

# Semiconductor Fundamentals

*Presented to*

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*Shiv Govind Singh*

*[sgsingh@iith.ac.in](mailto:sgsingh@iith.ac.in)*

*Professor, Electrical Engineering  
IIT Hyderabad, India*



भारतीय प्रौद्योगिकी संस्थान हैदराबाद  
Indian Institute of Technology Hyderabad

*Lecture 11*

# Course information

- ❖ Semiconductors Materials - Types of Solids, Space lattice, Atomic Bonding,
- ❖ Introduction to quantum theory, Schrodinger wave equation, Electron in free space, Infinite well, and step potentials, Allowed and forbidden bands
- ❖ Electrical conduction in solids, Density of states functions, Fermi-Dirac distribution in Equilibrium,
- ❖ Valence band and Energy band models of intrinsic and extrinsic Semiconductors. Degenerate and non degenerate doping
- ❖ Thermal equilibrium carrier concentration, charge neutrality
- ❖ Carrier transport – Mobility, drift, diffusion.

# Reference

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## Text Book:

1. Physics of Semiconductor Devices, *S. M. Sze*, John Wiley & Sons (1981).
2. Solid State Electronics by *Ben G. Streetman and Sanjay Banerjee*, Prentice Hall International, Inc.
3. Semiconductor Physics and Devices, Donald A. Neamen, Tata Mcgraw-Hill Publishing company Limited.
4. Advanced Semiconductor Fundamentals by Pirret

## Reference Book:

1. Fundamentals of Solid-State Electronic Devices, *C. T. Sah*, Allied Publisher and World Scientific, 1991.
2. Complete Guide to Semiconductor Devices, *K. K. Ng*, McGraw Hill, 1995.
3. Solid state physics, Ashcroft & Mermins.
4. Introduction to Solid State Electronics, *E. F. Y. Waug*, North Holland, 1980.

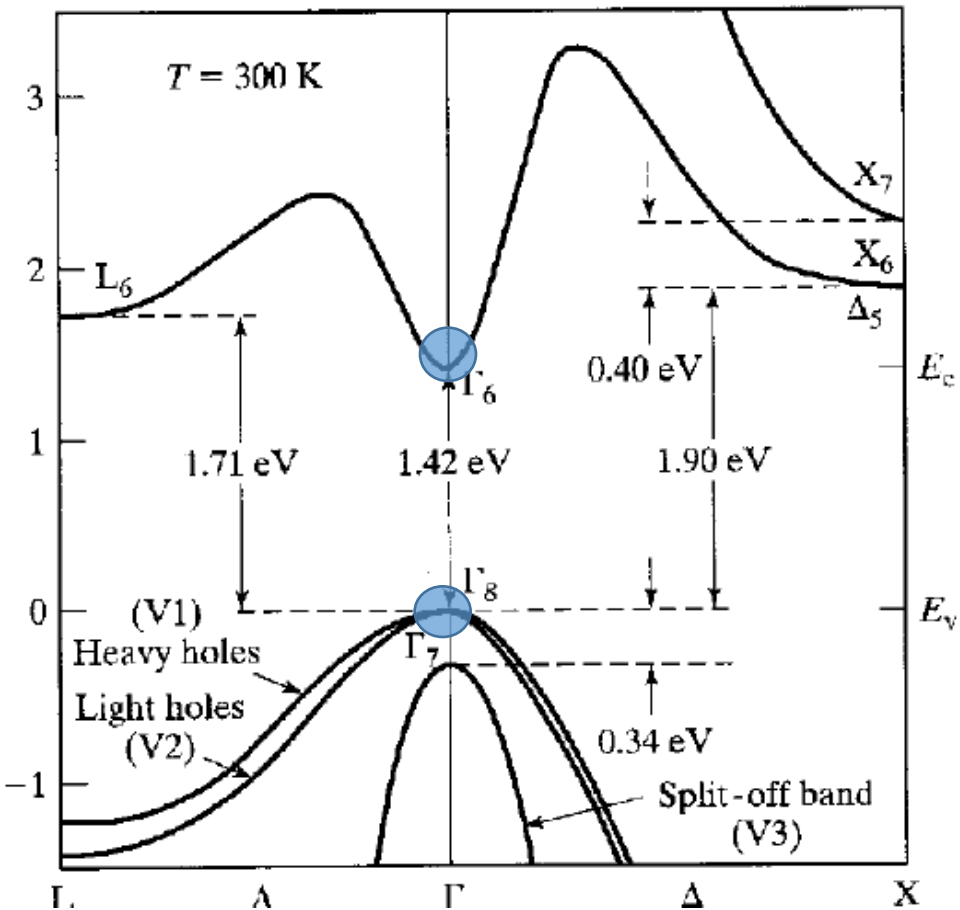
# Recap

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$$n_o = \int_0^{E_{top}} f(E) g(E) dE$$

$$DOS \text{ in } 3DCrystal, \quad g(E) = \frac{m^*}{2\pi^2 \hbar^3} \sqrt{2m^* (E - E_c)}$$

# DOS Si, Ge and GaAs



$$DOS = \frac{m_n^*}{2\pi^2\hbar^3} \sqrt{2m_n^* (E - E_c)}$$

$$DOS = \frac{m_n^*}{2\pi^2\hbar^3} \sqrt{2m_n^* (E_v - E)}$$

$$DOS = \frac{m_n^*}{2\pi^2\hbar^3} \sqrt{2m_n^* (E_v - E)}$$

# Effective mass in ellipsoidal band i.e Si, and Ge

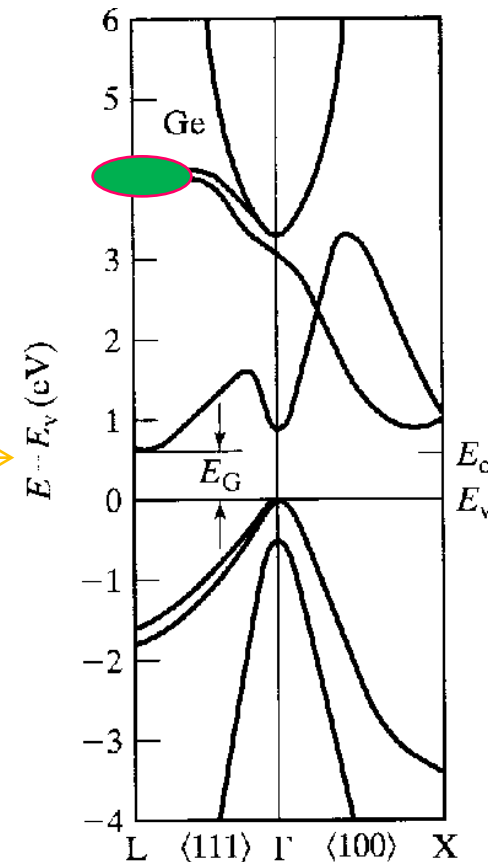
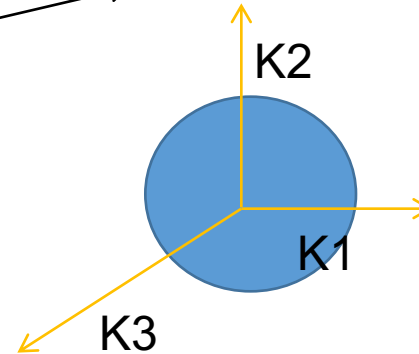
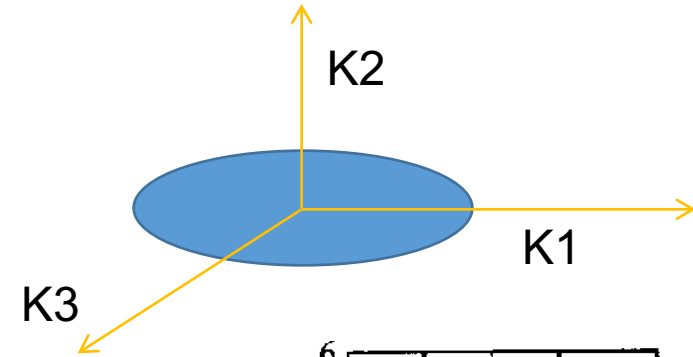
$$E - E_c = \frac{\hbar^2 k_1^2}{2m_l^*} + \frac{\hbar^2 k_2^2}{2m_t^*} + \frac{\hbar^2 k_3^2}{2m_t^*}$$

$$1 = \frac{k_1^2}{\frac{(E - E_c)2m_l^*}{\hbar^2}} + \frac{k_2^2}{\frac{(E - E_c)2m_t^*}{\hbar^2}} + \frac{k_3^2}{\frac{(E - E_c)2m_t^*}{\hbar^2}}$$

$\nearrow$   $a^2$                        $\nearrow$   $b^2$                        $\nearrow$   $\hbar^2$

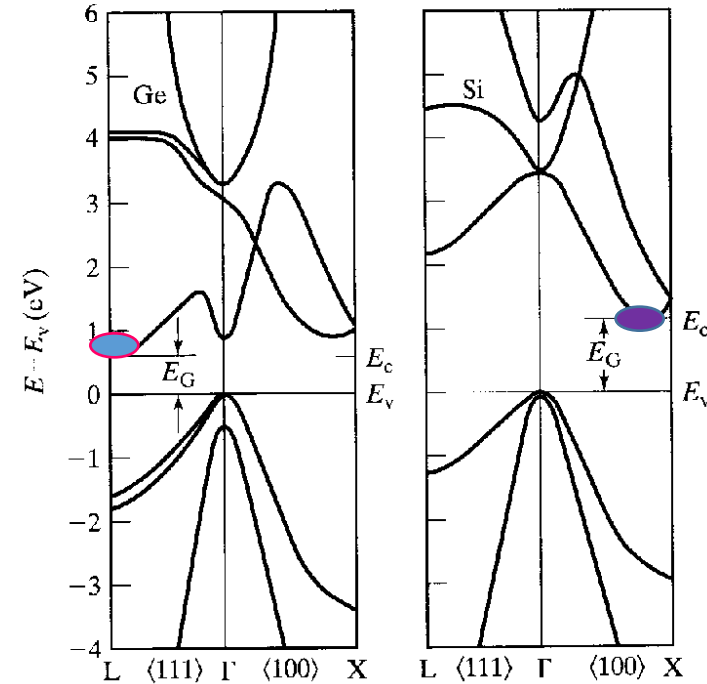
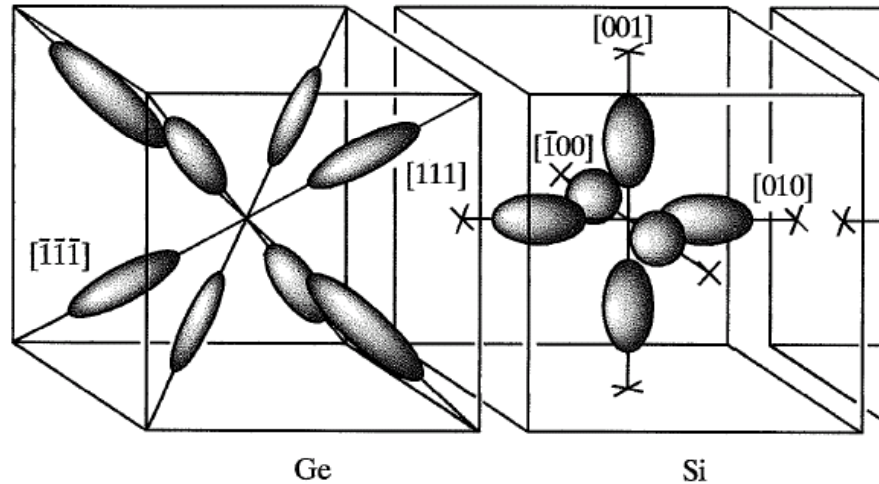
$$N_{el} \frac{4}{3} \pi a b^2 = \frac{4}{3} \pi k_{eff}^3$$

$$m_{eff}^* = N_{el}^{2/3} (m_l^* m_t^{*2})^{1/3}$$



# Density of states for case of Ge/Si in conduction band

$$g_c(E) = \frac{m_n^{*3/2}}{\hbar^3} \sqrt{\frac{2(E - E_c)}{2}}$$



$$m_{eff}^* = 4^{2/3} (m_l^* m_t^{*2})^{1/3} \text{ for..Ge}$$

$$m_{eff}^* = 6^{2/3} (m_l^* m_t^{*2})^{1/3} \text{ for..Si}$$

Density of electron/holes in energy interval  $dE$ -

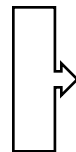
$$n_o = \int_0^{E_{top}} f(E)g(E)dE$$

$f(E)$  can obey MB distribution if electron behave as classical particle.

i.e.  $\sim Ae^{-E/kT}$

But Electron don't obey MB distribution as it is quantum particle and fermions', hence it obey Fermi Dirac distributions.

Classical MB  
Fermions FD  
Boson BE



QM particle





# Rule for filling the electron

**Pauli Principle: Only one electron per state**

**Total number of electrons is conserved**  $N_T = \sum_i N_i$

**Total energy of the system is conserved**  $E_T = \sum_i E_i N_i$

# Fermi- Dirac distribution function

When  $T > 0$

At thermal equilibrium, the electrons do not simply fill the lowest energy states first.

Fermi-Dirac statistics which gives the distribution of probability of an electron to have an energy  $E$  at temperature  $T$

$$f_e(E) = \frac{1}{\exp\left(\frac{E - E_F}{k_b T}\right) + 1}$$

$E_F$  is the Fermi energy and  $k_b$  is the Boltzmann constant.

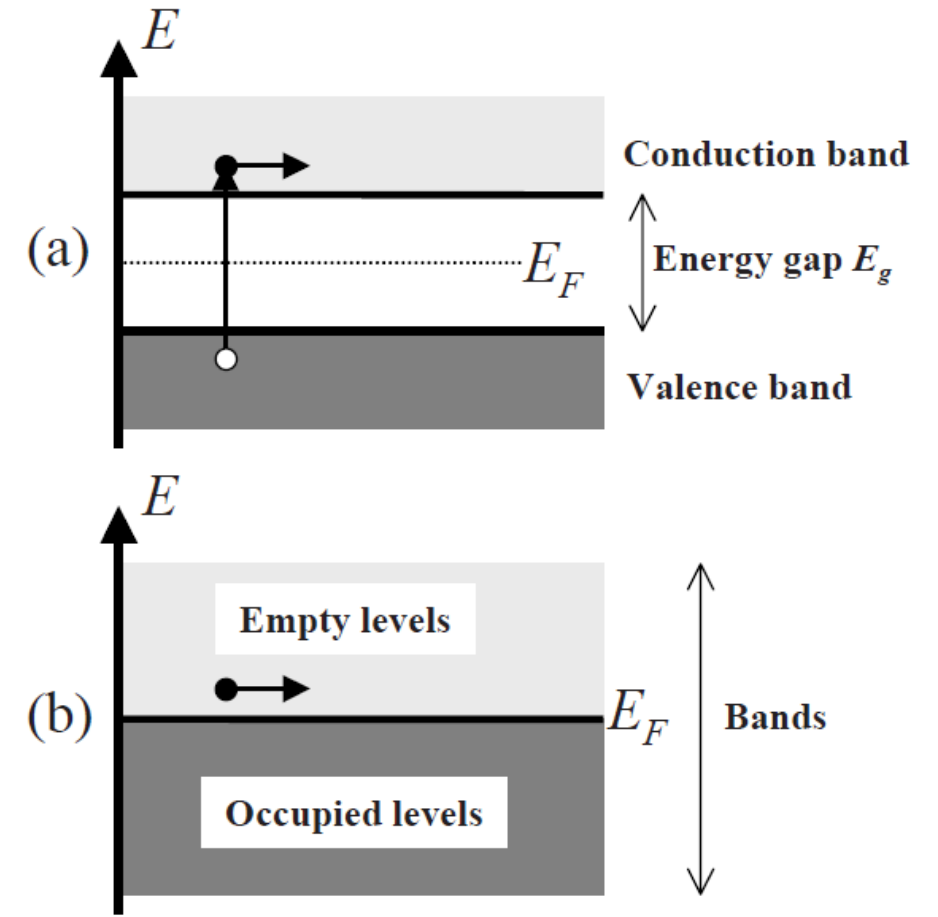
Fermi-Dirac statistics involves a chemical potential  $\mu$  instead of the Fermi energy  $E_F$ . *This chemical* potential depends on the temperature and any applied electrical potential. But in most cases of semiconductors, the difference between  $\mu$  and  $E_F$  is very small at the temperatures usually considered.

# Fermi Level

**Assumption:** solid where there are  $m$  energy levels and  $n$  electrons, and ( $m > n$ )

At equilibrium, when no electron is in an excited state (e.g. at the absolute zero temperature, 0 K), the lowest  $n$  energy levels will be occupied by electrons and the next remaining  $m-n$  energy levels remain empty.

If the **highest occupied state** is inside a band, the energy of this state is called the **Fermi level** and is denoted by  $E_F$ .



# More Insight

(1) At  $T = 0\text{ K}$  and  $E < E_F$

(a) Fermi-Dirac distribution is unity

At  $T = 0\text{ K}$  and  $E > E_F$

(b) Fermi-Dirac distribution  $[f_e(E)]$  is Zero

(2) At  $T > 0\text{ K}$

(a)  $E = E_F$ ,  $f_e(E) = 1/2$

(b)  $E > E_F + 3KT$ ,

$$\exp[(E - E_F) / kT] > 1 \Rightarrow f_e(E) \approx \exp - [(E - E_F) / kT]$$

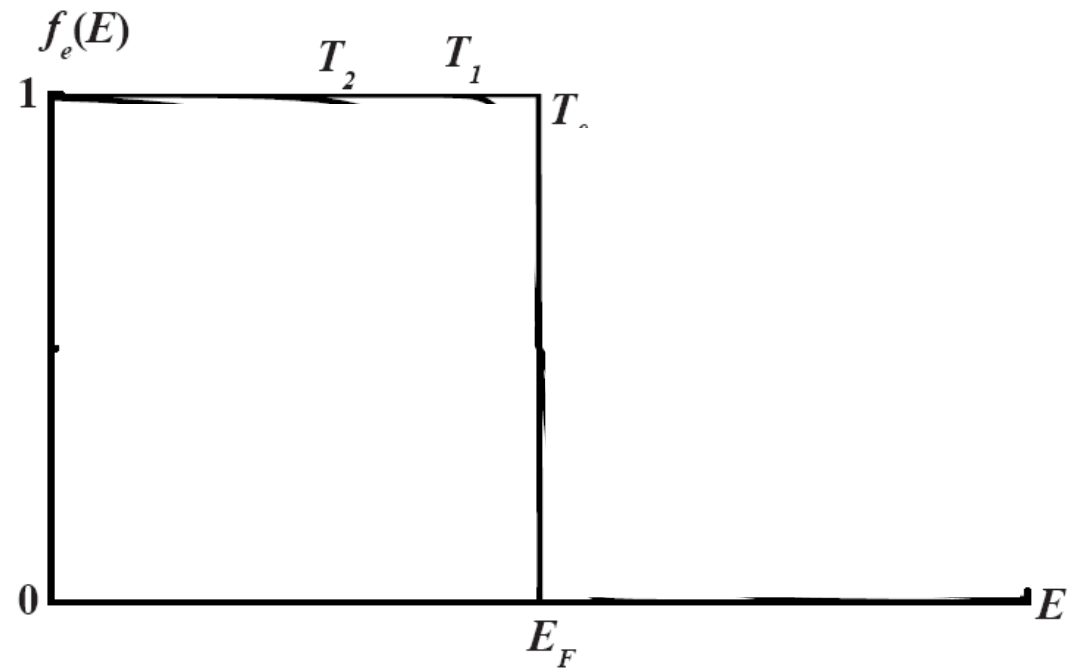
Most states with energy above  $3KT$  from  $E_F$  will be empty.

(c)  $E < E_F - 3KT$ ,

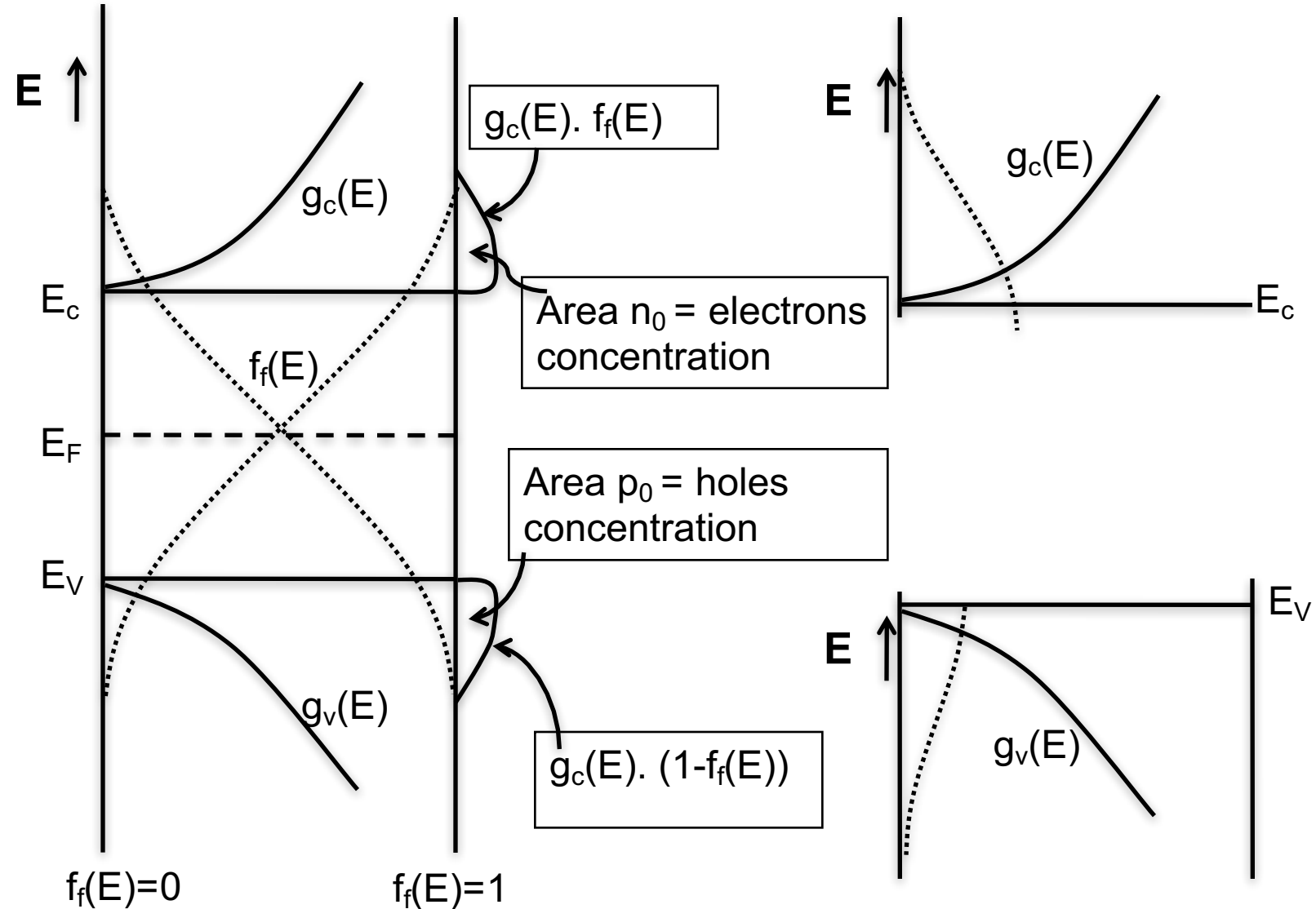
$$\exp[(E - E_F) / kT] < 1 \Rightarrow f_e(E) \approx 1 - \exp(E - E_F) / kT]$$

Most states with energy below  $3KT$  from  $E_F$  will be filled.

(d)  $E \gg 3KT$ , beyond  $E_f$  then MB and FD merged as in the case of Si



# Equilibrium Distribution of Electrons and Holes



Now again

$$n_o = \int_{E_c}^{E_{top}} f(E) g(E) dE$$

$$n_o = \int_{E_c}^{E_{top}} \frac{\frac{m_n^{*3/2}}{\hbar^3} \sqrt{(E - E_c)} dE}{1 + \exp((E - E_F) / kT)} \quad \text{May not be integrable}$$

Apply MB distribution as  $E_g/2 \sim 16kT$

$$n_o = \int_E^{\infty} \frac{\frac{m_n^{*3/2}}{\hbar^3} \sqrt{(E - E_c)} dE}{\exp((E - E_F) / kT)} = N_c e^{-\left(\frac{E_c - E_f}{kT}\right)}$$

$$N_c = 2 \left( \frac{m_n^* kT}{\pi \hbar^2} \right)^{3/2} \approx 10^{19} \quad \text{Effective density of states in CB}$$



# Heavily Doped Si

For heavily doped the semiconductor MB distribution does not hold so, need to solve Fermi integral for solution.

e.g let us take  $n=10^{20}$

We get  $E_F > E_c$  which is not possible

$$n_o = \int_E^{\infty} \frac{\frac{m_n^{*3/2}}{\hbar^3} \sqrt{(E - E_c)} dE}{\exp((E - E_F)/kT)} = N_c e^{-\left(\frac{E_c - E_f}{kT}\right)}$$

So

$$n_0 = \frac{2N_c}{\sqrt{\pi}} F_{1/2}\left(\frac{(E - E_c)}{kT}\right)$$



# Holes in Semiconductor

$$p_o = \int_{E_{bot}}^{E_v} \{ (1 - f(E)) \} g_v(E) dE$$
$$p_o = \int_E^{\infty} \frac{\frac{m_p^{*3/2}}{\hbar^3} \sqrt{(E_v - E)} dE}{\exp(E - E_F) / kT} = N_v e^{-\frac{(E_F - E_v)}{kT}}$$

$$N_v = 2 \left( \frac{m_v^* kT}{\pi \hbar^2} \right)^{3/2}$$

Since

$$m_n^* \neq m_p^*$$

$$N_c \neq N_v$$





# Fermi Level in Intrinsic Semiconductor at Equilibrium

$$m_{n_{dos}}^* \neq m_{p_{dos}}^*$$

$$N_c \neq N_v$$

But at Equilibrium  $n_0 = p_0$

$$N_c e^{-\frac{(E_c - E_F)}{kT}} = N_v e^{-\frac{(E_F - E_v)}{kT}}$$

$$E_F = \frac{E_C + E_V}{2} + kT \ln \frac{m_{p_{dos}}^*}{m_{n_{dos}}^*}$$

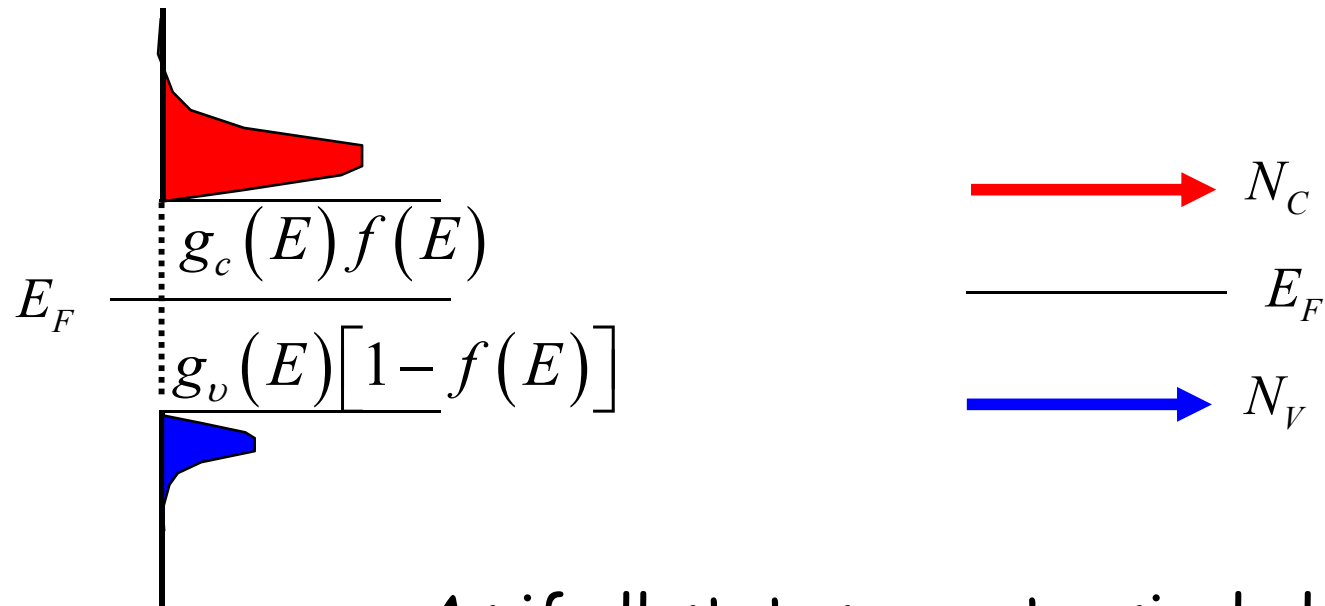
$$\text{if } m_{p_{dos}}^* = m_{n_{dos}}^*$$

$E_F$  will be in middle of band gap



# Effective Density of States

$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c) \rightarrow N_C e^{-\beta(E_c - E_F)} \quad \text{if} \quad \beta(E_c - E_F) > 3$$



As if all states are at a single level  $E_c$



# Intrinsic carrier concentration

Product of  $n_0$  and  $p_0$

$$p_o = N_v e^{\frac{-(E_F - E_v)}{kT}}$$

$$n_o = N_c e^{\frac{-(E_c - E_F)}{kT}}$$

$$n_o p_o = N_v e^{\frac{-(E_F - E_v)}{kT}} N_c e^{\frac{-(E_c - E_F)}{kT}}$$

$$n_o p_o = N_v N_c e^{\frac{-(E_c - E_v)}{kT}} = N_v N_c e^{\frac{-E_g}{kT}}$$

$$n_i^2 = N_v N_c e^{\frac{-E_g}{kT}} = n_o p_o$$

Where  $E_g$  is band gap,

At equilibrium  $n_o = p_o = n_i$

$n_i$  depends upon temperature and band gap

	$N_c$ (/cm <sup>3</sup> ) (10 <sup>18</sup> )	$N_v$ (/cm <sup>3</sup> ) (10 <sup>18</sup> )	$\frac{m_n^*}{m_o}$	$\frac{m_p^*}{m_o}$
Si	28	10.4	1.08	.56
Ge	10.9	6	.55	.37
GaAs	.47	7	.067	.48

	$E_g$ (eV)	Concentration (/cm <sup>3</sup> ) 300K	Concentration (/cm <sup>3</sup> ) 400K
Si	1.12	1.510 <sup>10</sup>	~10 <sup>13</sup>
Ge	.72	2.4X10 <sup>13</sup>	~10 <sup>16</sup>
GaAs	1.42	1.8X10 <sup>6</sup>	-
SiC	3.5	10	100