SOLUTIONS: ECE 305 Homework: Week 2

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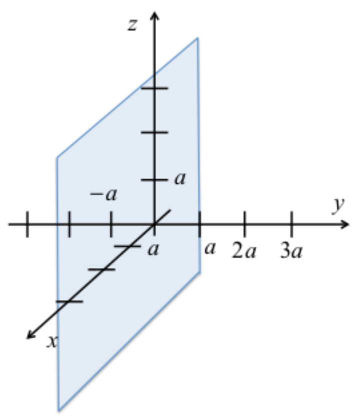
1) Sketch the following planes or directions.

a) (010)
b) $[010]$
c) (203)
d) $(\bar{1}1\bar{1})$

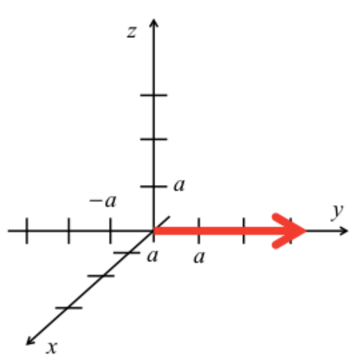
Solution:

1a) (010) is a plane.



Note that a similar plane with a y-intercept at 2a, or 3a would also be an (010) plane.

1b) [010] is a direction normal to the plane in 1a)



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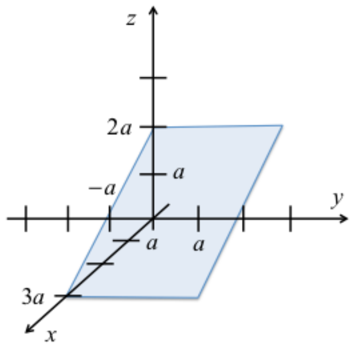
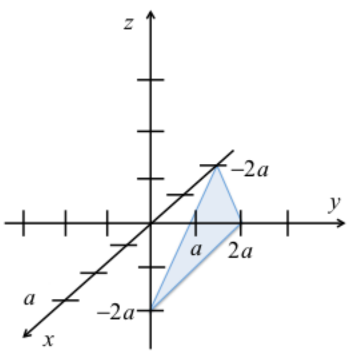
1c) (203) is a plane



1d) $\left(\bar{1}1\bar{1}\right)$ is a plane



Note that a similar plane with intercepts at $x=-a, y=a, z=-a$ would also be a $\left(\bar{1}1\bar{1}\right)$ plane. In general, {*h*, *k*, *l*} denotes a family of planes.

2) Consider a hydrogen atom (1 electron orbiting one proton). Suppose that we excite the electron from the ground state to the first excited state (see Fig. 2.1 in Pierret, SDF).

a) The electron relaxes to the ground state and emits a photon. What is the energy of this photon (in eV)?

b) What is the wavelength, λ, of the photon in vacuum? It may be helpful to recall that $E=h\nu$, where *h* is Planck's constant and ν is the frequency, and $c=\lambda\nu$, where c is the speed of light. **Pay attention to units.**

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Solution:

2a) From Fig. 2.1 in Pierret, SDF:
$E_0 = -13.6\text{ eV}$ (ground state)
$E_1 = -3.4\text{ eV}$ (first excited state)
(Note that $E = 0$ is the energy of the electron infinitely far away from the proton.

$E_{ph} = -3.4 - (-13.6) = 10.2\text{ eV}$ (photon energy) $\boxed{E_{ph} = 10.2\text{ eV}}$

2b) $\lambda = \dfrac{c}{\nu} = \dfrac{c}{E_{ph}/h} = \dfrac{hc}{E_{ph}}$

$$\lambda = \frac{hc}{E_{ph}} = \frac{\left(6.626\times10^{-34}\text{ J-s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{10.2\times\left(1.6\times10^{-19}\right)\text{ J}} = 1.22\times10^{-7}\text{ m}$$

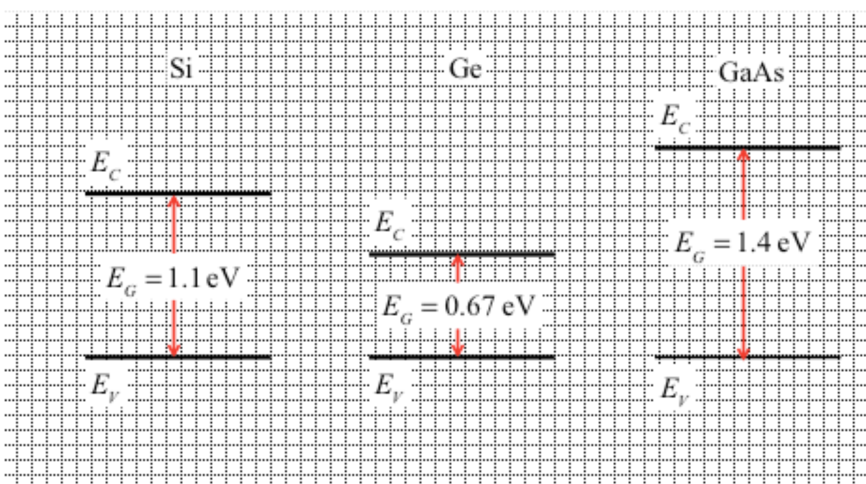
Careful: Be sure to use MKS (SI) units – meters for distance, meters/second for velocity, and Joules for energy.

$\boxed{\lambda_{ph} = 122\text{ nm}}$

3) Consider Si ($E_G = 1.1\text{ eV}$), Ge ($E_G = 0.67\text{ eV}$) and GaAs ($E_G = 1.4\text{ eV}$).

a) Using the same vertical scale , draw a simple energy band diagram (showing E_C and E_V) for each of the three materials.
b) On the energy band diagram for Si, illustrate an electron in the conduction band.
c) On the energy band diagram for Ge, illustrate a hole in the valence band.

Solution:
3a)



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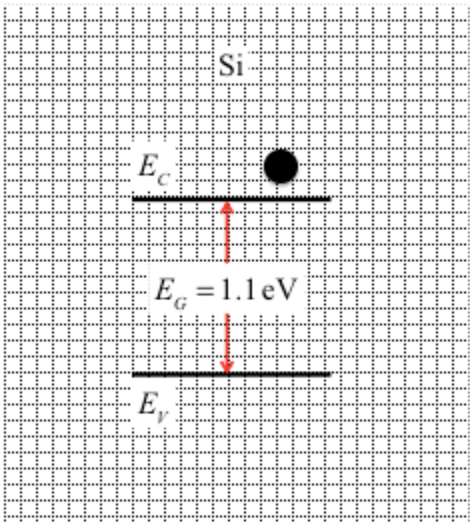
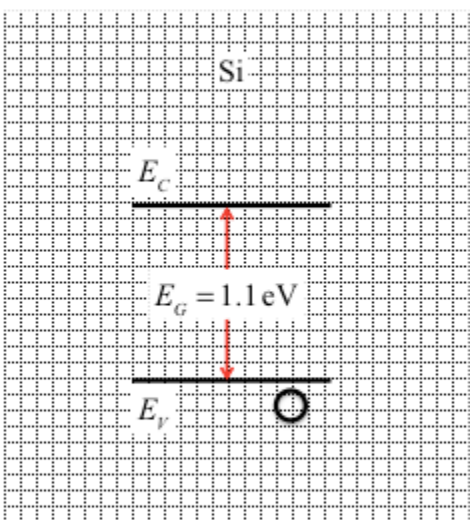
A comment on the sketch in Fig. 3a). Note that we arbitrarily aligned the valence bands of each of the three semiconductors. In general, these bands do not line up. The precise "band line-ups" are critical when making "heterojunctions", which put two different semiconductors together.

3b) An electron in the conduction band is a filled state in the conduction band. It is a carrier of negative charge.



3c) A hole in the valence band is an empty state in the valence band. It is a carrier of positive charge.



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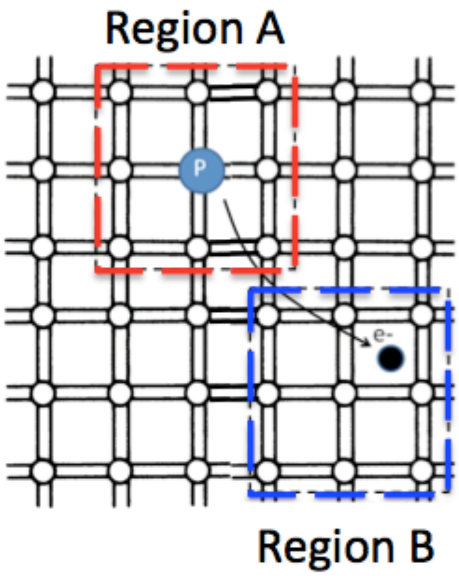
4) Consider a region of Si, which is a perfect single crystal except for the phosphorous atom shown in the figure below. Note that the P atom has donated one electron to the lattice (as shown).



a) Find the net charge within Region A.

b) Find the net charge within Region B.

c) Repeat parts a) and b), assuming that the P atom has been replaced with a B atom and that the B atom has accepted one electron, and that the corresponding hole is within region B.

Solutions:

4a) Phosphorus is from column V, so there are 5 valence electrons. Four of the valence electrons form covalent bonds with four Si nearest neighbors. The fifth electron is weakly bound. The thermal energy breaks the bond, so the Phosphorus has lost one electron and has a positive charge of q.

The charge in region A is $+q = 1.60 \times 10^{-19}$ C

4b) Region B was neutral, until the fifth electron from the phosphorus broke its bond and moved to region B. Region B now has one extra electron.

The charge in region B is $-q = -1.60 \times 10^{-19}$ C

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4c) Boron is from column III, so there are 3 valence electrons. An additional electron is obtained from the thermal energy that breaks on of the Si-Si bonds. Now there are four electrons for the Boron atom to form covalent bonds with four Si nearest neighbors. The Boron atom has acquired one additional electron, so it now has a negative charge of -*q*.

The charge in region A is $-q = -1.60 \times 10^{-19}$ C

Region B was neutral, until one of the Si bond was broken; the electron moved to region A. Region B now has one fewer electrons, so it has acquired a positive charge.

The charge in region B is $+q = +1.60 \times 10^{-19}$ C

5) Consider the conduction band of Si. Typically, only the states near the bottom of the conduction band are occupied with electrons. Assume that all states within 0.1 eV of the bottom of the band are occupied and answer the following questions.

5a) How many electrons are in the conduction band? Express your answer per cm^3.

5b) Compare this number to the atomic density of Si.

Solution:

5a) The density of states is $g_c(E) = \dfrac{m_n^* \sqrt{2m_n^*(E - E_C)}}{\pi^2 \hbar^3}$ (This expression assume that two electrons occupy each state.)

The number of filled states is

$$N_e = \int_{E_C}^{E_C + 0.1\,\text{eV}} g_c(E)\,dE = \frac{m_n^* \sqrt{2m_n^*}}{\pi^2 \hbar^3} \int_{E_C}^{E_C + 0.1\,\text{eV}} (E - E_C)^{1/2}\,dE = \frac{m_n^* \sqrt{2m_n^*}}{\pi^2 \hbar^3} \frac{2}{3} (E - E_C)^{3/2} \Big|_{E_C}^{E_C + 0.1\,\text{eV}}$$

$$N_e = \frac{m_n^* \sqrt{2m_n^*}}{\pi^2 \hbar^3} \frac{2}{3} \left(0.1 \times 1.6 \times 10^{-19}\right)^{3/2}$$ (convert eV to Joules)

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For Si, $m_n^* = 1.18 m_0 = 1.18 \times 9.11 \times 10^{-31}$ kg (Table 2.1, p. 34, SDF)

$$m_n^* = 1.07 \times 10^{-30} \text{ kg}$$

$$N_e = \frac{\sqrt{2}\left(m_n^*\right)^{3/2}}{\pi^2 \hbar^3} \frac{2}{3}\left(0.16 \times 10^{-19}\right)^{3/2} = \frac{1.414 \times \left(1.11 \times 10^{-45}\right)}{9.87 \times \left(1.055 \times 10^{-34}\right)^3} \times 0.667 \times \left(2.02 \times 10^{-30}\right)$$

$$N_e = \frac{1.414 \times 0.667}{9.87 \times 1.055 \times 10^{-34}} \times \frac{1.11 \times 10^{-45}}{1.055 \times 10^{-34}} \times \frac{2.02 \times 10^{-30}}{1.055 \times 10^{-34}} = 1.82 \times 10^{26} \text{ m}^{-3}$$

(Note the order of the divisions so that my calculator does not underflow.)

The answer is per cubic meter because we have done all the calculations in MKS units, but we are asked for a number per cubic centimeter.

$$N_e = 1.82 \times 10^{26} \text{ m}^{-3} \times \left(\frac{\text{m}}{10^2 \text{cm}}\right)^3 = 1.82 \times 10^{20} \text{cm}^{-3} \qquad \boxed{N_e = 1.82 \times 10^{20} \text{cm}^{-3}}$$

Note that this is considered to be a very large concentration of electrons in the conduction band.

5b) Silicon (Si) has a diamond crystal structure and a lattice spacing of a = 5.42 Angstroms.

The atomic density is:

$$N_a = \frac{8}{\left(5.42 \times 10^{-8}\right)^3} \text{ cm}^{-3} = 5.02 \times 10^{22} \text{cm}^{-3}$$

$$\frac{N_e}{N_a} = \frac{1.82 \times 10^{20}}{5.02 \times 10^{22}} = 0.004$$

$$\boxed{\frac{N_e}{N_a} = 0.04\%}$$

Note that a high concentration of electrons in the conduction band is only 0.04% of the atomic density.