Semiconductor Fundamentals

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EE2187 class in Semester 1 2019/20

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Lecture 8

Quiz 4 Time 8

1. At T > 0, Why there is electron and hole i.e how it is generated? 3

2. What is hole? 2

3. At room temperature considering the simplistic model find out number of particle required to remove electron from Ge atoms. 3
4. Draw a graph between intrinsic carrier density with temperature for Si and Ge. 3

Course information

- Semiconductors Materials - Types of Solids, Space lattice, Atomic Bonding,
- Introduction to quantum theory, Schrodinger wave equation, Electron in free space, Infinite well, and step potentials, Allowed and forbidden bands
- Electrical conduction in solids, Density of states functions, Fermi-Dirac distribution in Equilibrium,
- Valence band and Energy band models of intrinsic and extrinsic Semiconductors. Degenerate and non degenerate doping
- Thermal equilibrium carrier concentration, charge neutrality
- Carrier transport – Mobility, drift, diffusion, Continuity equation.

Reference

Text Book:

1. Physics of Semiconductor Devices, *S. M. Sze*, John Wiley & Sons (1981).
2. Solid State Electronics by *Ben G. Streetman and Sanjay Banerjee,* Prentice Hall International , Inc.
3. Semiconductor Physics and Devices, Donald A. Neamen, Tata Mcgraw-Hill Publishing company Limited.
4. Advanced Semiconductor Fundamentals by Pirret

Reference Book:

1. Fundamentals of Solid-State Electronic Devices, *C. T. Sah*, Allied Publisher and World Scientific, 1991.
2. Complete Guide to Semiconductor Devices, *K. K. Ng*, McGraw Hill, 1995.
3. Solid state physics, Ashcroft & Mermins.
4. Introduction to Solid State Electronics, *E. F. Y. Waug*, North Holland, 1980.

Recap

One-dimensional Kronig-Penney model

Assumption:

All previous assumption still hold

The crystal is infinitely large

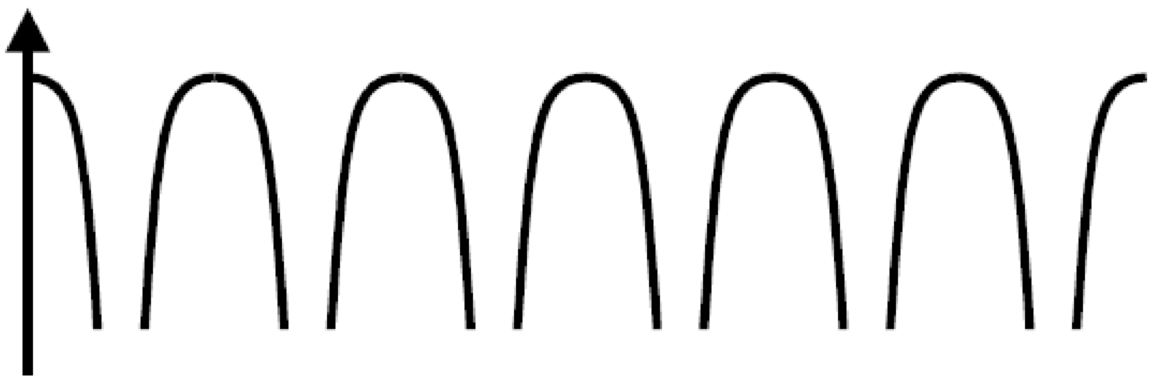
Potential energy of an electron has the form of periodic array of rectangular wells.

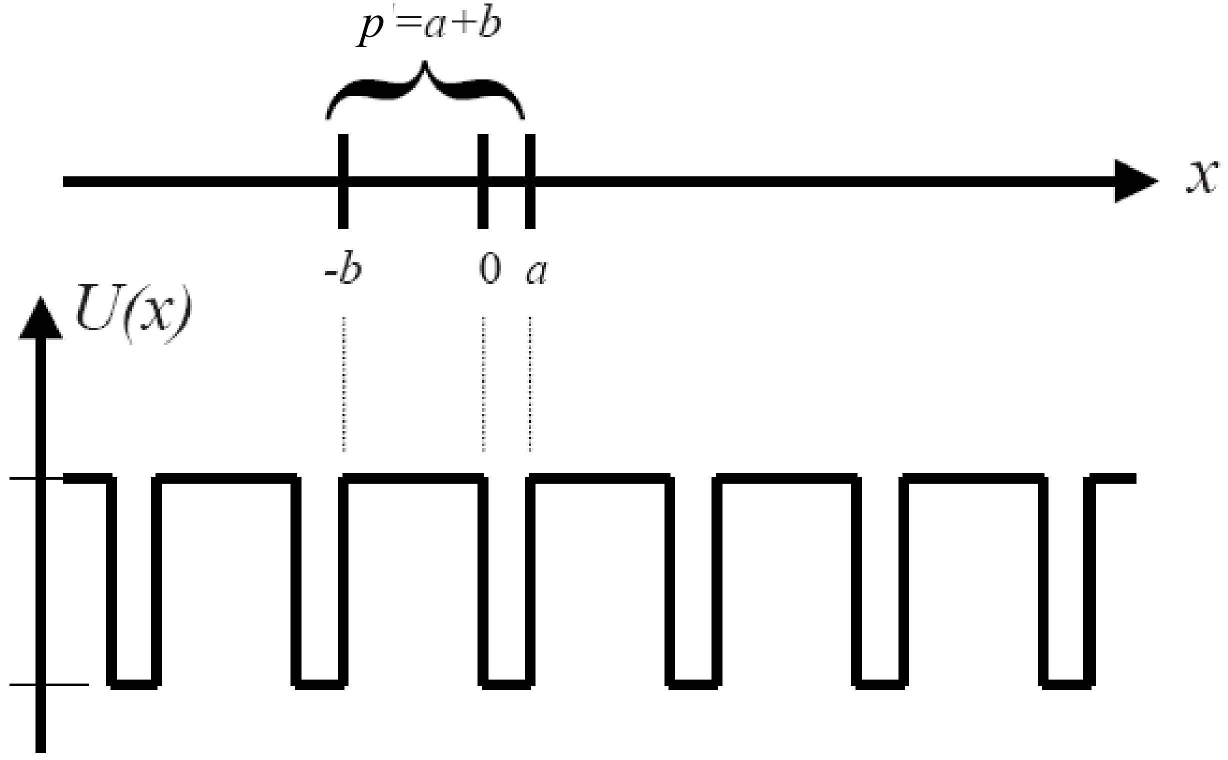
The period of potential is (a+b);

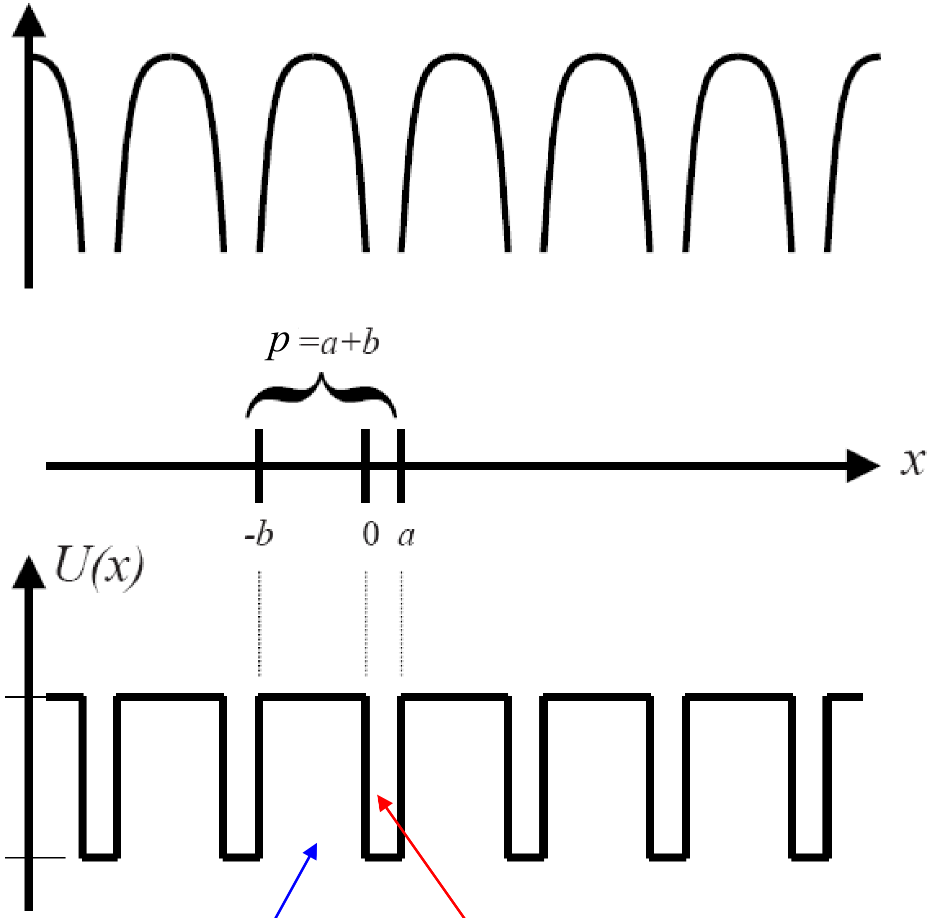
The Schrodinger equation in two region

$$\boxed{\begin{array}{ll} d^2\Psi(x)/dx^2+\alpha^2\Psi(x)=0 & 0<x<a \\ d^2\Psi(x)/dx^2+\beta^2\Psi(x)=0 & -b<x<0 \end{array}} \quad \begin{array}{l}(2)\\(3)\end{array}$$

$$\beta = \sqrt{\frac{2m(U_0-E)}{\hbar^2}} \qquad \alpha = \sqrt{\frac{2mE}{\hbar^2}}$$







$$\psi(x)=A_a\,Sin\,(\alpha x)+B_a\,Cos(\alpha x)$$

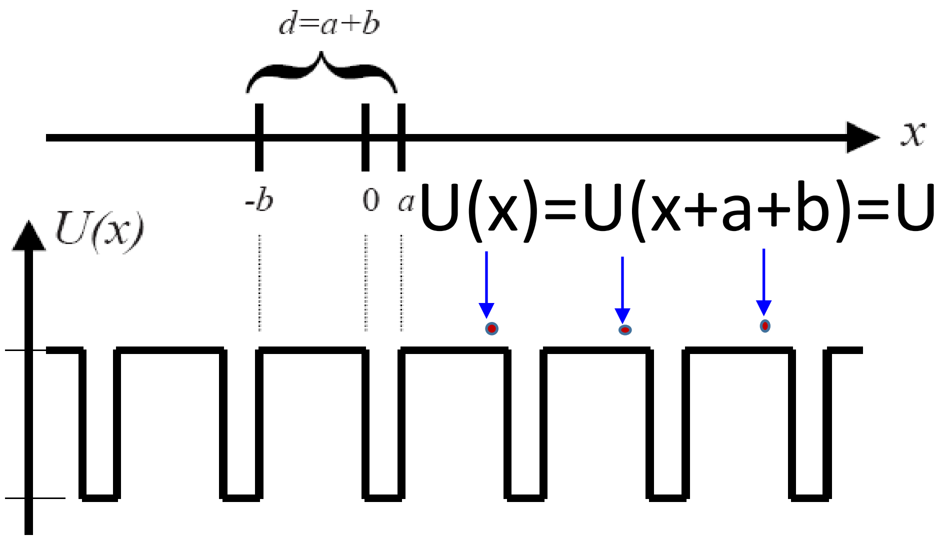
$$\psi(x)=A_b Sin\,(\beta x)+Bb\,Cos(\beta x)$$

N atom have 2N unknowns constants

Bloch's Theorem

The Bloch theorem provides a powerful mathematical simplification for the Wave-functions of particles evolving in a periodic potential. The solutions of the Schrodinger equation in such a potential are not pure plane waves as they were in the case of a free particle, but are waves which are modulated by a function having the periodicity of the potential or lattice. Such functions are then called Bloch wave functions and can be expressed as:

$$\boxed{\Psi(x)=e^{ikx}u_k(x) \text{ where } u_k(x)=u_k(x+a)}$$



U(x)=U(x+a+b)=U(x+p)

$$\psi(x+2p)=\psi(x+2p)e^{ikp}$$

$$\psi(x+2p)=\psi(x+p)e^{ikp}$$

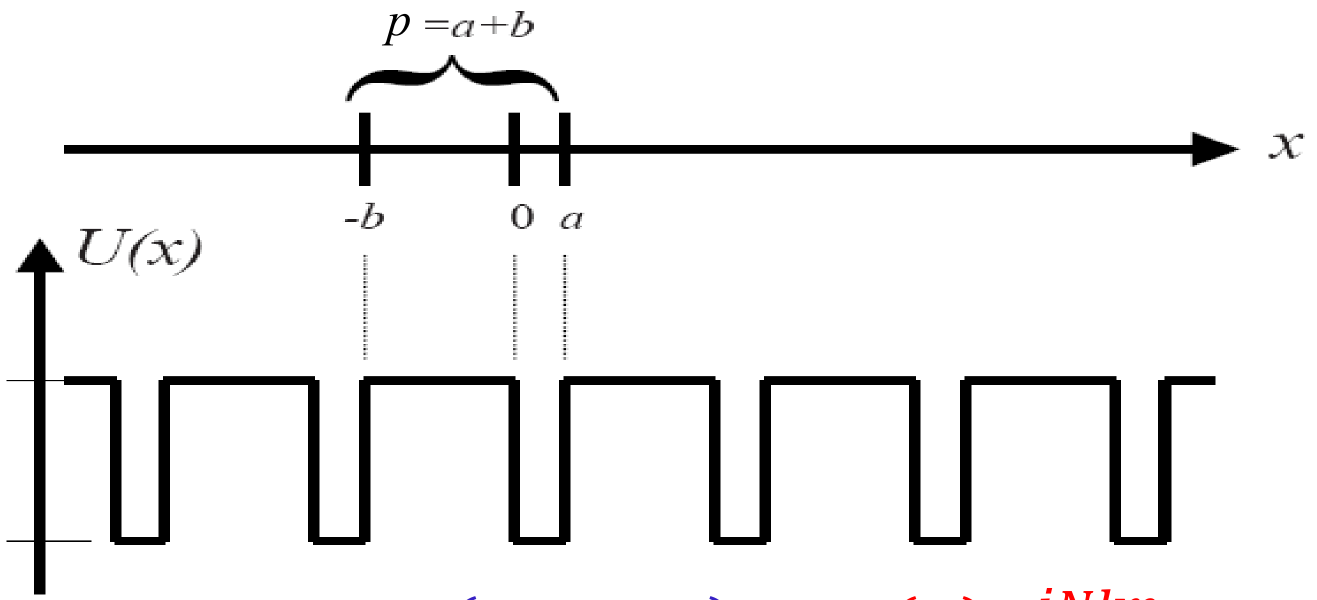
$$\psi(x+2p)=\psi(x)e^{ikp}\,e^{ikp}$$

$$\psi(x+2p)=\psi(x)e^{i2kp}$$

$$\psi(x+Np)=\psi(x)e^{iNkp}$$

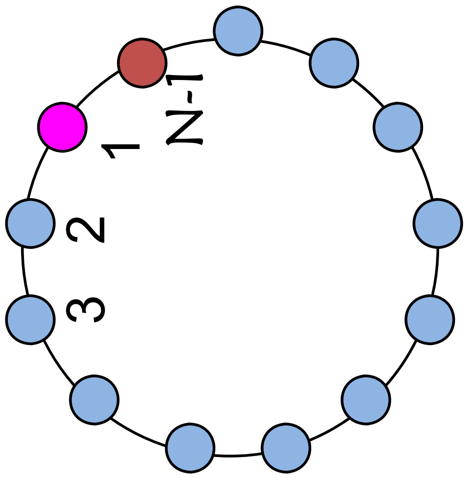
$$|\psi(x)|^2=|\psi(x+p)|^2 \Rightarrow \psi(x+p)=\psi(x)e^{ikp}$$

Periodic Boundary Condition



$$\psi(x+Np)=\psi(x)e^{iNkp}$$

$$e^{iNkp}=1\equiv e^{\pm i2\pi n}$$

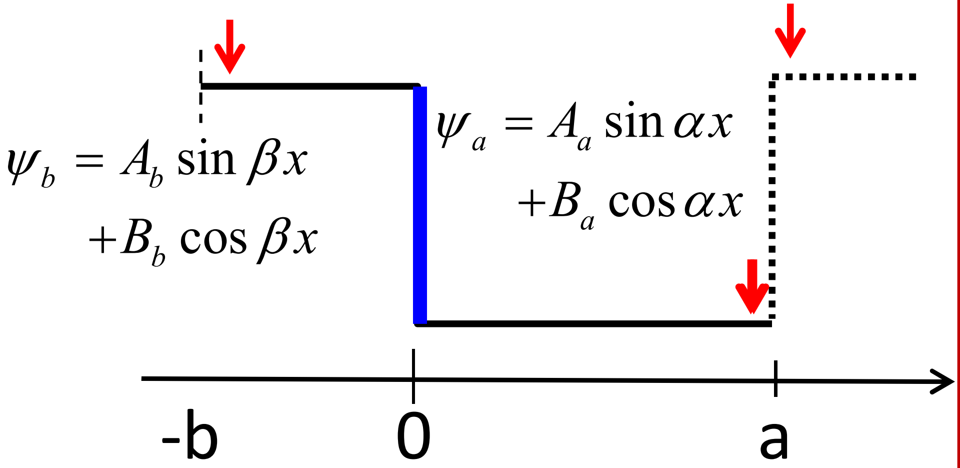


$$k=\pm\frac{2\pi n}{Np}=-\frac{N}{2}\ldots-1{,}01{,}\ldots N/2$$

$$k_{max}=\frac{\pi}{p} \qquad k_{min}=-\frac{\pi}{p}$$

BC

$$\alpha \equiv \sqrt{2mE/\hbar^2} \qquad \beta \equiv i\sqrt{2m(U_0 - E)/\hbar^2}$$



$$\psi\big|_{x=0^-} = \psi\big|_{x=0^+}$$

$$\left.\frac{d\psi}{dx}\right|_{x=0^-} = \left.\frac{d\psi}{dx}\right|_{x=0^+}$$

$$B_a = B_b$$

$$\alpha A_a = \beta A_b$$

$$A_a \sin \alpha a + B_a \cos \alpha a = e^{ik(a+b)}[-A_b \sin \beta b + B_b \cos \beta b]$$

$$\alpha A_a \sin \alpha a - \alpha B_a \cos \alpha a = e^{ik(a+b)}[\beta A_b \sin \beta b + \beta B_b \cos \beta b]$$

$$\psi_a\big|_{x=a} = \psi_b\big|_{x=-b}\, e^{ikp}$$

$$\left.\frac{d\psi_a}{dx}\right|_{x=a} = \left.\frac{d\psi_b}{dx}\right|_{x=-b} e^{ikp}$$

Solution

$$\boxed{\{(\beta^2-\alpha^2)/2\alpha\beta\}\sinh \beta b \sin \alpha a + \cos h\beta b \cos \alpha a = \cos k(a+b)}$$

For more convenient Kroning and Penney consider the case for which potential barrier become delta function i.e. $V\rightarrow\infty$ and $b \rightarrow 0$ but Vb is finite. Above equation reduces to-

$$\boxed{(P\sin \alpha a)/ \alpha a + \cos \alpha a = \cos ka}$$

Where $P=4\pi^2 mVba/h^2$

Vb measure of area of potential barrier

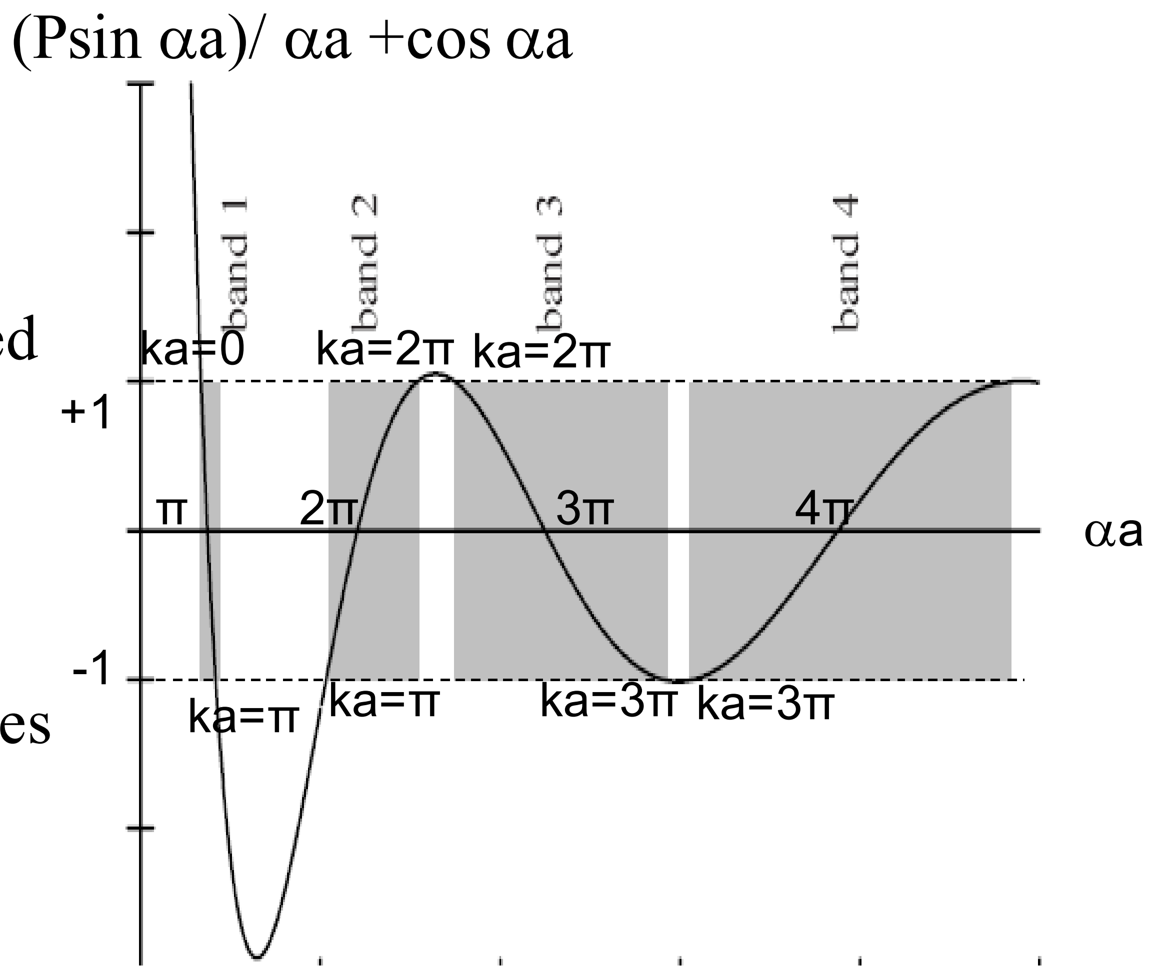
P is measure of binding a given electron to a particular potential barrier

$$\boxed{(P\sin \alpha a)/ \alpha a + \cos \alpha a = \cos ka}$$

From the figure following conclusion may be drawn

(a). Energy spectrum of electron consists of allowed energy bands separated by forbidden regions.

(b). As the energy increases the allowed energy band increases.



Cont.

(c). Width of particular allowed band decreases with increasing P.

For P$\rightarrow\infty$ allowed energy band become infinitely narrow. which corresponds to the case of an isolated atom with atomic spacing a$\rightarrow\infty$.

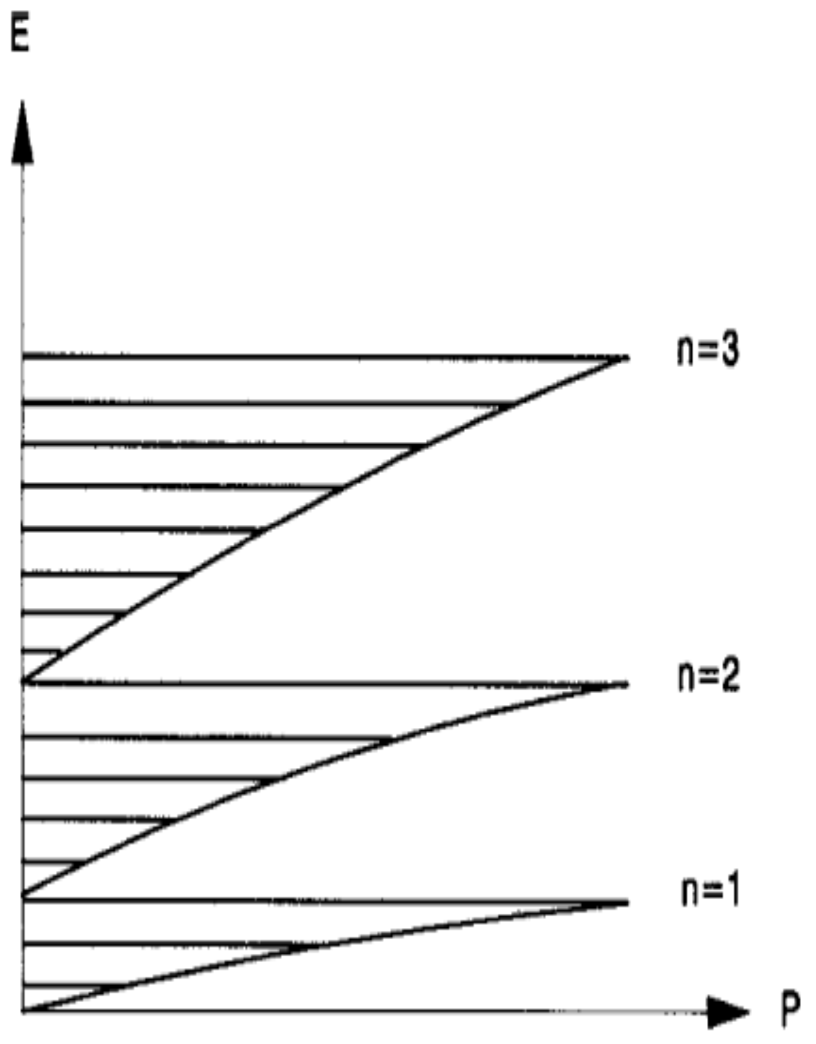
In that case $\sin\alpha a = o$

$\alpha a = \pm n\pi$, $E_n = n^2h^2/8ma^2$

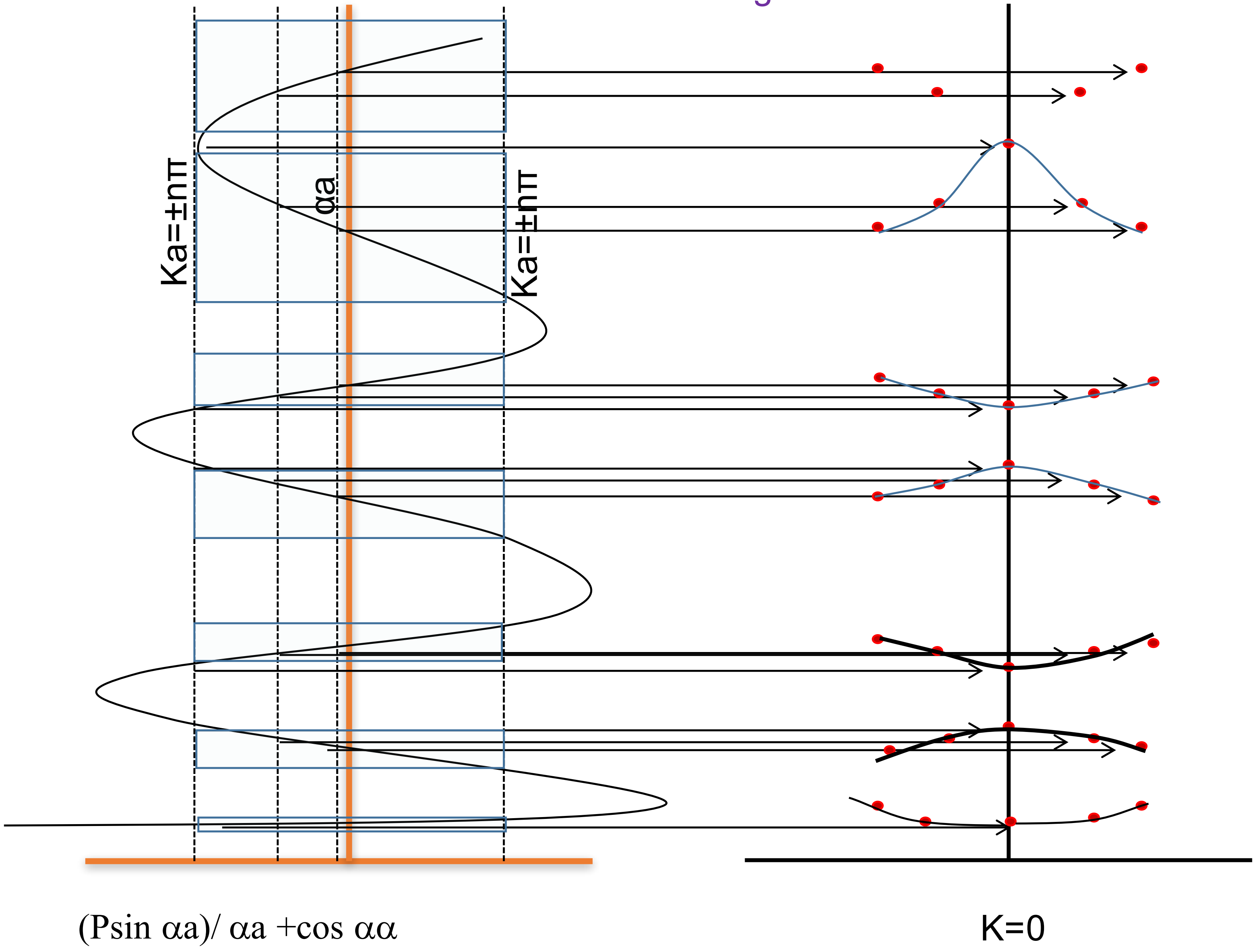
Energy levels of a particle in a box.

This we can see because ….

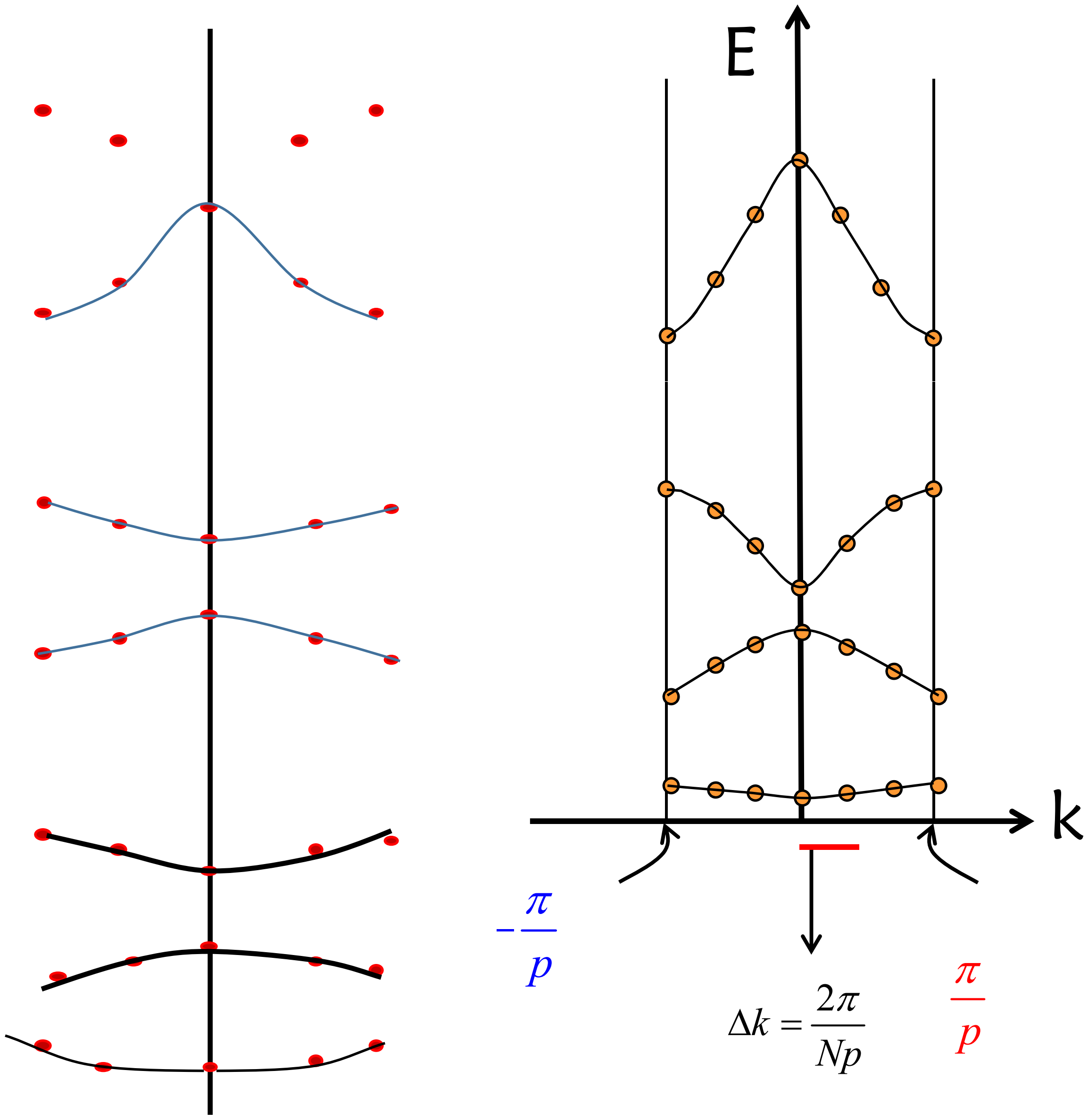
And for P$\rightarrow$0 we simply have free electron model and the energy spectrum is quasi continuous.



E-K diagram



Number of States



$$k = \pm\frac{2\pi n}{Np} \qquad n = -\frac{N}{2}.....-1,0,1,....\frac{N}{2}$$

$$\frac{States}{band} = \frac{k_{\max} - k_{\min}}{\Delta k} = \frac{2\pi/p}{2\pi/Np} = N$$

Motion of electron in in one dimension

What happened to electron when external electric field(F) is applied?

Assumption : One electron contain in Briliouin zone under consideration

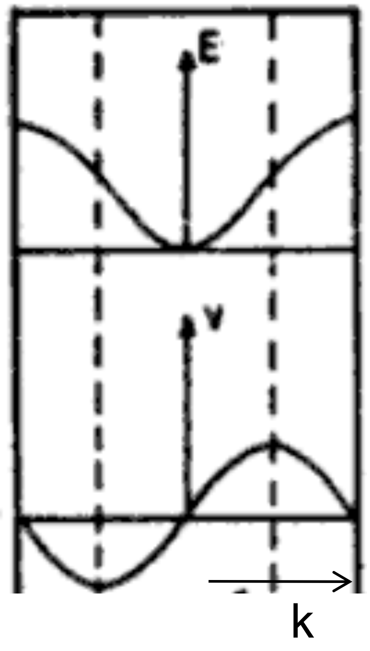
Suppose electron in k state initially. If field is acted for small time dt the electron gain energy

Particle velocity is equal to group velocity of the waves

Group velocity

$$v = \frac{d\omega}{dk}$$

$$v = \frac{1}{\hbar}\frac{dE}{dk} \qquad E = \hbar\omega$$



For free electron $\quad E = \hbar^2 k^2 / 2m$

$$v = \hbar k \;/ m$$

v= 0 for top and bottom of the band and
Achieve maxima for k=k_0 .
And then decrease with increasing energy
Which is altogether different from free electron.

Effective Mass

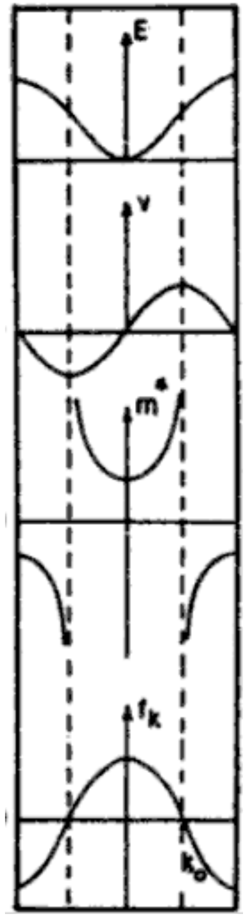
$$\frac{dv}{dt} = \frac{\frac{1}{\hbar}d\left(\frac{dE}{dk}\right)}{dt} = \frac{\frac{d}{dt}\left(\frac{dE}{dk}\right)}{\hbar}$$

$$= \frac{\left(\frac{d^2E}{dk^2}\right)\frac{dk}{dt}}{\hbar}$$

$$= \frac{\left(\frac{d^2E}{dk^2}\right)\frac{d(\hbar k)}{dt}}{\hbar^2}$$

F=m*dv/dt

$$m^* = \hbar^2 / (d^2E / dk^2)$$



Measure for the extent to which an electron in state k is free represented by f_k =m/m*

$$= m(d^2E / dk^2) / \hbar^2$$

More insight on Effective mass

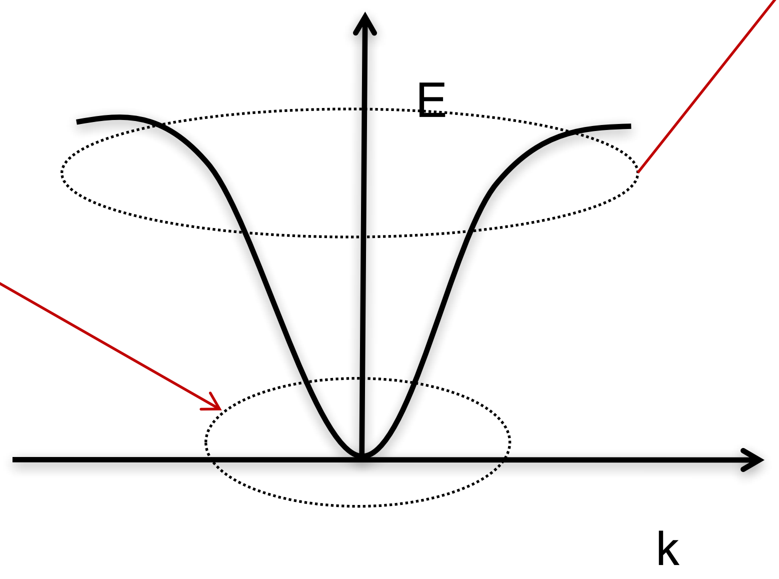
For free particle case, Energy E is-

$$E = \hbar^2 k^2 / 2m$$

m*=m (Newtonian mass)

Since mass of the electron depend upon the curvature, Newtonian mechanics does not hold

Curve is symmetric
i.e. $E=c_1k^2+c_2k^4+c_3k^6\ldots$
Effective mass is constant hence can be treated as Newtonian mass



Case1: Completely filled band

The electron current density

$J = -qnv = q/L \sum_{i=0}^{i=N} v_{\mathrm{i}}$

n=no. of electron per unit volume
q=electronic charge
v=group velocity
V=Volume of material
N=total no. of electron

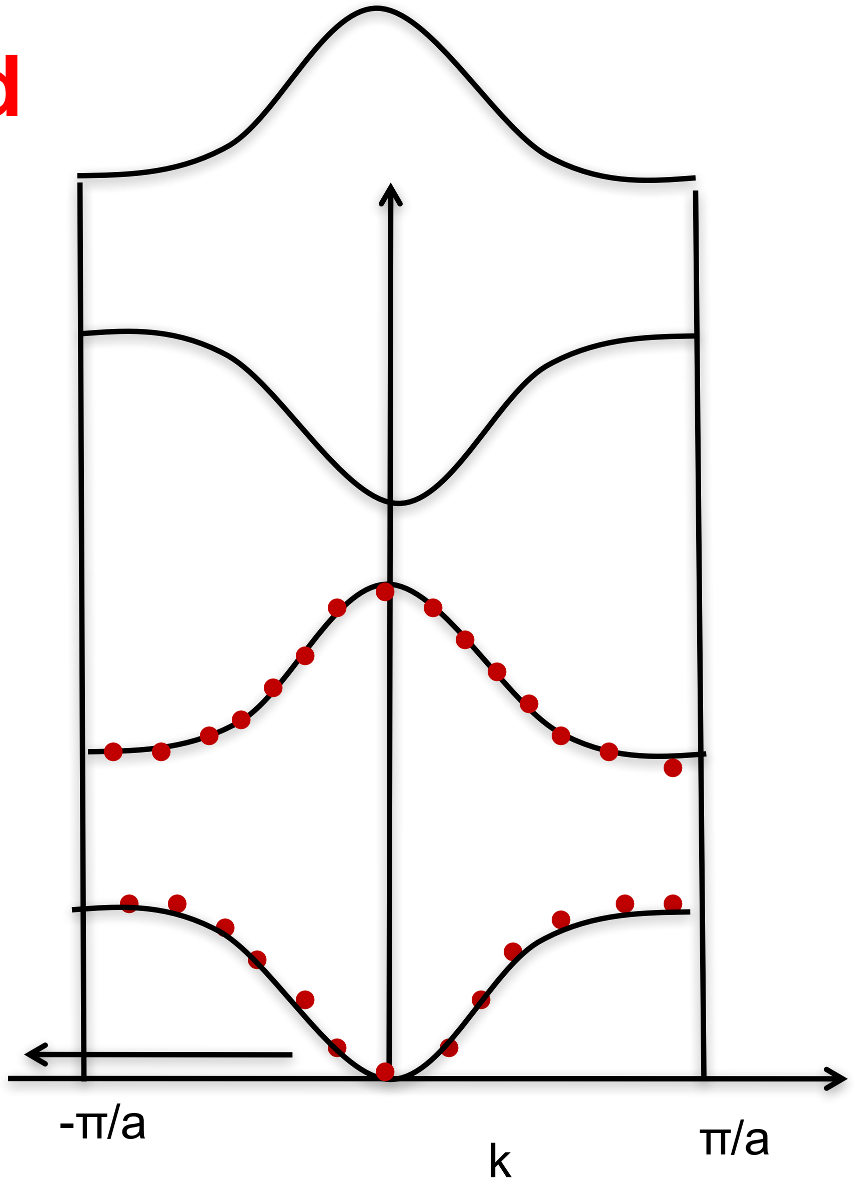
Fully empty there is no electron to conduct to J=0

Fully filled effective electron available -

$$J_2 = -\frac{q}{L}\sum_{i(filled)} \upsilon_i = -\frac{q}{L}\sum_{0}^{k_{max}} \upsilon_i - \frac{q}{L}\sum_{-k_{min}}^{0} -|\upsilon_i| = 0$$

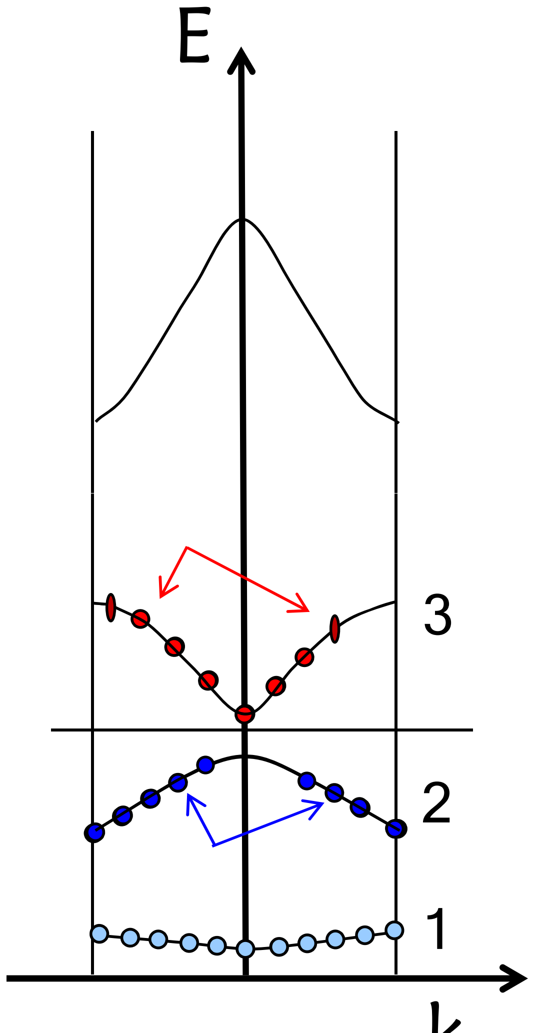
J=0 i.e no current at T=0K

Fully empty



Filled and empty band don't carry any current

Case2: Partially filled (Electron and holes)



Filling of electron in band

Follows the Pauli exclusive principal and Fermi Dirac Distribution.

Group 4 element as Si, Ge or C have a even no of electron. At T=0K , the band or fully filled. Hence these are non-conducting at T=0K and are insulator. But T>0K there is strong possibility to go next band as band gap is less so start conducting. While in SiO2 like insulator band gap is high so almost no probability of electron to go next band……

Group 1element as Na, K have a one electron. At T=0K , the band or partially filled. Hence these are conducting at T=0K and are conductor.