

SOLUTIONS: ECE 305 Homework: Week 3

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- 1) Assume Silicon (bandgap 1.12 eV) at room temperature (300 K) with the Fermi level located **exactly** in the middle of the bandgap. Answer the following questions.
 - a) What is the probability that a state located at the bottom of the conduction band is **filled**?
 - b) What is the probability that a state located at the top of the valence band is **empty**?

Solution:

1a) Begin with the Fermi function: $f(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}}$

For our problem: $E = E_C$

$$f(E_C) = \frac{1}{1 + e^{(E_C-E_F)/k_B T}} \approx e^{-(E_C-E_F)/k_B T}$$

The approximation is extremely good, because the bottom of the conduction band is very far above the Fermi level. The semiconductor is said to be **nondegenerate**.

$$E_F = \frac{E_C + E_V}{2} \quad (\text{The Fermi level is exactly in the middle of the bandgap.})$$

$$E_C - E_F = E_C - \frac{E_C + E_V}{2} = \frac{E_C - E_V}{2} = \frac{E_G}{2}$$

so the probability is

$$f(E_C) \approx e^{-(E_C-E_F)/k_B T} = e^{-E_G/2k_B T}$$

$$E_G = 1.12 \text{ eV} \quad k_B T = 0.026 \text{ eV} \quad E_G/2k_B T = 21.5$$

$$f(E_C) = e^{-E_G/2k_B T} = e^{-21.5} = 4.43 \times 10^{-10}$$

What if we had not made the non-degenerate approximation? Then

$$f(E_C) = \frac{1}{1 + e^{(E_C-E_F)/k_B T}} = \frac{1}{1 + e^{E_G/2k_B T}} = \frac{1}{1 + 2.26 \times 10^9} = 4.43 \times 10^{-10}$$

Same answer as with the non-degenerate assumption.

HW3 Solutions (continued):

1b) Begin again with the Fermi function: $f(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}}$

For this part: $E = E_V$

$$f(E_V) = \frac{1}{1 + e^{(E_V-E_F)/k_B T}}$$

The Fermi function gives the probability that a state is **full**. We want the probability that the state is empty, which is

$$1 - f(E_V) = 1 - \frac{1}{1 + e^{(E_V-E_F)/k_B T}} = \frac{e^{(E_V-E_F)/k_B T}}{1 + e^{(E_V-E_F)/k_B T}} = \frac{1}{e^{-(E_V-E_F)/k_B T} + 1}$$

$$1 - f(E_V) = \frac{1}{1 + e^{(E_F-E_V)/k_B T}} \approx e^{-(E_F-E_V)/k_B T}$$

The approximation is extremely good, because the Fermi level is very far above the top of the valence band. The **nondegenerate** approximation can be used for the valence band too.

$$E_F = \frac{E_C + E_V}{2} \quad (\text{The Fermi level is exactly in the middle of the bandgap.})$$

$$E_F - E_V = \frac{E_C + E_V}{2} - E_V = \frac{E_C - E_V}{2} = \frac{E_G}{2}$$

so the probability is

$$1 - f(E_V) = e^{-(E_F-E_V)/k_B T} = e^{-E_G/k_B T}$$

$$\boxed{1 - f(E_V) = e^{-E_G/2k_B T} = 4.43 \times 10^{-10}}$$

2) For Si at room temperature, calculate the following quantities. (Numerical answers required, and don't forget to include units with your answer).

- The density of states in the conduction band, $g_C(E)$, at an energy 26 meV above E_C .
- The density of states in the valence band, $g_V(E)$, at an energy 26 meV below E_V .
- The effective density of conduction band states, N_C .
- The effective density of valence band states, N_V .
- Compute the ratio of the effective density of conduction band states to the atomic density of Si.

HW3 Solutions (continued):

Solution:

2a) Begin with the DOS expression (SDF, eqn.(2.6a), p. 41)

$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3} \quad E > E_c$$

$$g_c(E) = \frac{m_0 \sqrt{2m_0}}{\pi^2 \hbar^3} \left(\frac{m_n^*}{m_0} \right)^{3/2} \sqrt{E - E_c}$$

$$g_c(E) = \frac{9.11 \times 10^{-31} \sqrt{2 \times 9.11 \times 10^{-31}}}{3.14^2 \times (1.06 \times 10^{-34})^3} \left(\frac{m_n^*}{m_0} \right)^{3/2} \sqrt{E - E_c} \quad (\text{everything in MKS (SI) units})$$

$$g_c(E) = 1.05 \times 10^{56} \left(\frac{m_n^*}{m_0} \right)^{3/2} \sqrt{E - E_c}$$

According to SDF, p. 34, $m_n^*/m_0 = 1.18$

$$g_c(E) = 1.05 \times 10^{56} (1.18)^{3/2} \sqrt{E - E_c} = 1.35 \times 10^{56} \sqrt{E - E_c} (\text{J-m}^3)^{-1}$$

Note the units. This is the number of states per unit energy (in Joules) per unit volume (in cubic meters).

We are asked for the density of states 26 meV (milli electron volts) above the bottom of the conduction band. In Joules:

$$E - E_c = 0.026 \times 1.60 \times 10^{-19} = 4.16 \times 10^{-21} \text{ J}$$

$$g_c(0.026 \text{ eV} + E_c) = 1.35 \times 10^{56} \sqrt{4.16 \times 10^{-21}} \frac{1}{\text{J-m}^3} = 8.71 \times 10^{45} \frac{1}{\text{J-m}^3}$$

The answer is correct, but the units are inconvenient. Let's express the answer per eV per cubic cm, which is the more typical way of doing things for semiconductors.

$$g_c(0.026 \text{ eV} + E_c) = 8.71 \times 10^{45} \frac{1}{\text{J}} \times \frac{1.6 \times 10^{-19} \text{ J}}{\text{eV}} \frac{1}{\text{m}^3} \times \left(\frac{10^{-2} \text{ m}}{\text{cm}} \right)^3$$

$$g_c(0.026 \text{ eV} + E_c) = 1.39 \times 10^{21} \frac{1}{\text{eV} \cdot \text{cm}^3}$$

HW3 Solutions (continued):

Note: The total number of states in a range of energy of $k_B T = 0.026 \text{ eV}$ is approximately

$$g_C(0.026 \text{ eV} + E_C) \times 0.026 = 3.62 \times 10^{19} \frac{1}{\text{cm}^3},$$

which is a reasonable number, as we shall see below. This answer is approximate, because to get the total number of states in the bottom $k_B T$ of the conduction band, we should do the integral

$$N_{tot} = \int_{E_C}^{E_C+k_B T} g_C(E) dE .$$

A note about units. Calculations should be done in the MKS (also called the SI) system of units. All of the fundamental constants (e.g. Plank's constant, Boltzmann's constant, rest mass of an electron, etc.) are in these units. After we have an answer, it is often convenient to convert to different units. For example, in semiconductor work, we like to quote carrier and doping densities per cubic cm, not per cubic meter, which is the proper MKS unit.

Be careful about units!

2b) Begin with the DOS expression (SDF, eqn.(2.6b), p. 41)

$$g_V(E) = \frac{m_p^* \sqrt{2m_p^*(E_V - E)}}{\pi^2 \hbar^3} \quad E < E_V$$

$$g_V(E) = \frac{m_0 \sqrt{2m_0}}{\pi^2 \hbar^3} \left(\frac{m_p^*}{m_0} \right)^{3/2} \sqrt{E_V - E}$$

$$g_V(E) = \frac{9.11 \times 10^{-31} \sqrt{2 \times 9.11 \times 10^{-31}}}{3.14^2 \times (1.06 \times 10^{-34})^3} \left(\frac{m_p^*}{m_0} \right)^{3/2} \sqrt{E_V - E} \quad (\text{everything in MKS (SI) units})$$

$$g_V(E) = 1.05 \times 10^{56} \left(\frac{m_p^*}{m_0} \right)^{3/2} \sqrt{E_V - E}$$

According to SDF, p. 34, $m_p^*/m_0 = 0.81$

HW3 Solutions (continued):

$$g_V(E) = 1.05 \times 10^{56} (0.81)^{3/2} \sqrt{E_V - E} = 7.65 \times 10^{55} \sqrt{E_V - E} (\text{J}\cdot\text{m}^3)^{-1}$$

Note that the DOS in the valence band is a little smaller than in the conduction band because the hole effective mass is smaller than the electron effective mass.

We are asked for the density of states 26 meV (milli-electron volts) below the top of the valence band. In Joules

$$E_V - E = 0.026 \times 1.60 \times 10^{-19} = 4.16 \times 10^{-21} \text{ J}$$

$$g_C(E_V - 0.026 \text{ eV}) = 7.65 \times 10^{55} \sqrt{4.16 \times 10^{-21}} \frac{1}{\text{J}\cdot\text{m}^3} = 4.94 \times 10^{45} \frac{1}{\text{J}\cdot\text{m}^3}$$

The answer is correct, but the units are inconvenient. Let express the answer per eV per cubic cm, which is the more typical way of doing things for semiconductors.

$$g_V(E_V - 0.026 \text{ eV}) = 4.94 \times 10^{45} \frac{1}{\text{J}} \times \frac{1.6 \times 10^{-19} \text{ J}}{\text{eV}} \frac{1}{\text{m}^3} \times \left(\frac{10^{-2} \text{ m}}{\text{cm}} \right)^3$$

$$g_V(E_V - 0.026 \text{ eV}) = 7.90 \times 10^{20} \frac{1}{\text{eV} \cdot \text{cm}^3}$$

Note: The total number of states in a range of energy of $k_B T = 0.026 \text{ eV}$ is approximately

$$g_V(E_V - 0.026 \text{ eV}) \times 0.026 = 2.06 \times 10^{19} \frac{1}{\text{cm}^3}$$

which is a reasonable number, as we shall see below.

- 2c) This is a problem about the “effective density of states” which is different from the density-of-states. The effective density of states, is roughly the total number of states in an energy range of $k_B T$ near the bottom of the conduction band or top of the valence band. It is derived by integration as discussed in the text. The result is given in SDF on p. 51

$$N_C = 2.510 \times 10^{19} \left(\frac{m_n^*}{m_0} \right)^{3/2} \frac{1}{\text{cm}^3}$$

Using $m_n^* = 1.18 m_0$, we find

HW3 Solutions (continued):

$$N_c = 2.510 \times 10^{19} (1.18)^{3/2} \text{ cm}^{-3} = 3.22 \times 10^{19} \text{ cm}^{-3}$$

$N_c = 3.22 \times 10^{19} \text{ cm}^{-3}$

Note that this is close to the value we estimated in part 2a):

$$g_c (0.026 \text{ eV} + E_c) \times 0.026 = 3.62 \times 10^{19} \frac{1}{\text{cm}^3}$$

2d) Repeat 2c) but for the valence band.

$$N_v = 2.510 \times 10^{19} \left(\frac{m_p^*}{m_0} \right)^{3/2} \frac{1}{\text{cm}^3}$$

Using $m_n^* = 0.81 m_0$, we find

$$N_v = 2.510 \times 10^{19} (0.81)^{3/2} \text{ cm}^{-3} = 1.83 \times 10^{19} \text{ cm}^{-3}$$

$N_v = 1.83 \times 10^{19} \text{ cm}^{-3}$

Note that this is close to the value we estimated in part 2b):

$$g_v (E_v - 0.026 \text{ eV}) \times 0.026 = 2.06 \times 10^{19} \frac{1}{\text{cm}^3}$$

2e) Recall the atomic density of Si is $N_{Si} = 5.00 \times 10^{22} \text{ cm}^{-3}$ so the ratio

$$\frac{N_c}{N_{Si}} = \frac{3.22 \times 10^{19}}{5.00 \times 10^{22}} = 6.44 \times 10^{-4}$$

$\frac{N_c}{N_{Si}} = 0.06 \%$

The number of states near the bottom of the conduction band that can be occupied is a small fraction of the density of silicon atoms.

HW3 Solutions (continued):

- 3) Consider a region of Si at room temperature. For each of the following cases, calculate the equilibrium electron and hole concentrations (n and p). Assume that the dopants are fully ionized.
- Intrinsic material ($N_D = N_A = 0$)
 - $N_D = 1.00 \times 10^{13} \text{ cm}^{-3}$ $N_A = 0$
 - $N_D = 1.00 \times 10^{17} \text{ cm}^{-3}$ $N_A = 0$
 - $N_D = 0$ $N_A = 1.00 \times 10^{17} \text{ cm}^{-3}$
 - $N_D = 1.00 \times 10^{17} \text{ cm}^{-3}$ $N_A = 3.00 \times 10^{17} \text{ cm}^{-3}$

Solution:

- 3a) For intrinsic material, we know that $n = p = n_i$. According to SDF, p. 54,

$$n_i = 1.00 \times 10^{10} \text{ cm}^{-3} \text{ so}$$

$$n = p = n_i = 1.00 \times 10^{10} \text{ cm}^{-3}$$

- 3b) First, let's ask what to expect. The donor density is 1000 times the intrinsic density. All of the donor electrons will go in the conduction band, overwhelming the number of intrinsic carriers that were there. So we expect

$$n \approx N_D = 1.00 \times 10^{13} \text{ cm}^{-3}$$

We would determine the hole density from $np = n^2$, so

$$p = \frac{n^2}{N_D} \approx \frac{n^2}{10^{13}} = \frac{10^{20}}{10^{13}} = 1.00 \times 10^7 \text{ cm}^{-3}$$

Note that doping the semiconductor n-type means that we now have more electrons in the conduction band, and we have fewer holes in the valence band than for the intrinsic semiconductor.

How would we do this problem more accurately?

Begin by assuming that the semiconductor is neutral:

$$p - n + N_D^+ - N_A^- = 0$$

Assume that the dopants are fully ionized and use $p = n_i^2/n$

$$\frac{n_i^2}{n} - n + N_D - N_A = 0$$

HW3 Solutions (continued):

Solve the quadratic equation for n

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

Plug in numbers:

$$\begin{aligned} n &= \frac{10^{13} - 0}{2} + \sqrt{\left(\frac{10^{13}}{2}\right)^2 + 10^{20}} = 0.5 \times 10^{13} + \sqrt{0.25 \times 10^{26} + 10^{20}} \\ &= 0.5 \times 10^{13} + \sqrt{0.25 \times 10^{26} + 0.000001 \times 10^{26}} \end{aligned}$$

$$n = 0.5 \times 10^{13} + 0.5 \times 10^{13} = 1.0 \times 10^{13}$$

$$n = 1.00 \times 10^{13} \text{ cm}^{-3}$$

Just as expected. We find the hole density from

$$p = \frac{n^2}{n} \approx \frac{n^2}{N_D} = \frac{10^{20}}{10^{13}} = 1.00 \times 10^7 \text{ cm}^{-3}$$

$$p = 1.00 \times 10^7 \text{ cm}^{-3}$$

- 3c) First, let's ask what to expect. The donor density is 10^7 times the intrinsic density. All of the donor electrons will go in the conduction band, overwhelming the number of intrinsic carriers that were there. So we expect

$$n \approx N_D = 1.00 \times 10^{17} \text{ cm}^{-3}$$

We would determine the hole density from $np = n_i^2$, so

$$p = \frac{n^2}{n} \approx \frac{n^2}{N_D} = \frac{10^{20}}{10^{17}} = 1.00 \times 10^3 \text{ cm}^{-3}$$

Note that doping the semiconductor n-type means that we now have more electrons and we have fewer holes than the intrinsic semiconductor.

Now do this problem more accurately.

Use the quadratic equation for n :

HW3 Solutions (continued):

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

Plug in numbers:

$$\begin{aligned} n &= \frac{10^{17} - 0}{2} + \sqrt{\left(\frac{10^{17}}{2}\right)^2 + 10^{20}} = 0.5 \times 10^{17} + \sqrt{0.25 \times 10^{34} + 10^{20}} \\ &= 0.5 \times 10^{17} + \sqrt{0.25 \times 10^{34} + 0} \end{aligned}$$

$$n = 0.50 \times 10^{17} + 0.50 \times 10^{17} = 1.00 \times 10^{17}$$

$$n = 1.00 \times 10^{17} \text{ cm}^{-3}$$

Just as expected. Next, we find the hole density from

$$p = \frac{n^2}{n} \approx \frac{n_i^2}{N_D} = \frac{10^{20}}{10^{17}} = 1.00 \times 10^3 \text{ cm}^{-3}$$

$$p = 1.00 \times 10^3 \text{ cm}^{-3}$$

- 3d) Again, let's ask what to expect. The acceptor density is 10^7 times the intrinsic density. All of the acceptor holes will go in the valence band, overwhelming the number of intrinsic carriers that were there. So we expect

$$p \approx N_A = 1.00 \times 10^{17} \text{ cm}^{-3}$$

We would determine the electron density from $np = n_i^2$, so

$$n = \frac{n^2}{p} \approx \frac{n_i^2}{N_A} = \frac{10^{20}}{10^{17}} = 1.00 \times 10^3 \text{ cm}^{-3}$$

Note that doping the semiconductor p-type means that we now have more holes in the valence band, and we have fewer electrons in the conduction band than the intrinsic semiconductor.

Now do this problem more accurately.

Use the quadratic equation for p :

HW3 Solutions (continued):

$$p = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}$$

Plug in numbers:

$$\begin{aligned} p &= \frac{10^{17} - 0}{2} + \sqrt{\left(\frac{10^{17}}{2}\right)^2 + 10^{20}} = 0.5 \times 10^{17} + \sqrt{0.25 \times 10^{34} + 10^{20}} \\ &= 0.5 \times 10^{17} + \sqrt{0.25 \times 10^{34} + 0} \end{aligned}$$

$$p = 0.50 \times 10^{17} + 0.50 \times 10^{17} = 1.00 \times 10^{17}$$

$$p = 1.00 \times 10^{17} \text{ cm}^{-3}$$

Just as expected. We find the electron density from

$$n = \frac{n_i^2}{p} \approx \frac{n_i^2}{N_A} = \frac{10^{20}}{10^{17}} = 1.00 \times 10^3 \text{ cm}^{-3}$$

$$n = 1.00 \times 10^3 \text{ cm}^{-3}$$

- 3e) Since the net p-type doping is $N_A - N_D = 3 \times 10^{17} - 1 \times 10^{17} = 2 \times 10^{17}$ is much greater than the intrinsic density, we expect: $p = 2 \times 10^{17}$ and $n = n_i^2 / p = 0.5 \times 10^3 \text{ cm}^{-3}$. This is correct, as we can see by solving the problem properly.

$$p = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2} = \frac{3 \times 10^{17} - 1 \times 10^{17}}{2} + \sqrt{\left(\frac{2 \times 10^{17}}{2}\right)^2 + 10^{20}}$$

$$p = 1 \times 10^{17} + \sqrt{2 \times 10^{34} + 10^{20}} = 2 \times 10^{17}$$

$$p = 2 \times 10^{17} \text{ cm}^{-3}$$

$$n = \frac{n_i^2}{p} \approx \frac{n_i^2}{2 \times 10^{17}} = \frac{10^{20}}{2 \times 10^{17}} = 5.00 \times 10^2 \text{ cm}^{-3}$$

$$n = 5.00 \times 10^2 \text{ cm}^{-3}$$

HW3 Solutions (continued):

One might conclude from these examples, that there is no need to use space-charge neutrality and the quadratic equation. That would be wrong, as seen in the next problem. It only happened in this problem because in each case (except for case a)), the net doping was much larger than the intrinsic carrier density.

- 4) Assuming silicon with completely ionized dopants, compute n and p for the following case.

$$N_D = 5.00 \times 10^{16} \text{ cm}^{-3}$$

$$N_A = 0$$

$$T = 700 \text{ K}$$

Solution:

We expect the intrinsic carrier density to be MUCH larger at higher temperatures. Using the information on p. 57 of SDF, we have

$$n_i(700 \text{ K}) = 2.865 \times 10^{16} \text{ cm}^{-3}$$

This is close to the doping density, so we need to use the quadratic equation.

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

$$n = 2.5 \times 10^{16} + \sqrt{(2.5 \times 10^{16})^2 + (2.865 \times 10^{16})^2}$$

$$n = 2.5 \times 10^{16} + \sqrt{6.25 \times 10^{32} + 8.21 \times 10^{32}} = 6.3 \times 10^{16}$$

$n = 6.3 \times 10^{16} \text{ cm}^{-3}$

$$p = \frac{n_i^2}{n} = \frac{(2.865 \times 10^{16})^2}{6.3 \times 10^{16}} = \frac{8.21 \times 10^{32}}{6.3 \times 10^{16}} = 1.30 \times 10^{16} \text{ cm}^{-3}$$

$p = 1.30 \times 10^{16} \text{ cm}^{-3}$

HW3 Solutions (continued):

- 5) For each of the cases (a-e) in Prob. 3, calculate the Fermi level position, with respect to the **intrinsic level** $(E_F - E_i)$. Note that you need to consider sign.

Solution:

In problem 3), we computed n and p. For this problem, we need an expression that relates n and p to the Fermi level and intrinsic level. The expressions are on p. 53 of SDF:

$$n = n_i e^{(E_F - E_i)/k_B T}$$

$$p = n_i e^{(E_i - E_F)/k_B T}$$

Solving for $(E_F - E_i)$ from the first equation, we have

$$(E_F - E_i) = k_B T \ln\left(\frac{n}{n_i}\right)$$

or in electron volts

$$\frac{(E_F - E_i)}{q} = \frac{k_B T}{q} \ln\left(\frac{n}{n_i}\right) = 0.026 \ln\left(\frac{n}{10^{10}}\right)$$

3a) $n = p = n_i = 1.00 \times 10^{10} \text{ cm}^{-3}$

$$\frac{(E_F - E_i)}{q} = 0.026 \ln(1) = 0$$

$$\frac{(E_F - E_i)}{q} = 0$$

or we could say: $(E_F - E_i) = 0 \text{ eV}$

3b) $n = 1.00 \times 10^{13} \text{ cm}^{-3}$

$$\frac{(E_F - E_i)}{q} = 0.026 \ln\left(\frac{10^{13}}{10^{10}}\right) = 0.180$$

$$\frac{(E_F - E_i)}{q} = 0.180$$

HW3 Solutions (continued):

3c) $n = 1.00 \times 10^{17} \text{ cm}^{-3}$

$$\frac{(E_F - E_i)}{q} = 0.026 \ln\left(\frac{10^{17}}{10^{10}}\right) = 0.026 \ln(10^7) = 0.419$$

$$\frac{(E_F - E_i)}{q} = 0.419$$

Note: The larger the n-type doping, the higher the Fermi level is above the intrinsic level.

3d) $n = 1.00 \times 10^3 \text{ cm}^{-3}$

$$\frac{(E_F - E_i)}{q} = 0.026 \ln\left(\frac{10^3}{10^{10}}\right) = 0.026 \ln(10^{-7}) = -0.419$$

$$\frac{(E_F - E_i)}{q} = -0.419$$

Note the sign – the Fermi level is now **below the intrinsic level** because this is a p-type semiconductor.

3e) $n = 5.00 \times 10^2 \text{ cm}^{-3}$

$$\frac{(E_F - E_i)}{q} = 0.026 \ln\left(\frac{5 \times 10^2}{10^{10}}\right) = 0.026 \ln(5 \times 10^{-8}) = -0.437$$

$$\frac{(E_F - E_i)}{q} = -0.437$$

Also a p-type semiconductor, so the Fermi level is below the intrinsic level.

HW3 Solutions (continued):

- 6) For each of the cases (a-e) in Prob. 3, calculate the Fermi level position, with respect to the **bottom of the conduction band** ($E_F - E_C$). Note that you need to consider sign.

Solution:

This is basically the same problem as 5) – we just want to know the position of the Fermi level with respect to another energy, bottom of the conduction band instead of the intrinsic level.

To solve this problem, we need an expression that relates n to the Fermi level and bottom of the conduction band. The expression is on p. 52 of SDF:

$$n = N_C e^{(E_F - E_C)/k_B T}$$

Solving for $(E_F - E_C)$ we have

$$(E_F - E_C) = k_B T \ln\left(\frac{n}{N_C}\right)$$

or in electron volts

$$\frac{(E_F - E_C)}{q} = \frac{k_B T}{q} \ln\left(\frac{n}{N_C}\right) = 0.026 \ln\left(\frac{n}{N_C}\right)$$

We computed N_C in prob. 2c: $N_C = 3.22 \times 10^{19} \text{ cm}^{-3}$

$$\frac{(E_F - E_C)}{q} = 0.026 \ln\left(\frac{n}{3.22 \times 10^{19}}\right)$$

Note that this quantity will be negative for a non-degenerate semiconductor, because the Fermi level is always well below the conduction band.

For case 3a) $n = p = n_i = 1.00 \times 10^{10} \text{ cm}^{-3}$

$$\frac{(E_F - E_C)}{q} = 0.026 \ln\left(\frac{10^{10}}{3.22 \times 10^{19}}\right) = 0.569$$

$$\frac{(E_F - E_C)}{q} = -0.569$$

or we could say: $(E_F - E_C) = -0.569 \text{ eV}$

HW3 Solutions (continued):Case 3b) $n = 1.00 \times 10^{13} \text{ cm}^{-3}$

$$\frac{(E_F - E_C)}{q} = 0.026 \ln\left(\frac{10^{13}}{3.22 \times 10^{19}}\right) = -0.390$$

$$\boxed{\frac{(E_F - E_C)}{q} = -0.390}$$

Case 3c) $n = 1.00 \times 10^{17} \text{ cm}^{-3}$

$$\frac{(E_F - E_C)}{q} = 0.026 \ln\left(\frac{10^{17}}{3.22 \times 10^{19}}\right) = -0.150$$

$$\boxed{\frac{(E_F - E_C)}{q} = -0.150}$$

Note: The larger the n-type doping, the closer the Fermi level is to the conduction band.

3d) $n = 1.00 \times 10^3 \text{ cm}^{-3}$

$$\frac{(E_F - E_C)}{q} = 0.026 \ln\left(\frac{10^3}{3.22 \times 10^{19}}\right) = -0.988$$

$$\boxed{\frac{(E_F - E_C)}{q} = -0.988}$$

Note: This value is getting close to the bandgap of Si. The Fermi level is getting close to the top of the valence band.

3e) $n = 5.00 \times 10^2 \text{ cm}^{-3}$

$$\frac{(E_F - E_C)}{q} = 0.026 \ln\left(\frac{5 \times 10^2}{3.22 \times 10^{19}}\right) = -1.01$$

$$\boxed{\frac{(E_F - E_C)}{q} = -1.01}$$

Also a p-type semiconductor, so the Fermi level is close to the top of the valence band.

- 7) For the case of problem 4), calculate the Fermi level position, with respect to the **intrinsic level** $(E_F - E_i)$. Note that you need to consider sign.

Solution:

The electron density was $n = 6.3 \times 10^{16} \text{ cm}^{-3}$

and the intrinsic carrier concentration was $n_i(700 \text{ K}) = 2.865 \times 10^{16} \text{ cm}^{-3}$

$$\frac{(E_F - E_i)}{q} = \frac{k_B T}{q} \ln\left(\frac{n}{n_i}\right)$$

HW3 Solutions (continued):

$$\text{at } T = 700 \text{ K: } \frac{k_B T}{q} = \frac{1.38 \times 10^{-23} \times 700}{1.6 \times 10^{-19}} = 0.0604 \text{ eV}$$

$$\frac{(E_F - E_i)}{q} = 0.060 \ln \left(\frac{6.3 \times 10^{16}}{2.865 \times 10^{16}} \right) = 0.048$$

$$\boxed{\frac{(E_F - E_i)}{q} = 0.048}$$

The Fermi level is almost in the middle of the gap – very close to the intrinsic level. This happens because $n = 6.3 \times 10^{16} \text{ cm}^{-3}$ is only a little bigger than $n_i(700 \text{ K})$.

- 8) Your textbook defines a non-degenerate semiconductor as one for which the Fermi level is at least $3k_B T$ below the conduction band edge and at least $3k_B T$ above the valence band edge. What is the largest electron density for non-degenerate Si?

Solution:

$$\text{Require: } (E_C - E_F) \geq 3k_B T$$

$$n = N_C e^{(E_F - E_C)/k_B T}$$

$$n \leq N_C e^{(-3k_B T)/k_B T} = N_C e^{-3}$$

$$n \leq (3.22 \times 10^{19}) \times e^{-3} = 1.60 \times 10^{18}$$

$\boxed{n = 1.60 \times 10^{18} \text{ cm}^{-3}}$ is the largest electron density in Si at room temperature for which we can use non-degenerate expressions.

Note: Your book is fairly conservative on this. Many people would use non-degenerate equations when

$$n \leq N_C = 3.22 \times 10^{19} \text{ cm}^{-3}$$

Some might even use the assumption for higher doping densities, but they understand what they are doing and what the errors are.

HW3 Solutions (continued):

- 9) For each case below, explain physically (i.e. in words in no equations) whether the intrinsic level is above or below the middle of the gap, then compute the location of the intrinsic level relative to the middle of the bandgap, $E_{mid} = (E_C + E_V)/2$.
- Silicon at T = 300K with $m_n^* = 1.18m_0$ and $m_p^* = 0.81m_0$.
 - GaAs at T = 300K with $m_n^* = 0.067m_0$ and $m_p^* = 0.524m_0$.

Solution:

a) Silicon:

The carrier density is basically the number of states times the probability that the states are occupied. The density of states varies as effective mass to the 3/2 power, so the conduction band density of states is larger than the valance band density of states for Si. To get equal numbers of electrons and holes, **the Fermi level (intrinsic level) will need to be a little below the middle of the bandgap.** This will make the probability that conduction band states are occupied a little smaller than the probability that valence band states are empty, and compensate for the larger DOS in the conduction band.

To compute $E_i - E_{mid}$ we use eqn. (2.36) in p. 62 in SDF and find

$$\frac{E_i - E_{mid}}{q} = \frac{3}{4} \frac{k_B T}{q} \ln\left(\frac{m_p^*}{m_n^*}\right) = 0.75 \times 0.026 \times \ln\left(\frac{0.81}{1.18}\right) = -0.007 \text{ eV}$$

$$\frac{E_i - E_{mid}}{q} = -0.007 \text{ eV}$$

The intrinsic level is 7 meV **below** the middle of the gap in Si.

b) GaAs:

For GaAs, the conduction band density of states is smaller than the valance band density of states (just the opposite of Si). Using the same reasoning as in 9a), to get equal numbers of electrons and holes, **the Fermi level (intrinsic level) will need to be a little above the middle of the bandgap.** This will make the probability that conduction band states are occupied a little largerthan the probability that valence band states are empty, and compensate for the smaller DOS in the conduction band,

HW3 Solutions (continued):

To compute $E_i - E_{mid}$ we use eqn. (2.36) in p. 62 in SDF and find

$$\frac{E_i - E_{mid}}{q} = \frac{3}{4} \frac{k_B T}{q} \ln\left(\frac{m_p^*}{m_n^*}\right) = 0.75 \times 0.026 \times \ln\left(\frac{0.524}{0.067}\right) = +0.040 \text{ eV}$$

$$\frac{E_i - E_{mid}}{q} = +0.040 \text{ eV}$$

The intrinsic level is 40 meV **above** the middle of the gap in GaAs.

This example shows that the intrinsic level is always close to the middle of the bandgap -- even when the effective masses of electrons and holes are quite different.

- 10) A semiconductor (not necessarily Si) is doped at $N_D = 2 \times 10^{14} \text{ cm}^{-3}$ and $N_A = 1.2 \times 10^{14} \text{ cm}^{-3}$. The thermal equilibrium electron concentration is found to be $n = 1.1 \times 10^{14} \text{ cm}^{-3}$. Assume complete ionization of dopants, and compute the intrinsic carrier concentration and the equilibrium hole concentration.

Solution:

Begin with the general expression:

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

$$1.1 \times 10^{14} = \frac{2.0 \times 10^{14} - 1.2 \times 10^{14}}{2} + \sqrt{\left(\frac{0.8 \times 10^{14}}{2}\right)^2 + n_i^2} = 0.4 \times 10^{14} + \sqrt{0.16 \times 10^{28} + n_i^2}$$

Solve for n_i^2 :

$$\sqrt{0.16 \times 10^{28} + n_i^2} = 0.7 \times 10^{14} \quad 0.16 \times 10^{28} + n_i^2 = 0.49 \times 10^{28} \quad n_i^2 = 0.33 \times 10^{28}$$

$$n_i = 5.7 \times 10^{13} \text{ cm}^{-3}$$

HW3 Solutions (continued):

$$np = n_i^2 \rightarrow p = \frac{n_i^2}{n} = \frac{0.33 \times 10^{28}}{1.1 \times 10^{14}} = 3.00 \times 10^{13}$$

$p = 3.00 \times 10^{13} \text{ cm}^{-3}$

11) Answer the following questions about resistivity at $T = 300\text{K}$.

- a) **Compute** the resistivity of intrinsic Si, Ge, and GaAs.
- b) **Compute** the resistivity of n-type Si, Ge, and GaAs doped at $N_D = 10^{19} \text{ cm}^{-3}$.
Assume complete ionization of dopants.

Solution:

According to eqn. (3.7) on p. 85 of SDF

$$\rho = \frac{1}{nq\mu_n + pq\mu_p} \Omega\text{-cm}$$

Units (MKS (SI)): What are the units of $1/nq\mu_n$ or $1/pq\mu_p$?

$$\frac{1}{nq\mu} \rightarrow \frac{1}{\text{m}^{-3}} \times \frac{1}{\text{C}} \times \frac{1}{\text{m}^2/(\text{V}\cdot\text{s})} = \frac{\text{V}}{(\text{C}/\text{s})} - \text{m} = \frac{\text{V}}{\text{A}} - \text{m} = \Omega\text{-m} \quad \text{MKS or SI}$$

but we are usually given carrier densities per cm^3 and mobility in $\text{cm}^2/(\text{V}\cdot\text{s})$

$$\frac{1}{nq\mu} \rightarrow \frac{1}{\text{cm}^{-3}} \times \frac{1}{\text{C}} \times \frac{1}{\text{cm}^2/(\text{V}\cdot\text{s})} = \frac{\text{V}}{(\text{C}/\text{s})} - \text{cm} = \frac{\text{V}}{\text{A}} - \text{cm} = \Omega\text{-cm}$$

so then the answer comes out in $\Omega\text{-cm}$.

11a) We need the carrier densities and mobilities:

$$n = p = n_i$$

From Fig. 3.5, p. 80 of SDF for silicon:

$$\mu_n \approx 1400 \frac{\text{cm}^2}{\text{V}\cdot\text{s}} \quad \mu_p \approx 460 \frac{\text{cm}^2}{\text{V}\cdot\text{s}}$$

HW3 Solutions (continued):

$$\rho = \frac{1}{nq\mu_n + pq\mu_p} = \frac{1}{n_i q (\mu_n + \mu_p)} \Omega\text{-cm}$$

$$\rho = \frac{1}{10^{10} \times 1.6 \times 10^{-19} \times (1400 + 460)} = 3.4 \times 10^5 \Omega\text{-cm}$$

$\rho = 3.4 \times 10^5 \Omega\text{-cm}$ (silicon)

This is a very large resistivity – not as large as an insulator, but very large for a semiconductor.

For Ge:

$$\rho = \frac{1}{n_i q (\mu_n + \mu_p)} \Omega\text{-cm}$$

From Fig. 3.5, p. 80 of SDF for Ge:

$$\mu_n \approx 4000 \frac{\text{cm}^2}{\text{V}\cdot\text{s}} \quad \mu_p \approx 2000 \frac{\text{cm}^2}{\text{V}\cdot\text{s}}$$

From Fig. 2.20, p. 54 of SDF for Ge:

$$n_i(300 \text{ K}) = 2 \times 10^{13} \text{ cm}^{-3}$$

$$\rho = \frac{1}{2 \times 10^{13} \times 1.6 \times 10^{-19} \times (4000 + 2000)} = 5.2 \times 10^1 \Omega\text{-cm}$$

$$\rho = 5.2 \times 10^1 \Omega\text{-cm} \text{ (Ge)}$$

Intrinsic Ge is fairly conductive! This happens because the mobilities are higher, but mostly because the bandgap is much lower, so the intrinsic carrier concentration is much larger.

For GaAs:

$$\rho = \frac{1}{n_i q (\mu_n + \mu_p)} \Omega\text{-cm}$$

From Fig. 3.5, p. 80 of SDF for GaAs:

$$\mu_n \approx 8500 \frac{\text{cm}^2}{\text{V-s}} \quad \mu_p \approx 430 \frac{\text{cm}^2}{\text{V-s}}$$

HW3 Solutions (continued):

From Fig. 2.20, p. 54 of SDF for Ge:

$$n_i(300 \text{ K}) = 2.25 \times 10^6 \text{ cm}^{-3}$$

$$\rho = \frac{1}{2.25 \times 10^6 \times 1.6 \times 10^{-19} \times (8500 + 430)} = 3.1 \times 10^8 \Omega\text{-cm}$$

$$\boxed{\rho = 3.1 \times 10^8 \Omega\text{-cm}} \quad (\text{GaAs})$$

Intrinsic GaAs has a resistivity that is orders of magnitude larger than Si! This happens even though the electron mobility is much larger than Si because the bandgap is much larger, so the intrinsic carrier concentration is orders of magnitude smaller.

11b) Since we are heavily doped n-type, we can ignore the contribution from holes.

$$\rho = \frac{1}{nq\mu_n} \Omega\text{-cm}$$

For Si:

From Fig. 3.5, p. 80 of SDF for Si:

$$\mu_n \approx 110 \frac{\text{cm}^2}{\text{V-s}}$$

$$\rho = \frac{1}{1 \times 10^{19} \times 1.6 \times 10^{-19} \times 110} = 5.7 \times 10^{-3} \Omega\text{-cm}$$

$$\boxed{\rho = 5.7 \times 10^{-3} \Omega\text{-cm}} \quad (\text{Si})$$

This is about eight orders of magnitude lower than the intrinsic resistivity of Si. The ability to control the resistivity over orders of magnitude is what makes semiconductors so useful.

For Ge:

From Fig. 3.5, p. 80 of SDF for Si:

$$\mu_n \approx 900 \frac{\text{cm}^2}{\text{V-s}}$$

HW3 Solutions (continued):

$$\rho = \frac{1}{1 \times 10^{19} \times 1.6 \times 10^{-19} \times 900} = 6.9 \times 10^{-4} \Omega\text{-cm}$$

$$\boxed{\rho = 6.9 \times 10^{-4} \Omega\text{-cm}} \quad (\text{Ge})$$

Even lower than Si because the mobility is higher.

For GaAs:

From Fig. 3.5, p. 80 of SDF for GaAs:

$$\mu_n \approx 3200 \frac{\text{cm}^2}{\text{V-s}}$$

$$\rho = \frac{1}{1 \times 10^{19} \times 1.6 \times 10^{-19} \times 3200} = 2.0 \times 10^{-4} \Omega\text{-cm}$$

$$\boxed{\rho = 2.0 \times 10^{-4} \Omega\text{-cm}} \quad (\text{GaAs})$$

Even lower than Ge because the mobility is higher.

Comments: For Si, we found that the range of possible resistivities is

$$3.0 \times 10^5 \Omega\text{-cm} < \rho < 6 \times 10^{-3} \Omega\text{-cm}$$

about eight orders of magnitude. In practice, we could never have pure, intrinsic Si, there will always be some unintentional dopants no matter how pure we try to make Si, so the resistivities would not be as high as indicated here, but it's common to find resistivities in the 1000's. Also, one can dope Si to 10^{20} cm^{-3} , so resistivities 10 times lower than indicated above can be obtained.

How do these limits compare to insulators and metals? A Google search shows

Resistivity of copper: $\rho_{Cu} = 1.68 \times 10^{-8} \Omega\text{-cm}$

Resistivity of diamond: $\rho_{diamond} = 10^{13} - 10^{20} \Omega\text{-cm}$

So semiconductors are not great metals and not great insulators – their usefulness comes from the ability to vary their resistivity controllably with doping.

HW3 Solutions (continued):

12) You are given a 10 Ohm-cm silicon wafer at 300 K.

12a) If it is n-type, What is the electron density?

12b) If it is p-type, what is the hole density?

Solution:

From Fig. 3.8 on p. 86 of SDF, we can read off the doping density for a given resistivity.

12a) 10 Ohm-cm n-type corresponds to $N_D \approx 5 \times 10^{14} \text{ cm}^{-3}$. Under these conditions:

$$n \approx N_D \approx 5 \times 10^{14} \text{ cm}^{-3}$$

12b) 10 Ohm-cm p-type corresponds to $N_A \approx 1.3 \times 10^{15} \text{ cm}^{-3}$. Under these conditions:

$$p \approx N_A \approx 1.3 \times 10^{15} \text{ cm}^{-3}$$

13) Assume a hypothetical semiconductor with the following properties:

$$E_G = 1 \text{ eV} \quad N_C = N_V = 1 \times 10^{19} \text{ cm}^{-3}$$

$$E_C - E_F = 0.4 \text{ eV}$$

$$T = 600 \text{ K}$$

Answer the following questions.

- 13a) What is the equilibrium electron density?
- 13b) What is the equilibrium hole density?
- 13c) What is the intrinsic carrier concentration, n_i ?
- 13d) What is the net doping density, $N_D - N_A$, assuming that the dopants are fully ionized?

- 13e) Where is the intrinsic level located with respect to the conduction band?
 (Note: You can simply write down the answer without any work, if you explain your reasoning.)

HW3 Solutions (continued):

Solutions:

- 13a) What is the equilibrium electron density?

$$n = N_C e^{(E_F - E_C)/k_B T}$$

$$k_B T \text{ (eV)} = \frac{1.38 \times 10^{-23} \times 600}{1.6 \times 10^{-19}} = 0.052$$

$$n = 10^{19} e^{-(0.4)/0.052} = 4.56 \times 10^{15} \text{ cm}^{-3}$$

$$n = 4.56 \times 10^{15} \text{ cm}^{-3}$$

- 13b) What is the equilibrium hole density?

$$p = N_V e^{(E_V - E_F)/k_B T}$$

$$E_V - E_F = (E_V - E_C) + (E_C - E_F) = -E_G + 0.4 = -1.0 + 0.4 = -0.6$$

$$p = N_V e^{(E_V - E_F)/k_B T} = 10^{19} e^{-0.6/0.052} = 9.75 \times 10^{13} \text{ cm}^{-3}$$

$$p = 9.75 \times 10^{13} \text{ cm}^{-3}$$

- 13c) What is the intrinsic carrier concentration, n_i ?

$$np = 4.56 \times 10^{15} \times 9.75 \times 10^{13} = 4.45 \times 10^{29} = n_i^2$$

$$n_i = \sqrt{n_i^2} = 6.67 \times 10^{14} \text{ cm}^{-3}$$

- 13d) What is the net doping density, $N_D - N_A$, assuming that the dopants are fully ionized?

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

Solve for $N_D - N_A$

$$N_D - N_A = n - \frac{n_i^2}{n}$$

$$N_D - N_A = 4.56 \times 10^{15} - \frac{4.45 \times 10^{29}}{4.56 \times 10^{15}} = 4.66 \times 10^{15}$$

$$N_D - N_A = 4.66 \times 10^{15} \text{ cm}^{-3}$$

HW3 Solutions (continued):

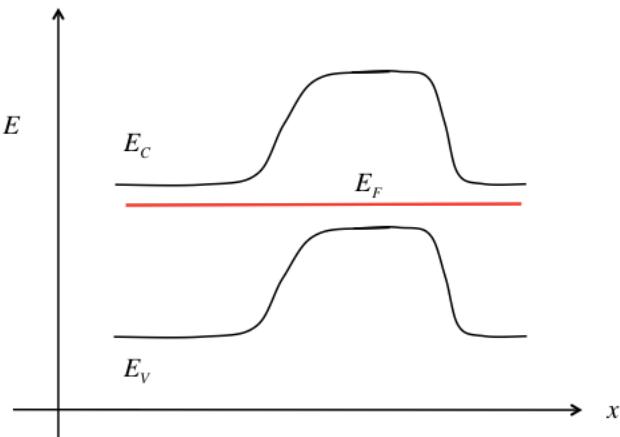
- 13e) Where is the intrinsic level located with respect to the conduction band?
 (Note: You can simply write down the answer without any work, if you explain your reasoning.

Since $N_C = N_V$ putting the Fermi level exactly at the middle of the bandgap will produce equal numbers of electrons and holes.

Exactly at mid-gap.

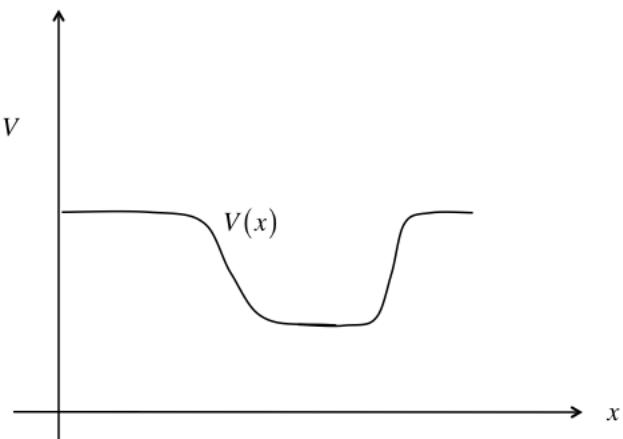
- 14) For the energy band diagram sketched below, answer the following questions.

- 14a) Sketch the electrostatic potential, $V(x)$, vs. position, x .
- 14b) Sketch the electric field, $E(x)$, vs. position, x .
- 14c) Sketch the electron density, $n(x)$ vs. position, x .
- 14d) Sketch the hole density, $p(x)$, vs. position, x .

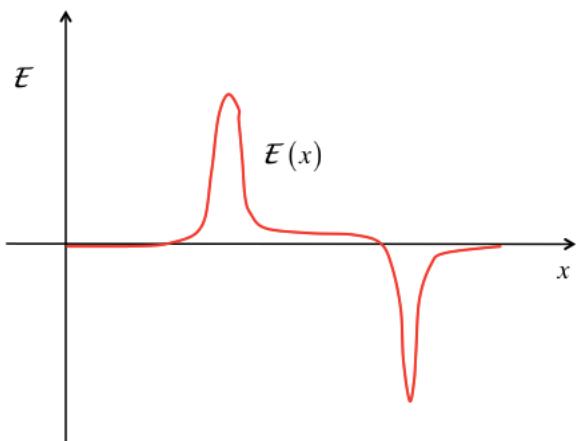


HW3 Solutions (continued):**Solution:**

- 14a) Sketch the electrostatic potential,
- $V(x)$
- , vs. position,
- x
- .

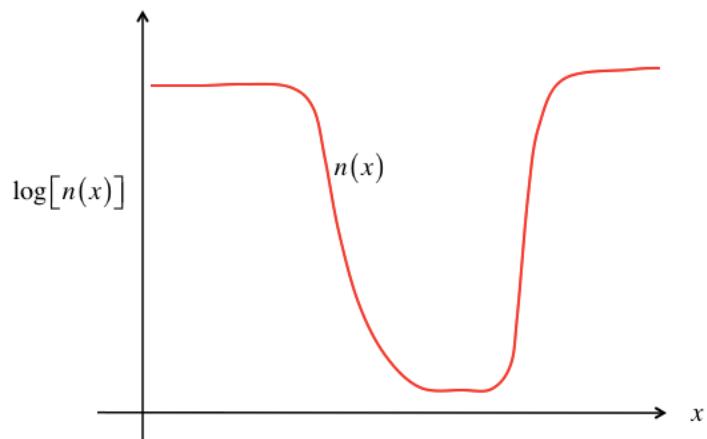


- 14b) Sketch the electric field,
- $E(x)$
- , vs. position,
- x
- .



HW3 Solutions (continued):

14c) Sketch the electron density, $n(x)$ vs. position, x .



14d) Sketch the hole density, $p(x)$, vs. position, x .

