## FEM_Description

Brief Summary

The following code generates a finite element based structural model of the BFF Aircraft, given a set of parameter values. The parameters represent the structural properties of various parts of the aircraft and the mass / inertia distribution. The list of parameters for the FE model can be found in the reference.

For a given set of parameters, the code generates a FE based structural grid of the aircraft as shown in the figure. Spatial domain mass and stiffness matrices of corresponding to the heave, bend and twist states of the gridpoints are generated. These matrices are used to generate modal domain data including modal frequencies and mode shapes. A state space description, with force as an input and acceleration as output at gridpoints is also generated. The structural model can be generated by running the FEModelBFF.m script.

The code could also be used to update the FE model to match with experimental results. The code can be used with any suitable optimization algorithm to find the optimal parameter values. The optimization algorithm reduces a cost function, representing the separation between the output of the FE model and observation from ground vibration test (experimental data). The set of optimized parameters could be substituted back into the following code to get the final FE model.
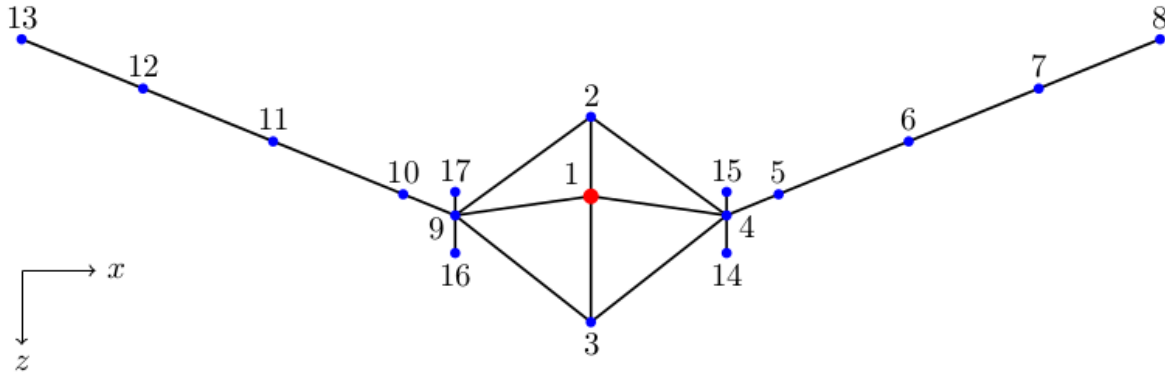
Reference

Gupta, A., Moreno, C. P., Pfifer, H., Balas, G. J., "Updating a Finite Element Based Structural Model of a Small Flexible Aircraft", AIAA, 2015

List of Functions

a) BFFParameters2FEM
[Griddata] = BFFParameters2FEM(Params)

This function takes in specific parameter values of the FE model and returns a complete (geometric and material property) description of the FE model for the Body Freedom Flutter aircraft.

<u>Inputs:</u>
-Params: Structure of parameter values used to model / optimize the BFF Aircraft. It has following fields:
- Params.MuFl: Mass per unit length of fuselage elements (Kg/m).
- Params.E: Young's Modulus of the elements constituting the wings (Gpa).
- Params.G: Torsional Modulus of the elements constituting the wings (Mpa).
- Params.MuW: Mass per unit length of wing elements (Kg/m).
- Params.Mass: An eight element vector representing the point masses at gridpoints 1-8. As the mass distribution on the aircraft is assumed to be symmetric, the point masses at other gridpoints are automatically known from this vector. For example: Point mass at gridpoint-13 is equal to point mass at gridpoint-8 (Kg).
- Params.Inertia: An 8-by-2 element matrix where each row represents the point [roll inertia, pitch inertia] at gridpoints 1-8. As the mass distribution on the aircraft is assumed to be symmetric, the point inertias at other gridpoints are automatically known from this vector. For example: Point inertia at gridpoint-13 is equal to point pitch inertia at gridpoint-8 (Kg-m^2).

<u>Outputs:</u>
-Griddata: Structure containing matrices that describe the geometry and material properties of the FE model, describing the FE model completely. The fields of Griddata are:
- Griddata.Np: Number of gridpoints in the FE model.
- Griddata.Nel: Number of elements in the FE model.
- Griddata.Cons: Np-by-3 matrix where the ith row represents the presence of constraints on heave, bend and roll of the ith gridpoint respectively. Presence of constraint is denoted by a 1 and absence by 0.
- Griddata.GridP: Np-by-2 matrix where the ith row is [X, Y]. (X,Y) is the coordinates of the ith grid point about the nose of the aircraft (m) .
- Griddata.MatProp: Ns-by-5 matrix where the ith row contains the section properties: Young's modulus, rigidity modulus, width, height, mass/length of a particular material. Here, Ns is the number of different materials used in the FE model.
- Griddata.El: Nel-by-3 matrix where the ith row is [P1, P2, Index]. P1 and P2 are the end gridpoints of the ith element. Index specifies the material properties of the element by specifying the row number of MatProp matrix corresponding to the material of the ith element.

- Griddata.MPt: Np-by-1 vector where the ith element contains the additional point mass at the ith gridpoint of the FE model (Kg).
- Griddata.IPt: Np-by-2 matrix where i^{th} row is [Iz, Ix]. Iz is point roll inertia at i^{th} gridpoint and Ix is point pitch inertia at i^{th} gridpoint (Kg-m^2).

b) GetSpatial

[MB,CB,KB,FEModelSS] = GetSpatial(Griddata,Zeta)

This function takes in the FE description of the aircraft structure and generates the spatial level mass, damping and stiffness matrices corresponding the to heave, bend and twist states of the gridpoints and the structural model in state space format.

Inputs:

-Griddata: Structure containing matrices that describe the geometry and material properties of the FE model, describing the FE model completely. The fields of Griddata are:

- Griddata.Np: Number of gridpoints in the FE model.
- Griddata.Nel: Number of elements in the FE model.
- Griddata.Cons: Np-by-3 matrix where the ith row represents the presence of constraints on heave, bend and roll of the ith gridpoint respectively. Presence of constraint is denoted by a 1 and absence by 0.
- Griddata.GridP: Np-by-2 matrix where the ith row is [X, Y]. (X,Y) is the coordinates of the ith grid point about the nose of the aircraft (m) .
- Griddata.MatProp: Ns-by-5 matrix where the ith row contains the section properties: Young's modulus, rigidity modulus, width, height, mass/length of a particular material. Here, Ns is the number of different materials used in the FE model.
- Griddata.El: Nel-by-3 matrix where the ith row is [P1, P2, Index]. P1 and P2 are the end gridpoints of the ith element. Index specifies the material properties of the element by specifying the row number of MatProp matrix corresponding to the material of the ith element.
- Griddata.MPt: Np-by-1 vector where the ith element contains the additional point mass at the ith gridpoint of the FE model (Kg).
- Griddata.IPt: Np-by-2 matrix where i^{th} row is [Iz, Ix]. Iz is point roll inertia at i^{th} gridpoint and Ix is point pitch inertia at i^{th} gridpoint (Kg-m^2).

-Zeta: An m element vector containing the modal damping coefficients of the modes of the aircraft. Here, m is equal to the number of modes required to be output by the FE model.

Outputs:

-MB: A (3*(Np-1))-by-(3*(Np-1)) mass matrix of the aircraft (spatial). The first three rows correspond to the heave, bend and twist states at first node, the next three rows for the second node and so on for Np-1 nodes. Here, Np-1 is the number of free nodes.

-CB: A (3*(Np-1))-by-(3*(Np-1)) damping matrix of the aircraft (spatial). The first three rows correspond to the heave, bend and twist states at first node, the next three rows for the second node and so on for Np-1 nodes. Here, Np-1 is the number of free nodes.

-KB: A (3*(Np-1))-by-(3*(Np-1)) mass matrix of the aircraft (spatial). The first three rows correspond to the heave, bend and twist states at first node, the next three rows for the second node and so on for Np-1 nodes. Here, Np-1 is the number of free nodes.

-FEModelSS: A state space system representation of the structural model of the aircraft. It has $3*(Np-1)$ inputs: corresponding to force, bending moment and twisting moments at $(Np-1)$ free points, $3*(Np-1)$ outputs corresponding to acceleration in heave, bend and twist at $(Np-1)$ free points and $6*(Np-1)$ states corresponding to heave, bend, twist and their corresponding velocities at $(Np-1)$ points. Here, $Np-1$ is the number of free nodes.

c) GetModal
[Modes,AircraftData,Griddata] = GetModal(MB,KB,Griddata,Zeta,AircraftData)

This function takes in the spatial domain mass, damping and stiffness
matrices, the and the modal damping ratios and outputs the modal domain data.

Inputs:
-MB: A (3*(Np-1))-by-(3*(Np-1)) mass matrix of the aircraft (spatial). The first three rows
correspond to the heave, bend and twist states at first node, the next three rows for the second
node and so on for Np-1 nodes. Here, Np-1 is the number of free nodes.

-KB: A (3*(Np-1))-by-(3*(Np-1)) mass matrix of the aircraft (spatial). The first three rows
correspond to the heave, bend and twist states at first node, the next three rows for the second
node and so on for Np-1 nodes. Here, Np-1 is the number of free nodes.

-Griddata: Structure containing matrices that describe the geometry and material properties of
the FE model, describing the FE model completely. The fields of Griddata are:
 ● Griddata.Np: Number of gridpoints in the FE model.
 ● Griddata.Nel: Number of elements in the FE model.
 ● Griddata.Cons: Np-by-3 matrix where the ith row represents the presence of constraints
   on heave, bend and roll of the ith gridpoint respectively. Presence of constraint is
   denoted by a 1 and absence by 0.
 ● Griddata.GridP: Np-by-2 matrix where the ith row is [X, Y]. (X,Y) is the coordinates of the
   ith grid point about the nose of the aircraft (m) .
 ● Griddata.MatProp: Ns-by-5 matrix where the ith row contains the section properties:
   Young's modulus, rigidity modulus, width, height, mass/length of a particular material.
   Here, Ns is the number of different materials used in the FE model.
 ● Griddata.El: Nel-by-3 matrix where the ith row is [P1, P2, Index]. P1 and P2 are the end
   gridpoints of the ith element. Index specifies the material properties of the element by
   specifying the row number of MatProp matrix corresponding to the material of the ith
   element.
 ● Griddata.MPt: Np-by-1 vector where the ith element contains the additional point mass at
   the ith gridpoint of the FE model (Kg).
 ● Griddata.IPt: Np-by-2 matrix where i^{th} row is [Iz, Ix]. Iz is point roll inertia at i^{th} gridpoint
   and Ix is point pitch inertia at i^{th} gridpoint (Kg-m^2).

-Zeta: An m element vector containing the modal damping coefficients of the modes of the
aircraft. Here, m is equal to the number of modes required to be output by the FE model.

-AircraftData (Optional): A structure containing the high level data about the aircraft. It contains
the following fields:
 ● AircraftData.mAC: Total mass of the aircraft (Kg).
 ● AircraftData.IB: Total inertias of the aircraft (Kg-m^2).
 ● AircraftData.s: Wing area of the aircraft (m^2).

- AircraftData.c: Reference chord of the aircraft (m).
- AircraftData.cg: Position of the center of gravity of the aircraft (m).
- AircraftData.b: Reference span of the aircraft (m).

Outputs:
- Modes: A structure containing the matrices describing the final structural model of the aircraft. It contains modal and spatial level matrices like:
  - Modes.Phif: An (Np-1)*3-by-m matrix where the ith column contains the mode shape of ith mode. The first three rows correspond to the heave, bend and twist at first node, the next three rows for the second node and so on for Np-1 free nodes.
  - Modes.Phib: An (Np-1)*3-by-6 matrix where the columns contains the rigid mode shape The columns correspond to x, y, z displacement and x, y, z rotation in the body fixed frame while the rows correspond to the heave, bend and pitch of Np-1 free nodes. The first three rows correspond to the heave, bend and twist at first node, the next three rows for the 2nd node and so on for Np-1 free nodes.
  - Modes.Omegaf: An m element matrix containing the calculated modal frequencies for the first m modes (rad/s).
  - Modes.Zeta: An m element matrix containing the modal damping ratios for the first m modes.
  - Modes.Mff: An m-by-m modal mass matrix of the aircraft. Here, m is the number of modes considered.
  - Modes.Bff: An m-by-m modal damping matrix of the aircraft. Here, m is the number of modes considered.
  - Modes.Kff: An m-by-mx modal stiffness matrix of the aircraft. Here, m is the number of modes considered.
  - Modes.MB: An (3*(Np-1))-by-(3*(Np-1)) mass matrix of the aircraft (spatial). The first three rows correspond to the heave, bend and twist states at first node, the next three rows for the 2nd node and so on for Np-1 free nodes. Here, Np-1 is the number of free nodes.
  - Modes.KB: An (3*(Np-1))-by-(3*(Np-1)) stiffness matrix of the aircraft (spatial). The first three rows correspond to the heave, bend and twist states at first node, the next three rows for the 2nd node and so on for Np-1 free nodes. Here, Np-1 is the number of free nodes.

-AircraftData: A structure containing the high level data about the aircraft. If the optional input, AircraftData is not provided, the output AircraftData has the following fields:
  - AircraftData.mAC: Total mass of the aircraft (Kg).
  - AircraftData.IB: Total inertias of the aircraft (Kg-m^2).
  - AircraftData.cg: Position of the center of gravity of the aircraft (m).
If AircraftData is provided as an input, the function overwrites these fields with values obtained from the FE model

-Griddata: Structure containing matrices that describe the geometry and material properties of the FE model, describing the FE model completely. The fields of Griddata are:

- Griddata.Np: Number of gridpoints in the FE model.
- Griddata.Nel: Number of elements in the FE model.
- Griddata.Cons: Np-by-3 matrix where the ith row represents the presence of constraints on heave, bend and roll of the ith gridpoint respectively. Presence of constraint is denoted by a 1 and absence by 0.
- Griddata.GridP: Np-by-2 matrix where the ith row is [X, Y]. (X,Y) is the coordinates of the ith grid point about the center of gravity of the aircraft (m) .
- Griddata.MatProp: Ns-by-5 matrix where the ith row contains the section properties: Young's modulus, rigidity modulus, width, height, mass/length of a particular material. Here, Ns is the number of different materials used in the FE model.
- Griddata.El: Nel-by-3 matrix where the ith row is [P1, P2, Index]. P1 and P2 are the end gridpoints of the ith element. Index specifies the material properties of the element by specifying the row number of MatProp matrix corresponding to the material of the ith element.
- Griddata.MPt: Np-by-1 vector where the ith element contains the additional point mass at the ith gridpoint of the FE model (Kg).
- Griddata.IPt: Np-by-2 matrix where i^{th} row is [Iz, Ix]. Iz is point roll inertia at i^{th} gridpoint and Ix is point pitch inertia at i^{th} gridpoint (Kg-m^2).

d) GetCGloc
[CGloc] = GetCGloc(Griddata)

This function calculates the center of gravity of the aircraft for a given finite element griddata.

Inputs:
-Griddata: Structure containing matrices that describe the geometry and material properties of the FE model, describing the FE model completely. The fields of Griddata are:
- Griddata.Np: Number of gridpoints in the FE model.
- Griddata.Nel: Number of elements in the FE model.
- Griddata.Cons: Np-by-3 matrix where the ith row represents the presence of constraints on heave, bend and roll of the ith gridpoint respectively. Presence of constraint is denoted by a 1 and absence by 0.
- Griddata.GridP: Np-by-2 matrix where the ith row is [X, Y]. (X,Y) is the coordinates of the ith grid point (m) .
- Griddata.MatProp: Ns-by-5 matrix where the ith row contains the section properties: Young's modulus, rigidity modulus, width, height, mass/length of a particular material. Here, Ns is the number of different materials used in the FE model.
- Griddata.El: Nel-by-3 matrix where the ith row is [P1, P2, Index]. P1 and P2 are the end gridpoints of the ith element. Index specifies the material properties of the element by specifying the row number of MatProp matrix corresponding to the material of the ith element.
- Griddata.MPt: Np-by-1 vector where the ith element contains the additional point mass at the ith gridpoint of the FE model (Kg).
- Griddata.IPt: Np-by-2 matrix where i^{th} row is [Iz, Ix]. Iz is point roll inertia at i^{th} gridpoint and Ix is point pitch inertia at i^{th} gridpoint (Kg-m^2).

Outputs:
-CGloc: A variable which specifies the z coordinate of the centre of gravity of the aircraft (m).

<u>Pseudo-code</u>:

1) A matlab script to generate the FE model of the aircraft will have the following structure.

%%%

```
% Set Parameter values
Params.{field}={value};
Zeta = {values};

% Optional Aircraft_data
load Aircraftdata.mat

% Generate FE description of the aircraft based
% Obtain griddata
[Griddata] = BFFParameters2FEM(Params);

% Obtain spatial matrices and state space description
[MB,CB,KB,FEMModelSS] = GetSpatial(Griddata,Zeta);

% Obtain modes and global aircraft mass and inertia from the FE Model
[Modes,AircraftData,Griddata] = GetModal(MB,KB,Griddata,Zeta,AircraftData);
or
[Modes,AircraftData,Griddata] = GetModal(MB,KB,Griddata,Zeta);
```
%%%