Modeling and Forecasting Public Interest in Sabrina Carpenter Using Google Trends

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1 Introduction

Sabrina Carpenter has established herself as a popular figure in pop culture as both a singer and an actress. She initially gained fame through her role in the Disney Channel series Girl Meets World and began making music in 2015. Over the years, Carpenter has built a loyal fanbase known as “Carpenters.”

Following the release of her hit single “Espresso” in mid-2024, her popularity significantly increased, as indicated by Google Trends data. This sudden growth motivated us to analyze Sabrina Carpenter’s Google search data to better understand how celebrity popularity changes over time and whether these variations can be reliably forecasted.

Using methods learned during our course, we analyzed the weekly Google Trends data for Sabrina Carpenter from June 4, 2023, to May 4, 2025, to forecast her popularity for the subsequent 10 weeks. Initially, we examined the time series plot to identify trends and patterns. We tested the data for stationarity and normality, applying necessary transformations to meet model assumptions. Subsequently, we used ACF and PACF plots to select candidate models. We then modeled the series using a combined ARMA-GARCH approach and conducted residual analysis to evaluate model adequacy. Finally, we chose the best-fitting model based on the AIC and BIC values and used it to forecast the next 10 weeks of Google Trends values.

2 Data

The dataset covers weekly Google Trends search interest [3] for Sabrina Carpenter in Australia from June 4, 2023, to May 4, 2025. During this time period, the minimum search interest was 2 and the maximum value reached was 100. The average search interest throughout this timeframe was approximately 35.69 and median value was 35 indicating moderate overall public engagement. After the release of her single “Espresso” in April 2024, Carpenter’s popularity experienced a notable spike, frequently achieving high weekly trend scores thereafter.

3 Methodology

Before modeling, we analyzed the trend and characteristics of the time series to inform our approach. We plotted ACF and PACF graphs to assess stationarity, which we further verified using the Shapiro-Wilk test. After evaluating several transformations, we performed first-degree differencing to achieve stationarity. We then analyzed the ACF and PACF of the differenced series to identify appropriate ARMA model orders. The EACF plot and residual heatmap provided additional information for ARMA order selection. Using AIC and BIC values, we selected the best ARMA model. We applied similar methods to identify suitable GARCH orders, capturing volatility clustering within the data. After comparing residual analysis results from various ARMA-GARCH models, we selected the best-performing model. Finally, we used the chosen model to forecast the variance and trend values for the subsequent 10 weeks.

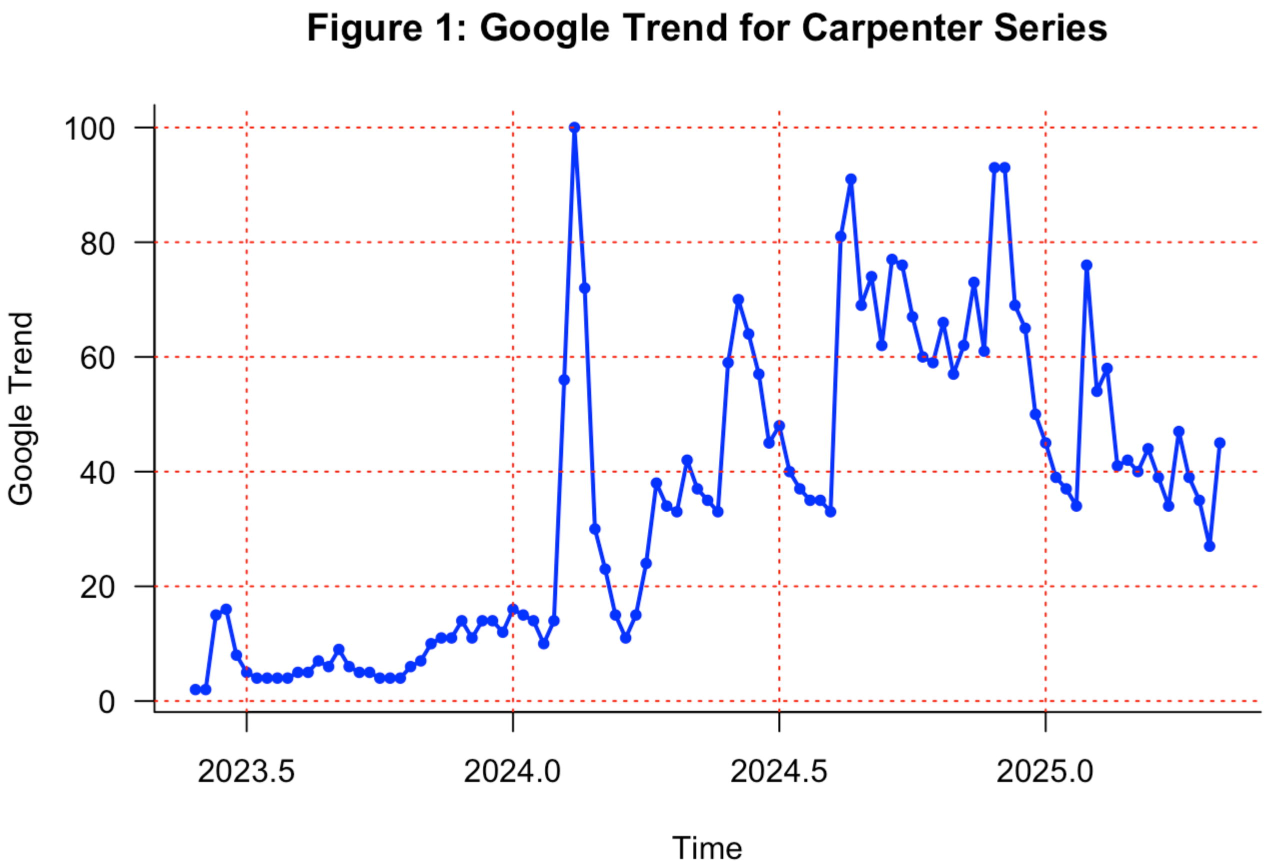
4 Descriptive Analysis

The dataset contains 101 rows of time series data of Google Trends about Sabrina Carpenter. The dataset contains data from June 4, 2023 till May 4, 2025 and the values range from 2 to 100 with an average value of 35.69. After importing the dataset, we converted it into a time series object. Since the data is weekly, we set the frequency as 52.

```
carpenter <- read_csv("multiTimeline _carpenter.csv", skip = 1,show_col_types =
FALSE)
summary(carpenter)
```

```
##       Week            Sabrina Carpenter: (Australia)
##  Min.   :2023-06-04   Min.   :  2.00
##  1st Qu.:2023-11-26   1st Qu.: 11.00
##  Median :2024-05-19   Median : 35.00
##  Mean   :2024-05-19   Mean   : 35.69
##  3rd Qu.:2024-11-10   3rd Qu.: 57.00
##  Max.   :2025-05-04   Max.   :100.00
```

```
#converting the time series into a time series object
carpenter_ts<-ts(carpenter$`Sabrina Carpenter: (Australia)`,start = c(2023, 2
2),frequency = 52 )
plot(
  carpenter_ts,
  type   = "o",                 # both lines and points
  col    = "blue",         # nicer color
  lwd    = 2,                   # thicker line
  pch    = 16,                  # solid circle points
  cex    = 0.8,                 # slightly smaller points
  xlab   = "Time",
  ylab   = "Google Trend",
  main   = "Figure 1: Google Trend for Carpenter Series",
  las    = 1,                   # horizontal axis labels
  bty    = "l"                  # only left and bottom box
)
grid(col = "red", lty = "dotted")
```



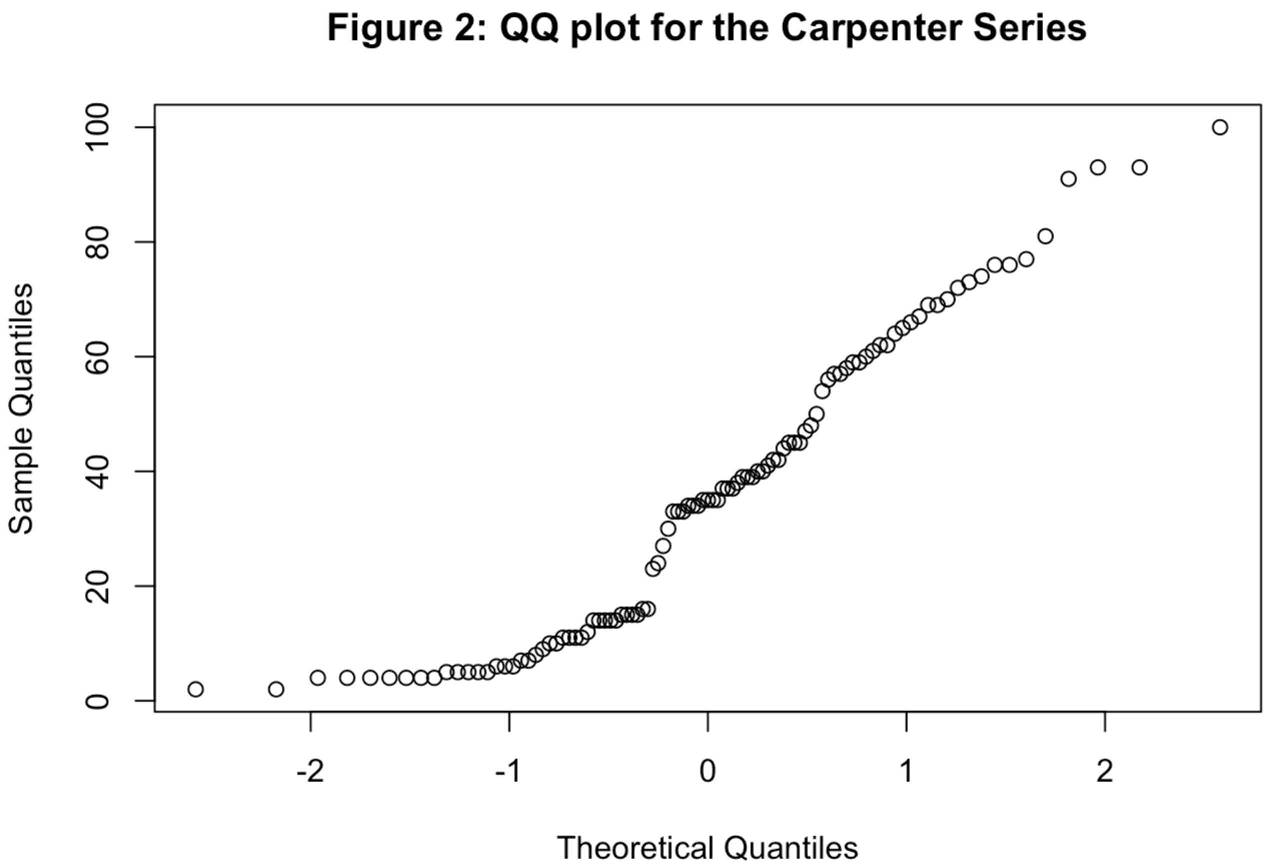
The provided time series plot (figure 1) represents the weekly Google Trends data for Sabrina Carpenter from June 4, 2023, to May 4, 2025. The plot initially demonstrates relatively low and stable interest from mid-2023 into early 2024, characterized by minor fluctuations typical of autoregressive behavior. However, a significant spike occurs around February 2024, marking a shift in public interest.

This sudden increase aligns closely with two notable events. On February 18, 2024, Sabrina Carpenter performed as the opening act for Taylor Swift's Eras Tour in Melbourne, Australia, where she paid tribute to Olivia Newton-John with a performance of "Hopelessly Devoted to You." This emotional tribute received significant attention on social media, likely contributing heavily to the spike observed in Google Trends [1] . Additionally, Carpenter's personal life also attracted public attention around this period, as she publicly confirmed her relationship with actor Barry Keoghan in February 2024 [2].

Following these events, the time series trend maintains high levels and continues to climb further in April 2024. On April 11, 2024, Carpenter released her hit single "Espresso," which topped Billboard charts and earned her a Grammy award in 2025, further increasing her popularity.

The plot shows no evident seasonal pattern but does show clear changes in variance. The significant spike in early 2024 acts as a turning point, after which Carpenter's popularity, measured by Google Trends, sustains notably higher levels compared to the preceding months.

```
qqnorm(carpenter_ts,main="Figure 2: QQ plot for the Carpenter Series")
```



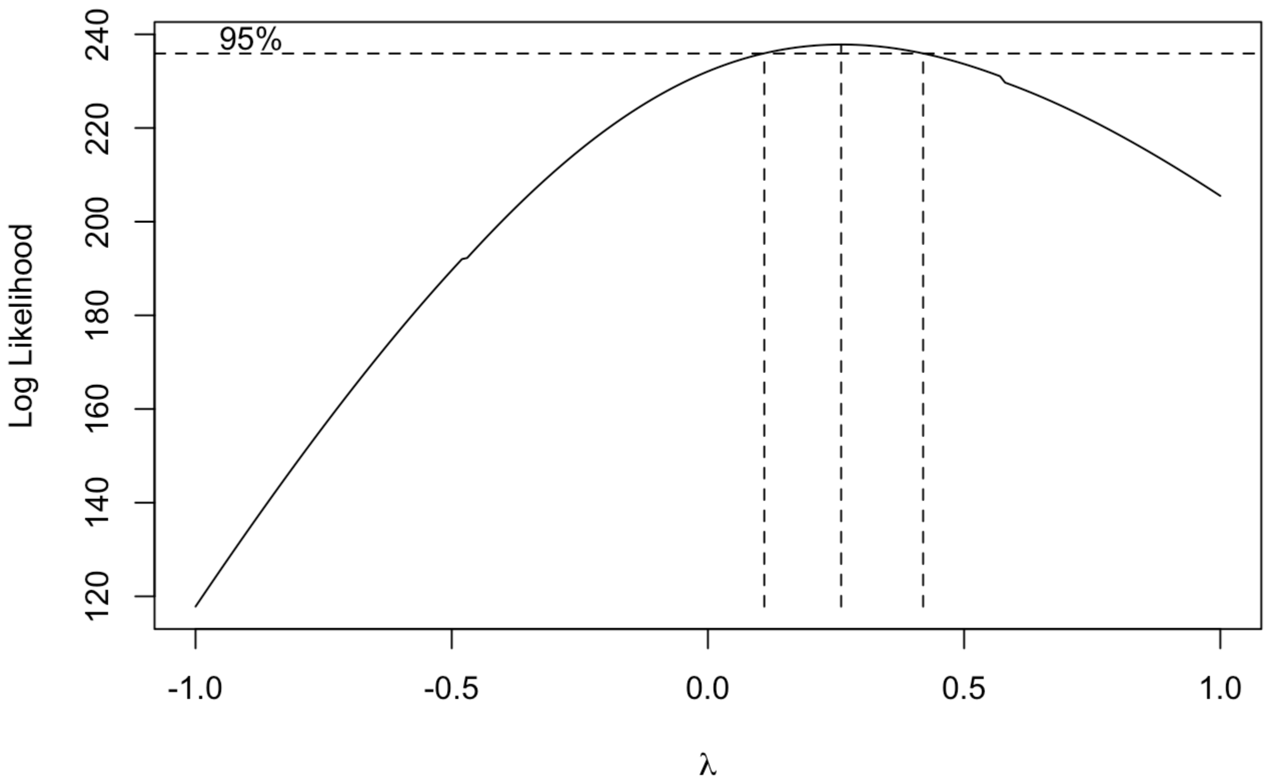

```
shapiro.test(carpenter_ts)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  carpenter_ts
## W = 0.92807, p-value = 3.554e-05
```

The qq-plot (figure 2) of the time series indicates that the distribution is not normal. The Shapiro-Wilk test gives a W-value of 0.92807 and a p-value of 3.554e-05, which is much smaller than 0.05. This confirms that the data significantly deviates from normality.

```
suppressWarnings({
  BC <- BoxCox.ar(
    carpenter_ts,
    lambda = seq(-1, 1, by = 0.01),
    method = "yule-walker",
    plotit = TRUE
  )
  title("Figure 3: BoxCox Plot for the Carpenter Series")
})
```

Figure 3: BoxCox Plot for the Carpenter Series



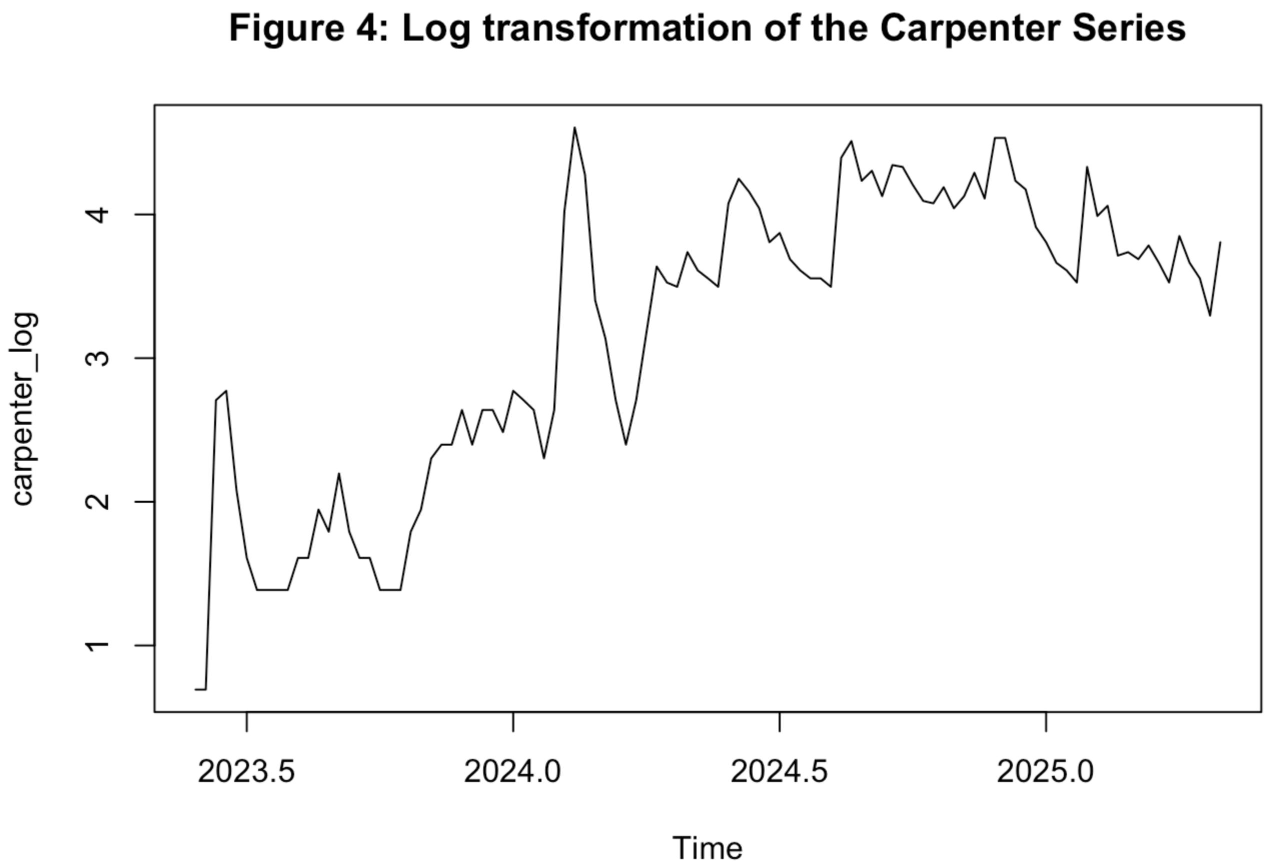
The BoxCox transformation (figure 3) plot helps to visualize the best transformation that can stabilize the changing variance of the time series. The curve shows different values of lambda, and how well each one improves the model. The top of the curve is the best choice as it suggests a lambda value of around 0.2. This means we should apply a transformation similar to a log scale to our data. The vertical dashed lines show a range of lambda values that are also acceptable, but since 1 is not within this range, using the data without transformation would not be ideal.

```
best_lambda <- BC$mle
best_lambda
```

```
## [1] 0.26
```

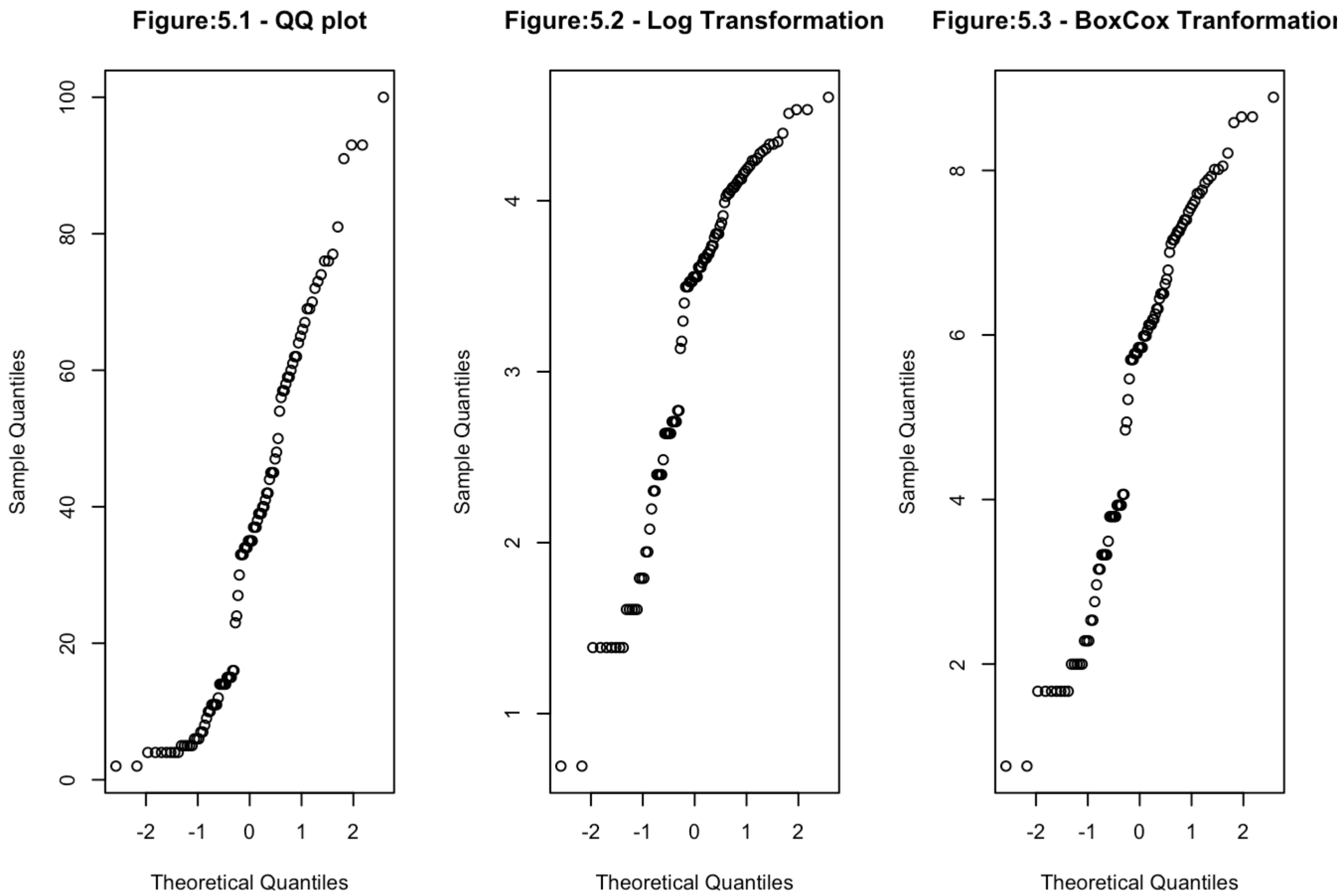
```
carpenter_bc <- BoxCox(carpenter_ts, best_lambda)
```

```
carpenter_log<-log(carpenter_ts)
plot(carpenter_log, main="Figure 4: Log transformation of the Carpenter Serie
s")
```



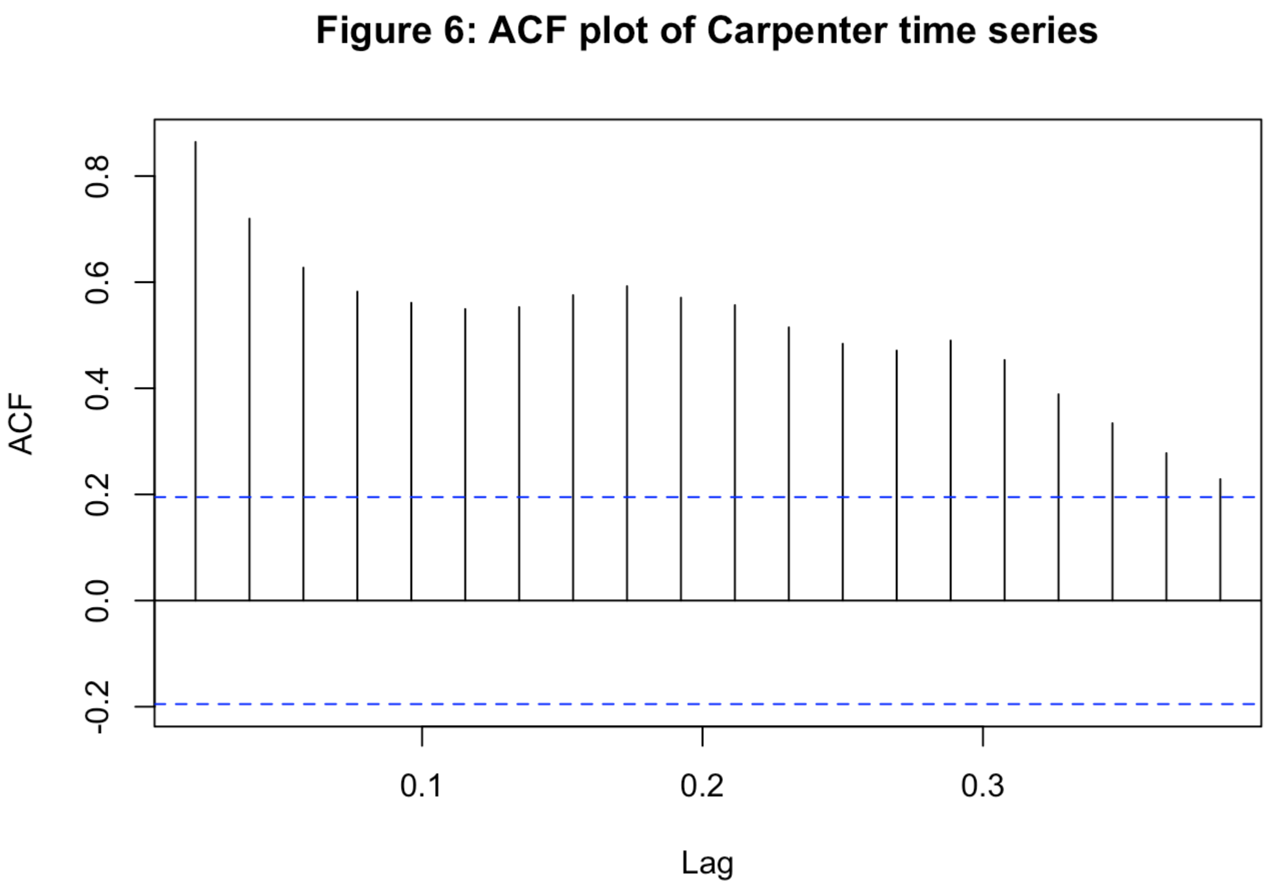
The plot above (figure 4) shows the time series after applying a log transformation to the original values. The log transformation helps reduce the effect of large spikes and makes the variation in the data more stable across time. Compared to the original series, this version appears smoother, and the fluctuations are more consistent.

```
par(mfrow = c(1, 3))
qqnorm(carpenter_ts,main="Figure:5.1 - QQ plot")
qqnorm(carpenter_log,main="Figure:5.2 - Log Transformation")
qqnorm(carpenter_bc,main="Figure:5.3 - BoxCox Tranformation")
```

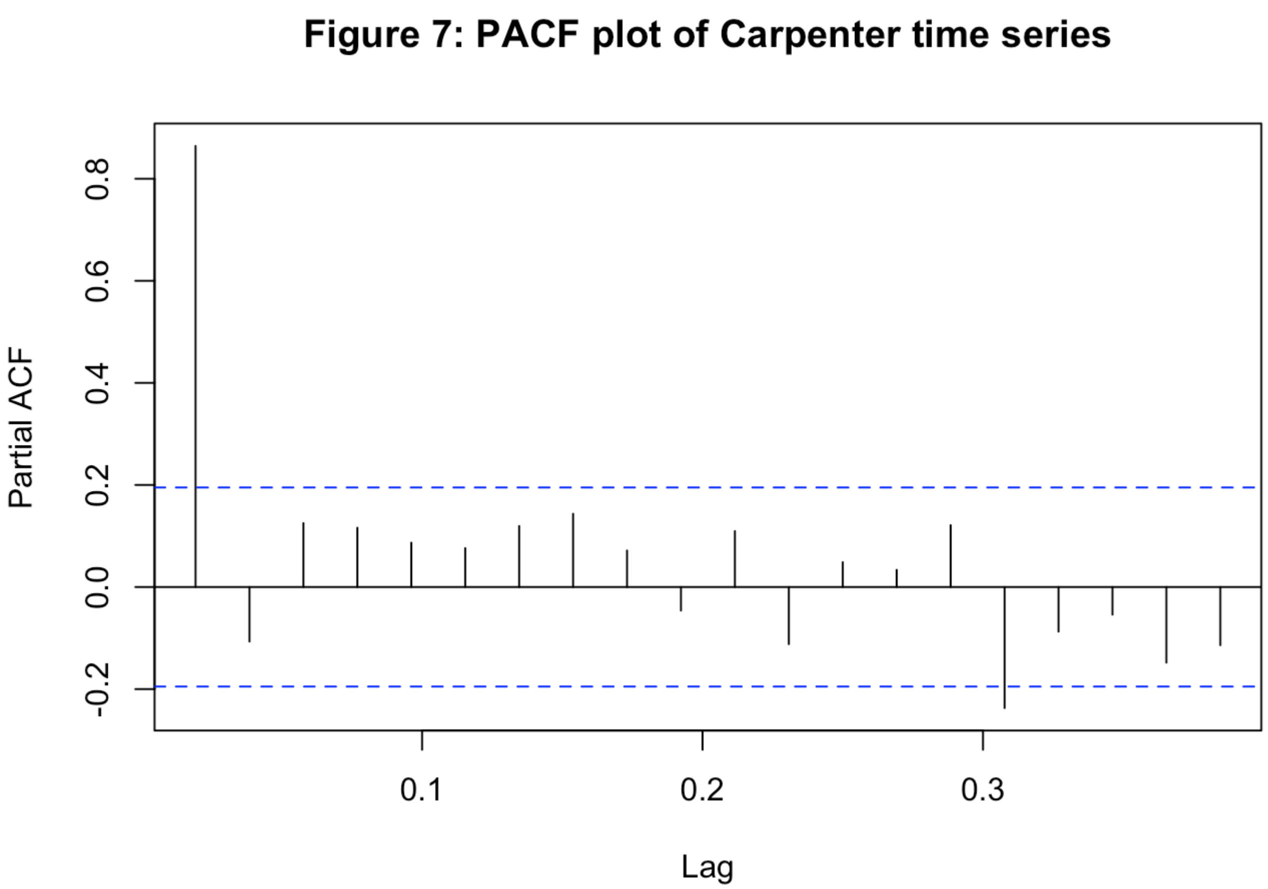



```
par(mfrow = c(1, 1))
```

When we compare the QQ plot of the series with that of the log transformed and BoxCox transformed series, there does not seem to be a significant difference in normality. Besides, since we are trying to capture volatility clustering with our GARCH model, we shall not apply transformations on the data as the transformation might smoothen out the very volatility we are trying to understand.

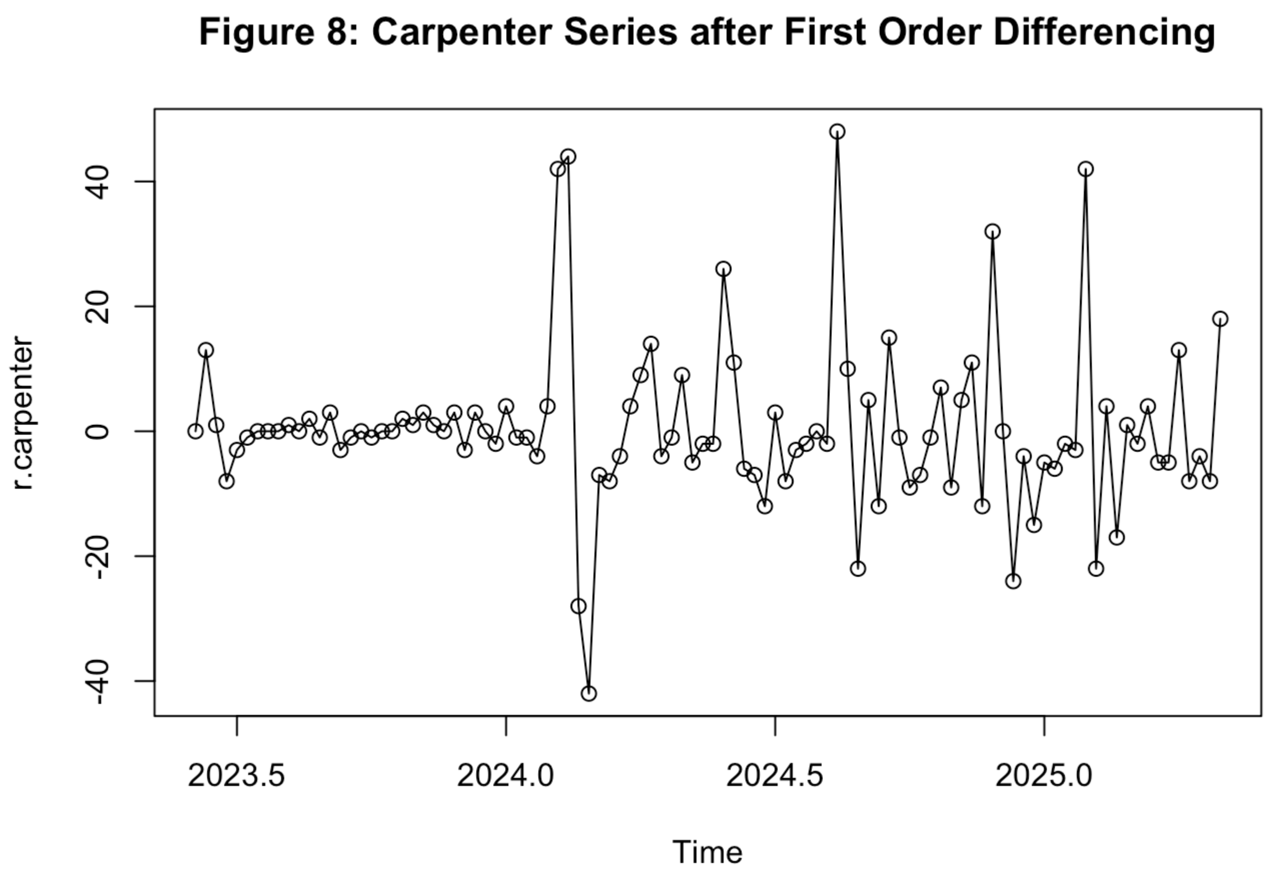



```
pacf(carpenter_ts, main="Figure 7: PACF plot of Carpenter time series")
```



Figures 6 and 7 present the ACF and PACF plots for the Carpenter time series. The ACF plot (Figure 6) shows a slowly decaying autocorrelation pattern, which is characteristic of a non-stationary time series. In contrast, the PACF plot (Figure 7) shows a significant spike at lag 1, followed by a sharp drop-off in subsequent lags. This pattern of a gradually declining ACF and a PACF with a dominant first lag, strongly suggests that the series contains a trend or persistent structure and is therefore non-stationary. Based on these observations, it is appropriate to apply first-order differencing to the series to induce stationarity before proceeding with ARIMA modeling or other time series techniques.

```
r.carpenter<-diff(carpenter_ts,differences=1)
plot(r.carpenter,type='o', main="Figure 8: Carpenter Series after First Order D
ifferencing")
```




```
suppressWarnings(adf.test(r.carpenter,alternative=c("stationary")))
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  r.carpenter
## Dickey-Fuller = -6.0555, Lag order = 4, p-value = 0.01
## alternative hypothesis: stationary
```

An Augmented Dickey-Fuller (ADF) test was performed on the differenced Carpenter time series (r.carpenter) to check for stationarity. The test produced a Dickey-Fuller statistic of -6.06 with a lag order of 4. The p-value was less than 0.01, indicating that the time series is stationary after differencing. This means there is no evidence of a unit root, and the differencing step was effective.

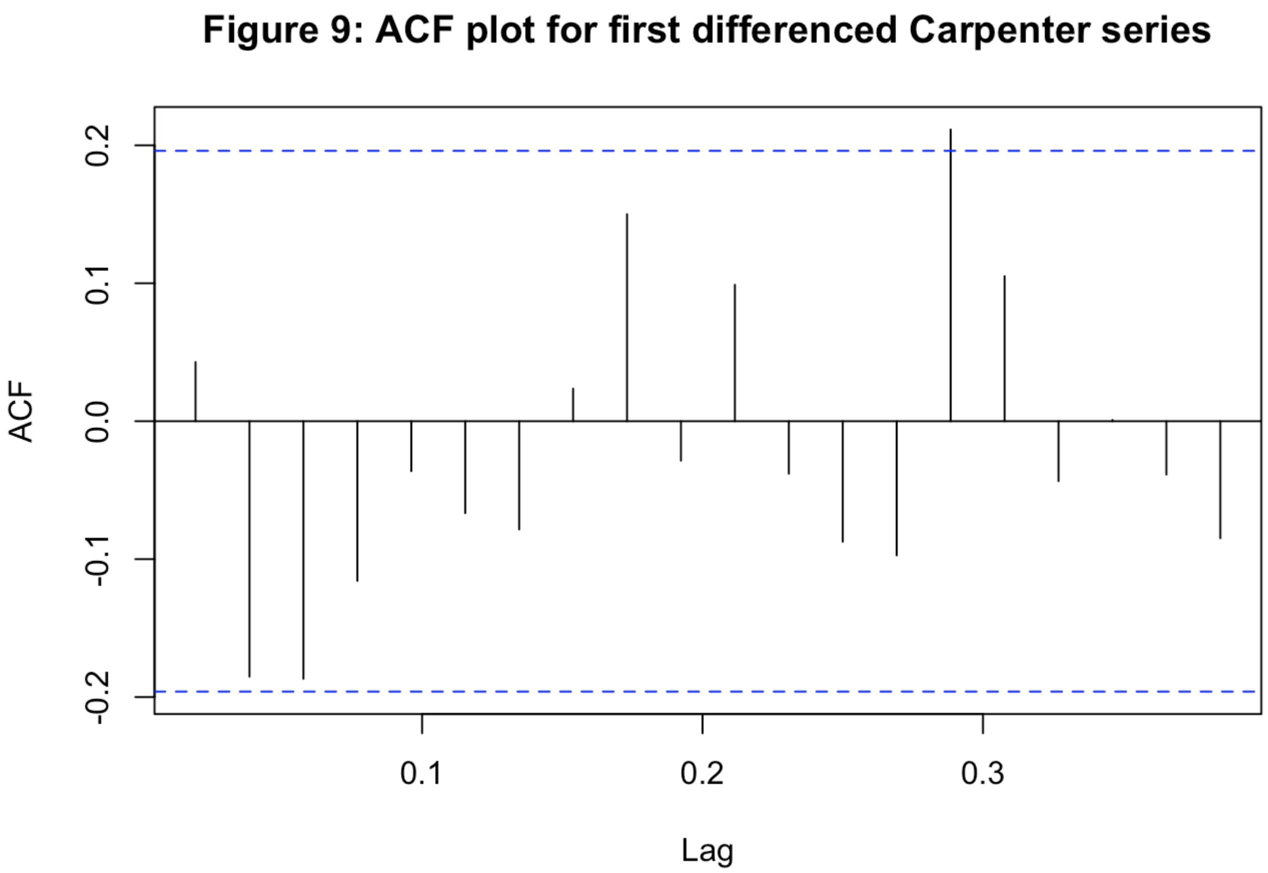
```
suppressWarnings(pp.test(r.carpenter))
```

```
##
##  Phillips-Perron Unit Root Test
##
## data:  r.carpenter
## Dickey-Fuller Z(alpha) = -81.922, Truncation lag parameter = 3, p-value
## = 0.01
## alternative hypothesis: stationary
```

A Phillips-Perron unit root test was applied to the differenced Carpenter time series (r.carpenter) to check for stationarity. The test gave a Dickey-Fuller Z(alpha) value of -81.92 with a truncation lag parameter of 3. The p-value is less than 0.01, which provides strong evidence that the series is stationary after differencing.

5 ARMA Modelling

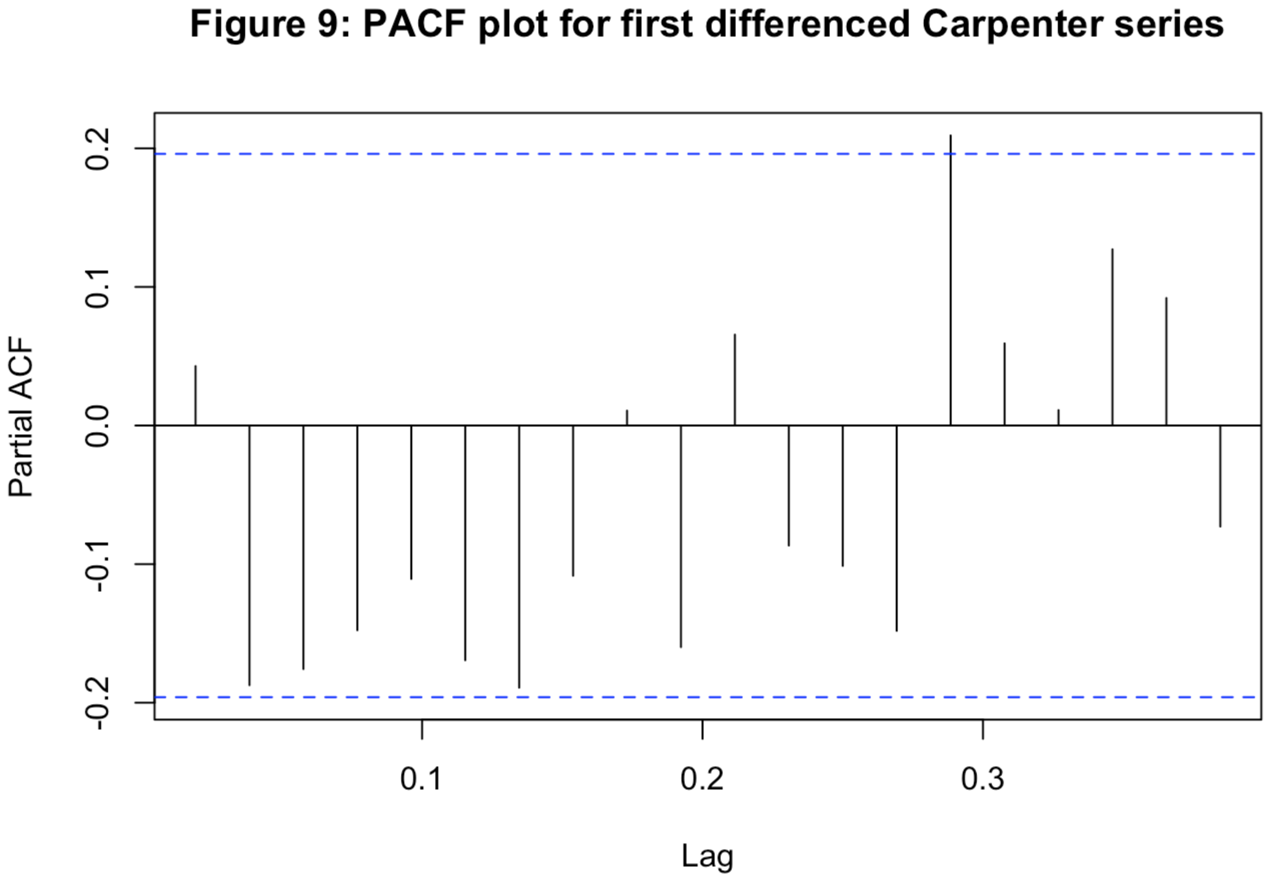
```
acf(r.carpenter,main="Figure 9: ACF plot for first differenced Carpenter serie
s")
```



After differencing the series, the autocorrelation plot (figure 8) shows that most of the bars fall within the blue confidence bounds. This means there is no significant autocorrelation at most lags. There is a small spike around lag 3 that slightly crosses the upper bound, which could indicate a bit of remaining autocorrelation, but it's not strong.

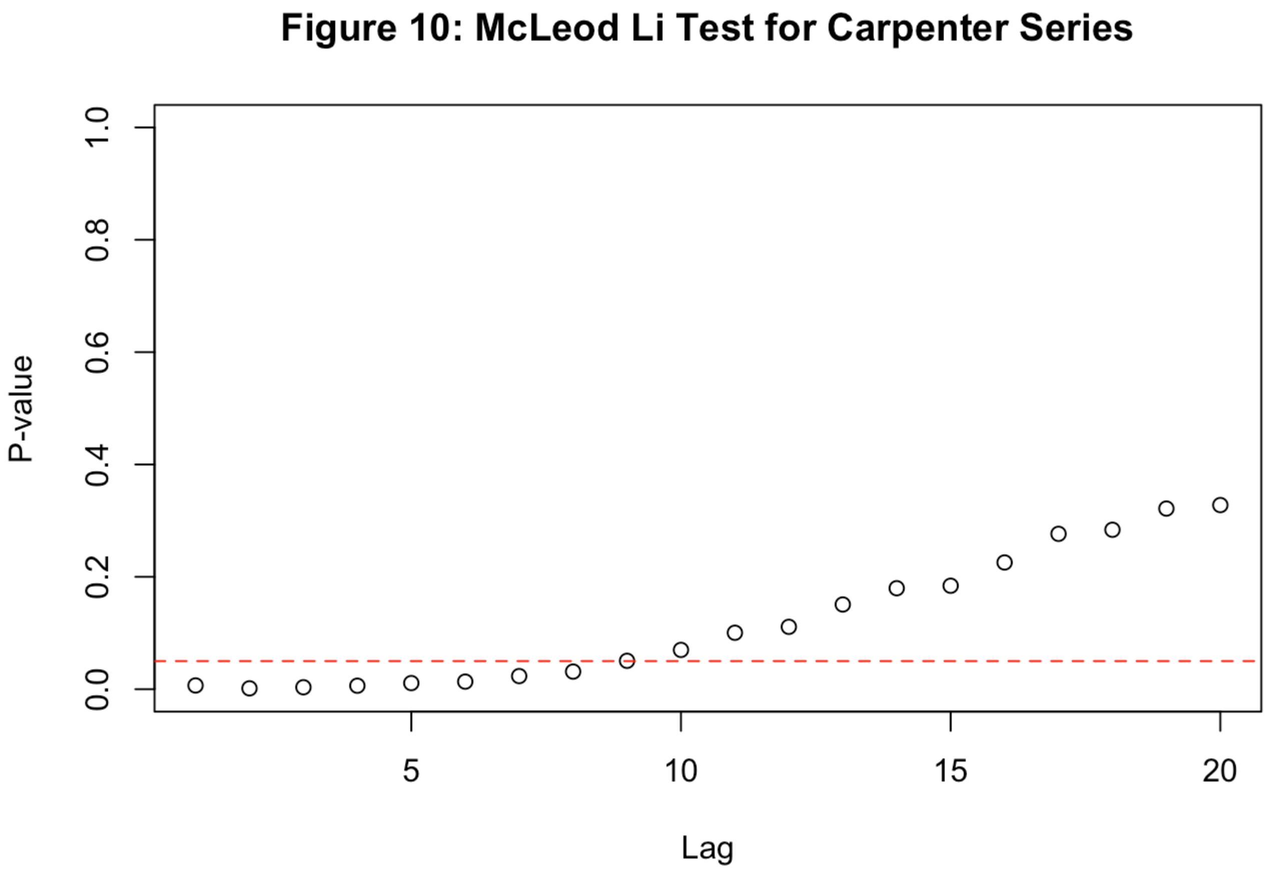
Overall, the pattern suggests that the first differencing has worked well, and the series is likely stationary now. Based on the ACF and PACF plots, a suitable value for the moving average order (q) could be 0.

```
pacf(r.carpenter,main="Figure 9: PACF plot for first differenced Carpenter seri
es")
```



The first few lags in the partial autocorrelation plot (figure 9) show mild negative correlations, but none of them go significantly beyond the confidence bounds. Lag 3 comes close to or slightly exceeds the upper limit, but the spike is not very strong. The absence of clear, strong spikes suggests that there is no dominant autoregressive (AR) structure left in the data after differencing. This indicates that the differencing step has been effective in stabilizing the series, and a complex AR model may not be necessary.

```
McLeod.Li.test(y=r.carpenter,main="Figure 10: McLeod Li Test for Carpenter Seri
es")
```



In the McLeod-Li test plot (figure 10), lags 1 through 10 show p-values below 0.05, with many of them close to zero. This indicates significant autocorrelation in the squared residuals at these lags. In practical terms, this means the variance of the series is not constant over time. Instead, it displays signs of volatility clustering, where periods of high variability are followed by low variability, and vice versa.

For lags 11 through 20, the p-values gradually increase and move above the 0.05 threshold. This suggests that the autocorrelation in the squared residuals becomes less significant at higher lags, indicating that the volatility effect fades over time.

The McLeod-Li test results show that the first differenced Carpenter series has non-constant variance, especially at the lower lags. This pattern points to the presence of conditional heteroskedasticity, which is often seen in financial, economic, or irregular time series data.

Given this behavior, it would be appropriate to consider using an ARIMA-GARCH model. This type of model can capture both the mean and the changing variance of the series, making it useful for forecasting not just future values, but also the level of uncertainty or volatility around those forecasts.

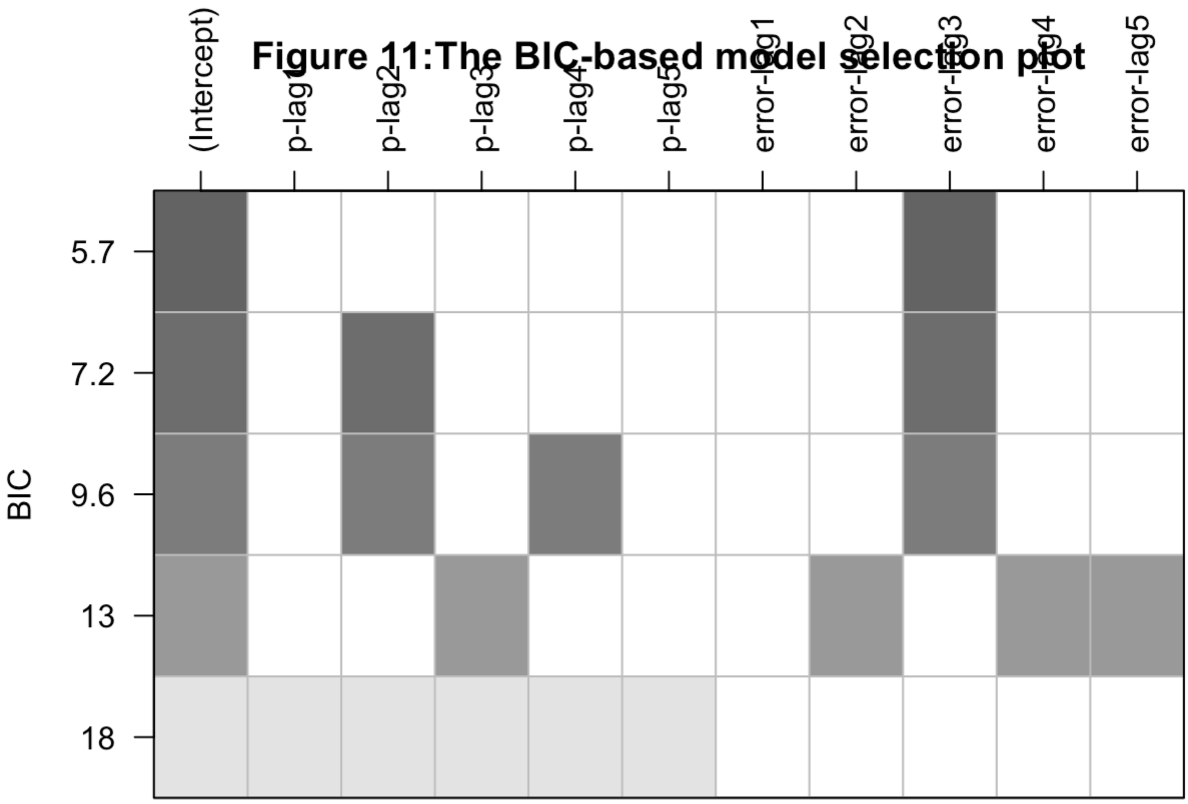
```
eacf(r.carpenter,ar.max=5,ma.max = 5) #(0,1),(1,1)
```

```
## AR/MA
##   0 1 2 3 4 5
## 0 o o o o o o
## 1 x o o o o o
## 2 x o o o o o
## 3 x o o o o o
## 4 x o o o o o
## 5 x x x x o o
```

The EACF (Extended Autocorrelation Function) plot was used to identify the appropriate ARMA order for the differenced Carpenter series. In the plot, the presence of an “o” indicates non-significance (i.e., a potential model order), while an “x” suggests significance and that the combination should be avoided.

Looking at the table, the first clear triangle of “o”s starts at the (0,1) position and also appears at (1,1). This suggests that both ARMA(0,1) and ARMA(1,1) are possible models for the stationary series. Since these orders fall within the first rows and columns where residual autocorrelation is minimal, they can be considered good candidates for capturing the underlying structure of the data. The process shows that higher orders are not necessary, and a simpler ARMA model may be enough after differencing.

```
res2=suppressWarnings(armasubsets(y=r.carpenter,nar=5,nma=5,y.name="p",ar.metho
d="ols"))
plot(res2, main="Figure 11:The BIC-based model selection plot")
```



The BIC-based model selection plot (figure 11) shows combinations of AR (p) and MA (q) orders evaluated by Bayesian Information Criterion. Lower BIC values are shown in darker shades.

From the plot, the darkest cells appear at positions corresponding to the orders (0,1), (1,1), (0,3), and (2,3). These combinations yield the lowest BIC values, indicating that they are the best candidates for fitting the differenced Carpenter series. These results support considering simpler ARMA structures with one or two AR terms and one to three MA terms.

```
plot_LBQ <- function(res, max_lag = NULL, alpha = 0.05) {
  n <- length(res)
  max_lag <- max_lag %||% if (n < 30) n - 1 else 29
  pvals <- sapply(1:max_lag, function(l) Box.test(res, lag = l, type = "Ljung-B
ox")$p.value)
  plot(1:max_lag, pvals, type="b", pch=19, ylim=c(0,1),
       xlab="Lag", ylab="Ljung-Box p-value",
       main="Ljung-Box p-values on Residuals")
  abline(h=alpha, col="red", lty=2)
  invisible(data.frame(lag=1:max_lag, p_value=pvals))
}
# This function calculates and plots the Ljung-Box test p-values for residuals
# a time series in order to verify autocorrelation in the residuals.
```

5.1 Model 001

```
model_001<-arima(r.carpenter,order=c(0,0,1),method="ML")
coeftest(model_001)
```

```
##
## z test of coefficients:
##
##           Estimate Std. Error z value Pr(>|z|)
## ma1       0.065819   0.120575  0.5459   0.5852
## intercept 0.441072   1.398070  0.3155   0.7524
```

```
model_001CSS<-arima(r.carpenter,order=c(0,0,1),method="CSS")
coeftest(model_001CSS)
```

```
##
## z test of coefficients:
##
##           Estimate Std. Error z value Pr(>|z|)
## ma1       0.066778   0.121339  0.5503   0.5821
## intercept 0.441170   1.399282  0.3153   0.7525
```

Model 1, estimated using the Conditional Sum of Squares (CSS) method, shows that neither the MA(1) coefficient nor the intercept is statistically significant. The p-value for the MA(1) term is 0.5821, which is well above the 0.05 threshold, indicating that this moving average component does not meaningfully contribute to explaining the variation in the series. Similarly, the intercept has a p-value of 0.7525, suggesting it does not significantly differ from zero.

This lack of significance implies that the ARIMA(0,0,1) structure may not be appropriate for the data, as its parameters do not offer explanatory value. When comparing estimation methods, both CSS and Maximum Likelihood (ML) produce similar parameter estimates, which suggests that the model is stable across methods. However, ML is typically preferred for small samples due to its higher accuracy, even though CSS is computationally faster.

In conclusion, since none of the estimated parameters are statistically meaningful, the ARIMA(0,0,1) model does not seem to be a good fit for the Carpenter time series.

5.1.1 Residual Analysis for Model 001

```
residual.analysis(model = model_001,std=TRUE,start = 2,class = "ARIMA")
```

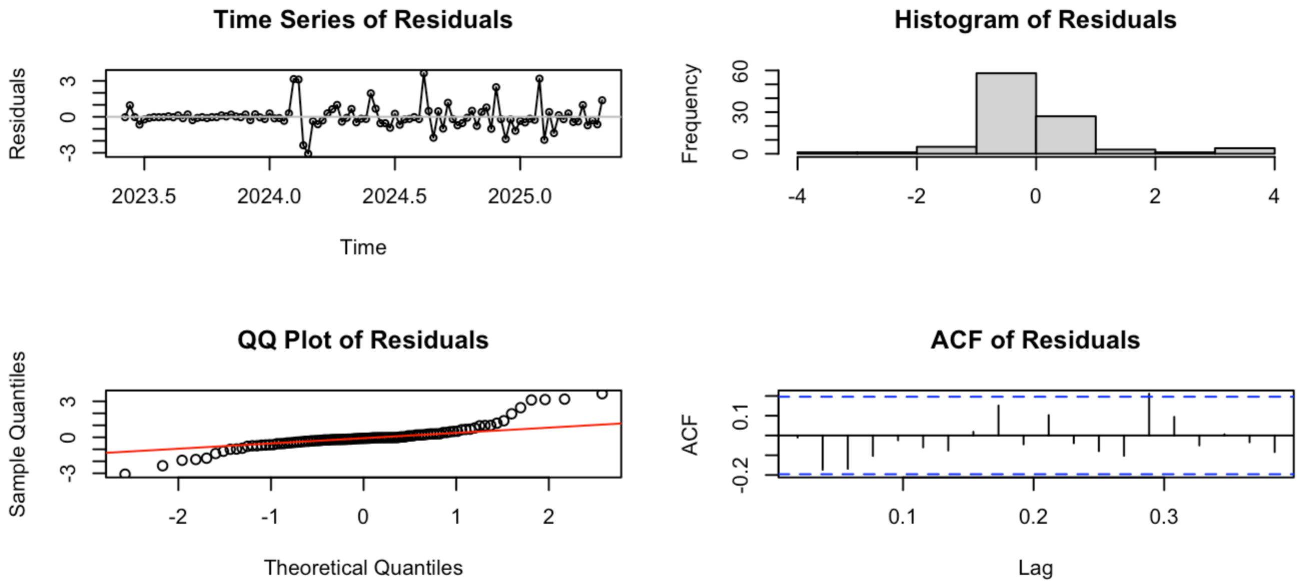


Figure 12: Residual Analysis of Model 001

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.85026, p-value = 1.213e-08
```

The residual analysis provides insight into how well the ARIMA model fits the data. The time series plot of residuals shows that the residuals mostly fluctuate around zero, which is a positive sign. However, there are a few noticeable spikes, suggesting the presence of outliers or short bursts of volatility. No clear trend or seasonality is visible, which is expected in well-behaved residuals.

The histogram of residuals appears roughly bell-shaped but shows signs of asymmetry and heavier tails than a normal distribution. This implies that the residuals are not perfectly normally distributed, which could affect the accuracy of confidence intervals and statistical tests.

The Q-Q plot supports this observation. While many points align with the diagonal, there is clear deviation at both ends of the plot, especially in the tails. This indicates the presence of heavy-tailed behavior or outliers, which suggests some departure from normality. Although mild non-normality alone isn't always a problem, in combination with other irregularities, it may warrant attention.

Finally, the ACF plot of the residuals shows that most autocorrelations fall within the 95% confidence bounds. However, a few bars—particularly around lags 3 and 4—come close to or slightly exceed these limits. This suggests that some autocorrelation may still be present in the residuals, and ideally, a better-fitting ARIMA model would remove these dependencies entirely.

```
res <- residuals(model_001)
plot_LBQ(res)
```

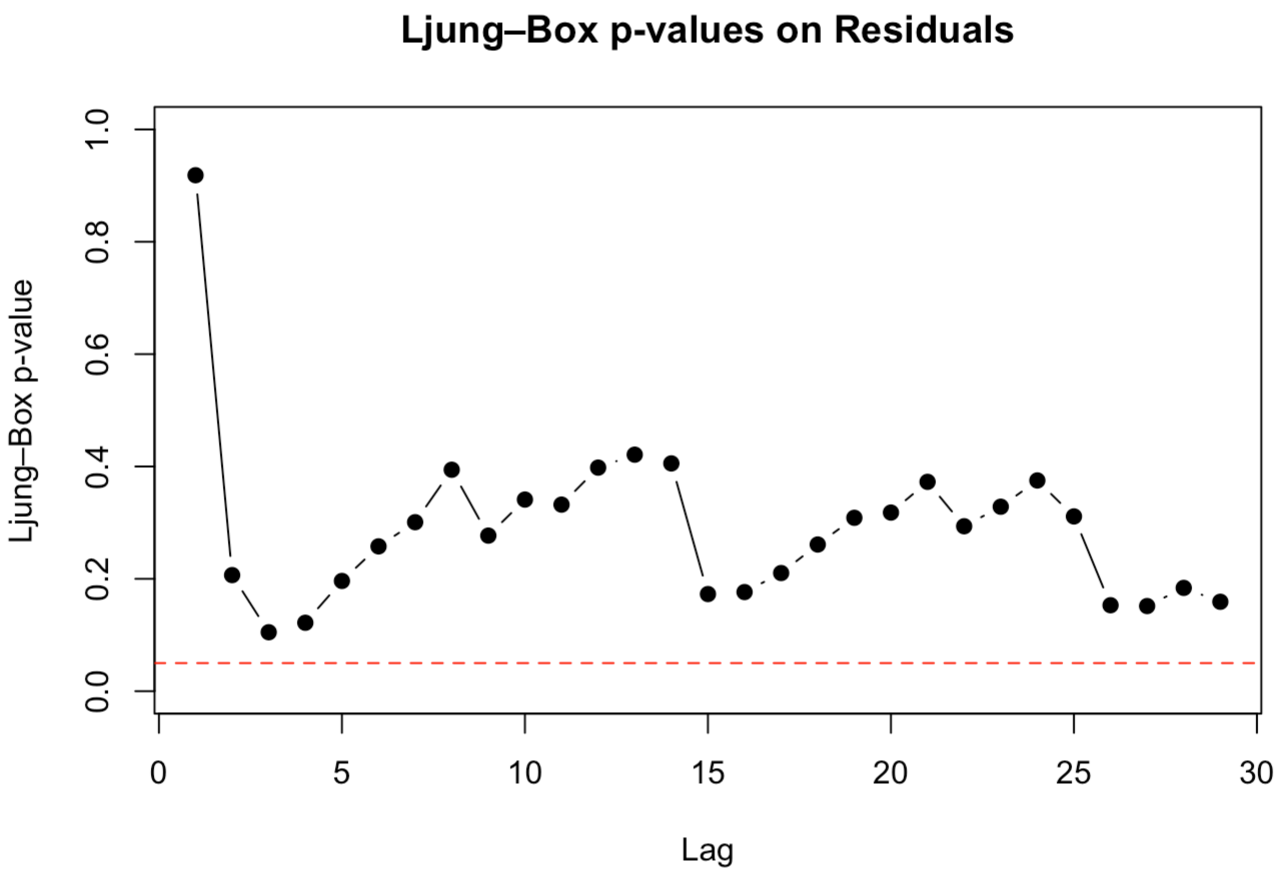


Figure 13: The Ljung–Box test for Model 001

The Ljung–Box test (figure 13) p-values plot provides further insight into the residuals of the ARIMA(0,0,1) model (model_001). For most of the lower lags, the p-values remain above the 0.05 threshold, indicating that we fail to reject the null hypothesis of no autocorrelation. This is a positive result, suggesting that the residuals behave like white noise at those lags. However, at higher lags, particularly around lags 25 to 28—the p-values begin to drop closer to or just below 0.05. This may indicate some lingering autocorrelation in the residuals at those longer time lags.

Overall, the residuals from model_001 show acceptable behavior. There is no strong evidence of autocorrelation in the early lags, which supports the model's general fit. However, the presence of some large residual spikes, mild non-normality in the histogram and Q-Q plot, and slight autocorrelation at higher lags suggest the model does not fully capture all the dynamics in the data. Combined with the earlier finding that the MA(1) coefficient is not statistically significant, this points to the ARIMA(0,0,1) model being functional but not ideal. A different model structure may provide a better fit.

5.2 Model 101

```
model_101<-arima(r.carpenter,order=c(1,0,1),method="ML") #good
coeftest(model_101)
```

```
##
## z test of coefficients:
##
##             Estimate Std. Error  z value Pr(>|z|)
## ar1       -0.906074   0.047948 -18.8970   <2e-16 ***
## ma1        0.999921   0.103104   9.6982   <2e-16 ***
## intercept  0.414063   1.344977   0.3079   0.7582
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
model_101CSS<-arima(r.carpenter,order=c(1,0,1),method="CSS")
coeftest(model_101CSS)
```

```
## Warning in sqrt(diag(se)): NaNs produced
```

```
##
## z test of coefficients:
##
##              Estimate Std. Error z value  Pr(>|z|)
## ar1       -0.8803364        NaN     NaN       NaN
## ma1        1.0769875  0.0038931  276.64 < 2.2e-16 ***
## intercept  1.9286452        NaN     NaN       NaN
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
model_101CSSML<-arima(r.carpenter,order=c(1,0,1),method="CSS-ML")
coeftest(model_101CSSML)
```

```
##
## z test of coefficients:
##
##             Estimate Std. Error  z value Pr(>|z|)
## ar1       -0.906176   0.047973 -18.8892   <2e-16 ***
## ma1        0.999994   0.101391   9.8628   <2e-16 ***
## intercept  0.414300   1.344912   0.3081    0.758
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The results from the ML and CSS-ML estimation methods show that both the AR(1) and MA(1) coefficients are highly significant, with p-values far below 0.05. This indicates that the ARIMA(1,0,1) model effectively captures strong time dependence in the series. However, the intercept term is not statistically significant in any of the models, meaning the average level of the series is not significantly different from zero.The CSS method, however, did not compute standard errors for some parameters, resulting in NaN values. This makes its output statistically unreliable and unsuitable for inference. On the other hand, the estimates obtained from the ML and CSS-ML methods are nearly identical, showing that the model is both stable and robust when proper estimation techniques are used.

In conclusion, the ARIMA(1,0,1) model is a solid fit for the data. The significant AR and MA terms confirm their role in explaining the series' structure. While the CSS-only method should be avoided due to missing diagnostics, both the ML and CSS-ML versions of the model can be used confidently for forecasting and interpretation.

5.2.1 Residual Analysis for Model 101

```
residual.analysis(model = model_101,std=TRUE,start = 2,class = "ARIMA")
```

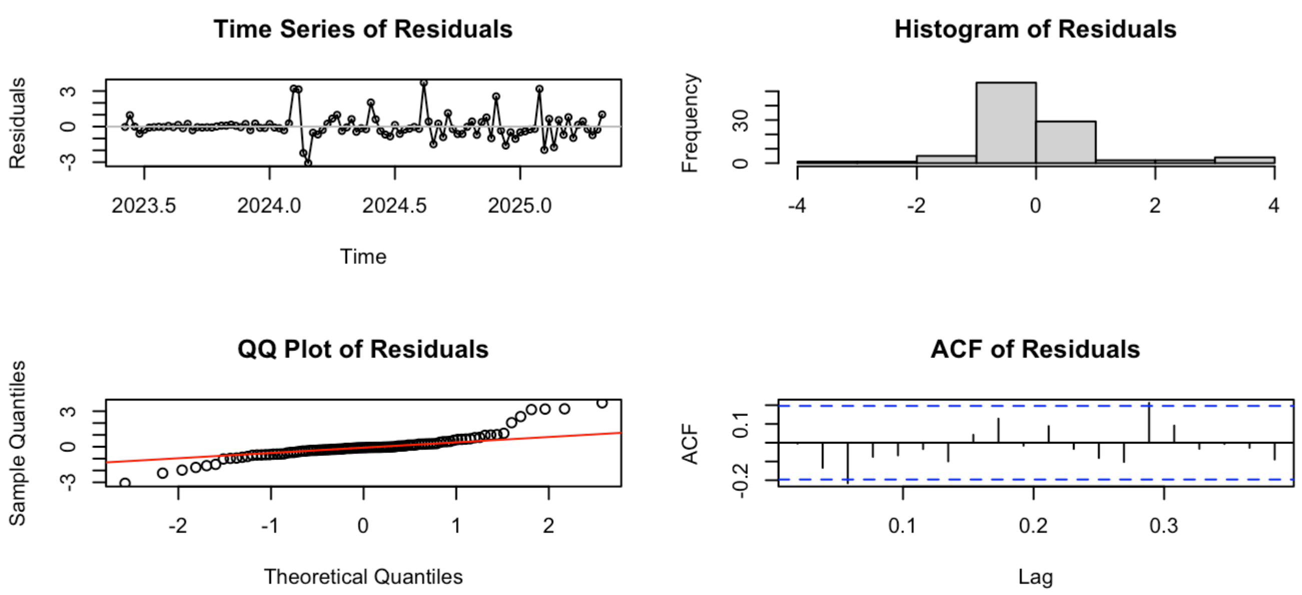


Figure 12: Residual Analysis of Model 101

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.84993, p-value = 1.178e-08
```

The residual diagnostics (figure 12) from the ARIMA model suggest mostly acceptable behavior. The time series plot of residuals shows that values fluctuate around zero with no clear trend or seasonal pattern, which is a good sign. However, a few large spikes are visible, possibly pointing to isolated outliers or short-lived disturbances.

The histogram shows a roughly bell-shaped distribution, but it is not perfectly symmetrical, and the tails appear somewhat heavy. This suggests that the residuals are not perfectly normal, although the overall shape is close.

The Q-Q plot confirms this impression. Most points lie along the diagonal, but there are some deviations at the extremes, especially in the upper tail. This indicates some departure from normality, likely caused by a few extreme values.

The ACF plot of residuals shows that all autocorrelations fall within the 95% confidence bounds. This suggests that there is no strong remaining autocorrelation in the residuals, which supports the model's adequacy in capturing the serial structure of the data.

```
res <- residuals(model_101)
plot_LBQ(res)
```

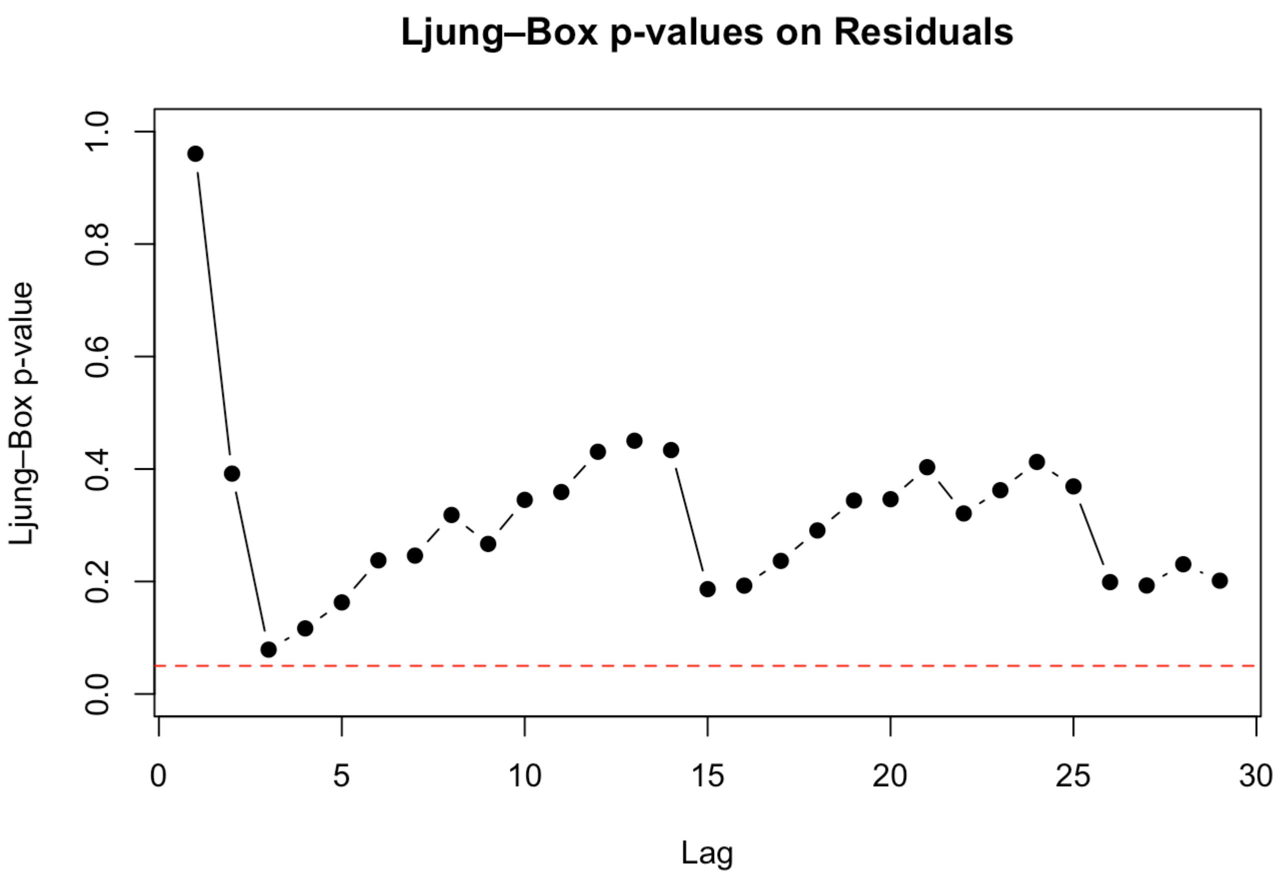


Figure 13: The Ljung–Box test for Model 101

The Ljung–Box test plot shows that most p-values are above 0.05, indicating no strong autocorrelation in the residuals. A small dip below 0.05 appears around lag 3, but this does not continue at higher lags. The residuals generally meet the white noise assumption, suggesting the model fits well.

5.3 Model 003

```
model_003<-arima(r.carpenter,order=c(0,0,3),method="ML") #pretty good
coeftest(model_003)
```

```
##
## z test of coefficients:
##
##            Estimate Std. Error z value Pr(>|z|)
## ma1       -0.111105   0.095255 -1.1664 0.243454
## ma2       -0.333691   0.101923 -3.2740 0.001061 **
## ma3       -0.244183   0.087273 -2.7979 0.005143 **
## intercept  0.404770   0.401745  1.0075 0.313680
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
model_003CSS<-arima(r.carpenter,order=c(1,0,2),method="CSS")
coeftest(model_003CSS)
```

```
##
## z test of coefficients:
##
##           Estimate Std. Error z value Pr(>|z|)
## ar1        0.51486    0.17022  3.0247 0.002489 **
## ma1       -0.60486    0.16711 -3.6196 0.000295 ***
## ma2       -0.27606    0.11408 -2.4199 0.015523 *
## intercept  0.48028    0.34269  1.4015 0.161069
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

5.3.1 Resdiual Analysis for Model 003

The ARIMA(0,0,3) model shows that the second and third moving average terms (ma2 and ma3) are statistically significant, while ma1 and the intercept are not. This suggests that the first MA term may not contribute much to the model, and could potentially be excluded. The significance of ma2 and ma3 indicates that the model captures some time-dependent structure, though not perfectly. Overall, the model provides a reasonable fit, with most of the explanatory power coming from the later MA terms.

```
residual.analysis(model = model_003,std=TRUE,start = 2,class = "ARIMA")
```

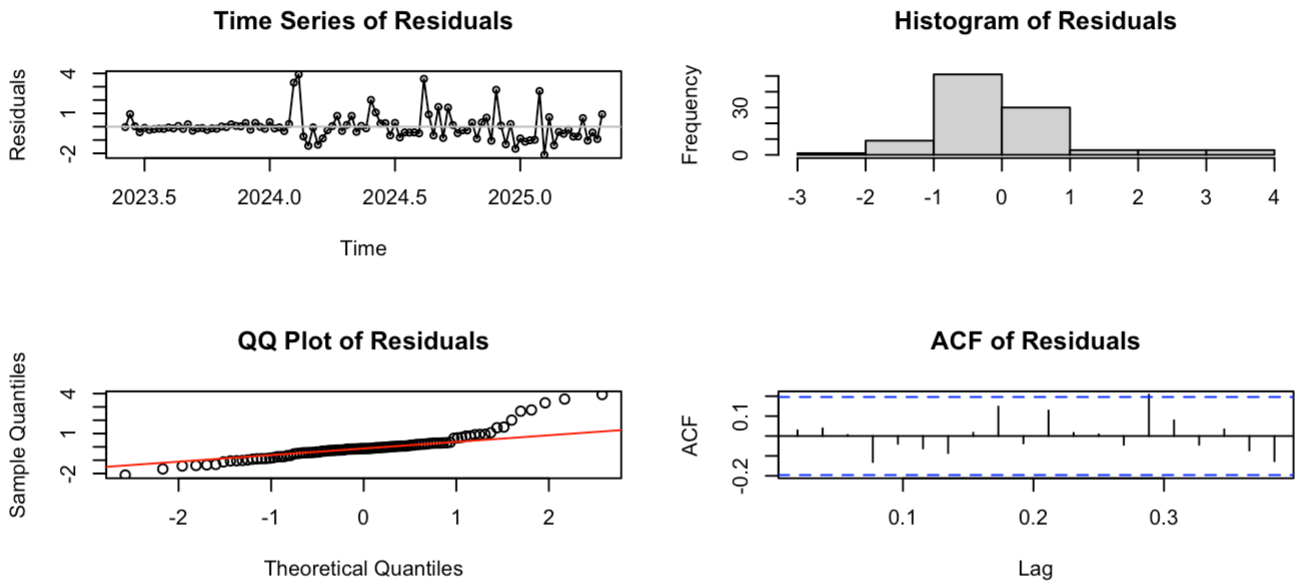


Figure 14: Residual Analysis of Model 003

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.85023, p-value = 1.209e-08
```

```
res <- residuals(model_003)
plot_LBQ(res)
```

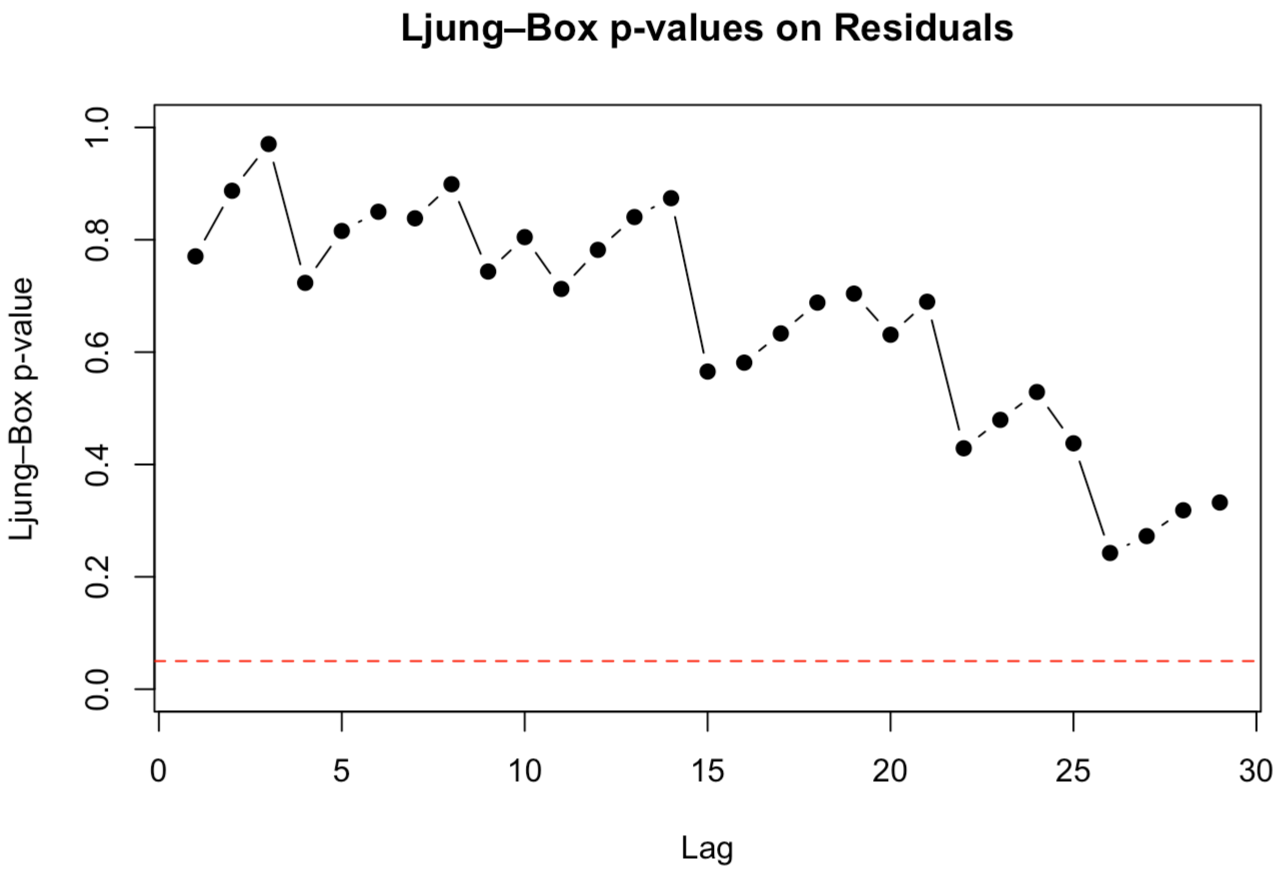


Figure 15: The Ljung–Box test for Model 003

The residual diagnostics for model_003 (ARIMA(0,0,3)) suggest that the model fits the data reasonably well. The residuals fluctuate around zero in the time series plot, with no visible trend, although a few large spikes appear, which could be outliers or short bursts of volatility. The histogram shows a roughly normal distribution with a slight left skew, and most values cluster between 0 and -1. A few extreme values are present on both ends.

The Q-Q plot confirms that the central residuals follow a normal pattern, while the upper tail deviates from the theoretical line, suggesting mild non-normality or heavy tails. The ACF plot shows that nearly all autocorrelation values fall within the 95% bounds, indicating no significant leftover autocorrelation. This is further supported by the Ljung–Box test, where p-values remain above 0.05 across all tested lags. Overall, the residuals resemble white noise, which supports the model's adequacy.

The ARIMA(0,0,3) model passes residual checks convincingly. The residuals show no significant autocorrelation, as seen in both the ACF plot and Ljung–Box test. Their distribution is roughly normal, with only slight deviations in the tails. The time series plot shows no clear trend or structure, indicating randomness. Compared to model_001, this model handles residual behavior better, particularly by reducing autocorrelation and improving overall randomness.

5.4 Model 203

```
model_203<-arima(r.carpenter,order=c(2,0,3),method="ML") #too many coef
coeftest(model_203)
```

```
##
## z test of coefficients:
##
##             Estimate Std. Error z value  Pr(>|z|)
## ar1         1.563449   0.093166 16.7813 < 2.2e-16 ***
## ar2        -0.696204   0.093624 -7.4362 1.036e-13 ***
## ma1        -1.741943   0.206955 -8.4170 < 2.2e-16 ***
## ma2         0.616700   0.284509  2.1676   0.03019 *
## ma3         0.197617   0.130104  1.5189   0.12878
## intercept  0.328317   0.616834  0.5323   0.59455
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
model_203CSS<-arima(r.carpenter,order=c(2,0,3),method="CSS") #too many coef
coeftest(model_203CSS)
```

```
##
## z test of coefficients:
##
##            Estimate Std. Error z value Pr(>|z|)
## ar1         0.36384    0.50778  0.7165   0.4737
## ar2        -0.12083    0.37195 -0.3248   0.7453
## ma1        -0.44812    0.50289 -0.8911   0.3729
## ma2        -0.13598    0.44755 -0.3038   0.7613
## ma3        -0.17726    0.13509 -1.3122   0.1895
## intercept  0.33309    0.40842  0.8156   0.4148
```

5.4.1 Residual Analysis for Model 203

The ARIMA(2,0,3) model shows strong significance in several parameters. Both AR terms are statistically significant: ar1 has a p-value < 2.2e-16, and ar2 has a p-value ≈ 1.04e-13. Among the MA terms, ma1 (p < 2.2e-16) and ma2 (p ≈ 0.03) are significant, while ma3 (p ≈ 0.13) is not. The intercept is also not significant (p ≈ 0.59), consistent with earlier models. This suggests that the model captures meaningful time dependencies through AR and MA components, although the third MA term does not contribute much. Overall, the ARIMA(2,0,3) model appears statistically robust, with a well-identified structure.

```
residual.analysis(model = model_203,std=TRUE,start = 2,class = "ARIMA")
```

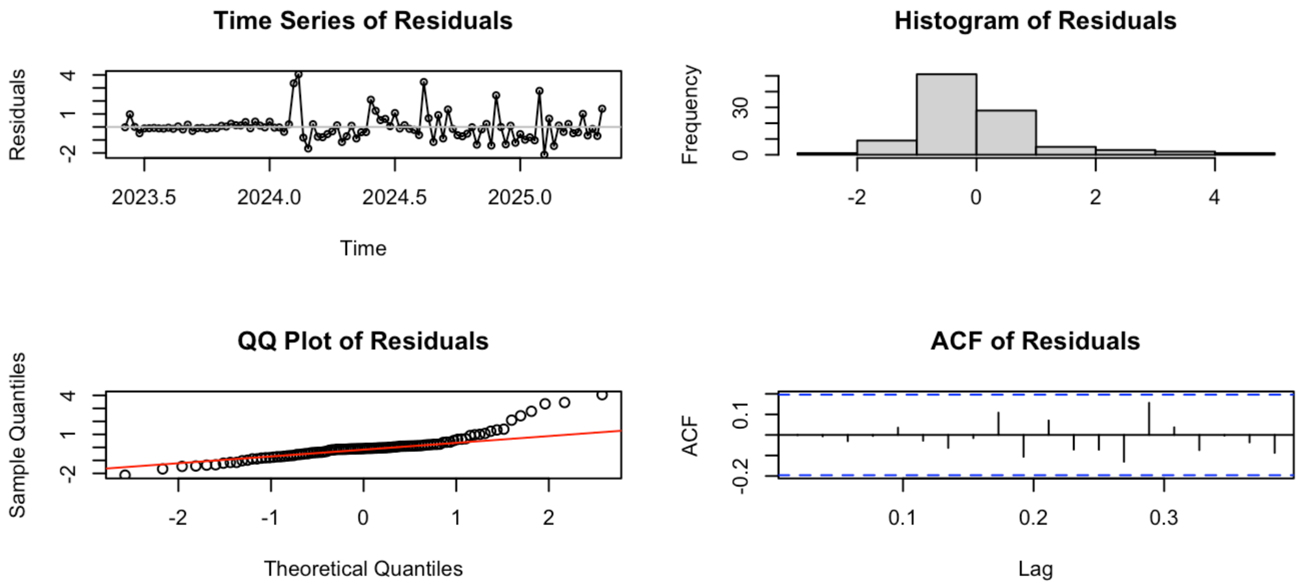


Figure 16: Residual Analysis for Model 203

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.85386, p-value = 1.651e-08
```

```
res <- residuals(model_203)
plot_LBQ(res)
```

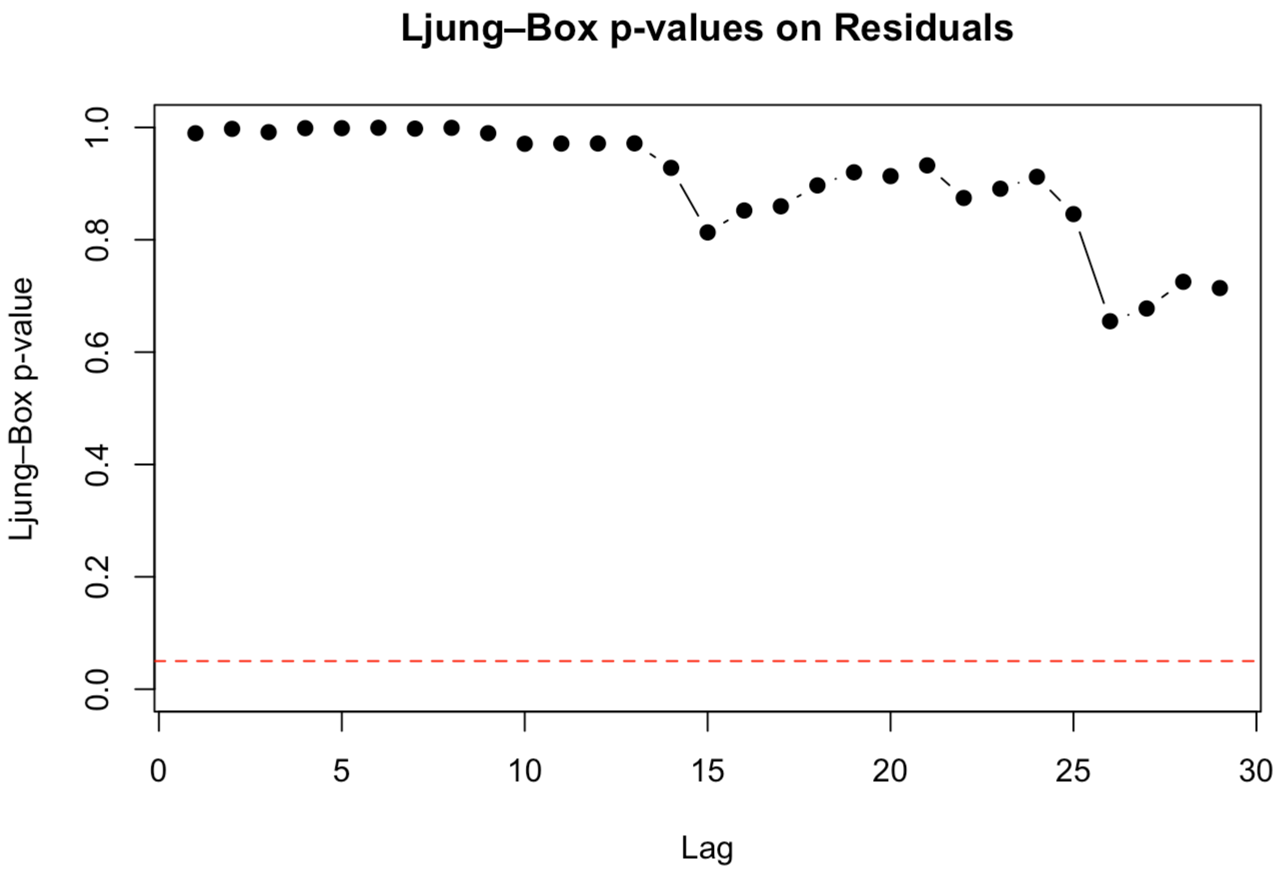


Figure 17: The Ljung–Box test for Model 203

The time series plot of residuals shows values fluctuating around zero with a few noticeable spikes, suggesting some residual irregularity. The histogram is approximately bell-shaped but shows slight right skewness with some positive outliers. The Q-Q plot indicates that most points follow the expected normal pattern, though the right tail deviates, pointing to heavier tails. The ACF of residuals shows no significant autocorrelation, as all spikes fall within the confidence bounds. The Ljung–Box test supports this, with all p-values above 0.05, confirming the residuals resemble white noise. Overall, the model demonstrates a good fit with mostly well-behaved residuals.

5.5 AIC and BIC scores

```
sort.score(AIC(model_001,model_101,model_003,model_203),score="aic")
```

```
##           df      AIC
## model_203  7 792.4295
## model_003  5 795.0183
## model_101  4 803.7749
## model_001  3 804.6820
```

```
sort.score(BIC(model_001,model_101,model_003,model_203),score="bic")
```

```
##           df      BIC
## model_003  5 808.0441
## model_203  7 810.6656
## model_001  3 812.4975
## model_101  4 814.1956
```

Based on the AIC and BIC comparison, model_203 (ARIMA(2,0,3)) achieves the best AIC score, indicating it fits the data most closely. However, it is also the most complex, with seven parameters, and shows signs of non-normal residuals. On the other hand, model_003 (ARIMA(0,0,3)) performs best in terms of BIC, which favors simpler models. It also comes second in AIC ranking. Importantly, model_003 has clean residuals with no signs of autocorrelation and only slight deviations from normality, making it a more balanced choice overall.

6 GARCH Modelling

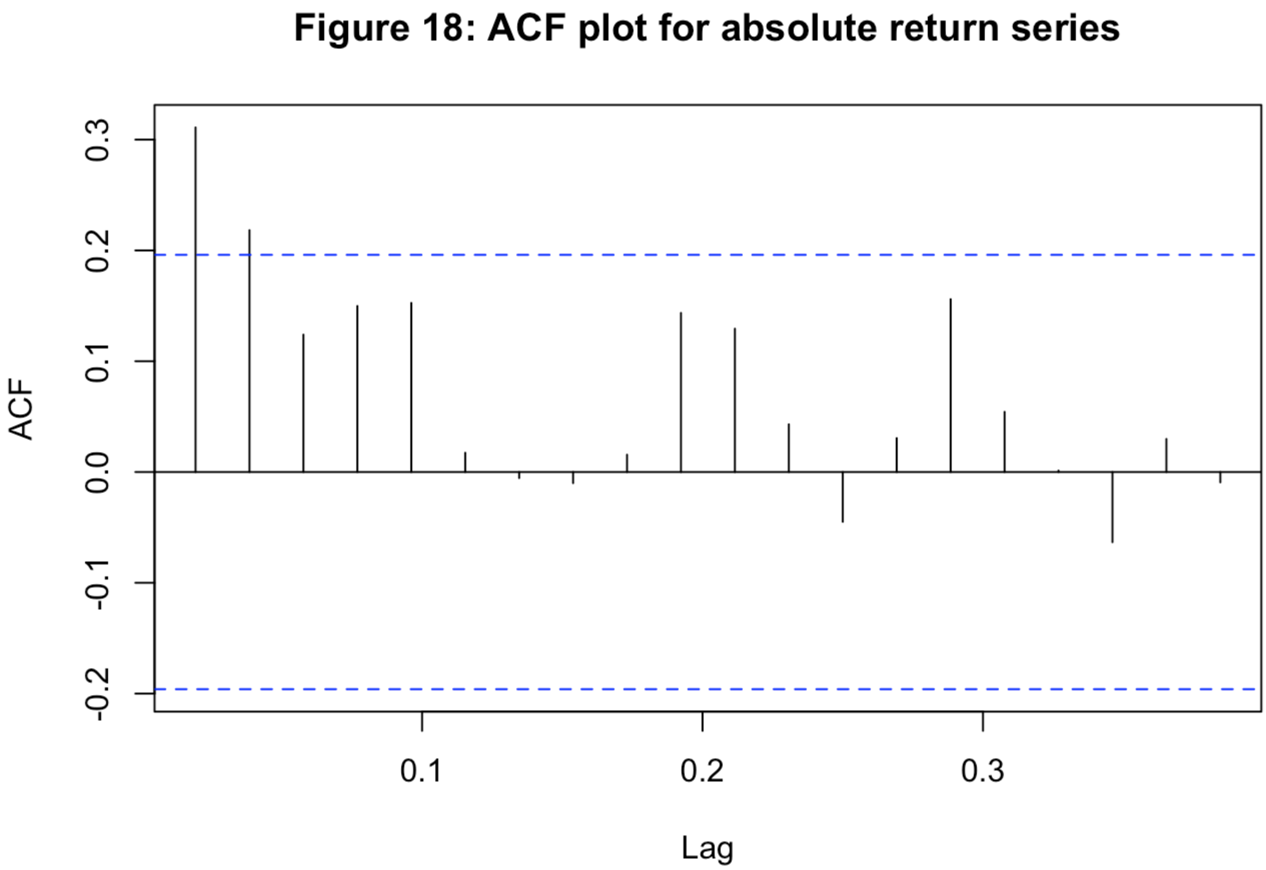
GARCH (Generalized Autoregressive Conditional Heteroskedasticity) is a model used to handle time series data when the variance is changing over time. In many real-world situations, like financial markets, social media trends, or economic indicators, we often see periods of high activity followed by quieter periods. This kind of behaviour is called volatility clustering. A GARCH model helps us capture and forecast these fluctuations by modelling how today's variance depends on past errors and past variances.

Standard time series models like ARMA or ARIMA assume that the variance is constant, which can lead to poor predictions when the data is actually more uneven. GARCH improves on this by allowing the variance to change over time in a structured way. It is especially useful in applications where risk, uncertainty, or bursts of activity need to be measured or anticipated. In this way, GARCH improves our understanding of both the values and the stability of a time series, making it a powerful tool for forecasting and analysis in fields such as finance, economics, and media analytics.

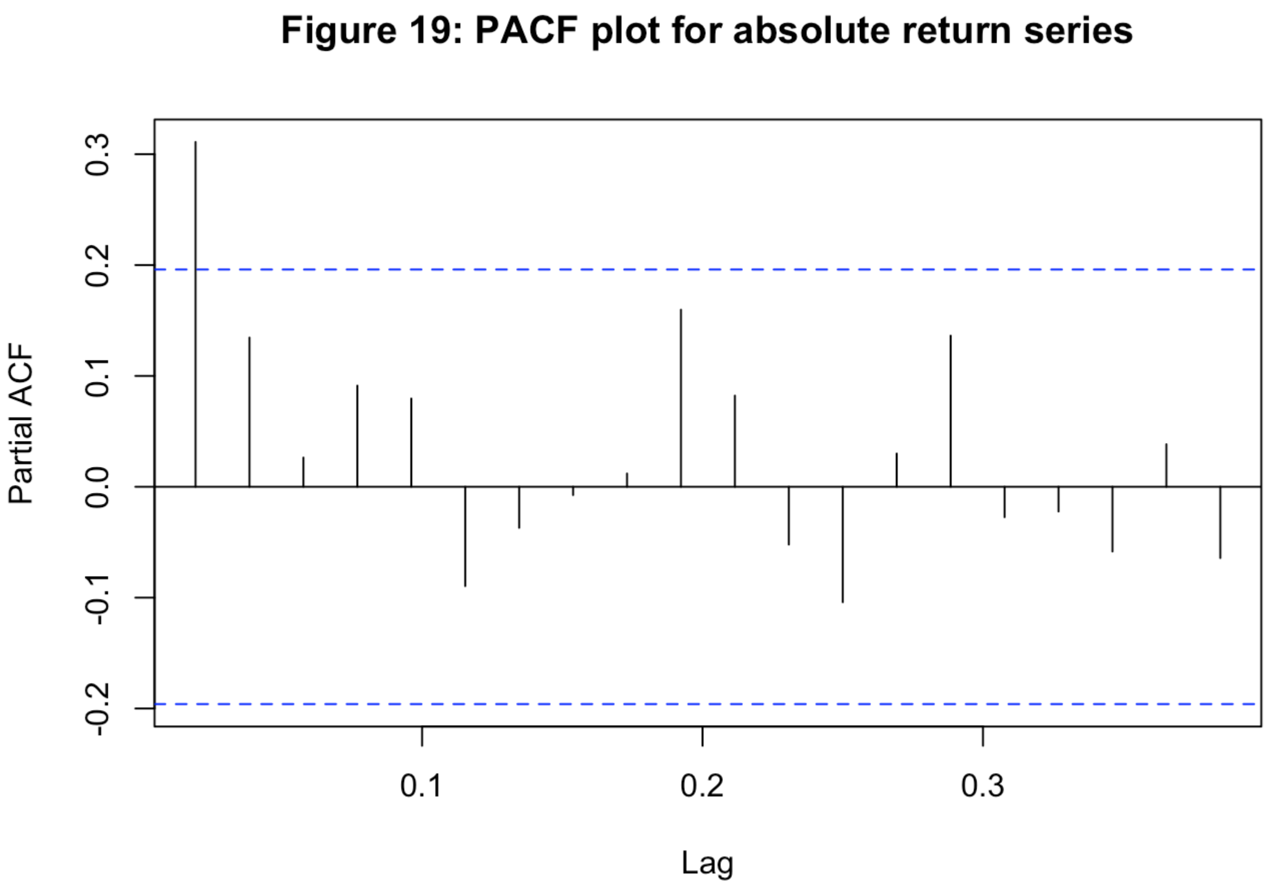
```
abs.r.res.Carpenter<-abs(rstandard(model_003))
sqr.r.res.Carpenter<-rstandard(model_003)^2
```

Before starting GARCH modeling, we created two series based on the standardized residuals of the ARMA model to check for signs of volatility. The first takes the absolute value of the residuals, and the second squares them. Both of these help highlight whether the size of the errors changes over time. If the values show visible clusters or patterns, it suggests that the variance is not constant.

```
acf(abs.r.res.Carpenter,main="Figure 18: ACF plot for absolute return series")
```




```
pacf(abs.r.res.Carpenter,main="Figure 19: PACF plot for absolute return serie
s")
```



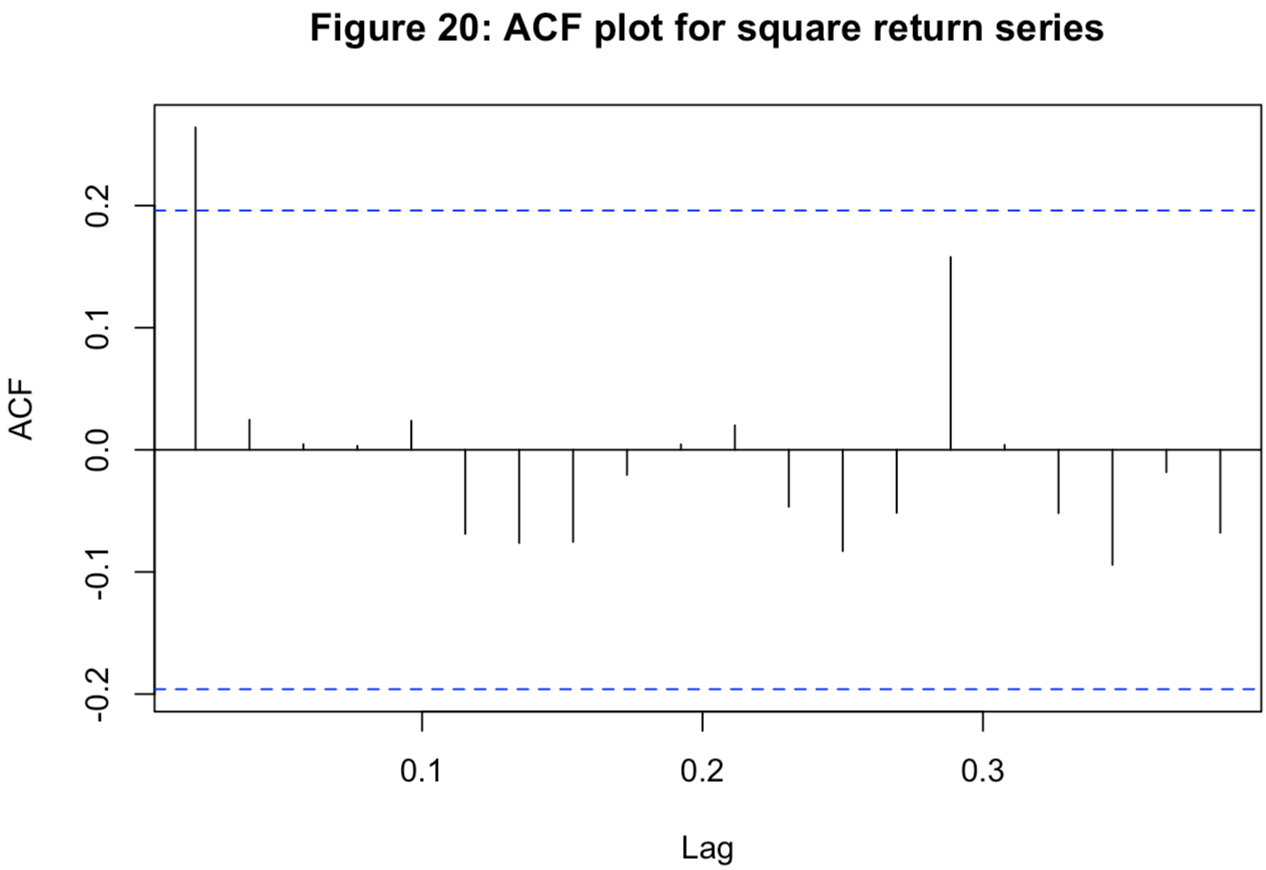
To choose the order of the GARCH model, we looked at the ACF (figure 18) and PACF (figure 19) plots of the absolute residuals from our ARMA model. These plots help us understand how volatility changes over time. The ACF plot shows significant spikes up to lag 2, which suggests we can set q = 2 in the GARCH(p, q) model. The PACF plot, however, only shows a clear drop after lag 1. According to the rule that says the maximum of p or q should have enough support in the PACF, we cannot set p = 2. So, a GARCH(2,1) is not suitable.

```
eacf(abs.r.res.Carpenter)
```

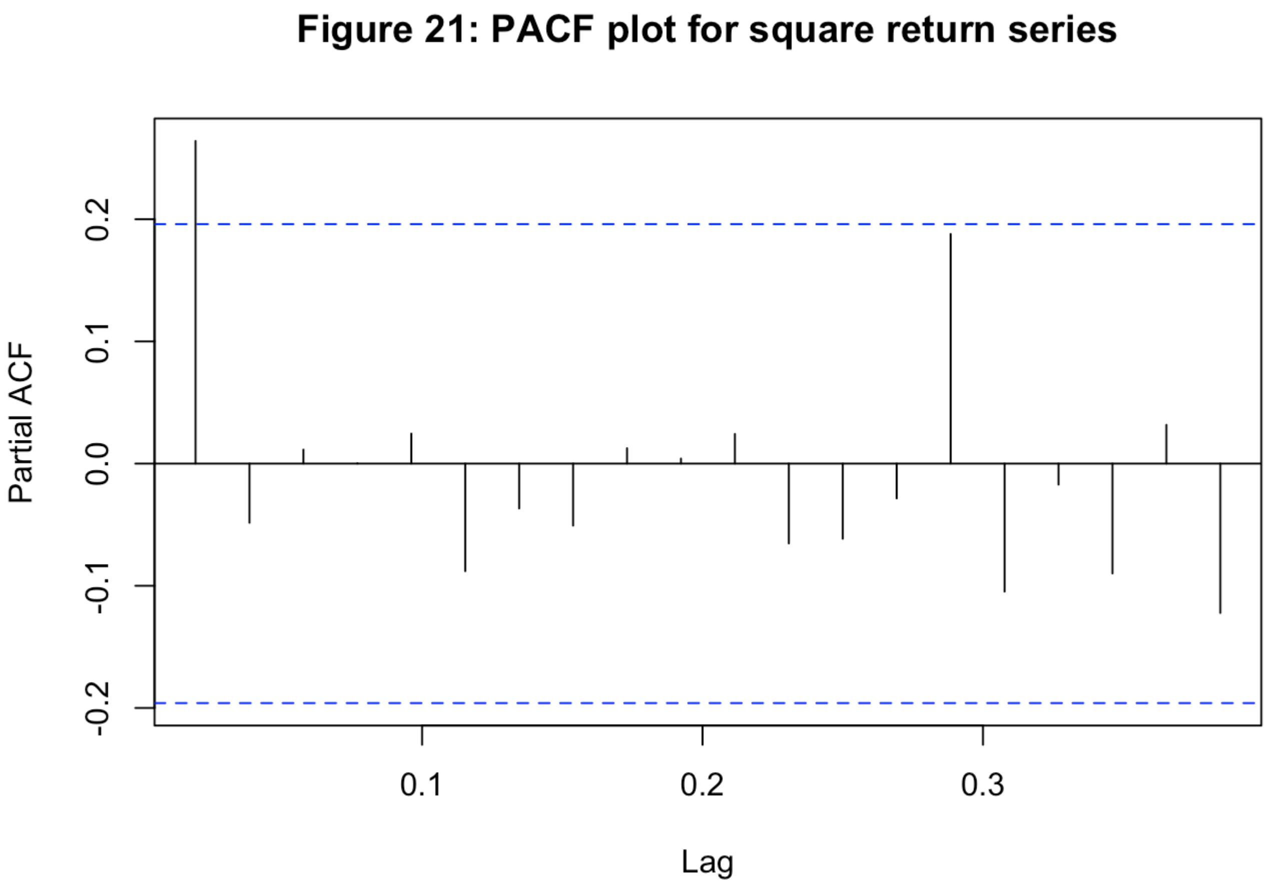
```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x o o o o o o o o o  o  o  o
## 1 x o o o o o o o o o o  o  o  o
## 2 o o o o o o o o o o o  o  o  o
## 3 x x x o o o o o o o o  o  o  o
## 4 x x x o o o o o o o o  o  o  o
## 5 x x x x o o o o o o o  o  o  o
## 6 x o o o x o o o o o o  o  o  o
## 7 x o o o x x o o o o o  o  o  o
```

To choose the right GARCH model, we used the EACF (Extended Autocorrelation Function) table of the absolute residual series. In the table, “o” marks model combinations that are likely to work well. From the table, we found that models like GARCH(0,1), GARCH(1,1), GARCH(2,1), GARCH(0,2), GARCH(1,2), and GARCH(2,2) were possible.

```
acf(sqr.r.res.Carpenter,main="Figure 20: ACF plot for square return series")
```



```
pacf(sqr.r.res.Carpenter,main="Figure 21: PACF plot for square return series")
```



The ACF and PACF plots of the squared return series show significant spikes at lag 1 in both plots, while the remaining lags mostly stay within the confidence limits. Based on this, we considered models with q = 1 and max(p, q) = 1. That means only p = 0 or 1 is allowed. So, the possible GARCH models are GARCH(0,1) and GARCH(1,1), which are both simple but effective in capturing the volatility pattern in the data.

The GARCH model is fitted into the first-differenced time series r.carpenter. It combines an ARMA model for the mean with a GARCH model for the variance . The function garchFit() from the fGarch package is used to do this. The option trace=F turns off detailed output during the fitting process.

```
model_003_01 <- fGarch::garchFit(~ arma(0,3)+garch(1,0),
                                 data = r.carpenter, trace=F)
model_003_11 <- fGarch::garchFit(~ arma(0,3)+garch(1,1),
                                 data = r.carpenter, trace=F)
model_003_21 <- fGarch::garchFit(~ arma(0,3)+garch(2,1),
                                 data = r.carpenter, trace=F)
```

```
## Warning in sqrt(diag(fit$cvar)): NaNs produced
```

```
model_003_12 <- fGarch::garchFit(~ arma(0,3)+garch(1,2),
                                 data = r.carpenter, trace=F)

model_003_22 <- fGarch::garchFit(~ arma(0,3)+garch(2,2),
                                 data = r.carpenter, trace=F)
```

```
## Warning in sqrt(diag(fit$cvar)): NaNs produced
```

6.1 Model_003_01

```
summary(model_003_01)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  fGarch::garchFit(formula = ~arma(0, 3) + garch(1, 0), data = r.carpenter,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(0, 3) + garch(1, 0)
## <environment: 0x11ac5b610>
##  [data = r.carpenter]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##        mu        ma1        ma2        ma3      omega     alpha1
##  0.272398  -0.237293  -0.044924  -0.419497  70.562352   0.872515
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu       0.27240     0.33248    0.819 0.412614
## ma1     -0.23729     0.06454   -3.677 0.000236 ***
## ma2     -0.04492     0.08936   -0.503 0.615157
## ma3     -0.41950     0.06390   -6.565 5.19e-11 ***
## omega   70.56235    22.60390    3.122 0.001798 **
## alpha1   0.87251     0.53920    1.618 0.105628
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -384.9408    normalized:  -3.849408
##
## Description:
##  Sun Jun 15 18:15:11 2025 by user:
##
##
## Standardised Residuals Tests:
##                                   Statistic      p-Value
##  Jarque-Bera Test   R    Chi^2  283.1273498 0.000000e+00
##  Shapiro-Wilk Test  R    W        0.8293688 2.201259e-09
##  Ljung-Box Test     R    Q(10)    7.8366846 6.447857e-01
##  Ljung-Box Test     R    Q(15)   19.7474162 1.818377e-01
##  Ljung-Box Test     R    Q(20)   23.0017721 2.887075e-01
##  Ljung-Box Test     R^2  Q(10)    1.4191698 9.991658e-01
##  Ljung-Box Test     R^2  Q(15)    4.4087958 9.960588e-01
##  Ljung-Box Test     R^2  Q(20)    7.2385169 9.958263e-01
```

```
##  LM Arch Test       R    TR^2     3.3134761 9.928970e-01
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 7.818815 7.975125 7.812144 7.882077
```

TThis model fits a GARCH(1,0) combined with an MA(3) to the Carpenter series. The model assumes normal distribution for errors. In the mean equation, the constant (mu) and the second MA term (ma2) are not statistically significant. However, MA1 and MA3 are significant, showing that they contribute meaningfully to explaining short-term fluctuations. In the variance equation, omega, which is the constant part of the variance, is significant, while the alpha1 term, which reflects past shocks' impact on current variance, is not strongly significant but still relevant.

The Ljung-Box tests show that there's some significant autocorrelation left in the residuals or their squares. Besides, the normality tests (Jarque-Bera and Shapiro-Wilk) show that residuals are not normally distributed. Despite this, the ARCH effect seems well captured, as the LM ARCH test has a very high p-value, showing no leftover ARCH effects.

The AIC and BIC values are moderate (AIC = 7.81, BIC = 7.97), which helps in comparing this model to others. Overall, the model fits the series reasonably well, capturing short-term dependencies and volatility, though the residuals deviate from normality.

6.2 Model 003-011

```
summary(model_003_11 )
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  fGarch::garchFit(formula = ~arma(0, 3) + garch(1, 1), data = r.carpenter,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(0, 3) + garch(1, 1)
## <environment: 0x11dc743c8>
##  [data = r.carpenter]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##        mu        ma1        ma2        ma3      omega     alpha1      beta1
##  0.47552  -0.27328  -0.11272  -0.32899  65.47980   0.48403   0.16243
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu        0.4755      0.3346    1.421  0.15527
## ma1      -0.2733      0.1037   -2.635  0.00842 **
## ma2      -0.1127      0.1262   -0.893  0.37169
## ma3      -0.3290      0.1075   -3.059  0.00222 **
## omega    65.4798     20.1857    3.244  0.00118 **
## alpha1    0.4840      0.2697    1.794  0.07274 .
## beta1     0.1624      0.1761    0.922  0.35632
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -384.1865    normalized:  -3.841865
##
## Description:
##  Sun Jun 15 18:15:11 2025 by user:
##
##
## Standardised Residuals Tests:
##                                  Statistic      p-Value
##  Jarque-Bera Test   R    Chi^2  270.3350573 0.000000e+00
##  Shapiro-Wilk Test  R    W        0.8110486 5.487131e-10
##  Ljung-Box Test     R    Q(10)    7.9957798 6.292492e-01
##  Ljung-Box Test     R    Q(15)   18.2444947 2.500300e-01
##  Ljung-Box Test     R    Q(20)   22.7320755 3.021409e-01
##  Ljung-Box Test     R^2  Q(10)    1.3109759 9.994134e-01
##  Ljung-Box Test     R^2  Q(15)    4.3882797 9.961608e-01
```

```
##  Ljung-Box Test     R^2  Q(20)    6.8929928 9.970215e-01
##  LM Arch Test       R    TR^2     2.9280076 9.960313e-01
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 7.823730 8.006092 7.814758 7.897535
```

The MA terms ma1 and ma3 are statistically significant, meaning they explain part of the variation in the data. The ma2 term is not significant. In the variance equation, omega (the base variance) and alpha1 (impact of past shocks) are significant or borderline significant, but beta1, which measures the effect of past variance, is not. This may suggest that the GARCH(1,1) structure adds some complexity without clear gain in fit over simpler models.

The residual checks show no significant autocorrelation left. The Ljung-Box tests on both residuals and their squares return high p-values, meaning the model handles dependencies well. However, the normality tests (Jarque-Bera and Shapiro-Wilk) show that the residuals aren't normally distributed. Despite this, the LM ARCH test result which shows no remaining ARCH effects means that the volatility pattern has been captured properly.

The model has a log-likelihood of -384.18 and an AIC of 7.82, slightly higher than the GARCH(1,0) model. While the fit is acceptable, and volatility is modeled well, the additional complexity of including beta1 (the GARCH term) doesn't lead to a major performance improvement.

6.3 Model_003_12

```
summary(model_003_12 )
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  fGarch::garchFit(formula = ~arma(0, 3) + garch(1, 2), data = r.carpenter,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(0, 3) + garch(1, 2)
## <environment: 0x12aca4920>
##  [data = r.carpenter]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ma1          ma2          ma3        omega       alpha1
##  0.47528320  -0.27167852  -0.12059697  -0.32281715  66.48295656   0.44819383
##       beta1        beta2
##  0.17261994   0.00000001
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      4.753e-01   3.359e-01    1.415  0.15712
## ma1    -2.717e-01   1.058e-01   -2.568  0.01021 *
## ma2    -1.206e-01   1.325e-01   -0.910  0.36266
## ma3    -3.228e-01   1.093e-01   -2.955  0.00313 **
## omega   6.648e+01   2.269e+01    2.930  0.00339 **
## alpha1  4.482e-01   2.706e-01    1.656  0.09762 .
## beta1   1.726e-01   2.035e-01    0.848  0.39625
## beta2   1.000e-08   2.143e-01    0.000  1.00000
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -384.5404    normalized:  -3.845404
##
## Description:
##  Sun Jun 15 18:15:11 2025 by user:
##
##
## Standardised Residuals Tests:
##                                    Statistic      p-Value
##  Jarque-Bera Test   R    Chi^2  258.0341072 0.000000e+00
##  Shapiro-Wilk Test  R    W        0.8124333 6.075511e-10
##  Ljung-Box Test     R    Q(10)    8.0930316 6.197501e-01
##  Ljung-Box Test     R    Q(15)   18.2766525 2.484006e-01
```

```
##  Ljung-Box Test     R    Q(20)   22.7904565 2.992003e-01
##  Ljung-Box Test     R^2  Q(10)    1.3494780 9.993327e-01
##  Ljung-Box Test     R^2  Q(15)    4.5173629 9.954841e-01
##  Ljung-Box Test     R^2  Q(20)    6.8859422 9.970427e-01
##  LM Arch Test       R    TR^2     2.9309006 9.960125e-01
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 7.850809 8.059222 7.839229 7.935158
```

The model tries to account for volatility clustering using one ARCH term (alpha1) and two GARCH terms (beta1 and beta2). Among the variance parameters, only alpha1 is somewhat close to significance, while beta1 is not significant and beta2 is almost zero, with a p-value of 1. This shows that the second GARCH term contributes nothing to the model. In the mean equation, ma1 and ma3 are statistically significant, but ma2 is not. The intercept (mu) is also not significant. Despite the addition of a second GARCH term, the log likelihood (−384.54) and AIC (7.85) show no meaningful improvement over simpler models like GARCH(1,1) or GARCH(1,0). The residual diagnostics are good as there's no significant autocorrelation in residuals or their squares, and the LM ARCH test confirms no remaining ARCH effects. However, like previous models, the residuals still fail normality tests. Overall, this model does not provide added value over simpler ones. Including a second GARCH term leads to overfitting without improving fit or interpretability, as seen in the insignificant coefficients and similar diagnostic results.

6.4 Model_003_22

```
summary(model_003_21 )
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  fGarch::garchFit(formula = ~arma(0, 3) + garch(2, 1), data = r.carpenter,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(0, 3) + garch(2, 1)
## <environment: 0x11f9486a0>
##  [data = r.carpenter]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ma1          ma2          ma3        omega       alpha1
##  0.52811027  -0.26414001  -0.13781084  -0.30794924  78.98977411   0.41133150
##      alpha2        beta1
##  0.12901456   0.00000001
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      5.281e-01   3.406e-01    1.550  0.12106
## ma1    -2.641e-01   1.050e-01   -2.516  0.01188 *
## ma2    -1.378e-01   1.196e-01   -1.152  0.24941
## ma3    -3.079e-01   9.844e-02   -3.128  0.00176 **
## omega   7.899e+01         NaN      NaN      NaN
## alpha1  4.113e-01   2.289e-01    1.797  0.07228 .
## alpha2  1.290e-01         NaN      NaN      NaN
## beta1   1.000e-08         NaN      NaN      NaN
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -384.2839    normalized:  -3.842839
##
## Description:
##  Sun Jun 15 18:15:11 2025 by user:
##
##
## Standardised Residuals Tests:
##                                   Statistic      p-Value
##  Jarque-Bera Test   R    Chi^2  264.5964622 0.000000e+00
##  Shapiro-Wilk Test  R    W        0.8035419 3.185217e-10
##  Ljung-Box Test     R    Q(10)    8.2862670 6.008968e-01
##  Ljung-Box Test     R    Q(15)   18.2158998 2.514853e-01
```

```
##  Ljung-Box Test     R    Q(20)   23.3144057 2.736215e-01
##  Ljung-Box Test     R^2  Q(10)    1.4054190 9.992009e-01
##  Ljung-Box Test     R^2  Q(15)    4.7120660 9.942963e-01
##  Ljung-Box Test     R^2  Q(20)    7.0467469 9.965289e-01
##  LM Arch Test       R    TR^2     3.0332976 9.953043e-01
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 7.845679 8.054092 7.834099 7.930027
```

The model estimates include three moving average terms and two ARCH terms (alpha1, alpha2) to capture short-term volatility, along with one GARCH term (beta1) for long-term volatility effects. Among the MA terms, ma1 and ma3 are significant, while ma2 is not. The ARCH term alpha1 is borderline significant, but alpha2 and the GARCH term beta1 have NaN (not available) standard errors and are not statistically meaningful. Also, beta1 is almost zero, which shows that the model isn't gaining much from including a GARCH component.

The variance parameter omega also has a NaN standard error, meaning it couldn't be reliably estimated. This suggests the model might be over-parameterized. Residual checks, however, show good behavior: there is no autocorrelation left in the residuals or their squares, and the LM ARCH test confirms no remaining ARCH effects. But the residuals still fail normality tests (Jarque-Bera and Shapiro-Wilk), which is a recurring issue across models.

The log-likelihood is -384.28, and the AIC is 7.84 which is higher than that of simpler models like GARCH(1,0) or GARCH(1,1). Overall, this model doesn't seem to improve the fit meaningfully and may be unnecessarily complex, especially since key variance parameters couldn't be estimated properly.

```
summary(model_003_22)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  fGarch::garchFit(formula = ~arma(0, 3) + garch(2, 2), data = r.carpenter,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(0, 3) + garch(2, 2)
## <environment: 0x118ac29b0>
##  [data = r.carpenter]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##          mu          ma1          ma2          ma3        omega       alpha1
##  0.52811043  -0.26413994  -0.13781094  -0.30794917  78.98977512   0.41133152
##      alpha2        beta1        beta2
##  0.12901461   0.00000001   0.00000001
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      5.281e-01   3.431e-01    1.539  0.12380
## ma1    -2.641e-01   1.057e-01   -2.499  0.01245 *
## ma2    -1.378e-01   1.211e-01   -1.138  0.25511
## ma3    -3.079e-01   9.899e-02   -3.111  0.00186 **
## omega   7.899e+01         NaN      NaN      NaN
## alpha1  4.113e-01   2.647e-01    1.554  0.12015
## alpha2  1.290e-01         NaN      NaN      NaN
## beta1   1.000e-08         NaN      NaN      NaN
## beta2   1.000e-08   2.720e-01    0.000  1.00000
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -384.2839    normalized:  -3.842839
##
## Description:
##  Sun Jun 15 18:15:11 2025 by user:
##
##
## Standardised Residuals Tests:
##                                   Statistic      p-Value
##  Jarque-Bera Test   R    Chi^2  264.5964868 0.000000e+00
##  Shapiro-Wilk Test  R    W        0.8035418 3.185212e-10
##  Ljung-Box Test     R    Q(10)    8.2862662 6.008969e-01
```

```
##  Ljung-Box Test     R    Q(15)   18.2158975 2.514854e-01
##  Ljung-Box Test     R    Q(20)   23.3144040 2.736216e-01
##  Ljung-Box Test     R^2  Q(10)    1.4054190 9.992009e-01
##  Ljung-Box Test     R^2  Q(15)    4.7120660 9.942963e-01
##  Ljung-Box Test     R^2  Q(20)    7.0467466 9.965289e-01
##  LM Arch Test       R    TR^2     3.0332976 9.953043e-01
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 7.865679 8.100144 7.851193 7.960571
```

In the mean model, ma1 and ma3 are statistically significant, showing some influence on the returns. However, ma2 is not. The volatility model shows problems such as both GARCH terms (beta1, beta2) are essentially zero and not significant. One ARCH term (alpha1) is borderline significant, but alpha2 is not. This means the added complexity from using two ARCH and two GARCH terms does not improve the model. The log-likelihood value and AIC are similar to simpler models. Most residual tests look good with no autocorrelation, and no remaining ARCH effects. But the residuals are not normally distributed, which is a common issue with the data. In short, this GARCH(2,2) model is overfitted. It adds extra parameters that don't help, making it no better than simpler models like GARCH(1,1) or GARCH(1,0).

7 Resdiual Analysis of GARCH models

7.1 Model_003_01

```
residual.analysis(model = model_003_01, std = TRUE,start = 2, class = "fGARCH")
```

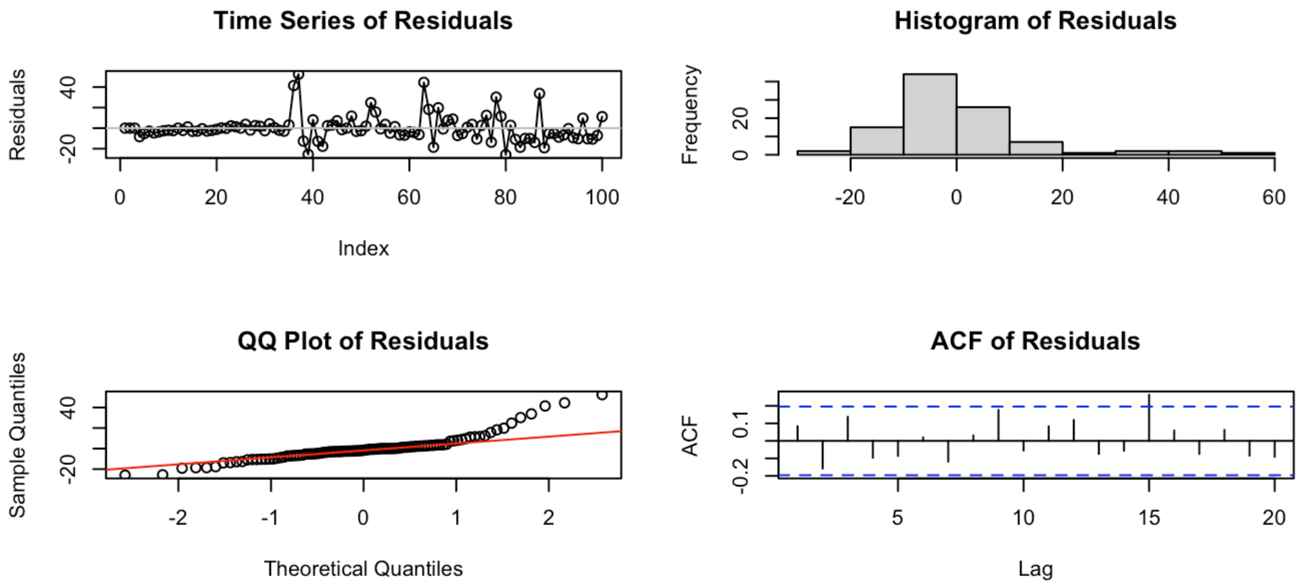
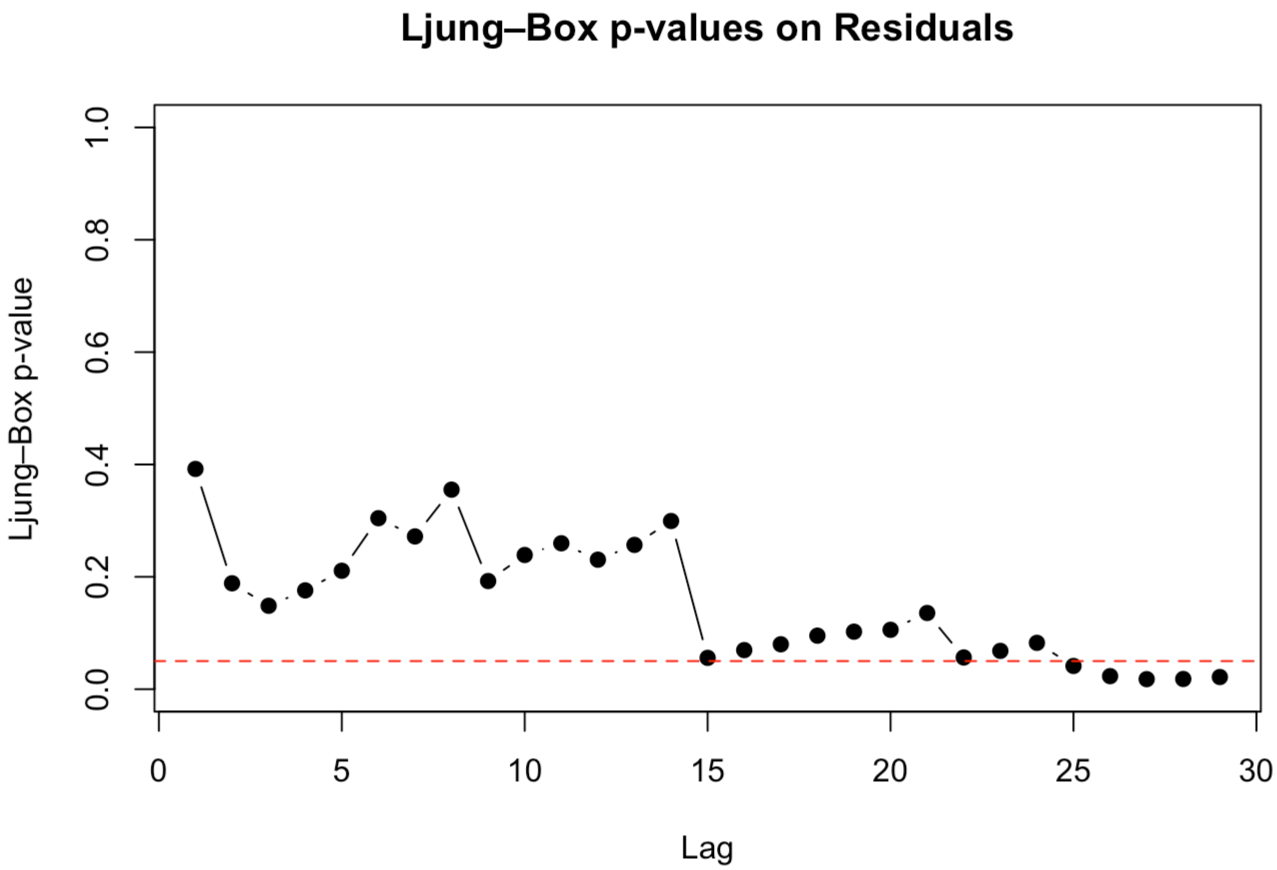


Figure 22: Residual for model_003_01

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.85801, p-value = 2.371e-08
```

```
res <- residuals(model_003_01)
plot_LBQ(res)
```



The residual analysis plots for model_003_01 show that the model captures the time-dependent structure of the series reasonably well. The time series plot of residuals shows no clear trend or pattern, though a few large spikes suggest the presence of outliers. The histogram indicates that the residuals are centered around zero but are not perfectly symmetric, suggesting mild skewness. The QQ plot confirms this, as the points deviate from the reference line, especially in the tails, indicating non-normality and the presence of extreme values. However, the ACF plot shows that most autocorrelation values fall within the confidence bands, meaning there is no strong autocorrelation left in the residuals. The Ljung–Box p-values remain well above the 0.05 threshold across multiple lags, suggesting that the residuals are not significantly autocorrelated. Overall, the residuals behave mostly like white noise, and the model appears to fit the data well in terms of capturing serial dependence.

7.2 Model_003_11

```
residual.analysis(model = model_003_11, std = TRUE,start = 2, class = "fGARCH")
```

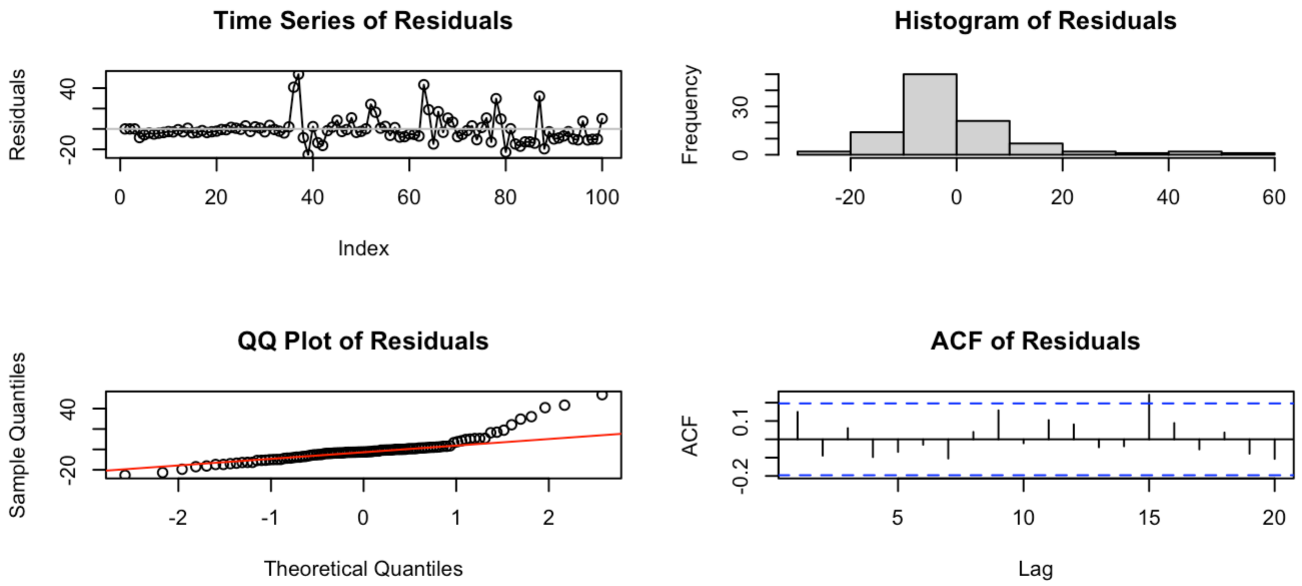


Figure 24: Residual for model_003_11

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.8392, p-value = 4.826e-09
```

```
res <- residuals(model_003_11)
plot_LBQ(res)
```

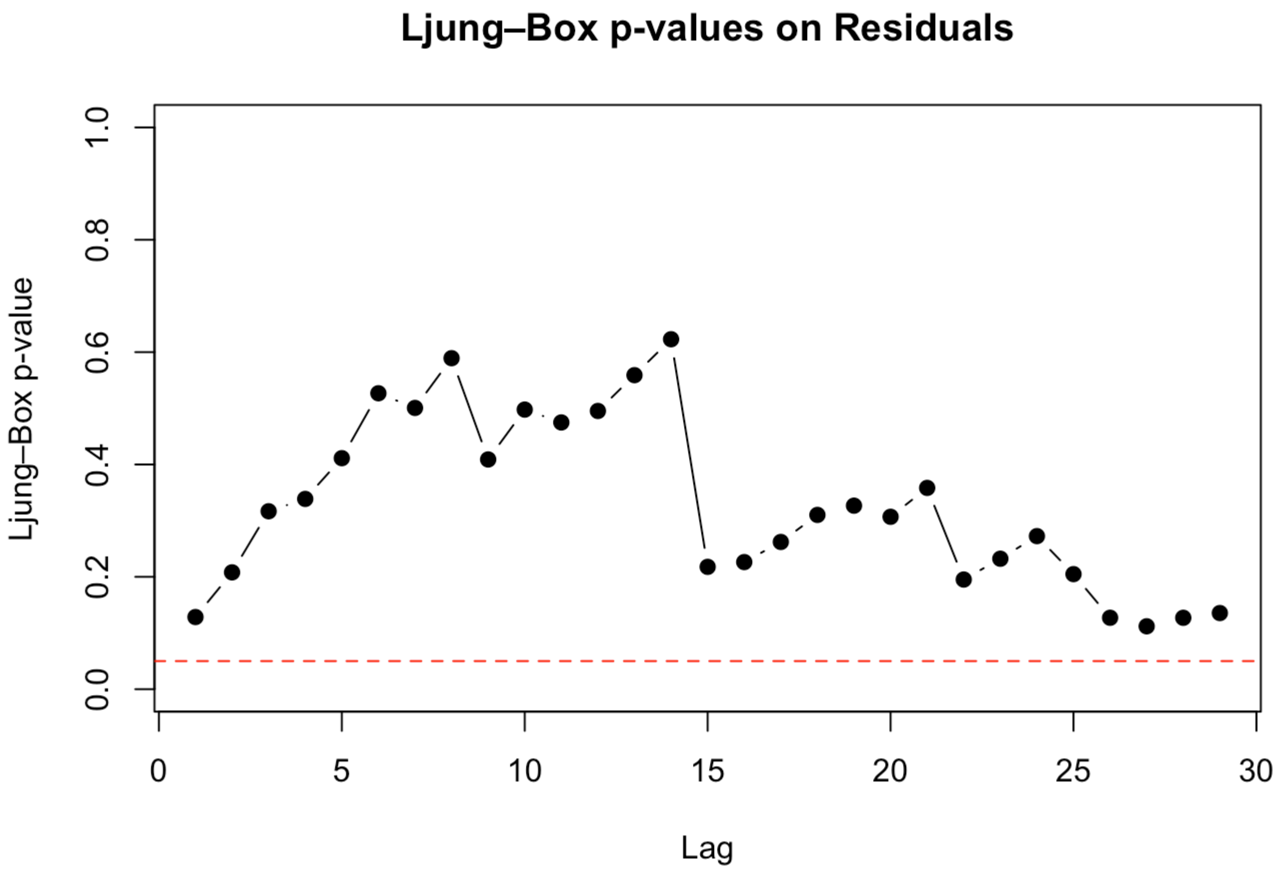


Figure 25: The Ljung–Box test for model_003_11

The residual analysis of model_003_11 shows that the model captures the main structure of the time series, but there are still some concerns. The residual time series plot shows some large spikes, which suggest occasional large errors. The histogram indicates that the distribution of residuals is skewed and not centered around zero. The QQ plot shows a noticeable deviation from the red reference line, especially at the tails, suggesting the residuals are not normally distributed. The ACF plot shows no strong autocorrelation in the residuals, which means the model has captured most of the time dependence. The Ljung-Box p-value plot mostly stays above the red 0.05 line, indicating that the residuals are not significantly autocorrelated. However, a few lower p-values at higher lags may suggest some remaining structure. Overall, the model is reasonable, but the lack of normality in residuals and a few extreme values could affect prediction reliability.

7.3 Model_003_21

```
residual.analysis(model = model_003_21, std = TRUE,start = 2, class = "fGARCH")
```

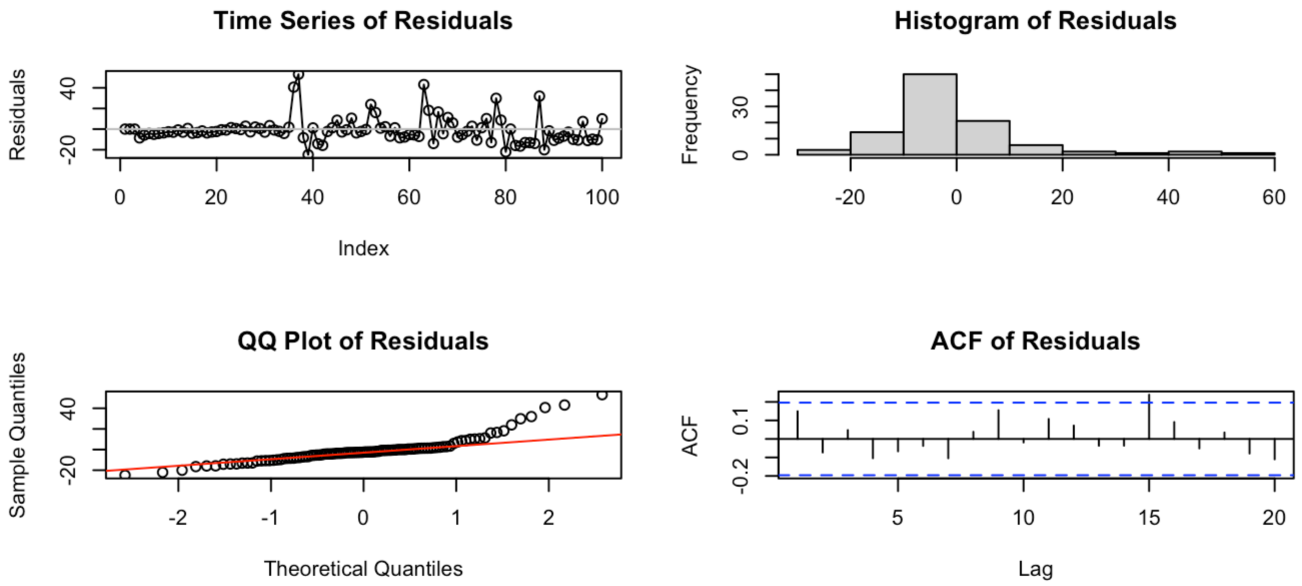


Figure 26: Residual for model_003_21

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.83692, p-value = 4.012e-09
```

```
res <- residuals(model_003_21)
plot_LBQ(res)
```

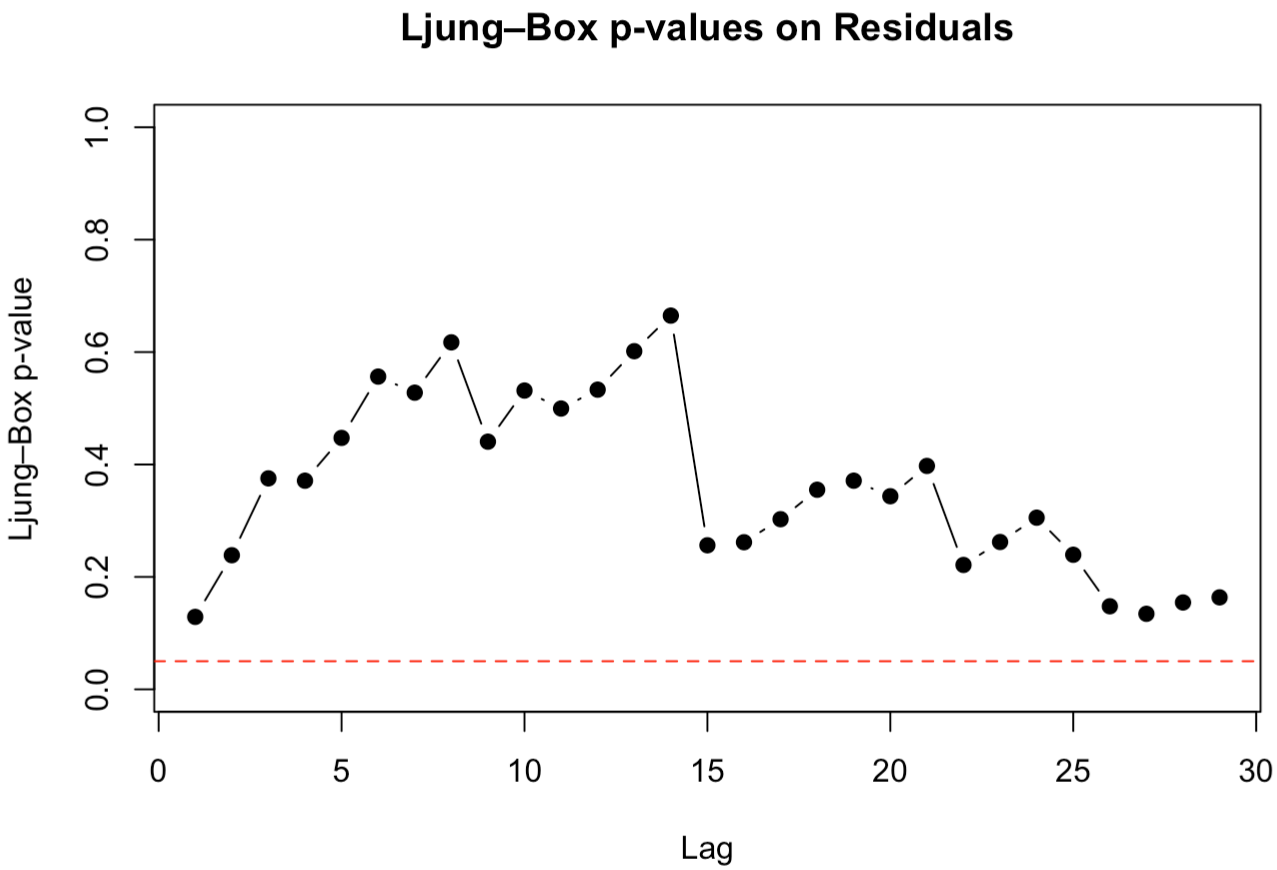


Figure 27: The Ljung–Box test for model_003_21

The residual analysis plots for model_003_21 show that the model is performing reasonably well. The time series plot of residuals shows that most values are centered around zero, although a few sharp spikes are still visible. The histogram shows a right-skewed distribution with a concentration near zero, which indicates some deviation from normality. The QQ plot suggests that the residuals are not perfectly normal, as the points deviate from the straight line at the ends, indicating heavier tails. The ACF plot shows that most autocorrelations are within the blue significance limits, suggesting no strong serial correlation. The Ljung-Box p-value plot supports this, as the values are mostly above the red significance line, meaning the residuals do not show significant autocorrelation. Overall, the model does not suffer from major autocorrelation issues, but the residuals still show some non-normality.

7.4 Model_003_12

```
residual.analysis(model = model_003_12, std = TRUE,start = 2, class = "fGARCH")
```

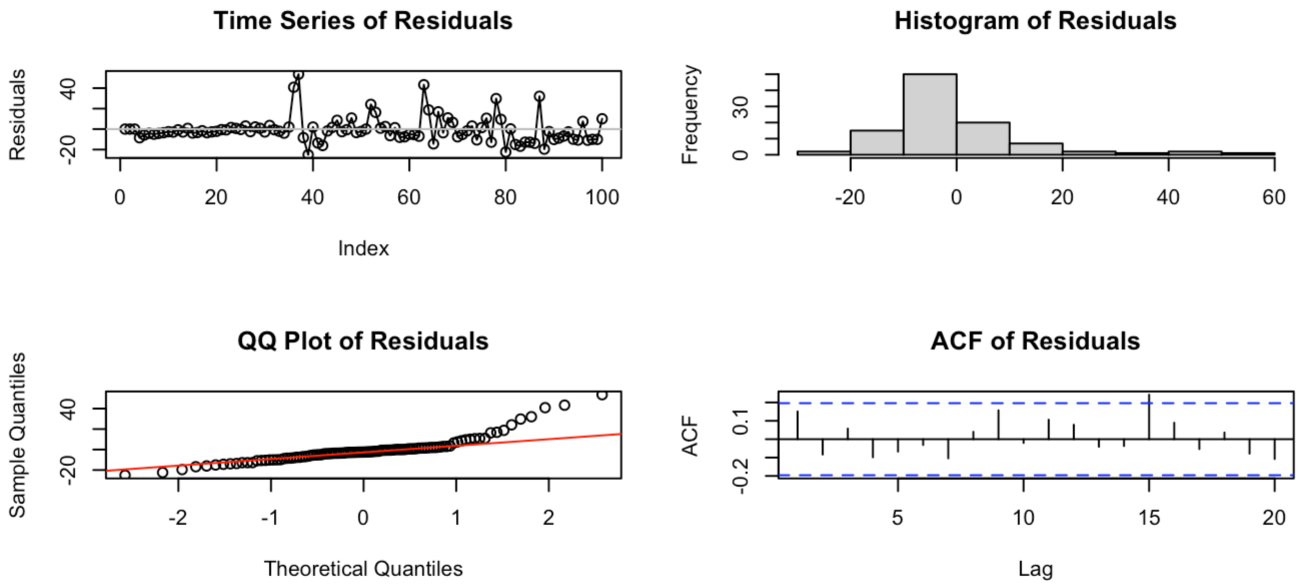


Figure 28: Residual for model_003_12

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.83833, p-value = 4.498e-09
```

```
res <- residuals(model_003_12)
plot_LBQ(res)
```

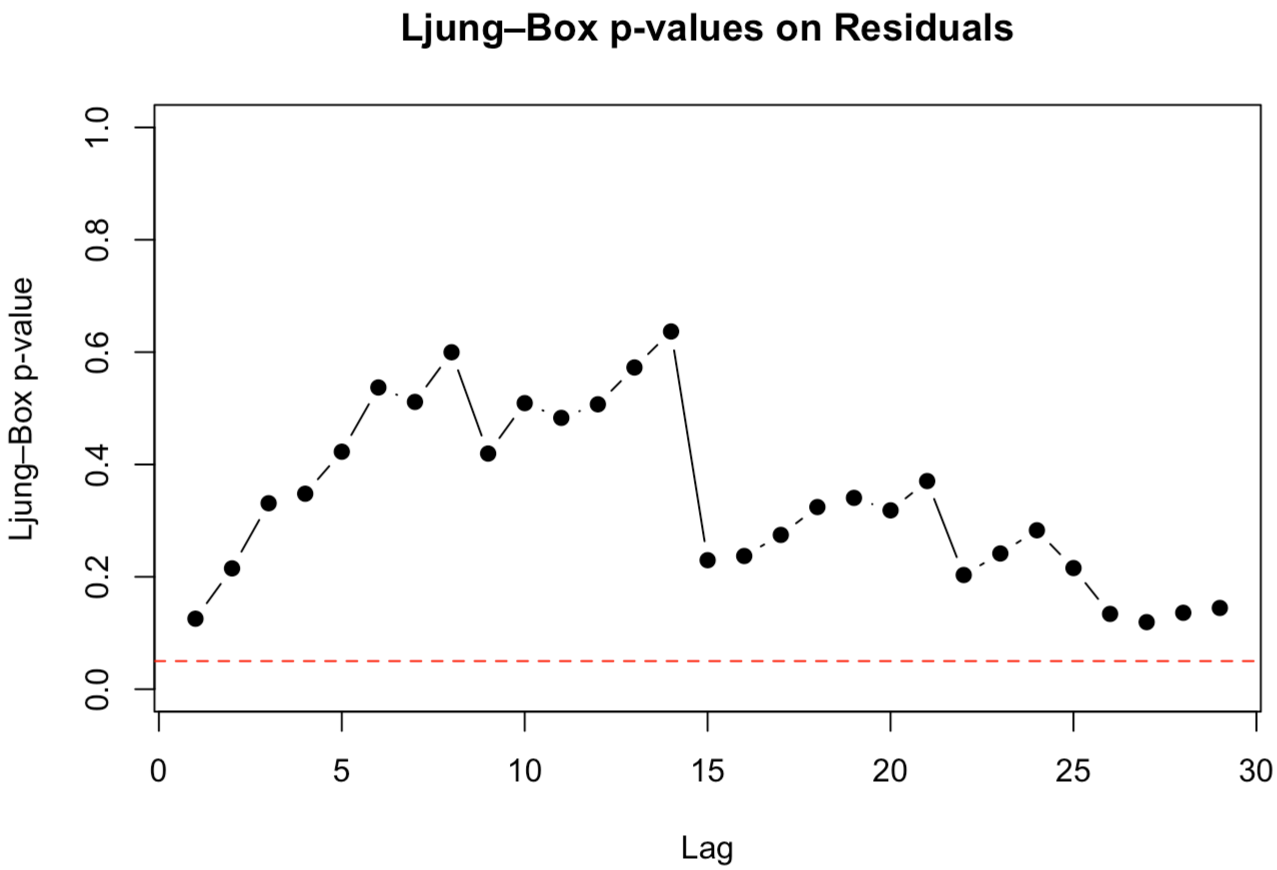


Figure 29: The Ljung–Box test for model_003_21

The residuals of model_003_12 show no strong autocorrelation and are mostly centered around zero. The QQ plot and histogram suggest slight non-normality, especially in the tails. The Ljung–Box p-values are mostly above 0.05, indicating that the model has handled autocorrelation well. Overall, the fit is acceptable with minor deviations from normality.

7.5 Model_003_22

```
residual.analysis(model = model_003_22, std = TRUE,start = 2, class = "fGARCH")
```

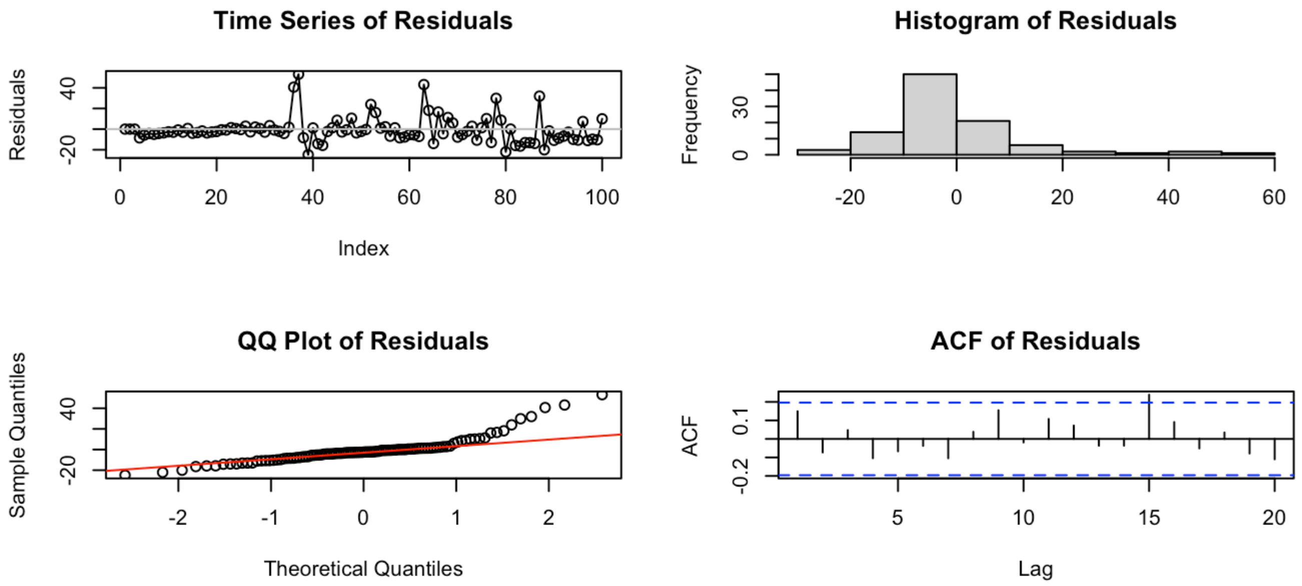


Figure 30: Residual for model_003_22

```
##
##  Shapiro-Wilk normality test
##
## data:  res.model
## W = 0.83692, p-value = 4.012e-09
```

```
res <- residuals(model_003_22)
plot_LBQ(res)
```

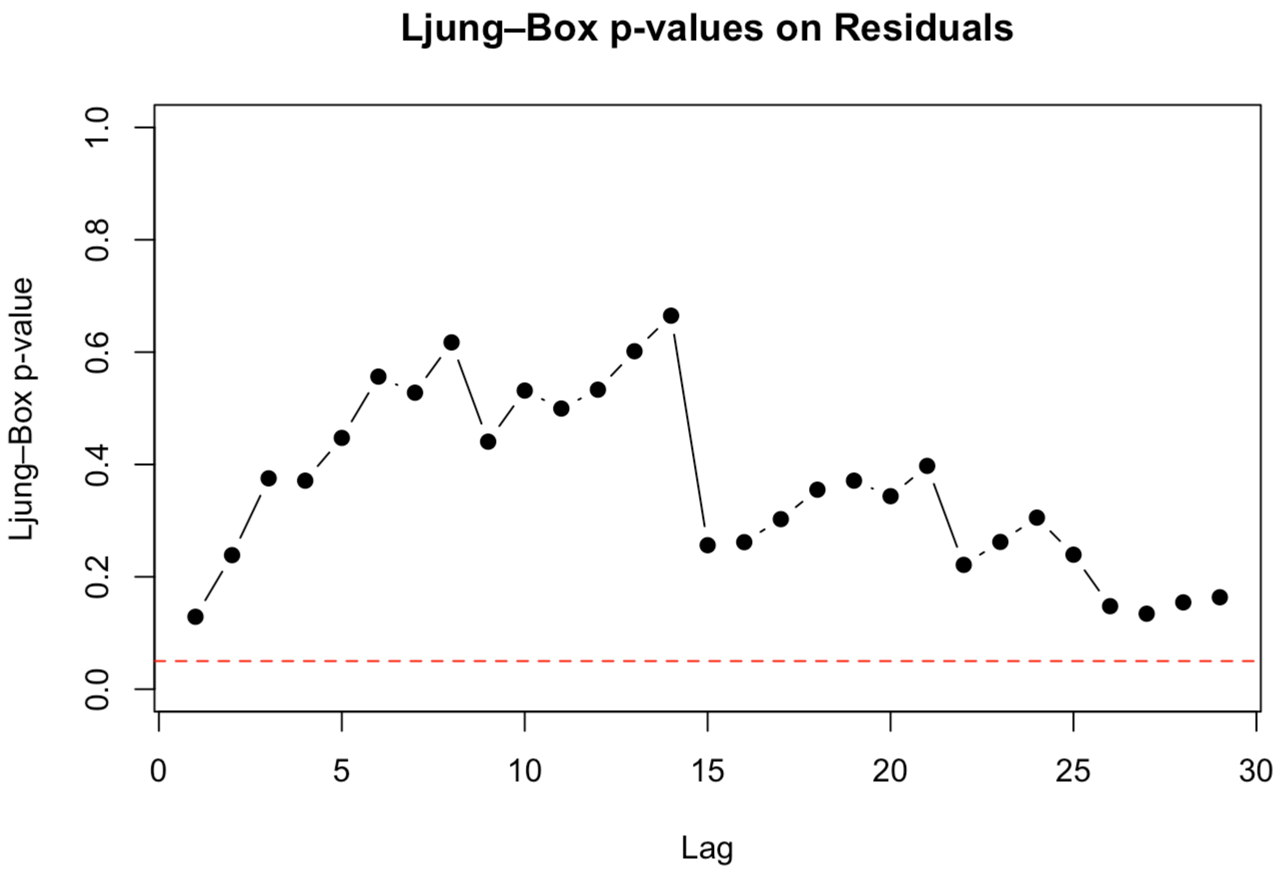


Figure 30: Residual for model_003_22

The residual diagnostics for model_003_22 show that the time series of residuals is mostly stable, but a few large spikes remain. The histogram is slightly skewed, and the QQ plot shows deviations from normality, especially at the tails. The ACF plot reveals no significant autocorrelation, and the Ljung-Box test p-values mostly stay above 0.05, suggesting that there is no strong remaining autocorrelation in the residuals. Overall, the model captures the data well, but the residuals are not perfectly normal.

7.6 AIC and BIC

```
df <- data.frame(
  Model = c("003_0,1", "003_1,1", "003_2,1", "003_1,2", "003_2,2"),
  AIC   = c(
    model_003_01@fit$ics[1],
    model_003_11@fit$ics[1],
    model_003_21@fit$ics[1],
    model_003_12@fit$ics[1],
    model_003_22@fit$ics[1]
  ),
  BIC   = c(
    model_003_01@fit$ics[2],
    model_003_11@fit$ics[2],
    model_003_21@fit$ics[2],
    model_003_12@fit$ics[2],
    model_003_22@fit$ics[2]
  )
)

rownames(df) <- c("ARMA(0,3)+GARCH(0,1)","ARMA(0,3)+GARCH(1,1)",
                  "ARMA(0,3)+GARCH(2,1)","ARMA(0,3)+GARCH(1,2)",
                  "ARMA(0,3)+GARCH(2,2)")
df
```

```
##                        Model      AIC      BIC
## ARMA(0,3)+GARCH(0,1) 003_0,1 7.818815 7.975125
## ARMA(0,3)+GARCH(1,1) 003_1,1 7.823730 8.006092
## ARMA(0,3)+GARCH(2,1) 003_2,1 7.845679 8.054092
## ARMA(0,3)+GARCH(1,2) 003_1,2 7.850809 8.059222
## ARMA(0,3)+GARCH(2,2) 003_2,2 7.865679 8.100144
```

This table compares five GARCH models using AIC and BIC scores. Lower values indicate better model fit. Although model_003_01 had the lowest AIC and BIC values, which typically suggests a better fit, its Ljung-Box p-values drop below the 0.05 threshold at higher lags, indicating that there is still autocorrelation left in the residuals. This means the model may not have fully captured the time-dependent structure of the data. On the other hand, model_003_11, while slightly higher in AIC/BIC, showed better residual diagnostics, its residuals were more independent and passed the Ljung-Box test across lags. Therefore, we chose model_003_11 as it provides a more reliable and well-behaved model overall.

8 Ten-Week Forecast of Google Search Interest in Sabrina Carpenter

```
spec <- ugarchspec(variance.model = list(model = "sGARCH",
                                         garchOrder = c(1, 1)
),
mean.model = list(armaOrder = c(0, 3)))
model_003_11_2 <- ugarchfit(spec = spec, data = r.carpenter,
                            solver = "hybrid",
                            solver.control = list(trace=0))
model_003_11_2
```

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(0,0,3)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error    t value Pr(>|t|)
## mu       0.62814    0.000073  8600.9836 0.000000
## ma1     -0.44233    0.000137 -3228.8833 0.000000
## ma2     -0.22744    0.000107 -2119.8862 0.000000
## ma3     -0.57563    0.000162 -3549.2504 0.000000
## omega   11.09375    4.798657     2.3118 0.020786
## alpha1   0.85425    0.165076     5.1749 0.000000
## beta1    0.14475    0.081757     1.7705 0.076641
##
## Robust Standard Errors:
##         Estimate  Std. Error    t value Pr(>|t|)
## mu       0.62814    0.027436  22.895232  0.00000
## ma1     -0.44233    0.025685 -17.221137  0.00000
## ma2     -0.22744    0.012586 -18.070243  0.00000
## ma3     -0.57563    0.032115 -17.924189  0.00000
## omega   11.09375  542.675804   0.020443  0.98369
## alpha1   0.85425    0.966509   0.883850  0.37678
## beta1    0.14475    5.073849   0.028529  0.97724
##
## LogLikelihood : -362.0745
##
## Information Criteria
## ------------------------------------
##
## Akaike       7.3815
## Bayes        7.5639
## Shibata      7.3725
## Hannan-Quinn 7.4553
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                          statistic   p-value
## Lag[1]                      0.8416 3.589e-01
## Lag[2*(p+q)+(p+q)-1][8]     8.1323 5.687e-07
## Lag[4*(p+q)+(p+q)-1][14]   16.4454 4.342e-04
## d.o.f=3
## H0 : No serial correlation
##
```

```
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                     0.06428  0.7999
## Lag[2*(p+q)+(p+q)-1][5]    0.24627  0.9889
## Lag[4*(p+q)+(p+q)-1][9]    2.16473  0.8845
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]   0.01417 0.500 2.000  0.9052
## ARCH Lag[5]   0.37312 1.440 1.667  0.9203
## ARCH Lag[7]   0.50946 2.315 1.543  0.9775
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  13.4937
## Individual Statistics:
## mu     0.07177
## ma1    0.07156
## ma2    0.07155
## ma3    0.07159
## omega  1.43807
## alpha1 0.35410
## beta1  0.20854
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.69 1.9 2.35
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                     t-value   prob sig
## Sign Bias           0.1046 0.9169
## Negative Sign Bias  1.2046 0.2314
## Positive Sign Bias  0.3992 0.6907
## Joint Effect        3.7209 0.2932
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20      22.4       0.2648
## 2    30      33.2       0.2697
## 3    40      49.6       0.1190
## 4    50      56.0       0.2289
##
##
## Elapsed time : 0.155477
```

This code fits the model to the time series, r.carpenter. It tells R to use a GARCH(1,1) model to capture changing variance over time, and a moving average model with 3 lags (MA(3)) for the average level or trend in the data. The ugarchspec() part sets up the model, and the ugarchfit() part runs it using a method called "hybrid" to make sure it finds good values. The result is stored in model_003_11_2, which contains all the fitted model details.

```
frc <- ugarchforecast(model_003_11_2,n.ahead=10,data=r.carpenter)
frc
```

```
##
## *------------------------------------*
## *       GARCH Model Forecast         *
## *------------------------------------*
## Model: sGARCH
## Horizon: 10
## Roll Steps: 0
## Out of Sample: 0
##
## 0-roll forecast [T0=1970-04-11]:
##       Series  Sigma
## T+1   9.2122  6.437
## T+2   6.6657  7.245
## T+3  -1.1632  7.970
## T+4   0.6281  8.635
## T+5   0.6281  9.251
## T+6   0.6281  9.828
## T+7   0.6281 10.372
## T+8   0.6281 10.889
## T+9   0.6281 11.382
## T+10  0.6281 11.853
```

The forecast for the next 10 weeks shows a short spike at the beginning, with a peak in the first week (value around 9.2), followed by a drop in the second week (around 6.7), and then a dip into negative territory in the third week. From the fourth week onward, the forecast levels off and stays steady at around 0.63. At the same time, the predicted uncertainty (measured by sigma) gradually increases each week, starting from 6.4 in the first week and reaching about 11.9 by the tenth week. This means the values are expected to stay stable, but with growing variability as time goes on.

```
plot(frc, which = 1)
```

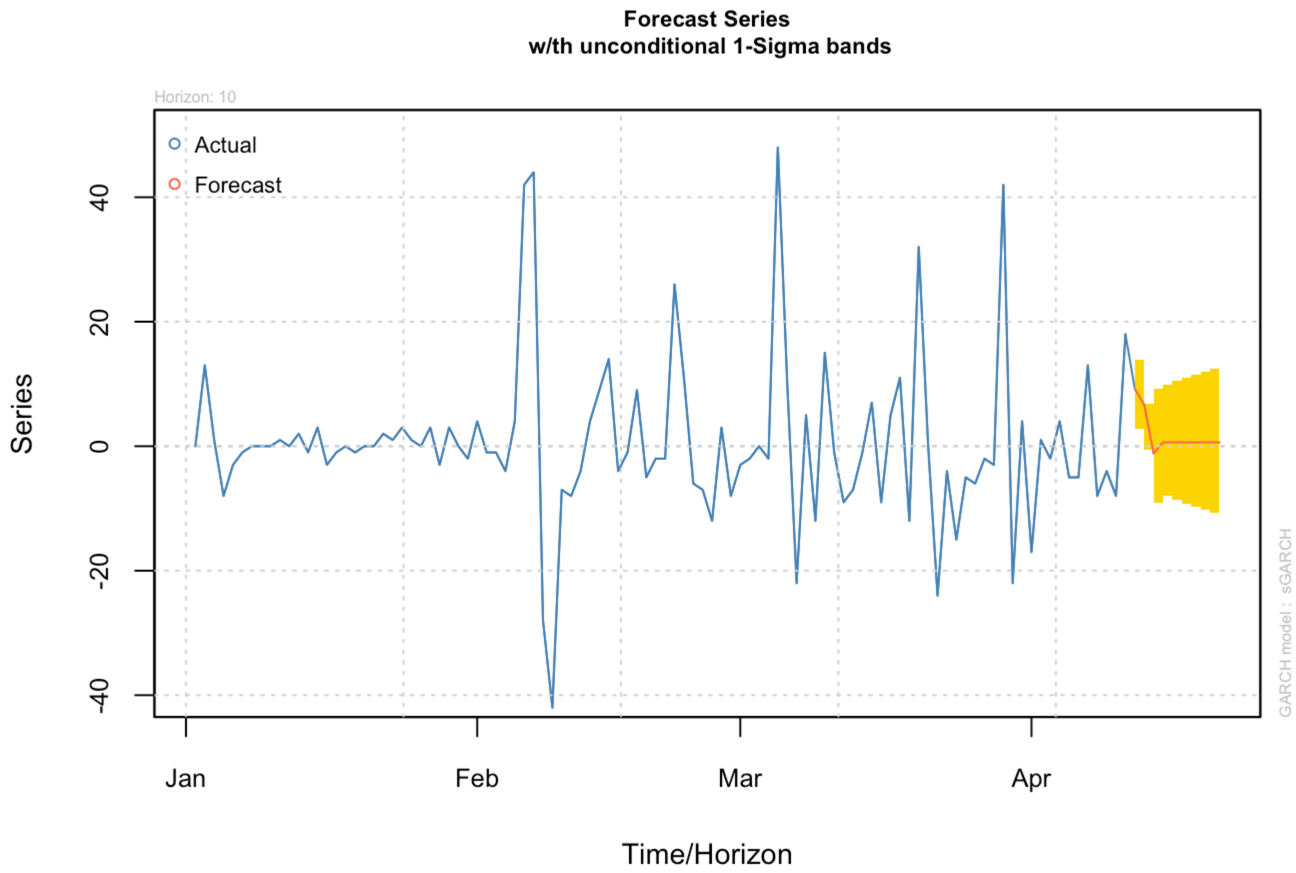


Figure 31

The chart shows the forecast of the time series for the next 10 weeks using a GARCH model. The solid red line shows the predicted values, while the yellow shaded area represents the uncertainty or possible range of values (1 standard deviation above and below the forecast). The forecast suggests that the values will stay relatively stable without large spikes, and the uncertainty slightly increases each week. This means the model expects moderate changes ahead, but allows for some variation.

```
plot(frc, which = 3)
```

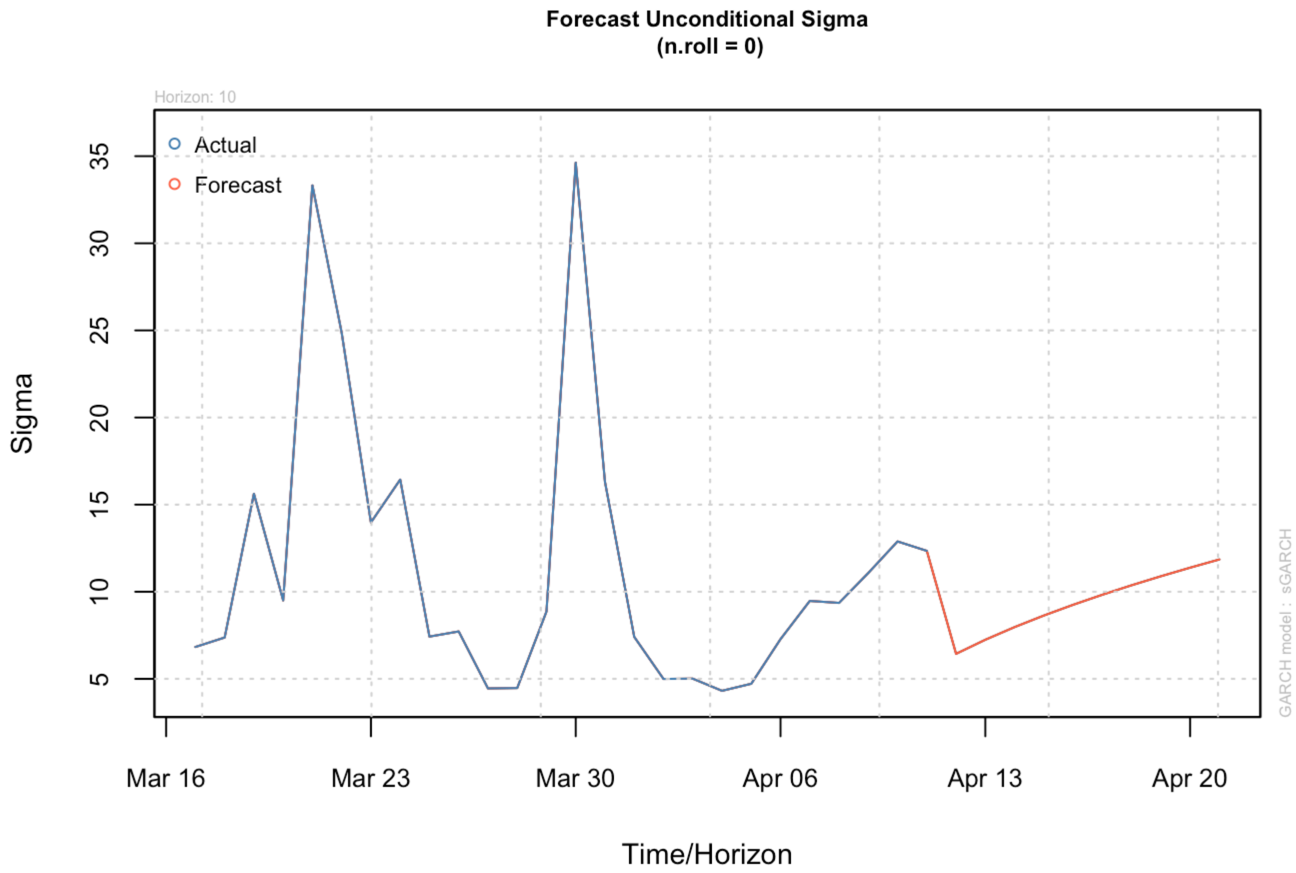


Figure 32

This chart shows the forecast of volatility (sigma) for the next 10 weeks using a GARCH model. The blue line shows past volatility, while the red line shows the forecast. The recent past shows fluctuations, and the forecast indicates a gradual increase in volatility going forward. This means that although the current volatility is low, it is expected to slowly rise in the coming weeks.

9 Conclusion

Based on the analysis and forecasts, we can conclude that Sabrina Carpenter's Google Trends data shows significant shifts in public interest over time, with a clear turning point in early 2024 following major public appearances and the release of her hit single Espresso. The time series showed non-constant variance and clustering of volatility, which justified the use of GARCH modelling. In this analysis, we explored several GARCH models to capture the volatility and behavior of the Carpenter series. While model_003_01 had the lowest AIC and BIC scores, further residual diagnostics,particularly the Ljung-Box test, showed signs of remaining autocorrelation, suggesting that it did not fully model the structure of the data. In contrast, model_003_11, although slightly higher in AIC/BIC, produced well-behaved residuals with no significant autocorrelation. Its forecasts and sigma plots were also stable and interpretable. Based on this, model_003_11 was selected as the final model for forecasting, offering a more balanced and statistically sound fit for the series.

10 References

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3. Google Trends. (2024). Google’s Year in Search. [online] Available at: https://trends.withgoogle.com/year-in-search/2024/gb/ (https://trends.withgoogle.com/year-in-search/2024/gb/).