Semiconductor Fundamentals

Presented to

EE2187 class in Semester 1 2019/20

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Lecture 6

Course information

- Semiconductors Materials - Types of Solids, Space lattice, Atomic Bonding,
- Introduction to quantum theory, Schrodinger wave equation, Electron in free space, Infinite well, and step potentials, Allowed and forbidden bands
- Electrical conduction in solids, Density of states functions, Fermi-Dirac distribution in Equilibrium,
- Valence band and Energy band models of intrinsic and extrinsic Semiconductors. Degenerate and non degenerate doping
- Thermal equilibrium carrier concentration, charge neutrality
- Carrier transport – Mobility, drift, diffusion, Continuity equation.

Reference

Text Book:

1. Physics of Semiconductor Devices, *S. M. Sze*, John Wiley & Sons (1981).
2. Solid State Electronics by *Ben G. Streetman and Sanjay Banerjee*, Prentice Hall International , Inc.
3. Semiconductor Physics and Devices, Donald A. Neamen, Tata Mcgraw-Hill Publishing company Limited.
4. Advanced Semiconductor Fundamentals by Pirret

Reference Book:

1. Fundamentals of Solid-State Electronic Devices, *C. T. Sah*, Allied Publisher and World Scientific, 1991.
2. Complete Guide to Semiconductor Devices, *K. K. Ng*, McGraw Hill, 1995.
3. Solid state physics, Ashcroft & Mermins.
4. Introduction to Solid State Electronics, *E. F. Y. Waug*, North Holland, 1980.

Recap

Energy Band Model

How the electrons are distributed over a range of energies at any given temperature

i.e. if distribution is known then one can get fraction of free and bound electron population at any temperature.

Three Basic Fundamentals of this model are

[1]. Allowed energy states

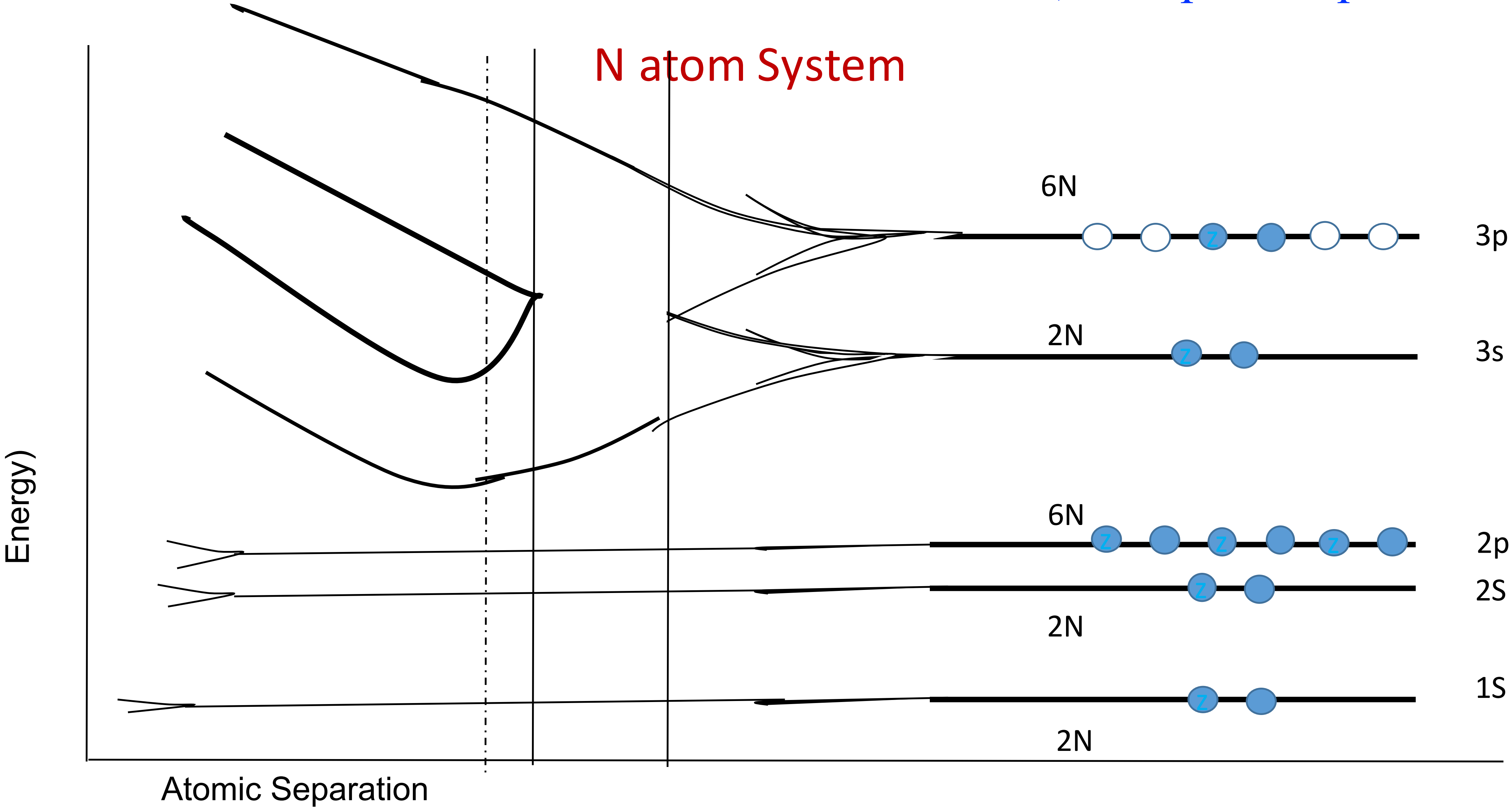
a. *Electrons , allowed only certain energy states.*
b. *Electrons occupy the lowest energy states available.*
c. *No two electrons can occupy the same energy state.*

[2]. Distribution of allowed states over energy, N(E).

[3]. Fraction of occupied states.

Allowed Energy State of e$^-$ in Si Crystal

Si atomic number is 14, Electron distribution $1s^2$, $2s^2$ $2p^6$ $3s^2$ $3p^2$



SE solution in Periodic Potential

One dimensional Schrodinger equation

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left[E - U(x,y,z)\right]\psi = 0,$$ Where U(x) is periodic potential i.e. U(x)=U(x+a)

One-dimensional Kronig-Penney model

Assumption:

All previous assumption still hold

The crystal is infinitely large

Potential energy of an electron has the form of periodic array of rectangular wells.

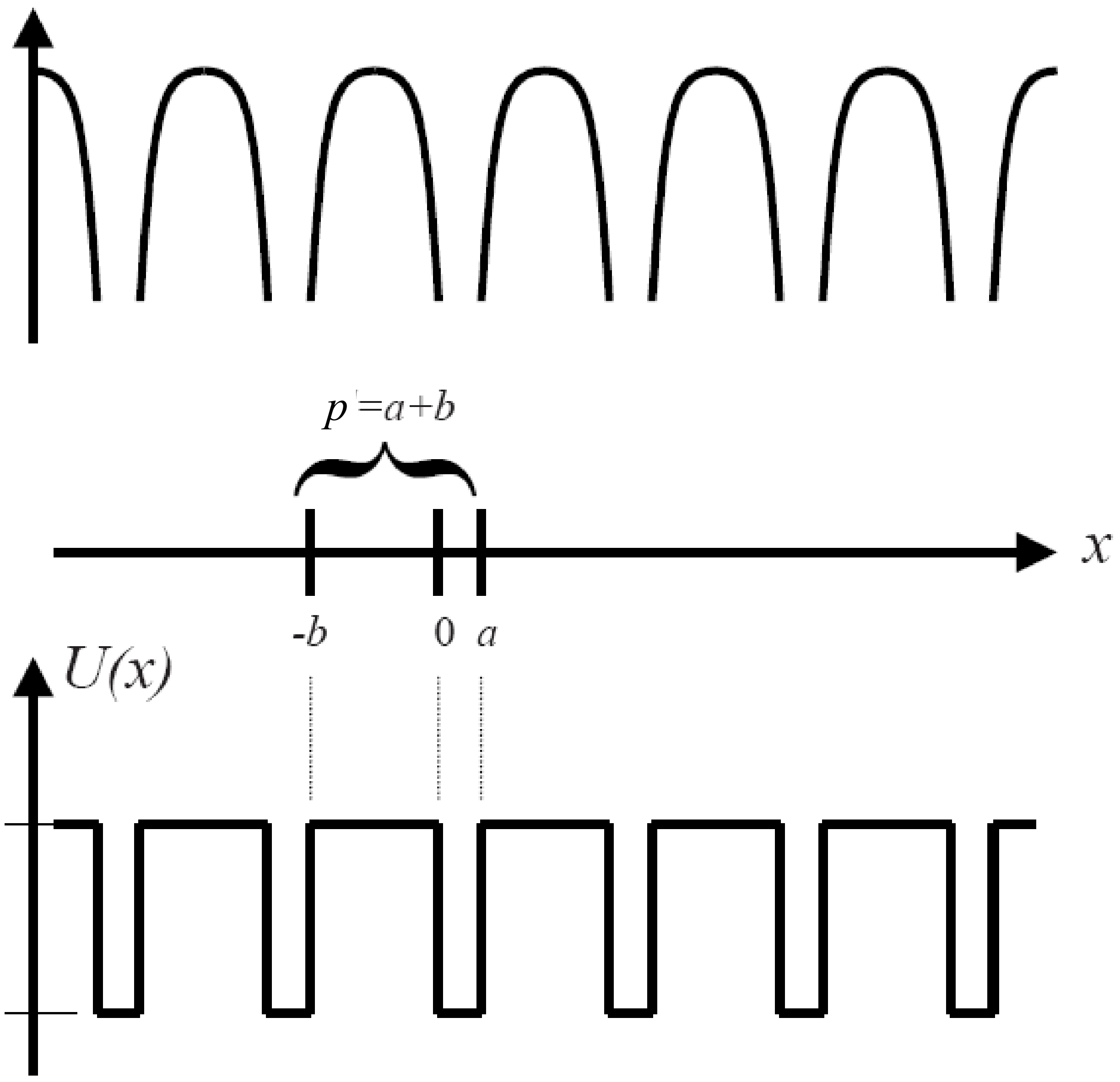
The period of potential is (a+b);

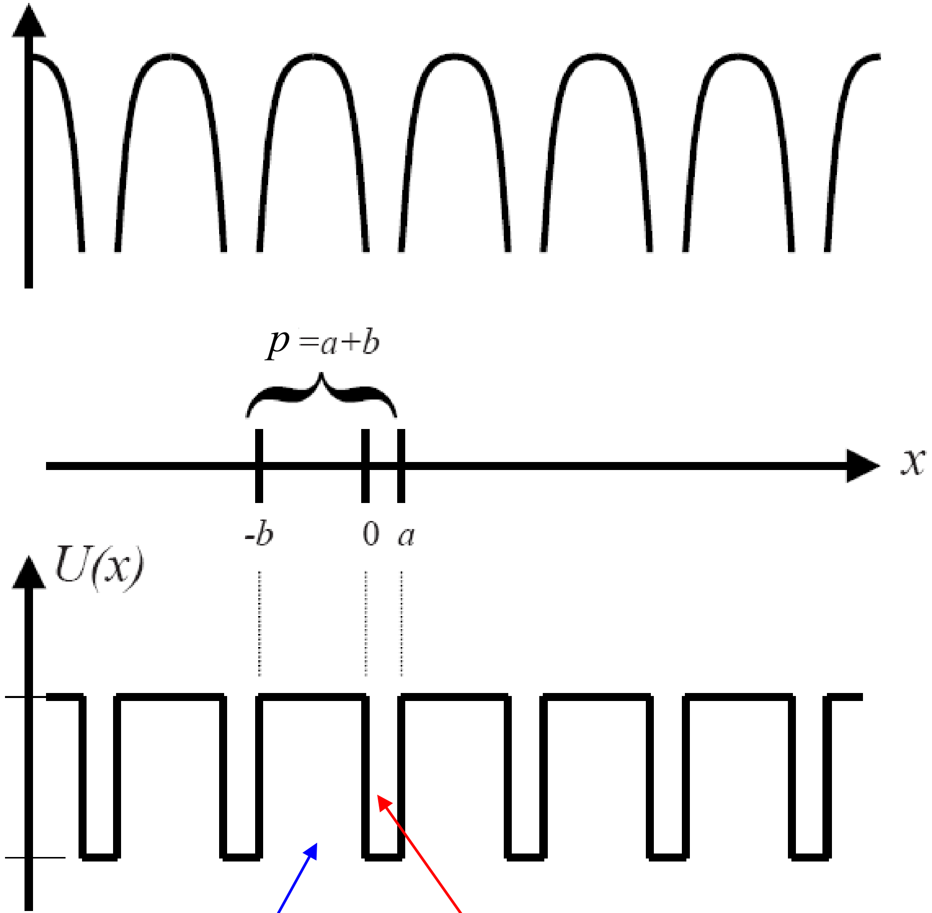
The Schrodinger equation in two region

$$d^2 \Psi(x)/dx^2+\alpha^2 \Psi(x)=0 \quad 0<x<a \tag{2}$$

$$d^2 \Psi(x)/dx^2+\beta^2 \Psi(x)=0 \quad -b<x<0 \tag{3}$$

$$\beta = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} \qquad \alpha = \sqrt{\frac{2mE}{\hbar^2}}$$





$$\psi(x)=A_a\,Sin\,(\alpha x)+B_a\,Cos(\alpha x)$$

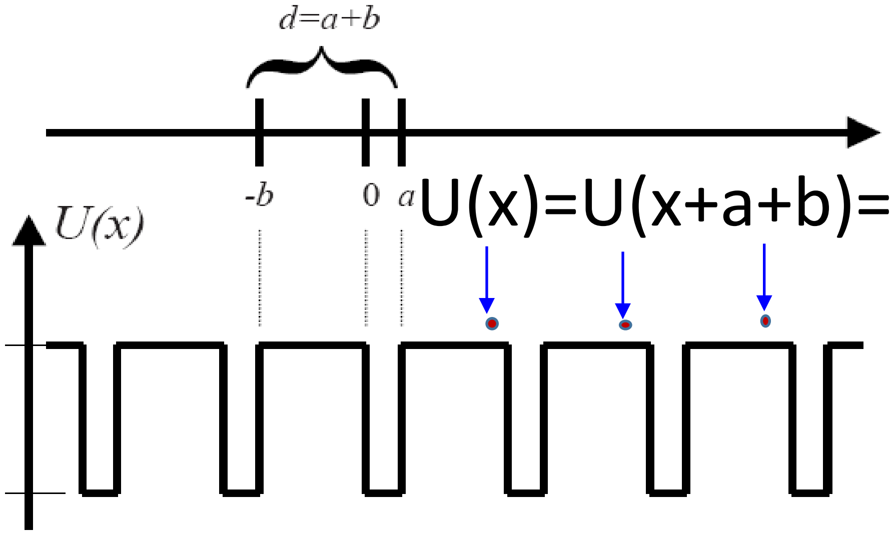
$$\psi(x)=A_b Sin\,(\beta x)+Bb\,Cos(\beta x)$$

N atom have 2N unknowns constants

Bloch's Theorem

The Bloch theorem provides a powerful mathematical simplification for the Wave-functions of particles evolving in a periodic potential. The solutions of the Schrodinger equation in such a potential are not pure plane waves as they were in the case of a free particle, but are waves which are modulated by a function having the periodicity of the potential or lattice. Such functions are then called Bloch wave functions and can be expressed as:

$$\boxed{\Psi(x)=e^{ikx}u_k(x) \text{ where } u_k(x)=u_k(x+a)}$$



$$U(x)=U(x+a+b)=U(x+p)$$

$$\psi(x+2p)=\psi(x+2p)e^{ikp}$$

$$\psi(x+2p)=\psi(x+p)e^{ikp}$$

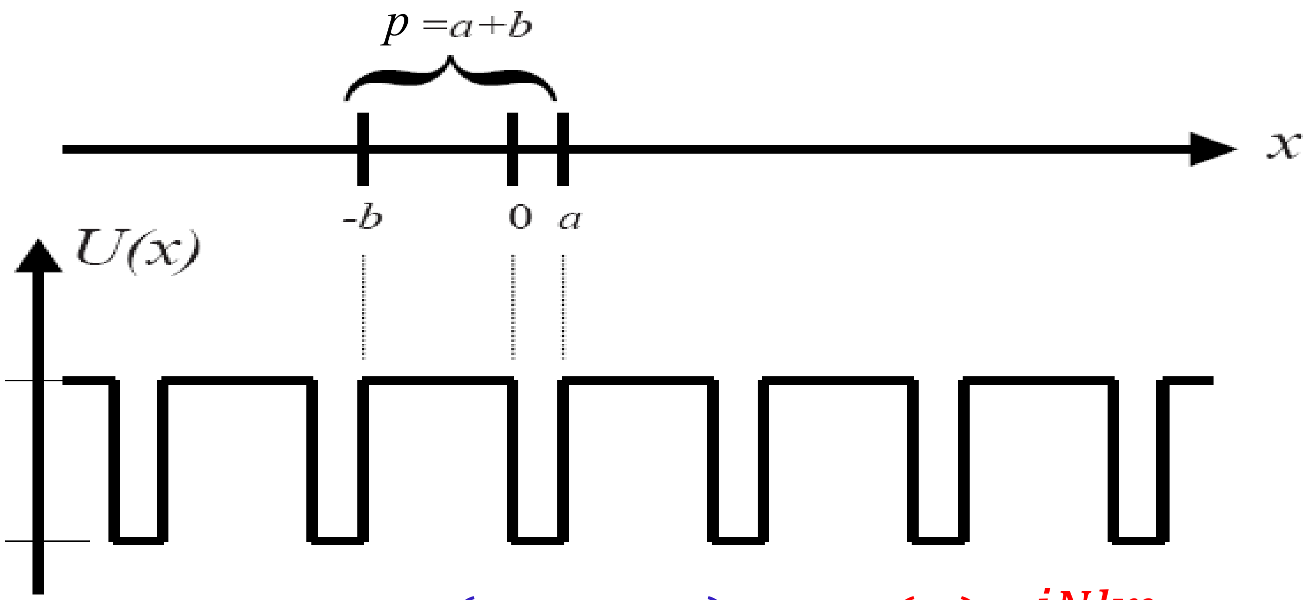
$$\psi(x+2p)=\psi(x)e^{ikp}\,e^{ikp}$$

$$\psi(x+2p)=\psi(x)e^{i2kp}$$

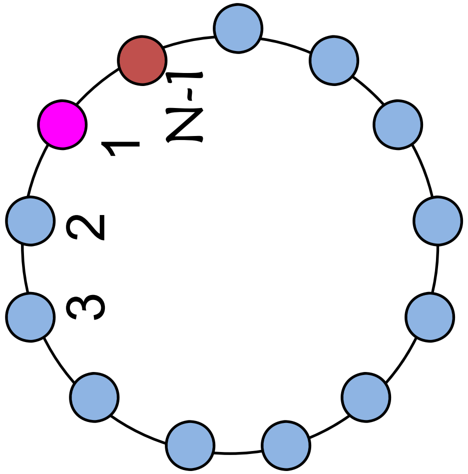
$$\psi(x+Np)=\psi(x)e^{iNkp}$$

$$|\psi(x)|^2=|\psi(x+p)|^2 \Rightarrow \psi(x+p)=\psi(x)\mathrm{e}^{ikp}$$

Periodic Boundary Condition



$$\psi(x+Np)=\psi(x)e^{iNkp}$$



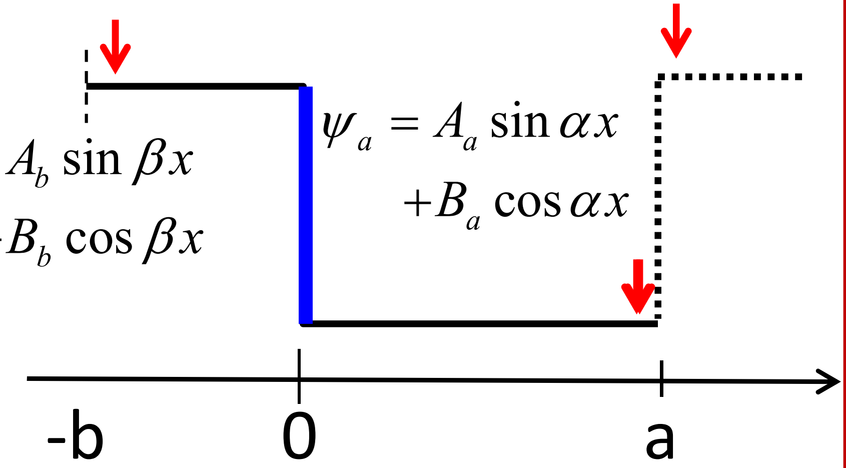
$$e^{iNkp}=1\equiv e^{\pm i2\pi n}$$

$$k=\pm\frac{2\pi n}{Np}=-\frac{N}{2}\ldots-1,01,\ldots N/2$$

$$k_{max}=\frac{\pi}{p} \qquad k_{min}=-\frac{\pi}{p}$$

BC

$$\alpha \equiv \sqrt{2mE/\hbar^2} \qquad \beta \equiv i\sqrt{2m(U_0 - E)/\hbar^2}$$



$$\psi_b = A_b \sin \beta x + B_b \cos \beta x$$

$$\psi_a = A_a \sin \alpha x + B_a \cos \alpha x$$

$$\psi\big|_{x=0^-} = \psi\big|_{x=0^+}$$

$$\left.\frac{d\psi}{dx}\right|_{x=0^-} = \left.\frac{d\psi}{dx}\right|_{x=0^+}$$

$$B_a = B_b$$

$$\alpha A_a = \beta A_b$$

$$A_a \sin \alpha a + B_a \cos \alpha a = e^{ik(a+b)}[-A_b \sin \beta b + B_b \cos \beta b]$$

$$\alpha A_a \sin \alpha a - \alpha B_a \cos \alpha a = e^{ik(a+b)}[\beta A_b \sin \beta b + \beta B_b \cos \beta b]$$

$$\psi_a\big|_{x=a} = \psi_b\big|_{x=-b}\, e^{ik\,p}$$

$$\left.\frac{d\psi_a}{dx}\right|_{x=a} = \left.\frac{d\psi_b}{dx}\right|_{x=-b} e^{ik\,p}$$

Solution

$$\boxed{\{(\beta^2-\alpha^2)/2\alpha\beta\} \sinh \beta b \sin \alpha a + \cos h\beta b \cos \alpha a = \cos k(a+b)}$$

For more convenient Kroning and Penney consider the case for which potential barrier become delta function i.e. $V\rightarrow\infty$ and $b \rightarrow 0$ but Vb is finite. Above equation reduces to-

$$\boxed{(P\sin \alpha a)/ \alpha a + \cos \alpha a = \cos ka}$$

Where $P=4\pi^2 mVba/h^2$

Vb measure of area of potential barrier

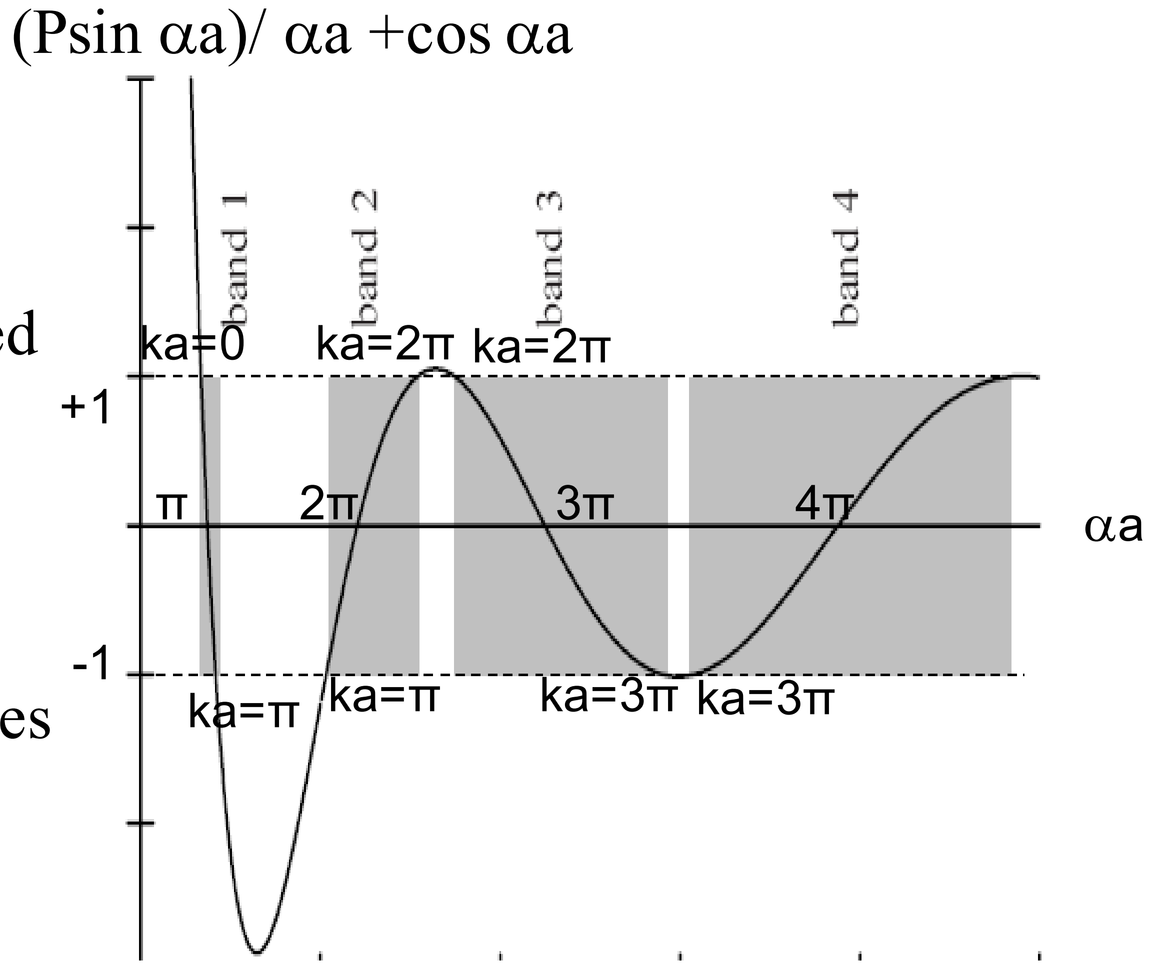
P is measure of binding a given electron to a particular potential barrier

$$(P\sin \alpha a)/\alpha a + \cos \alpha a = \cos ka$$

From the figure following conclusion may be drawn

(a). Energy spectrum of electron consists of allowed energy bands separated by forbidden regions.

(b). As the energy increases the allowed energy band increases.



Cont.

(c). Width of particular allowed band decreases with increasing P.

For $P \to \infty$ allowed energy band become infinitely narrow. which corresponds to the case of an isolated atom with atomic spacing $a \to \infty$.

In that case $\sin \alpha a = o$

$\alpha a = \pm n\pi$, $E_n = n^2h^2/8ma^2$

Energy levels of a particle in a box.

This we can see because ….

And for $P \to 0$ we simply have free electron model and the energy spectrum is quasi continuous.