

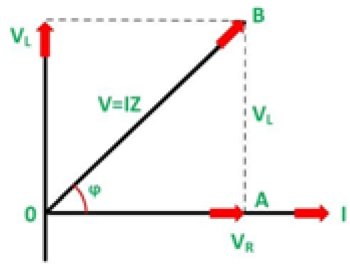
a) Drop across pure resistance, $V_R = I \times R$
b) Drop across pure inductance, $V_L = I \times X_L$ Where $X_L = 2\pi f L$

$$V = V_R + V_L$$
$$V = IR + IX_L$$

Phasor diagram :

The following steps are given below which are followed to draw the phasor diagram step by step:

- Current I is taken as a reference.
- The Voltage drop across the resistance V_R = IR is drawn in phase with the current I.
- The voltage drop across the inductive reactance V_L =IX_L is drawn ahead of the current I. As the current lags voltage by an angle of 90 degrees in the pure Inductive circuit.
- The vector sum of the two voltages drops V_R and V_L is equal to the applied voltage V.



Now, in right-angle triangle OAB
V_R = IR , V_L =IX_L

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (\mathrm{IX_L})^2}$$

$$V = I\sqrt{(R)^2 + (\mathrm{X_L})^2}$$

$$V = IZ$$

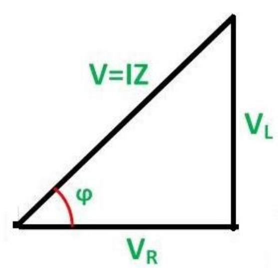
Where $Z = \sqrt{(R)^2 + (\mathrm{X_L})^2}$

From the phasor diagram the current passing through the circuit is lagging w.r.to voltage.

$$i = I_m(\sin\omega t - \phi)$$

Voltage Triangle:

From the phasor diagram triangle OAB is voltage triangle.

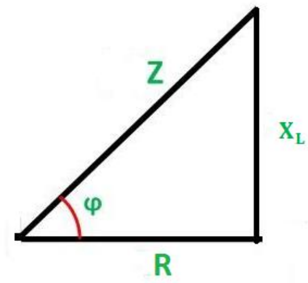


$$\cos\emptyset = \frac{V_R}{V}$$

$$\sin\emptyset = \frac{V_L}{V}$$

Impedance triangle:

If all the sides of voltage triangle are divided by current, we get triangle called impedance triangle.



$$\cos\emptyset = \frac{V_R}{V} = \frac{R}{Z}$$

$$\sin\emptyset = \frac{V_L}{V} = \frac{X_L}{Z}$$

$$\tan\emptyset = \frac{V_L}{V_R} = \frac{X_L}{R}$$

$$\boldsymbol{Z = \frac{V}{I}\ ,\ X_L = \frac{V_L}{I}\ ,\ R = \frac{V_R}{I}}$$

X component of impedance is R and is given by

$$R = Z\cos\emptyset$$

And Y component of impedance is $\mathrm{X_L}$ and is given by,

$$\mathrm{X_L} = \mathrm{Z}\sin\emptyset$$

In rectangular from the impedance is denoted as ,

$$Z = R + j\mathrm{X_L}\ \Omega$$

While in polar form, it is denoted as,

$$Z = |Z|\angle\phi\ \Omega$$

$$|Z| = \sqrt{(R)^2 + (\mathrm{X_L})^2},\ \ \phi = \tan^{-1}(\frac{X_L}{R})$$

Power:

The expression for the current in the series R-L circuit is

$$i = I_m(\sin\omega t - \phi\ )$$

The power is product of instantaneous values of voltage and current

$$\therefore P = v \times i = V_m \sin\omega t \times I_m(\sin\omega t - \phi)$$

$$= V_m I_m[(\sin\omega t)(\sin\omega t - \phi)]$$

$$(\sin a\ \ \sin b) = \frac{1}{2}[\cos(a-b) - (\cos a + b)]$$

$$= V_m I_m[\frac{\cos(\ \phi) - cos(2\omega t - \phi)}{2}]$$

$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$$

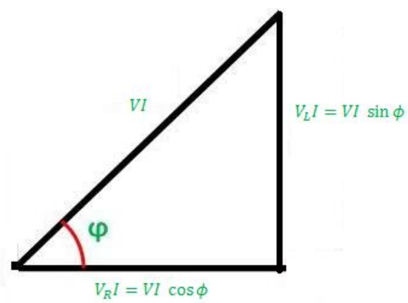
Now the second term is cosine tern whose average value over a cycle is zero. Hence, average power consumed is,

$$P_{av} = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

$$\boxed{\therefore P = VI \cos\phi}$$

Power Triangle:

If all the sides of Voltage triangle is multiplied by current I , we get Power triangle.



$$V = V_R + V_L$$

$$VI = VI\cos\phi + VI\sin\phi$$

So, three sides of this triangle are,

1) VI   2) $VI\cos\phi$ 3) $VI\sin\phi$

These terms can be defined as below.

Apparent power (S): it is defined as the product of r.m.s value of voltage (V) and current (I).

$$S = VI \quad VA$$

Real or True power(P): it is defined as the product of the applied voltage and active component of the current .

$$P = VI\cos\phi \; Watts$$

Reactive Power (Q): it is defined as product of the applied voltage and the reactive component of the current.

$$Q = VI\sin\phi \; VAR$$

Power Factor ($\cos\phi$): it is defined as factor by which the apparent power must be multiplied in order to obtain the true power.

$$\text{Power factor} = \frac{\text{True Power}}{\text{Apparent Power}} = \frac{VI\cos\phi}{VI} = \cos\phi$$

Problem:

Example 3.15.2 *A 20Ω resistance and 30 mH inductance are connected in series and the circuit is fed from a 230 V, 50 Hz AC supply. Find*

i) Reactance across the inductance, impedance, admittance, current

ii) Voltage across the resistance iii) Voltage across the inductance

iv) Real, reactive and active powers v) Power factor **May-08, Set-3, Marks 18**

Solution : The circuit is shown in the Fig. 3.15.6.

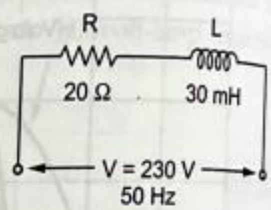


Fig. 3.15.6

i) $X_L = 2\pi f L = 2\pi \times 50 \times 30 \times 10^{-3} = \mathbf{9.4248\ \Omega}$

$\therefore\ Z = R + jX_L = 20 + j\,9.4248\ \Omega$

$= \mathbf{22.1094 \angle 25.2317^\circ\ \Omega}$ ... Impedance

$$Y = \frac{1}{Z} = \frac{1}{22.1094 \angle 25.2317^\circ}$$

$= \mathbf{0.04522 \angle -25.2317^\circ\ \mho}$ ... Admittance

$$\therefore\quad I = \frac{V}{Z} = \frac{230\angle 0^\circ}{22.1094\angle 25.2317^\circ} = \mathbf{10.4028\angle -25.2317^\circ\ A} \quad \text{... Current}$$

ii) $V_R = IR = 10.4028\angle -25.2317^\circ \times 20\angle 0^\circ = \mathbf{208.056\angle -25.2317^\circ\ V}$

iii) $V_L = I \times (j X_L) = 10.4028\angle -25.2317^\circ \times j\,9.4248$

$= 10.4028\angle -25.2317^\circ \times 9.4248\angle 90^\circ = \mathbf{98.0443 \angle 64.7683^\circ\ V}$

iv) $P = VI\cos\phi = 230 \times 10.4028 \times \cos(-25.2317^\circ) = \mathbf{2164.365\ W}$

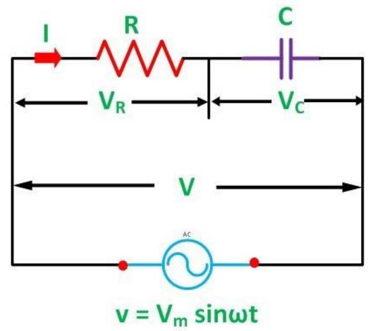
$Q = VI\sin\phi = 230 \times 10.4028 \times \sin(-25.2317^\circ) = \mathbf{-1019.9359\ VAR}$

The negative sign indicates lagging nature of reactive volt-amperes.

$S = VI = 230 \times 10.4028 = \mathbf{2392.644\ VA}$

v) Power factor= $\cos\emptyset = cos(-25.2317°) = 0.90459\ lagging$

2.8.2 Series R-C Circuit:



Consider a circuit consisting of pure resistance R ohms connected in series with a pure Capacitor of C farads. The series combination is connected across a.c supply given by

$$v = V_m \sin(\omega t)$$

Circuit draws a current 'I' then there are two voltage drops,

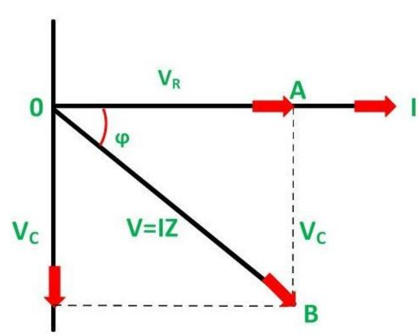
a) Drop across pure resistance, $V_R = I \times R$
b) Drop across pure inductance, $V_C = I \times X_C$ Where $X_C = \frac{1}{2\pi f C}$

$$V = V_R + V_L$$
$$V = IR + IX_C$$

Phasor diagram :

The following steps are given below which are followed to draw the phasor diagram step by step:

- Take the current I (r.m.s value) as a reference vector
- Voltage drop in resistance VR = IR is taken in phase with the current vector
- Voltage drop in capacitive reactance $V_C = IX_C$ is drawn 90 degrees behind the current vector, as current leads voltage by 90 degrees (in the pure capacitive circuit)
- The vector sum of the two voltage drops is equal to the applied voltage V (r.m.s value).



Now, in right-angle triangle OAB
V_R = IR , V_C =IX_C

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (\mathrm{I}\mathrm{X}_\mathrm{C})^2}$$

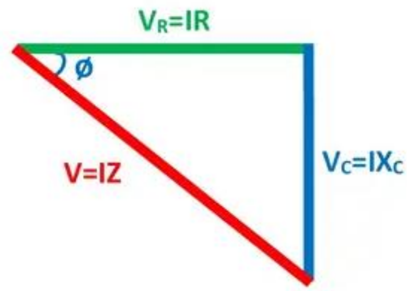
$$V = I\sqrt{(R)^2 + (\mathrm{X}_\mathrm{C})^2}$$

$$V = IZ$$

Where $Z = \sqrt{(R)^2 + (\mathrm{X}_\mathrm{C})^2}$

Voltage Triangle:

From the phasor diagram triangle OAB is voltage triagle.

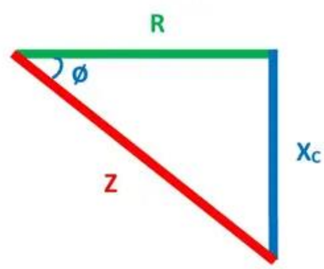


$$\cos\emptyset = \frac{V_R}{V}$$

$$\sin\emptyset = \frac{V_C}{V}$$

Impedance triangle:

If all the sides of voltage triangle are divided by current, we get triangle called impedance triangle.



$$\cos\emptyset = \frac{V_R}{V} = \frac{R}{Z}$$

$$\sin\emptyset = \frac{V_C}{V} = \frac{X_C}{Z}$$

$$\tan\emptyset = \frac{V_C}{V_R} = \frac{X_C}{R}$$

$$Z = \frac{V}{I}\ ,\ X_C = \frac{V_C}{I}\ ,\ R = \frac{V_R}{I}$$

X component of impedance is R and is given by

$$R = Z\cos\emptyset$$

And Y component of impedance is $\mathrm{X_L}$ and is given by,

$$\mathrm{X_C} = \mathrm{Z}\sin\emptyset$$

In rectangular from the impedance is denoted as ,

$$Z = R - j\mathrm{X_C}\ \Omega$$

While in polar form, it is denoted as,

$$Z = |Z|\angle - \phi\ \Omega$$

$$|Z| = \sqrt{(R)^2 + (\mathrm{X_L})^2},\ \ \phi = \tan^{-1}(\frac{-X_C}{R})$$

Power:

The expression for the current in the series R-C circuit is

$$i = I_m(\sin\omega t + \phi\ )$$

The power is product of instantaneous values of voltage and current

$$\therefore P = v \times i = V_m \sin\omega t \times I_m(\sin\omega t + \phi)$$

$$= V_m I_m[(\sin\omega t)(\sin\omega t + \phi)]$$

$$(\sin a\ \ \sin b) = \tfrac{1}{2}[\cos(a-b) - (\cos a + b)]$$

$$= V_m I_m[\frac{\cos(-\phi) - cos(2\omega t + \phi)}{2}]$$

$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi)$$

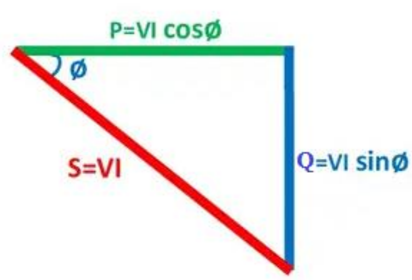
Now the second term is cosine tern whose average value over a cycle is zero. Hence, average power consumed is,

$$P_{av} = \tfrac{V_m I_m}{2}\cos\phi\ = \tfrac{V_m}{\sqrt{2}}\tfrac{I_m}{\sqrt{2}}\cos\phi$$

$$\therefore P = VI\ \cos\phi$$

Power Triangle:

If all the sides of Voltage triangle is multiplied by current I , we get Power triangle.



$$V = V_R + V_L$$

$$VI = VI\cos\phi + VI\sin\phi$$

So, three sides of this triangle are,

1) VI 2) $VI\cos\phi$ 3) $VI\sin\phi$

These terms can be defined as below.

Apparent power (S): it is defined as the product of r.m.s value of voltage (V) and current (I).

$$S = VI \quad VA$$

Real or True power(P): it is defined as the product of the applied voltage and active component of the current .

$$P = VI\,cos\,\phi \;\; Watts$$

Reactive Power (Q): it is defined as product of the applied voltage and the reactive component of the current.

$$Q = VI\,sin\,\phi\; VAR$$

Problem:

Example 3.16.1 *A capacitor having a capacitance of 10 μF is connected in series with a non inductive resistance of 120 Ω across 100 V, 50 Hz. Calculate the power, current and the phase difference between current and voltage.*

Jan.-10, Set-1, June-11, Set-2, Marks 8

Solution : The circuit is shown in the Fig. 4.62

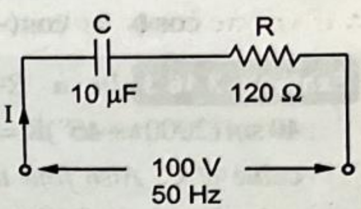


Fig. 3.16.6

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}} = 318.3098\ \Omega$$

$$Z = R - j X_C = 120 - j\,318.3098\ \Omega = 340.178\ \angle -69.344°\ \Omega$$

$$\therefore \quad I = \frac{V}{Z} = \frac{100\angle 0°}{340.178\ \angle -69.344°} = \mathbf{0.2939\ \angle\ 69.344°\ A}$$

Phase difference between V and I = ϕ = **69.344° leading**

$$\therefore \quad P = VI\cos\phi = 100 \times 0.2939 \times \cos(69.344°) = \mathbf{10.3697\ W}$$

$$\text{or} \quad P = I^2 R = (0.2939)^2 \times 120 = \mathbf{10.369\ W}$$

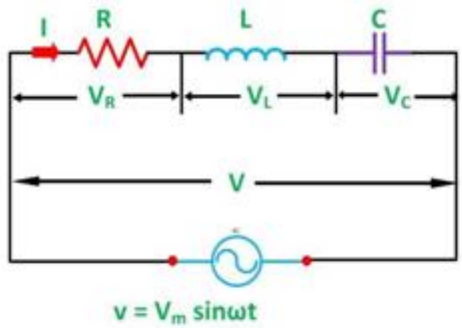
2.8.3 Series R-L-C Circuit:

Consider a circuit consisting of pure resistance R ohms connected in series with a Inductance L henries and Capacitence of C farads. The series combination is connected across a.c supply given by

$$v = V_m \sin(\omega t)$$

Circuit draws a current 'I' then there are two voltage drops,

a) Drop across pure resistance, $V_R = I \times R$
b) Drop across pure inductance, $V_L = I \times X_L$ Where $X_L = 2\pi f L$
c) Drop across pure inductance, $V_C = I \times X_C$ Where $X_C = \frac{1}{2\pi f C}$



$$V = V_R + V_L + V_C$$
$$V = IR + IX_L + IX_C$$

Phasor diagram :

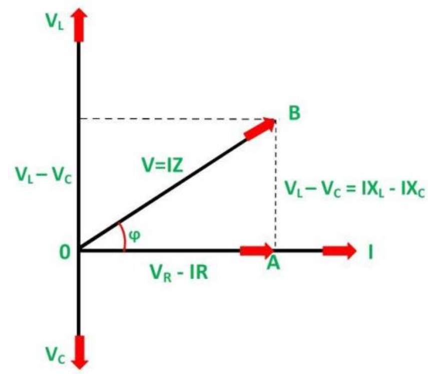
The following steps are given below which are followed to draw the phasor diagram step by step:

- V_R = IR that is the voltage across the resistance R and is in phase with the current I.
- V_L = IX_L that is the voltage across the inductance L and it leads the current I by an angle of 90 degrees.
- V_C = IX_C that is the voltage across capacitor C and it lags the current I by an angle of 90 degrees.

The phasor diagram depends on the conditions of the magnitudes of V_L and V_C which ultimately depends on the values of X_L and X_C. Let us consider the different cases.

$V_L > V_C$ or $X_L > X_C$:

The phasor diagram of the RLC series circuit when the circuit is acting as an inductive circuit that means ($V_L > V_C$)



From the voltage triangle,

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (\mathrm{IX_L} - \mathrm{IX_C})^2}$$

$$V = I\sqrt{(R)^2 + (\mathrm{X_L} - \mathrm{X_C})^2}$$

$$\boxed{V = IZ}$$

Impedance:

In rectangular from the impedance is denoted as ,

$$Z = R + j\mathrm{X}\,\Omega$$

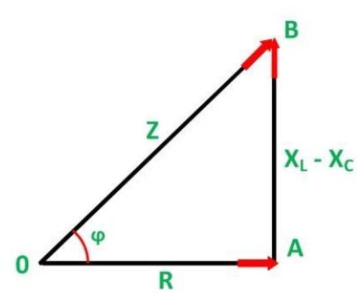
$$where\ X = \mathrm{X_L} - \mathrm{X_C}$$

While in polar form, it is denoted as,

$$Z = |Z|\angle\phi\ \Omega$$

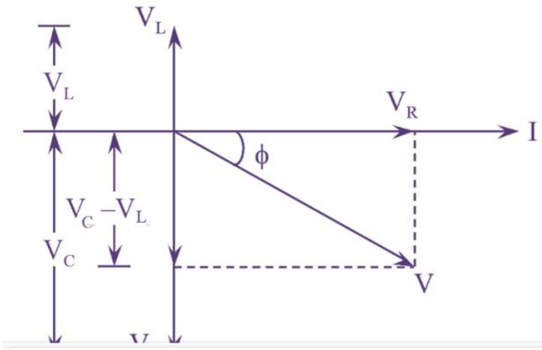
$$|Z| = \sqrt{(R)^2 + (\mathrm{X_L} - \mathrm{X_C})^2},\ \ \phi = \tan^{-1}(\frac{\mathrm{X_L} - \mathrm{X_C}}{R})$$

Impedance Triangle:



$V_L < V_C$ or $X_L < X_C$:

The phasor diagram of the RLC series circuit when the circuit is acting as an capacitive circuit that means ($V_L < V_C$)



From the voltage triangle,

$$V = \sqrt{(V_R)^2 + (V_C - V_L)^2} = \sqrt{(IR)^2 + (\mathrm{IX_C} - \mathrm{IX_L})^2}$$

$$V = I\sqrt{(R)^2 + (\mathrm{X_C} - \mathrm{X_L})^2}$$

$$\boxed{V = IZ}$$

Impedance:

In rectangular from the impedance is denoted as ,

$$Z = R + j\mathrm{X}\,\Omega$$

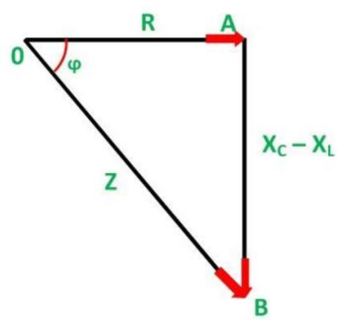
$$where\ X = \mathrm{X_C} - \mathrm{X_L}$$

While in polar form, it is denoted as,

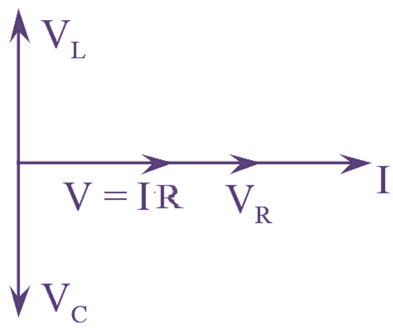
$$Z = |Z|\angle\phi\ \Omega$$

$$|Z| = \sqrt{(R)^2 + (\mathrm{X_C} - \mathrm{X_L})^2},\ \ \phi = \tan^{-1}\left(\frac{\mathrm{X_C} - \mathrm{X_L}}{R}\right)$$

Impedance Triangle:



V_L=V_C or X_L= X_C:



From the phasor diagram,

$$V = IR$$

$$X_L = X_C\ , \qquad Z = R$$

Power:

The average power consumed by the circuit is,

$$P_{av} = Average\ power\ consumed\ by\ R + Average\ power\ consumed\ by\ L + Average\ power\ consumed\ by\ C$$

But pure L and C never sonsume any power.

$$P_{av} = Power\ taken\ by\ R = I^2R = I(IR) = IV_R$$

$$but\ V_R = V\cos\emptyset$$

$$\boxed{P_{av} = VI\cos\emptyset\ watts}$$

2.9 Concept of resonance:

Electrical resonance is defined as the condition when the magnitude of capacitive reactance becomes equal to that of inductive reactance. As a result of resonance, maximum current flows through the RLC circuit.

2.9.1 Resonant frequency:

$$X_L = X_C.$$

Substituting the values of X_L and X_C,

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$f_0^{\ 2} = \frac{1}{2\pi^2 LC}$$

Solving this expression for f_0 yields,

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

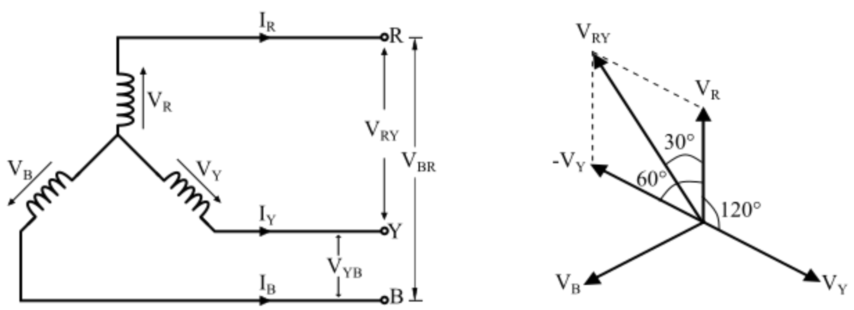
Where f_0= Resonant frequency

2.10 Three-phase balanced circuits-voltage and current relations in star and delta connections:

2.10.1 Star (Wye) Connected System:

Let V_R, V_Y and V_B represents the three phase voltages while V_{RY}, V_{YB} and V_{BR} represents the line voltages. Assume that the system is balanced, so

$$|V_R| = |V_Y| = |V_B| = |V_{ph}|$$



From the circuit and phasor diagram of star connected load, it can be observed that the line voltage V_{RY} is a vector difference of V_R and V_Y or the vector sum of V_R and $-V_Y$, i.e.

$$V_{RY} = V_R + (-V_Y) = V_R - V_Y$$

Applying parallelogram law to obtain the magnitude of this, we get,

$$V_{RY} = \sqrt{{V_R}^2 + {V_Y}^2 + 2V_R V_Y \cos 60^0}$$

$$V_{RY} = \sqrt{{V_{ph}}^2 + {V_{ph}}^2 + 2V_{ph} V_{ph} \cos 60^0} = \sqrt{3}V_{ph}$$

Similarly,

$$V_{YB} = V_Y - V_B = \sqrt{3}V_{ph}$$

$$V_{BR} = V_B - V_R = \sqrt{3}V_{ph}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L = Line\ voltage$$

$$\therefore V_L = \sqrt{3}V_{ph}$$

Therefore, in a star connected system,

$$\boxed{\boldsymbol{Line\ Voltage = \sqrt{3} \times Phase\ Voltage}}$$

In a star connection, the line current in each line is equal to the current in the corresponding phase winding.

Let I_R, I_Y and I_B being the currents in R, Y and B lines respectively. Since, the load is balanced, therefore,

$$I_R = I_Y = I_B = I_{ph}$$

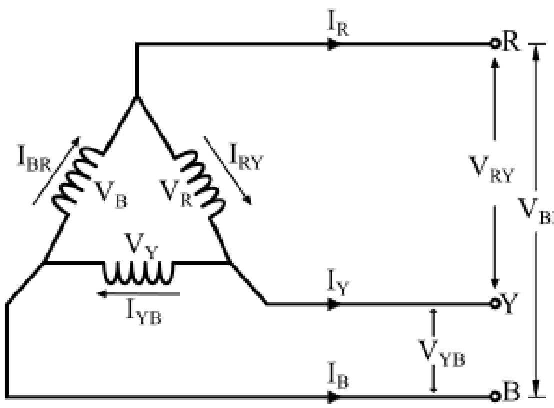
Then,

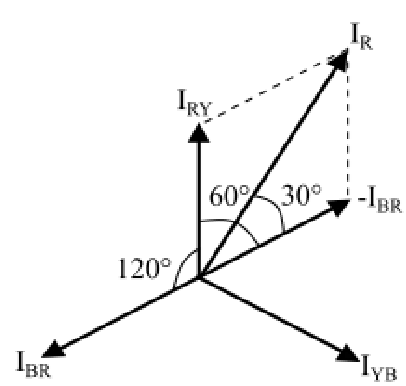
$$I_L = I_{ph}$$

$$\boxed{\boldsymbol{Line\ Current = Phase\ Current}}$$

2.10.2 Delta Connected System:

Let I_{RY}, I_{YB} and I_{BR} are the phase current in delta connected system while I_R, I_Y and I_B being the line currents.





By referring the circuit and phasor diagram, it can be seen that current in each line is the vector difference of corresponding phase currents and are given as,

$$I_R = I_{BR} - I_{RY}$$

$$I_Y = I_{RY} - I_{YB}$$

$$I_B = I_{YB} - I_{BR}$$

Now, the magnitude of current IR can be obtained by parallelogram law of vector addition, as follows,

$$\mathrm{I}_R = \sqrt{{I_{BR}}^2 + {I_{RY}}^2 + 2I_{BR}I_{RY}\cos 60^0}$$

Assume the system is balanced, therefore,

$$I_{RY} = I_{BR} = I_{YB} = I_{ph}$$

$$\mathrm{I}_R = \sqrt{{I_{ph}}^2 + {I_{ph}}^2 + 2I_{ph}I_{ph}\cos 60^0} = \sqrt{3}I_{ph}$$

Similarly,

$$\mathrm{I}_Y = \sqrt{3}I_{ph}$$

$$\mathrm{I}_B = \sqrt{3}I_{ph}$$

Since the system is balanced, therefore the current through each line will be the same, i.e.

$$I_R = I_Y = I_B = I_L = Line\ curent$$

$$\therefore I_L = \sqrt{3}I_{ph}$$

$$\boxed{\boldsymbol{Line\ Current\ = \sqrt{3} \times\ Phase\ Current}}$$

Since, neutral does not exist in a delta connected system, thus the phase voltage and line voltage are same. Refer the circuit diagram,

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

$$\boxed{\boldsymbol{V_L = V_{ph}}}$$