Practical 1

(1) $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

(2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

(3) $\lim_{x \to \frac{\pi}{6}} \left[\frac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

(4) $\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

(5) Examine the continuity of following function at given points

(i) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \frac{\pi}{2}$
$= \frac{\cos x}{\pi - 2x}$, for $\frac{\pi}{2} < x < \pi$ } at $x = \frac{\pi}{2}$

(ii) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$
$= x+3$ $\quad 3 \le x < 6$
$= \frac{x^2-9}{x+3}$ $\quad 6 \le x < 9$ } at $x=3$ & $x=6$

(6) Find value of k, so that the function $f(x)$ is cts at the indicated point

(i) $f(x) = \frac{1-\cos 4x}{x^2}$ $\quad x<0$
$\quad x=0$ } at $x=0$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \ne 0$
$= k$ $\quad x=0$ } at $x=0$

(iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$
$= K \quad x = \pi/3$ $\Big\}$ at $x = \pi/3$

① $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$= \lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \dfrac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$

$= \lim_{x \to a} \dfrac{(a+2x-3x) \cdot (\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$

$= \dfrac{1}{3} \lim_{x \to a} \dfrac{(a-x) \cdot (\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$

$= \dfrac{1}{3} \dfrac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$

$\dfrac{1}{3} \times \dfrac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\dfrac{1}{3} \times \dfrac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\dfrac{1}{3} \times \dfrac{4\sqrt{a}}{2\sqrt{3}\sqrt{a}}$

$= \dfrac{2}{3\sqrt{3}}$

Q.2 $\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$

$\lim_{y\to 0}\frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$\lim_{y\to 0}\frac{y}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$\frac{1}{\sqrt{a+0}\sqrt{a+0}+\sqrt{a}}$

$\frac{1}{\sqrt{a}(\sqrt{a}+\sqrt{a})}$

$\frac{1}{\sqrt{a}(2\sqrt{a})}=\frac{1}{2a}$

(3) $\lim_{x\to\pi/6}\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}$

By substituting $x-\frac{\pi}{6}=h$

$x=h+\pi/6$

where $h\to 0$

$\lim_{h\to 0}\frac{\cos(h+\pi/6)-\sqrt{3}\sin(h+\pi/6)}{\pi-6(h+\pi/6)}$ using $\cos(A+B)=\cos A\cdot\cos B\cdot$ $\sin A\cdot\sin B$

$\lim_{h\to 0}\frac{\cos h\cdot\cos\pi/6-\sin h\cdot\sin\pi/6-\sqrt{3}\sin\cos\pi/6+\cos h\cdot\sin\pi/6}{\pi-6\left(\frac{6h+\pi}{6}\right)}$

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$$\lim_{h\to 0} \frac{\cosh\frac{\sqrt{3}}{2} - \sin h\frac{1}{2} - \sqrt{3}\left(\sin h\frac{\sqrt{3}}{2} + \cos h\cdot\frac{1}{2}\right)}{\pi - 6h + \pi}$$

$$\lim_{h\to 0} \frac{\cos\frac{\sqrt{3}}{2}h - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cos\frac{\sqrt{3}h}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{-\sin 4h/2}{-6h}$$

$$\lim_{h\to 0} \frac{\sin 4h}{\cancel{8}12h}$$

$$\frac{1}{3}\lim_{h\to 0} \frac{\sin h}{h}$$

$$= \frac{1}{3} \times 1$$

$$= \frac{1}{3}$$

(4) $$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$$

By rationalizing numerator and denominator both

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3} + \sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5} + \sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty} \frac{8(\sqrt{x^2+3} + \sqrt{x^2+1})}{2(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$4\lim_{x\to\infty} \frac{\sqrt{x^2(1+\frac{3}{x^2})} + \sqrt{x^2(1+\frac{1}{x^2})}}{\sqrt{x^2(1+\frac{5}{x^2})} + \sqrt{x^2(1-3/x^2)}}$$

After applying limit we get.
$= 4$

⑤ ① $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \leq \pi/2$ $\Big\}$ at $x = \pi/2$

$= \frac{\cos x}{\pi - 2x}$ for $\pi/2 < x < \pi$

$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\quad f(\pi/2) = 0$

f at $x = \pi/2$ define.

$\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} \frac{\cos 2x}{\pi - 2x}$

$x - \pi/2 = h$

$x = h + \pi/2$

where $h \to 0$

$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{\pi - 2[h + \pi/2]}$

$\lim_{h \to 0} \frac{\cos(h+\pi/2)}{\pi - 2\left[\frac{2h+\pi}{2}\right]}$

$\lim_{h \to 0} \frac{\cos[h + \pi/2]}{-2h}$

$$\lim_{h\to 0} \cosh \cdot \frac{-\sinh}{-2h}$$

$$= \frac{1}{2}\lim_{h\to 0}\frac{\sinh}{h}$$

$$= 1/2$$

$$\lim_{x\to \pi/2} f(a) = \lim_{x\to \pi/2} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$$

$$\lim_{x\to \pi/2} f(x) = \lim_{x\to \pi/2} \frac{2\sin x \cdot \cos x}{\sqrt{2\sin^2 x}}$$

$$= \lim_{x\to \pi/2} \frac{2\cos x}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} \lim \cos x$$

LH $\neq$ R.H.S.

f is not continuous at $x = \pi/2$

(11) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \leq x \leq 6$

$= \frac{x^2-9}{x+3}$ $\quad 6 \leq x < 9$

at $x=3$ and $x=6$

at $x=3$

$$f(3) = \frac{x^2-9}{x-3} = 0$$

f at $x=3$ define

(I) $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x=3$ define

(II) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+3) = 6$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{(x-3)}$

LHS = RHS

f is continuous at $x=3$

for $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$

(6) (i) $f(x) = \left.\begin{array}{ll} \frac{1-\cos 4x}{x^2} & x<0 \\ = K & x=0 \end{array}\right\}$ at $x=0$

soln:- f is continuous at $x=0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = K$

$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = K$

$2 \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = K$

$2(2)^2 = K$

(II) $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$ $\}$ at $x = 0$

$= K$ $x = 0$

st

soln: $f(x) = (\sec^2 x)^{\cot^2 x}$

$$\lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$$

we know that

$$\lim_{x \to 0} (1 + px)^{\frac{1}{px}} = e$$

$= e$

$K = e$

(III) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $x \neq \pi/3$ $\}$ at $x = \pi/3$

$= K$ $x = \pi/3$

$x - \pi/3 = h$

$x = h + \pi/3$

where $h \to 0$

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3} - \tan \pi/3 + \tan h}{1 - \tan \pi/3}}{\pi - \pi - 3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3}(1-\tan\pi/3\cdot\tan h)\cdot(\tan\pi/3+\tan h)}{1-\tan\pi/3\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3}-\sqrt{3}\times\sqrt{3}\cdot\tan h)-(\sqrt{3}+\tan h)}{1-\tan\pi/3\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3}-3\tan h-\sqrt{3}-\tan h)}{1-\sqrt{3}\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$\lim_{h\to 0} \frac{4\tan h}{3h(1-\sqrt{3}\tan h)}$$

$$4/3 \lim_{h\to 0} \frac{\tan h}{h} \cdot \lim_{h\to 0} \frac{2}{(1-\sqrt{3}\tan h)}$$

$$= \frac{4}{3}\cdot\frac{1}{1-\sqrt{3}(0)}$$

$$= \frac{4}{3}(1/1)$$

$$= 4/3$$

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(7) (1)

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ = 9 & x = 0 \end{cases} \text{ at } x = 0$$

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$\lim_{x \to 0} \frac{2\sin^2 3/2 x}{x\tan x}$$

$$\lim_{x \to 0} \frac{2\sin^2 \frac{3x}{2} \times x^2}{x \frac{\tan x}{x^2} \times x^2}$$

$$= 2 \lim_{x \to 0} \frac{(3/2)^2}{1} = 2 \times \frac{9}{4} = \frac{9}{2}$$

$$\lim_{x \to 0} f(x) = \frac{4}{2} \quad . \; 9 = f(a)$$

$\therefore$ f is not continuous at $x = 0$

Redefine function.

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ \dfrac{9}{2} & x = 0 \end{cases}$$

Now, $\lim_{x \to 0} f(x) = f(0)$

f has removable discontinous at $x = 0$,

(8) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x = 0$

is continuous at $x = 0$

<u>given:</u> f is continuous at $x=0$

$$\lim_{x \to 0} f(x) = f(0).$$

$$\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$\lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$\lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$$

$$\lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

$$\log e + \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2 \lim_{x \to 0} \left(\frac{\sin x/2}{x}\right)^2$$

multiple with Num and denominator

$$= 1 + 2 \times \frac{1}{4}$$

$$= \frac{3}{2}$$

$$= f(0)$$

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(9) $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $x \neq \pi/2$

f(0) is continuous at $x = \pi/2$

$$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x \to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x(\sqrt{2} + \sqrt{1+\sin x})}$$

$$\lim_{x \to \pi/2} \frac{1 + \sin x}{1 \pm \sin^2 x(\sqrt{2} + \sqrt{1+\sin x})}$$

$$\lim_{x \to \pi/2} \frac{1 + \sin x}{(1 \pm \sin x)(1 - \sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1}{(1 - \sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{9}{2(\sqrt{2} + \sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})}$$

$$= \frac{1}{4\sqrt{2}}$$

$$f(\pi/2) = \frac{1}{4\sqrt{2}}$$

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Practical 2

Topic: Derivative

Q.1) Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

1) $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to a$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{}$$

formula: $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A-B)(1 + \tan A \cdot \tan B)$

$$= \lim_{h \to 0} \frac{\tan(a-a-a) - (1 + \tan a \tan(a+h))}{h \times \tan(a+b)\tan a}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right) \sin -\frac{h}{2}}{\cos a \cdot \cos(a+h) \times -\frac{h}{2}} \times -\frac{1}{2}$$

$$= \frac{-1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+0)}$$

$$= \frac{-1}{2} \times \frac{2\sin a}{\cos a \times \cos a}$$

$$= \tan a \cdot \sec a$$

Q.2 If $f(x) = 4x+1,\ x \le 2$
$= x^2+5 \quad x > 0$ at $x=2$, then find function differentiable or not.

Solution:

LHD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4 \times 2+1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x+2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{x-2}$$

$$= 4$$

$$Df(2^-) = 4$$

RHD:

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{x-2}$$

$$= 2+2$$

$$= 4$$

$Df(2^+) = 4$

RHD = LHD

f is differentiable at $x=2$

Q3) If $f(x) = 4x+7$, $x < 3$

$= x^2 + 3x+$, $x \geq 3$

find f is differentiable or not

<u>solution</u>:

RHD

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3\times3+1)}{}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)} = 3+6 = 9$$

$Df(3^+) = 9$

$LHD = Df(3^-)$

$$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x \to 3} \frac{4(x-3)}{(x-3)}$$

$Df(3^+) = 4$

$RHD \neq LHD$

f is not differentiable at $x=3$

If $f(x) = 8x-5$, $x \leq 2$

$= 3x^2-4x+7$, $x>2$ at $x=2$, then

find f is differentiable or not.

RHD:-

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2-4x+7-11}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2-4x-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2-6x+2x-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$$

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$$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{x-2}$$

$$= 3 \times 2 + 2 = 8$$

$Df(2^+) = 8$

LHD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x-16}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$$

$$= 8$$

$Df(2^-) = 8$

LHD = RHD

f is differentiable at $x = 3$

Practical 3

Topic:- Application of Derivative

1) Find the interval in which function is increasing or decreasing

i) $f(x) = x^3 - 5x - 11$

ii) $f(x) = x^2 - 4x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

iv) $f(x) = x^3 - 27x + 5$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2) Find the intervals in which function is concave upwards

1) $y = 3x^2 - 2x^3$

2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

3) $y = x^3 - 27x + 5$

iv) $y = 69 - 24 - 9x^2 + 2x^3$

v) $y = 2x^3 + x^2 - 20x + 4$

Solution:-

Q.1 $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\because$ f is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

+ − +

$-\sqrt{5}/3$ $\sqrt{5}/3$

$x \in (-\infty, -\sqrt{5}/3) \cup (\sqrt{5}/3, \infty)$

and f is decreasing iff $f'(x)<0$

$3x^2-5<0$

$3(x^2-5/3)<0$

$(x-\sqrt{5/3})(x+\sqrt{5/3})<0$

$x\in(-\sqrt{5/3},\sqrt{5/3})$

$-\sqrt{5/3}$ $\sqrt{5/3}$

2) $f(x)=x^2-4x$

$f'(x)=2x-4$

$\therefore$ $f(x)$ is increasing iff $f'(x)>0$

$2x-4>0$

$2(x-2)>0$

$x-2>0$

$x\in(2,\infty)$

and f is decreasing iff $f'(x)<0$

$\therefore$ $2x-4<0$

$\therefore$ $2(x-2)<0$

$\therefore$ $x-2<0$

$x\in(-\infty,2)$

3) $f(x)=2x^3+x^2-20x+4$

$f'(x)=6x^2+2x-20$

f is increasing iff $f'(x)>0$

$6x^2+2x-20>0$

$2(3x^2+x-10)>0$

$3x^2+x-10>0$

$3x^2+6x-5x-10>0$

$(x+2)(3x-5)>0$

$x\in(-\infty,-2)\cup(5/3,\infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$(x+2)(3x-5) < 0$

$+ \quad - \quad +$

$-2 \quad 5/3$

$x \in (-2, 5/3)$

) $f(x) = 3x^2 + 27x + 5$

$f'(x) = 3x^2 - 27$

f is increasing iff $f'(x) > 0$

$3(x^2 - 9) > 0$

$(x-3)(x+3) > 0$

$+ \quad +$

$-3 \quad 3$

$(x-3)(x+3) > 0$

$x \in (-\infty, -3) \cup (3, \infty)$

$f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

f' is increasing iff $f'(x) > 0$

$6x^2 - 18x - 24 > 0$

$6(x^2 - 3x - 4) > 0$

$x^2 - 4x - x - 4 > 0$

$(x-4)(x+1) > 0$

$x \in (-\infty, -1) \cup (4, \infty)$

$+ \quad - \quad +$

$-1 \quad 4$

and f is decreasing iff $f'(x) < 0$

$6x^2 - 18x - 24 < 0$

$(x-4)(x+1) < 0$

$x \in (-1, 4)$

+ − +

−1 4

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Q 2)

1) $y = 3x^2 - 2x^3$

$f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upward if $f''(x) > 0$

$(6 - 12x) > 0$

$12\left(\frac{6}{12} - x\right) > 0$

$x - \frac{1}{2} > 0$

$x > \frac{1}{2}$

$f''(x) > 0$

$x \in \left(\frac{1}{2}, \infty\right)$

2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x - \cancel{36x} + 24$

f is concave upward if $f''(x) > 0$

$12x^2 - 36x + 24 > 0$

$12(x^2 - 3x + 2) > 0$

$(x-2)(x-1) > 0$

$x \in (-\infty, 1) \cup (2, \infty)$

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1 2

3) $f = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$6x > 0$

$x > 0$

$x \in (0, \infty)$

4) $y = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24 - 18x + 6x$

$f''(x) = 12 - 18$

f is concave upward iff $f''(x) > 0$

$12x - 18 > 0$

$x - 3/2 > 0$

$x > 3/2$

$x \in (3/2, \infty)$

5) $y = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$f''(x) > 0$

$12x + 2 > 0$

$12(x + 2/12) > 0$

$x + 1/6 > 0$

$x > -1/6$

$f''(x) > 0$

there exist on interval $(-1/6, \infty)$

21/12/17

Topic:- Application of derivatives and newton's method.

1) Find maximum and minimum value of following.

i) $f(x) = x^2 + 16/x^2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$ $[-1/2, 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ $[-2, 3]$

2) Find the root of the following equation by newton's (take 4 iteration only) correct upto 4 decimal:

i) $f(x) = x^3 - 3x^2 - 55x + 93$ (take $x_0 = 0$)

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ is $[1, 2]$

soln:-

1) $f(x) = x^2 + 16/x^2$

$f'(x) = 2x - 32/x^3$

Now, Consider $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$\therefore 2x = 32/x^3$

$\therefore x^4 = 32/2$

$\therefore x^4 = 16$

$\therefore x = \pm 2$

$f''(x) = 2 + 96/x^2$

$f''(2) = 2 + 96/2^4$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

f has minimum value at $x=2$

$$f(2) = 2^2 + \frac{16}{2^2}$$
$$= 4 + \frac{16}{4}$$
$$= 4+4$$
$$= 8$$

$$f''(-2) = 2 + 96(-2)^4$$
$$= 2 + 96/16$$
$$= 2+16$$
$$= 8 > 0$$

< f has minimum value at $x=-2$

$\therefore$ function reaches minimum value at $x=2$, and $x=-2$

2) $f(x) = 3 - 5x^3 + 3x^5$

$$f'(x) = -15x^2 + 15x^4$$

Consider $= 15x^2 + 15x^4 = 0$

$$15x^4 = 15x^2$$
$$x^2 = 1$$
$$x = \pm 1$$

$$f''(x) = -30x + 60x^3$$
$$\cancel{f(1)} = -30 + 66$$
$$= 30 > 0$$ < f has minimum value at $x=2$

$$< f(1) = 3 - 5(1)^3 + 3(1)^5$$
$$= 6 - 5 = 1$$

$$f''(-1) = -30(-1) + 60(-1)^3$$
$$= 30 - 60$$

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$= -30 < 0$

f has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

∠ f has maximum value at $x = -1$ and has the minimum value 1 at $x = 1$

8) $f(x) = x^3 - 3x^2 + 1$

∠ $f'(x) = 3x^2 - 6x$

consider $f'(x) = 0$

∠ $3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$ f has maximum value at 0

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

∠ $f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

∠ f has maxinum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

∠ f has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$

4) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$< f'(x) = 6x^2 - 6x - 12$

Consider $f'(x) = 0$

$< 6x^2 - 6x - 12 = 0$

$< 6(x^2 - x - 2) = 0$

$< x^2 - x - 2 = 0$

$< x^2 + x - 2x - 2 = 0$

$< x(x+1) - 2(x+1) = 0$

$< (x-2)(x+1) = 0$

$< x = 2$ or $x = -1$

$< f''(x) = 12x - 6$

$< f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$<$ f has minimum value at $x = 2$

$< f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$<$ f has maximum value at $x = -1$

$< f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$<$ f has maximum value 8 at $x = -1$ and f has minimum value -19 at $x = 2$

Q.2) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $x_0 = 0 \rightarrow$ given

$f'(x) = 3x^2 - 6x - 55$

by newton's method

$x_{n+1} = x_n - f(x_n') / f'(x_n)$

$\therefore x_1 = x_0 - f(x_0)/f'(x_0)$

$\therefore x_1 = 0 + 9.5/55$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 5.5$

$= -55.9467$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$\therefore x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712 + 0.0011/55.9393$

$= 0.1712$

$\therefore$ the root of the equation is 0.1712

2) $f(x) = x^3 - 4x - 9$ [2, 3]

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

let $x_0 = 3$ be the initial approximation
$\therefore$ by newton's method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 2.7392 - 6.596/18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

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$x_2 = x_1 - f(x_2)/f'(x_2)$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7158 - 10.806 - 9 = -0.0901$

$f(3) = 3(2.7015)^2 - 4 = 21.8943 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901/17.8943 = 2.7015 + 0.0050$

$= 2.7065$

(3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

let $x_0 = 2$ be initial approximation by newton's method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_2)/f'(x_2)$

$= 2 - 2.2/5.2$

$= 2.04230 = 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4608 - 5.6772 - 10$
$= -8.2164$

$x_2 = x_1 - f(x_1)/f'(x_1)$
$= 1.577 + 0.6755/8.2164$
$= 1.577 + 0.0822$
$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 4.5677 - 4.9553 - 16.592 + 17$
$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= 8.2588 - 5.97312 - 10$
$= -7.7143$

$x_3 = x_2 - f(x_2)/f(x_2)$
$= 1.6592 + 0.0204/7.7143$
$= 1.6592 + 0.0026$
$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$
$= 4.5892 - 4.9708 - 16.618 + 17$
$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= 8.2847 - 5.9824 - 10$
$= -7.6977$

$x_4 = x_3 - f(x_3)/f'(x_3)$
$= 1.618 + \frac{0.0004}{7.6977}$
$= 1.6618$

Practical 5

Topic :- Integration

1) solve the following integration

1) $\dfrac{dx}{\sqrt{x^2+2x-3}}$

2) $\int (4e^{3x}+1)\,dx$

3) $\int (2x^2-3\sin x+5\sqrt{x})\,dx$

4) $\int \dfrac{x^3+3x+4}{\sqrt{x}}\cdot dx$

5) $\int (-7\sin(2t^4)\,dt$

6) $\int \sqrt{x}\,(x^2-1)\,dx$

7) $\int \dfrac{1}{x^3}\sin\left(\dfrac{1}{x^2}\right)\cdot dx$

8) $\int \dfrac{\cos x}{\sqrt[3]{\sin 2x}}\cdot dx$

9) $\int e^{\cos^2 x}\sin 2x\,dx$

10) $\int \dfrac{(x^2-2x)}{(x^3-3x^2+1)}\cdot dx$

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1) $\int \frac{1}{x^2+2x-3} \cdot dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}} \cdot dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$a^2+2ab+b^2=(a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} \cdot dx$

substitut

put $x-1=t$

$dx = \frac{1}{t} \times dt$ where $t=1$ $t=x+1$

$\int \frac{1}{\sqrt{t^2-4}} \cdot dt$

using

$\int \frac{1}{\sqrt{x^2-a^2}} \cdot dx = \ln\left(|x+\sqrt{x^2-a^2}\right)$

$= \ln\left(|t+\sqrt{t^2-4}\right)$

$t=x+1$

$= \ln(|x+1 \sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)+C$

2) $\int (4e^{3x}+1)dx$

$= \int 4e^{3x}.dx + \int 1 dx$

$= 4\int e^{3x} dx + \int 1 dx$ # $\int e^{ax} dx = \frac{1}{a} \times e^{ax}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + C$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x} dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}. dx$ # $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2.dx - \int 3\sin(x).dx + \int 5x^{1/2}.dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + C$

$= \frac{2x^3+10x\sqrt{x}}{3} + 3\cos x + C$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}.dx$

$= \int \frac{x^3+3x+4}{x^{1/2}} dx$

split the denominator

34.

$$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}} \cdot dx$$

$$= \int x^{5/2} \cdot dx + \int 3x^{1/2} dx + \int \frac{4}{x^{1/2}} \cdot dx$$

$$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$$

5) $\int t^7 \times \sin(2t^4) \cdot dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$$= \int t^{\cancel{7}} \times \sin(2t^4) \times \frac{1}{2 \times 4t^3} \cdot du$$

$$= \int t^4 \sin(2t^4) \times \frac{1}{2 \times 4} \cdot du$$

$$= \int t^4 \cdot \sin(2t^4) \times \frac{1}{8} du$$

$$= \frac{t^4 \times \sin(2t^4)}{8} du$$

substitute t^4 with $\frac{u}{2}$

$$= \int \frac{u/2 \times \sin(u)}{8} \cdot du$$

$$= \int \frac{u \times \sin(u)}{\cancel{2}} / 8 \cdot du$$

$$= \int \frac{u \times \sin(u)}{16} \cdot du$$

$$= \frac{1}{16} \int u \times \sin(u) \cdot du$$

$\int u\,dv = uv - \int v\,du$
where $v = u$

$dv = \sin(u) \times du$

$du = 1du \quad v = -\cos(u)$

$$= \frac{1}{16} \int u \times (-\cos(u)) - \int -\cos(u) \cdot du$$

$$= \frac{1}{16} \times (u \times (-\cos u)) + \int \cos(u) \cdot du$$

$\int \cos x\,dx = \sin x$

$$= \frac{1}{16} \times (u \times (-\cos(u)) + \sin(u))$$

Return the substitution $u = 2+4$

$$= 1/16 \times (2+4 \times (-\cos(2+4))) + \sin(2+4))$$

$$= -\frac{+4 \times \cos(2+4)}{8} + \sin\frac{(2+4)}{16} + C$$

$$\int \sqrt{x}\,(x2-1) \cdot dx$$

$$= \int \sqrt{x}\, x2 - \sqrt{x}\, dx$$

$$= \int x^{1/2} \times x^2 - x^{1/2}\,dx$$

$$= \int x^{5/2} - x^{1/2}\,dx$$

$= I_1 \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + c$

(iii) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}} \cdot dx$

$= \int \frac{\cos x}{\sin x^{2/3}} dx$

put $t = \sin(x)$

$t = \cos(x)$

$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos x \cdot dt}$

$= \frac{1}{\sin_x^{x/2}} \cdot dt$

$= \frac{1}{t^{2/3}} \cdot dt$

$I = \int \frac{1}{t^{2/3}} \cdot dt = \frac{-1}{(2/3-1)\, t^{2/3-1}}$

$= \frac{-1}{-\frac{1}{3}t^{\frac{2}{3}-1}} = \frac{1}{\frac{1}{3}t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$ 48

$= 3\sqrt[3]{t}$

return substitution $t = \sin(x)$

$= 3\sqrt[3]{\sin(x)} + C$

(x) $\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$

put $x^3-3x^2+1 = dt$

$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3\times 2x} \cdot dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x} \cdot dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)} \cdot dt$

$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3} \cdot dt$

$= \int \frac{1}{3(x^3-3x^2+1)} \cdot dt$

$= \int \frac{1}{3t} \cdot dt$

$= \frac{1}{3}\int \frac{1}{t} \cdot dt \qquad \int \frac{1}{x} \cdot dx = \ln|x|$

$= \frac{1}{3} \times \ln|t| + C$

$= \frac{1}{3} \times \ln(|x^3-3x^2+1|) + C$

AV
1/2020

Practical no.6

Topic:- Application of Integration and numeric of Integration

1) Find the length of the following curve

$x = t - \sin t$, $y = 1 - \cos t$, $t = [0, 2\pi]$

$$l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$$

$$= \int_0^{2\pi} \sqrt{(1-\cos t)^2 + (\sin t)^2} \cdot dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t} \cdot dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\ dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t} \cdot dt$$

$$= \int_0^{2\pi} 2 \left|\sin \frac{t}{2}\right| dt \quad \ldots \sin^2 \frac{t}{2} = \frac{1 - \cos t}{2}$$

$$= \int_0^{2\pi} \cancel{2\sin \frac{t}{2}} \cdot dt$$

$$= \left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$$

$$= 4 + 4$$
$$= 8$$

2) $y = \sqrt{4-x^2} \qquad x \in [-2,2]$

soln: $\frac{dx}{dr} = \frac{-2x}{2\sqrt{4-x^2}} = \frac{x}{\sqrt{4-x^2}}$

$$l = 2\int_0^2 \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\,.dx$$

$$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4-x^2}}\,.dx$$

$$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\,.dx$$

$$= 4\left(\sin^{-1}\left(\frac{x}{2}\right)\right)_0^2$$

$$= 2\pi$$

(3) $y = x^{3/2}$ in $[0,4]$

soln: $f'(x) = \frac{3}{2}x^{1/2}$

$$[f'(x)]^2 = \frac{9}{4}x$$

$$l = \int_0^3 \sqrt{1 + [f'(x)]^2}\,.dx$$

$$= \int_0^3 \sqrt{1 + \frac{9}{4}x}\,.dx$$

put $u = 1 + \frac{9}{4}x$; $du = \frac{9}{4} \cdot dx$

$$l = \int^{1+\frac{9}{4}x} \frac{4}{9}\sqrt{u} \cdot du = \left[\frac{4}{9} \cdot \frac{2}{3}\left(u^{3/2}\right)\right]_1^{1+\frac{9}{4}x}$$

$$= \frac{8}{27}\left[\left(1 + \frac{9x}{4}\right) - 1\right]$$

(ii) $x = 3\sin t$, $y = 3\cos t$, $t \in (0, 2\pi)$

$$\frac{dx}{dt} = 3\cos t \cdot \frac{dy}{dt} = -3\sin t$$

$$l = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} \cdot dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2} \cdot dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t} \cdot dt$$

$$= \int_0^{2\pi} \sqrt{9\sin y} + 9\cos^2 t \cdot dt$$

$$= \int_0^{2\pi} 3\sqrt{x} \cdot dx$$

$$= 3\int_0^{2\pi} \cdot dt$$

$$= 3[x]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= l = 6 \cdot \pi \text{ units}$$

(5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in (1,2)$

$\rightarrow \therefore \frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$

$l = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \cdot dy$

$= \int_1^2 \sqrt{1 + \frac{(y^4-1)}{4y^4}} \cdot dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}} \cdot dy$

$= \int_1^2 \frac{y^4+1}{2y^2} \cdot dy$

$= \frac{1}{2}\int_1^2 y^2 \cdot dy + \frac{1}{2}\int_1^2 y^{-2} \cdot dy$

$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1}}{1}\right]_1^2$

$= \frac{1}{2}\left(\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right)$

$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{8}\right]$

$= \frac{1}{2}\left[\frac{17}{8}\right]$

$l = \frac{17}{12}$ units

Q7 using simpson's rule solve the following

) $\int_0^2 e^{x^2} \cdot dx$ with $n=4$

[0, 0.5], [0.5, 1], [1, 1.5] [1.5, 2]

$$\int_0^2 e^{x^2} \cdot dx = \frac{1/2}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$$

$$= \frac{1/2}{3}\left(e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + e^{(1)^2}\right)$$

By simpson's rule

$$\int_0^2 e^{x^2} \cdot dx = \frac{1/2}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$$

$$= \frac{1/2}{3}\left(e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + e^{(1)^2}\right)$$

$$= 17.35362645$$

$\int_0^4 x^2 \cdot dx \quad \cancel{x=4}$

$\cancel{h = \frac{4-0}{4} = 1}$

x	0	1	2	3	4
y	0	1	4	9	16

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$$\int_0^1 x^2 \cdot dx = \frac{1}{3}\left[(y_0 + y_4) + 4(y_1 + y_2) + 2y_2\right]$$

$$= \frac{1}{3}\left[0 + 16 + 4(1+9) + 2\times 4\right]$$

$$= \frac{1}{3}\left[16 + 4(10) + 8\right]$$

$$= \frac{64}{3}$$

$$\int_0^4 x^2 \cdot dx = 21.3333$$

iii) $\int_0^{\pi} \sqrt{\sin x} \cdot dx$ with $\pi = 6$

$$l = \frac{\pi/3 - 0}{6} = \pi/18$$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$
y	0	0.4167	0.5848	0.7071	0.8017	0.8152	0.4306

$$\int_0^{\pi/3} \sqrt{\sin x} \cdot dx = \frac{1}{3}\left(y_0 + y_0 + 4(y_1 + y_3 + y_5) + 2(y_1 + y_3)\right)$$

$$= \frac{\pi/18}{3}\left[0.4167 + 0.4306 + 2[0.487 + 0.7071 + 0.8752) + 2(0.5848 + 0.842)\right]$$

$$= \pi/54\left[1.3473 + 4(1.999) + 2(1.3865)\right]$$

$$= \frac{\pi}{54}\left[1.3473 + 7.196 + 2.773\right]$$

$$= \frac{\pi}{54} \times 12.1163$$

$$\int_0^{\pi/3} \sqrt{\sin x} = 0.7049$$

11/01/2020

Practical no. 7

Topic:- Differential equation.

Q.1]

1) $x \cdot \frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$P(x) = \frac{1}{x} \quad Q(x) = \frac{e^x}{x}$

$I.F = e^{\int 1/x \, dx}$

$= e^{\int 1/x \cdot dx}$

$= e^{\ln x}$

$I.F = x$

$y(I.F) = \int Q(x) \, (I.F) \cdot dx + c$

$= \int \frac{e^x}{x} \cdot x \cdot dx + c$

$= \int e^x \cdot dx + c$

$xy = e^x + c$

Q.2 $e^x \cdot \frac{dy}{dx} + 2e^x y = 1$

$\frac{dy}{dx} + \frac{2e^x y}{e^x} = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

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$p(x) = 2, \quad Q(x) = e^{-x}$

$I.f = e^{\int 2dx}$

$= e^{2x}$

$y \cdot If = \int Q(x) \cdot (I.f) \cdot dx + c$

$y \cdot e^{2x} = \int e^{-x} \cdot e^{2x} \cdot dx + c$

$= \int e^{x} dx + c$

$y \cdot e^{2x} = e^{x} + c$

(3) $x \cdot \frac{dy}{dx} + \frac{\cos x}{x} = 2y$

$x \cdot \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$p(x) = 2/x \quad Q(x) = \cos x / x^2$

$I.f = e^{\int p \cdot dx}$

$= e^{\int 2/x \cdot dx}$

$= e^{\ln |x^2|}$

$= x^2$

$y(If) = \int Q(x) \cdot (I.f) \cdot dx + c$

$= \int \frac{\cos x}{x^2} \times x^2 dx + c$

$= \int \cos x \cdot dx + c$

$x^2 y = \sin x + c$

(4) $\frac{x \cdot dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$p(x) = 3/x \quad Q(x) = \sin x / x^3$

I.f

$I.f = e^{\int p \cdot dx}$

$= e^{\int 3/x \cdot dx}$

$= e^{\ln |x^3|}$

$= x^3$

$y \cdot I.f = \int Q(x) \cdot I.f \, dx + c$

$= \int \frac{\sin x}{x^3} \cdot x^3 \cdot dx + c$

$x^3 y = -\cos x + c$

(5) $e^{2x} \frac{dy}{dx} + 2e^{2x} \cdot y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$p(x) = 2 \quad Q(x) = 2x/e^{2x}$

$If = e^{\int p \, dx}$

$= e^{\int 2 \cdot dx}$

$= e^{2x}$

$y(If) = \int Q(x)(I.f) dx + c$

$y \cdot e^{2x} = \int \frac{2x}{e^{2x}} \cdot e^{2x} \cdot dx + c$

$= \int 2x \cdot dx + c$

$e^{2x} \cdot y = x^2 + c$

(6) $\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \cdot dy = 0$

$\sec^2 x \cdot \tan y \cdot dx = -\sec^2 y \tan x \cdot dy$

$\frac{\sec^2 x}{\tan x} \cdot dx = -\frac{\sec^2 y}{\tan y} \cdot dy$

on Integrating we get

$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} \cdot dy$

$\therefore \log|\tan x| = -\log|\tan y| + C$

$\therefore \log|\tan x \cdot \tan y| = C$

$\therefore \tan x \cdot \tan y = e^C$

(7) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

Differentiating on both sides

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

on Integrating we get.

$\int \frac{v+2}{v+1} \cdot dv = \int 3\,dx$

$\int \frac{v+1}{v} \cdot dx + \int \frac{1}{v+1} \cdot dv = 3x$

$v + \log|v+1| = 3x + C$

$2x + 3y + \log|2x+3y+1| = 3x + C$

$3y = x - \log|2x+3y+1| + C$

11/01/2020

Practical no. 9

Topic: limit and partial order derivative.

(1) Evaluate the following limit

(i) $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$ (ii) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

(iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2-z^2}{x^3-x^2y^2}$

(2) Find f_x, f_y for each of the following function.

(i) $f(x,y) = xye^{x^2}+y^2$ (ii) $f(x,y) = e^x \cos y$

(iii) $f(x,y) = x^3y^2-3x^2y+y^3+1$

(3) Using definition find values of f_x, f_y at $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$

(4) Find all second order partial derivatives of f. also verify whether $f_{xy} = f_{yx}$

(i) $f(x,y) = \frac{y^2-xy}{x^2}$ (ii) $f(x,y) = x^3+3x^2y^2-\log(x^2+1)$

(iii) $f(x,y) = \sin(x,y)+e^{x+y}$

(5) Find the linearization at given point.

(i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

(ii) $f(x,y) = 1-x+y\sin x$ at $(\pi/2, 0)$

(iii) $f(x,y) = \log x+\log y$ at $(1,1)$

Q:- $\lim\limits_{(x,y)\to(-4,1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

At $(-4,-1)$ Denominator $\neq 0$

$\therefore$ By applying limit

$$= \frac{(-4)^3-3(-1)+(-1)^2-1}{-4(-1)+5}$$

$$= \frac{-64+3+1-1}{4+5}$$

$$= \frac{-61}{9}$$

ii) $\lim\limits_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

at $(2,0)$; Denomination $\neq 0$

$\therefore$ By applying limit

$$= \frac{(0+1)((2)^2+0-(2))}{2+0}$$

$$= \frac{1(4+0-8)}{2}$$

$$= \frac{-4}{2}$$

$$= -2$$

iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$

at (1,1,1) Denominator = 0

$\therefore \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{(x-yz)(x+yz)}{x^2(x-yz)}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$

on Aplying limit

$= \frac{1+1(1)}{(1)^2}$

$= 2$

Q.2 $f(x,y) = xy\cdot e^{x^2+y^2}$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(xye^{x^2+y^2})$

$= ye^{x^2+y^2}(2x$

$\therefore f_x = 2xye^{x^2+y^2}$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(xye^{x^2+y^2})$

$= x\cdot e^{x^2+y^2}(2y)$

$f_y = \cancel{2yx^2}\ 2yxe^{x^2+y^2}$

(ii) $f(x,y) = e^x \cos y$

$$f_x = \frac{\partial}{\partial x}(f(x,y))$$

$$= \frac{\partial}{\partial x}(e^x \cos y)$$

$$f_x = e^x \cos y$$

$$f_y = \frac{\partial}{\partial y}(f(x,y))$$

$$= \frac{\partial}{\partial y}(e^x \cos y)$$

$$f_y = -e^x \sin y$$

(iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$$f_x = \frac{\partial}{\partial x}(f(x,y))$$

$$= \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$$

$$f_x = 3x^2y^2 - 6xy$$

$$f_y = \frac{\partial}{\partial y}(f(x,y))$$

$$= \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1$$

$$\therefore f_y = 2x^3y - 3x^2 + 3y^2$$

(1) $f(x,y) = \frac{2x}{1+y^2}$

$f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x \frac{d}{dx}(1+y^2)}{(1+y^2)^2}$

$= \frac{2+2y^2 - d}{(1+y^2)^2}$

$= \frac{2(1+y^2)}{(1+y^2)(1+y^2)}$

$= \frac{2}{1+y^2}$

at $(0,0)$

$= \frac{2}{1+0}$

$= 2$

$f_y = \frac{d}{dy}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x \frac{d}{dx}(1+y^2)}{(1+y^2)^2}$

$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$

$= \frac{-4xy}{(1+y^2)^2}$

at $(0,0) = \frac{-4(0)(0)}{(1+0)^2}$

$= 0$

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Q.4 $f(x,y) = \dfrac{y^2 - xy}{x^2}$

$$= \frac{x^2 \frac{d}{dx}(y^2 - xy) - (y^2 - xy)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_x = \frac{d}{dx}\left(\frac{-x^2y - 2x(y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4\left(\frac{d}{dx}(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy^2 + 2x^2y)\frac{d}{dx}(x^4)}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(x^2y - 2xy + 2x^2y)}{x^6}$$

$$f_{yy} = \frac{d}{dy}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2 - 0}{x^2}$$

$$= \frac{2}{x^2}$$

$$f_{xy} = \frac{d}{dx}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$$

$$= \frac{-x^2-4xy+2x^2}{xy} \quad \text{—(iii)}$$

$$f_{yx} = \frac{d}{dx}\left(\frac{2y-x}{x^2}\right)$$

$$= \frac{x^2\frac{d}{dx}(2y-x)-(2y-x)\frac{d}{dx}(x^2)}{(x^2)^2} \quad \text{—(iv)}$$

from (iii) and (iii)

(ii) $f(x,y) = x^3+3x^2y^2-\log(x^2+1)$

$$f_x = \frac{d}{dx}\left(x^2+3x^2y^2-\log(x^2+1)\right)$$

$$f_y = \frac{d}{dy}\left(x^3+3x^2y^2\cdot\log(x^2+1)\right)$$

$$= 3x^2+6xy^2-\frac{2x}{x^2+1}$$

$$= 0+6x^2y-0$$

$$= 6x^2y$$

$$f_{xy} = \frac{d}{dx}\left(3x^2+6xy^2-\frac{2x}{x^2+1}\right)$$

$$= 0+12xy-0$$

$$= 12xy$$

$$f_{yx} = \frac{d}{dx}\left(6x^2y\right)$$

$$= 12xy$$

$$f_{xy} = f_{yx}$$

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(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy + e^{x+y})$

$f_y = x\cos(xy) + e^{x+y}$ — (i)

$f_y = x\cos(xy + e^{x+y})$

$\therefore f_{xx} = \frac{d}{dx}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)(y) + e^{xy}(1)$

$= -y^2\sin(xy) + e^{x+y}$

$\therefore f_{yy} = \frac{d}{dx}(x\cos(x,y) + e^{x+y})$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x\sin(xy + e^{x+y})$ — (ii)

$f_{xy} = \frac{d}{dx}(y\cos(xy) + e^{x+y})$

$= y^2\sin(x+y) + \cos(xy) + e^{x+y}$ — (iii)

$\therefore f_{yx} = \frac{d}{dx}(x\cos(xy) + e^{x+y})$

$= \cancel{x^2}\cos(xy) + \cos(xy) + e^{x+y}$

From (iii) and (iv)

$f_{xy} \neq f_{yx}$

5 (1) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\rightarrow f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$$f(x) = \frac{1}{2\sqrt{x^2+y^2}}(2x)$$

$$= \frac{x}{\sqrt{x^2+y^2}}$$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$

$$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$$

$$= \frac{y}{\sqrt{x^2+y^2}}$$

f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$$

$$= \frac{x+y}{\sqrt{2}}$$

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(ii) $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$f(\pi/2, 0) = 1 - \pi/2 + 0 = 1 - \pi/2$

$f_x = 0 - 1 + y\cos x$

$f_y = 0 - 0 + \sin x$

f_x at $(\pi/2, 0) = -1 + 0$

$= -1$

f_y at $(\pi/2, 0) = \sin \pi/2$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \pi/2 + (-1)\left(x - \frac{\pi}{2}\right) + 1(y - 0)$

$= 1 - \pi/2 - x + \pi/2 + y$

$= 1 - x + y$

(iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\quad f_y = 0 + \frac{1}{y}$

f_x at $(1,1) = 1$ $\quad f_y$ at $(1,1) = 1$

$\therefore L(x,y) = f(a,b) + f(a,b)(x-a) + f(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

Practical no. 8

Topic :- Euler's method

(1) $\frac{dy}{dx} = y + e^{x} - 2$ $y(0) = 2$, $h = 0.5$
find $y(2)$

(2) $\frac{dy}{dx} = 1 + y^2$ $y(0) = 0$ $h = 0.2$ find $y(1)$

(3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $h = 0.2$ find $y(1)$

(4) $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$
for $h = 0.5$ & $h = 0.25$

(5) $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1) = 1$ find $y(1.2)$ with $h = 0.2$

solution :-

(1) $y(0) = 2$ $x_0 = 0$ $h = 0.5$
find $y(2)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$y(2) = 9.8215$

② $\frac{dy}{dx} = 1+y^2$ $y(0)=0$ $h=0.2$
find $y(1)$

$y_0=0$ $x_0=0$ $h=0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1.04	0.2
1	0.2	0.2	1.1664	0.408
2	0.4	0.408	1.4111	0.6412
3	0.6	0.6412	↓	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

③ $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0)=1$ $h=0.2$ find $y(1)$

$x_0=0$, $y_0=1$ $h=0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0.	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7694	1.5051
5	1	1.5051		

$y(1) = 1.5051$

④ $\frac{dy}{dx} = 3x^2+1$ $y(1)=2$ find $y(2)$

$h=0.5$ & $h=0.25$

$y_0=2$ $x_0=1$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6509	19.3360
3	1.75	19.3360	112.6426	299.9960
4	2	299.996		

$y(2) = 299.996$

⑤ $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1)=1$ find $y(1.2)$

$h=0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = 3.6$

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25/01/2020

Practical 10

Q.1) Find the directional derivative of the following function at given points & in the direction of given vector.

(1) $f(x,y) = x+2y-3 \quad a = (1,-1) \quad u = 3i - j$

$|\bar{u}| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1-2-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1+\frac{3}{\sqrt{10}}\right), \left(-1-\frac{h}{\sqrt{10}}\right)-3$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right) + 2\left(-1-\frac{h}{\sqrt{10}}\right)-3$

$= 1+\frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{-4 + h\sqrt{10} + 4}{h}$$

$$D_u f(a) = \frac{1}{\sqrt{10}}$$

(iii) $f(x) = y^2 - 4x + 1 \quad a = (3,4) \quad u = i + 5j$

Here, $u = i + 5j$ is not a unit vector.

$|\vec{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f_{xy}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$= \frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$$

$$D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

iii) $2x+3y \quad a=(1,2) \quad u=(3i+4j)$

Here, $u = 3i+4j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2+(4)^2} = \sqrt{25} = 5$

unit vector along u is $\frac{v}{|u|} = \frac{1}{5}(3,4)$

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

$f(a) = f(3,2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$$= f\left(1+\frac{3h}{5}, 2+\frac{4}{5}\right)$$

$f(a+hu) = 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$

$$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$$

$$= \frac{18h}{5} + 8$$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + 8 - 8}{h}$$

$$= \frac{18h}{5}$$

Q2 Find gradient vector of the following function at given point.

i) $f(x,y) = x^y + y^x \Rightarrow a = (1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \cdot \log x + x y^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (y x^{y-1} + y^2 \log y, \; x^y \log x + x y^{x-1})$

$\nabla f(1,1) = (1+0, 1+0)$

$= (1,1)$

ii) $f(x,y) = (\tan^{-1} x) y^2 \quad a = (1,-1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x \right)$

$\nabla f(1,-1) = \left(\cancel{\frac{1}{2}, \tan^{-1}(1)}(-2) \right)$

$= \left(\frac{1}{2}, \frac{\pi}{4}(-2) \right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2} \right)$

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(iii) $f(x,y,z) = xyz - e^{x+y+z}$ $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$
$f_y = xz - e^{x+y+z}$
$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+5}, xy - e^{x+y+z}$

$f(1,-1,0) = ((-1)(0) - e^{(1+(-1))+0}, (1)(0) - e^{x+(-1)+0},$

$(1)(-1) - e^{1+(-1)+0}$

$= (0 - e^0, 0 - e^0, -1 - e^0)$
$= (-1, -1, -2)$

Q.3 Find the equation of tangent & normal to each of the following using curves at given points

(1) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$
$f_x = \cos y \, 2x + e^{xy} \cdot y$
$f_y = x^2(-\sin y) + e^{xy} \cdot y$
$(x_0, y_0) = (1,0)$ $\therefore$ $x_0 = 1, y_0 = 0$

equation of tangent,
$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \, 2(1) + e^0 \, 0$

$= 1(2) + 0$
$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$
$= 0 + 1 \cdot (1)$
$= 1$

$2(x-1) + 1(y-0) = 0$
$2x - 2 + y = 0$
$2x + y - 2 = 0$ $\therefore$ It is the required equation of tangent.

equation of normal
$= ax + by + c = 0$
$= bx + ay + d = 0$
$1(1) + 2(y) + d = 0$
$1 + 2y + d = 0$ at $(1, 0)$
$= 1 + 2(0) + d = 0$
$d + 1 = 0$
$\therefore d = -1$

1) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$
$f_x = 2x + 0 - 2 + 0 + 0$
$= 2x - 2$
$f_y = 0 + 2y - 0 + 3 + 0$
$= 2y + 3$
$(x_0, y_0) = (2, -2)$ $\therefore x_0 = 2, y_0 = -2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$
$f_y(x_0, y_0) = 2(-2) + 3 = -1$
equation of tangent
$f_x(x - x_0) + f_y(y - y_0) = 0$
$2(x-2) + (-1(y+2)) = 0$

$2x - 2 - y - 2 = 0$
$2x - y - 2 = 0$
It is required eqn of tangent.

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Equation of Normal

$= ax + by + c = 0$

$bx + ay + d = 0$

$= -1\ (x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2, -2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$d = 6$

Q.4 Find the equation of tangent and normal line to each of the following surface.

① $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2, 1, 0) \therefore x_0 = 2, y_0 = 1, z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

equation tangent

$f_x(x - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0 \rightarrow$ this is required equation of tangent.

Equation of normal at $(4, 3, -11)$

$$\frac{x-x_0}{fx} = \frac{y-y_0}{fy} = \frac{z-z_0}{fz}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

(ii) $3xyz - x - y + z = -4$ at $(1, -1, 2)$

$3xyz - x - y + z + 4 = 0$

$fx = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$fy = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$fz = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ $x_0 = 1$, $y_0 = -1$, $z_0 = 2$

$fx(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$fy(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$fz(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

equation of tangent.

$-7(x-1)+5(y-1)-2(z-2)=0$

$-7x+7+5y+5-2z+4=0$

$-7x+5y-2z+16=0 \rightarrow$ this is required equation of tangent

Equation of normal at $(-7,5,-2)$

$$\frac{x-x_0}{dx}=\frac{y-y_0}{dy}=\frac{z-z_0}{dz}$$

$$=\frac{x-1}{-7}=\frac{y+1}{5}=\frac{z-2}{-2}$$

Q.5 Find the local maxima and minima for the following function

1) $f(x,y)=3x^2+y^2-3xy+6x-4y$

$f_x=6x+0-3y+6-0$

$=6x-3y+6$

$f_y=0+2y-3x+0-4$

$=2y-3x-4$

$f_x=0$

$6x-3y+6=0$

$3(2x-y+2)=0$

$2x-y+2=0$

$2x-y=-2$

$f_y=0$

$2y-3x-4=0$

$2y-3x=4$ — (ii)

multiply eqn 1 with 2

$4x - 2y = -4$

$2y - 3x = 4$

$x = 0$

substitute value of x in eqn (1)

$2(0) - y = -2$

$-y = -2$

$\therefore$ critical points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

f has maximum at $(0, 2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4$

(11) $f(x, y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$3x^2 - 2y = 0$ (11)

multiply eq① with ③

② with 4

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$y = 0$

substitute value of y in equn①

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

Critical point is (0,0)

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at (0,0)

$= 24(0) + 6(0) = 0$

$r = 0$

$rt - s^2 = 0(-2) - (5)^2$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

(nothing to say)

(iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0$ $\quad 2x + 2 = 0$

$x = \frac{-2}{2}$ $\quad x = -1$

$dy = 0 \quad -2y + 8 = 0$

$y = \frac{-8}{-2}$

$y = 4$

critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x, y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70$

$= 33$

AB
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