Chi-Squared Test for Machine Learning

Supplementary Lecture

Outline

- What is Chi-square statistics
- Applications in Machine learning
- Example

Chi-square Statistics

- A chi-square statistic is one way to show a relationship between two categorical variables
- The chi-squared statistic is a single number that tells you how much difference exists between your observed counts and the counts you would expect if there were no relationship at all in the population.
- The chi-square distribution can be represented as

$$\chi_c^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

Chi-Square Statistics

The chi-square distribution can be represented as

$$\chi_c^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

- If there is no difference in observed and expected frequencies, then the chi-square value would be zero. If there is a difference, then the value of chi-square would be more than zero.
- The subscript c represents degree of freedom- the maximum number of independent values

Chi square statistics

- Chi-square test is used to test the goodness of fit
 - determines if a sample data matches a population
- A chi-square test for independence compares two variables in a contingency table to see if they are related
 - A very small chi square test statistic means that your observed data fits your expected data extremely well. In other words, there is a relationship.
 - A very large chi square test statistic means that the data does not fit very well. In other words, there isn't a relationship.

Chi-Square Statistics

- Whether input features are relevant to the outcome to be predicted
- Whether the class (target variable) is dependent or independent on the input features (variable)
 - O If independent, we can say the feature is irrelevant in modeling the target class and can be removed (Feature selection)

Contingency Table

- Pairs of categorical values can be summarized using contingency table
- It shows the distribution of one variable in rows and another in columns
- Example:

Exited\	Yes	No	Total
Gender			
Male	38	178	216
Female	44	140	184
Total	82	318	400

Consider a data-set where we have to determine why customers are leaving the bank,

Let's perform a Chi-Square test for two variables.

Gender of a customer with values as Male/Female as the predictor and Exited describes whether a customer is leaving the bank with values Yes/No as the response. In this test we will check is there any relationship between Gender and Exited.

Dataset

	Gender	 Exited
1	Male	 Yes
2	Male	 Yes
3	Female	 No
4	Male	 Yes
5	Male	 No
6	Male	 No
•		
•		
400	Female	 No

Steps to perform the Chi-Square Test:

- 1. Define Hypothesis.
- 2. Build a Contingency table.
- 3. Find the expected values.
- 4. Calculate the Chi-Square statistic.
- 5. Accept or Reject the Null Hypothesis

Define Hypothesis

- Null Hypothesis (H0): Two variables are independent.
- Alternate Hypothesis (H1): Two variables are not independent.

Contingency Table

Compute the contingency table

Exited\	Yes	No	Total
Gender			
Male	38	178	216
Female	44	140	184
Total	82	318	400



	Gender	 Exited
1	Male	 Yes
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400	Female	 No

• DoF = (2-1) =1

Find the Expected Value

- $E_{ij} = (T_i * T_j) / N$
 - \bigcap T_i = Total in ith row
 - \bigcirc T_i = Total in jth column
- \bullet E₁₁= (216 * 82)/400 = 44.24
- \bullet E₁₂= (216 * 318)/400 = 171.72
- \bullet E₂₁= (184 * 82)/400 = 37.72
- \bullet E₂₂= (184 * 318)/400 = 146.28

Exited\	Yes	No	Total
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Male	38	178	216
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Compute chi-square statistics

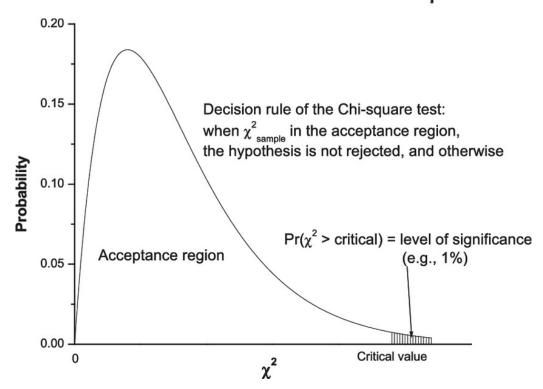
	O _i	E _i	(O _i -E _i) ² /E _i
1,1	38	44.24	0.880
1,2	178	171.72	0.23
2,1	44	37.72	1.045
2,2	140	146.28	0.27
	$\chi_c^2 = \sum \frac{(6)^2}{2}$	2.425	

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	Yes	NO
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Female	37.72	146.28

$$\chi_c^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$\chi_c^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$



Decision rule of the Chi-square test: when χ^2_{sample} in the acceptance region, the hypothesis is not rejected, and otherwise $\chi^2_{\text{const}} = \sum_{\text{constant of the constant of the significance}} \chi^2_{\text{constant of the constant of the constant of the Chi-square test: when <math>\chi^2_{\text{sample}} = \sum_{\text{constant of the constant of t$

Accept or reject the Hypothesis

Compute Chi-square_{tabulated}
from Chi-Square <u>table</u>, with
level of significance = 0.05
and DoF=1

Table of Chi-square statistics

t-statistics

F-statistics with other P-values: $\underline{P=0.05} \mid \underline{P=0.01} \mid \underline{P=0.001}$

df	P = 0.05	P = 0.01	P = 0.001
1	3.84	5.64	10.83
2	5.00	9.21	13.82
3	7.82	11.35	16.27
4	9.49	13.28	18.47
5	11.07	15.09	20.52
6	12.59	16.81	22.46
7	14.07	18.48	24.32
8	15.51	20.09	26.13
9	16.92	21.67	27.88
10	18.31	23.21	29.59

- Compute Chi-square_{tabulated} from Chi-Square <u>table</u>, with level of significance = 0.05 and DoF=1
- Chi-square_{tabulated} = 3.84
- Chi-square_{computed} = 2.425
- Chi-square computedChi-square tabulated

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Male	38	178	216
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		otai	02	310	400
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$$\chi_c^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

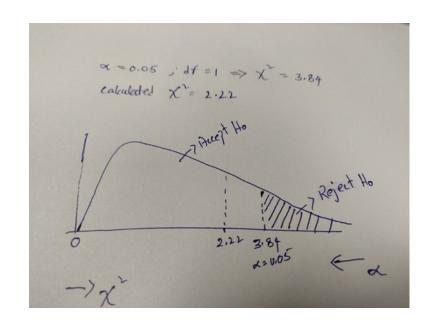
- Accepting the null hypothesis since the Chi-square_{computed} < Chi-square_{tabulated}
- That means, the variables are independent
- Gender variable cannot be selected for training the model.

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Male	38	178	216
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Total	82	318	400

	O _i	E _i	(O _i -E _i) ² /E _i
1,1	38	44.24	0.880
1,2	178	171.72	0.23
2,1	44	37.72	1.045
2,2	140	146.28	0.27
		Chi square	2.425

$$\chi_c^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

- Accepting the null hypothesis since the Chi-square_{computed}
 Chi-square_{tabulated}
- That means, the variables are independent
- Gender variable cannot be selected for training the model.



References

https://towardsdatascience.com/chi-square-test-for-feature-selection-in-machine-learning-206b1f0b8223

https://machinelearningmastery.com/chi-squared-test-for-machine-learning/