CVPR 2014 Visual SLAM Tutorial
Efficient Inference

Michael Kaess

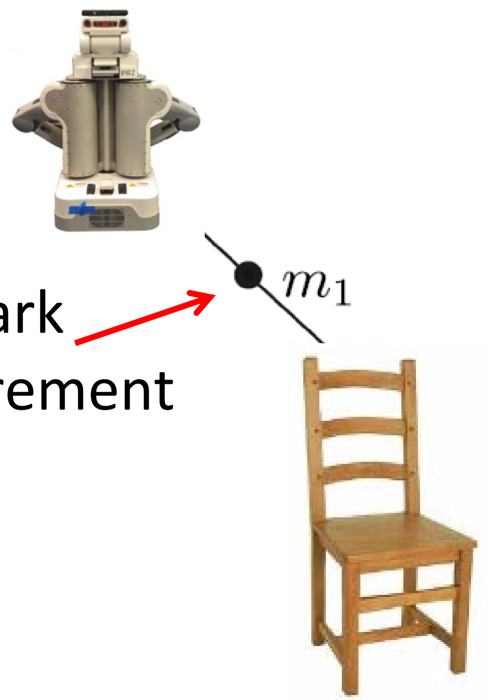
kaess@cmu.edu

The Robotics Institute

Carnegie Mellon University

The Mapping Problem (t=0)

Robot



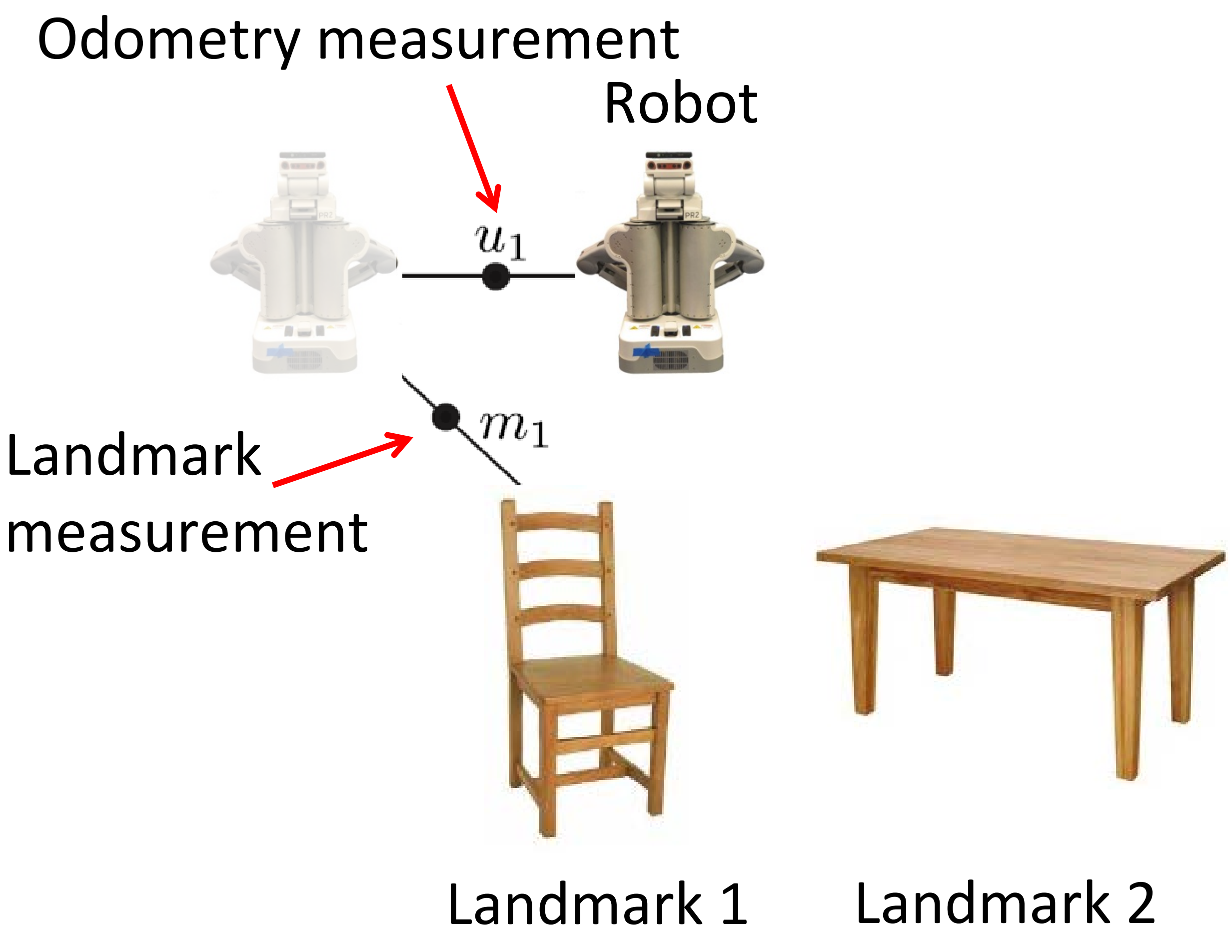
Landmark
Measurement

Landmark

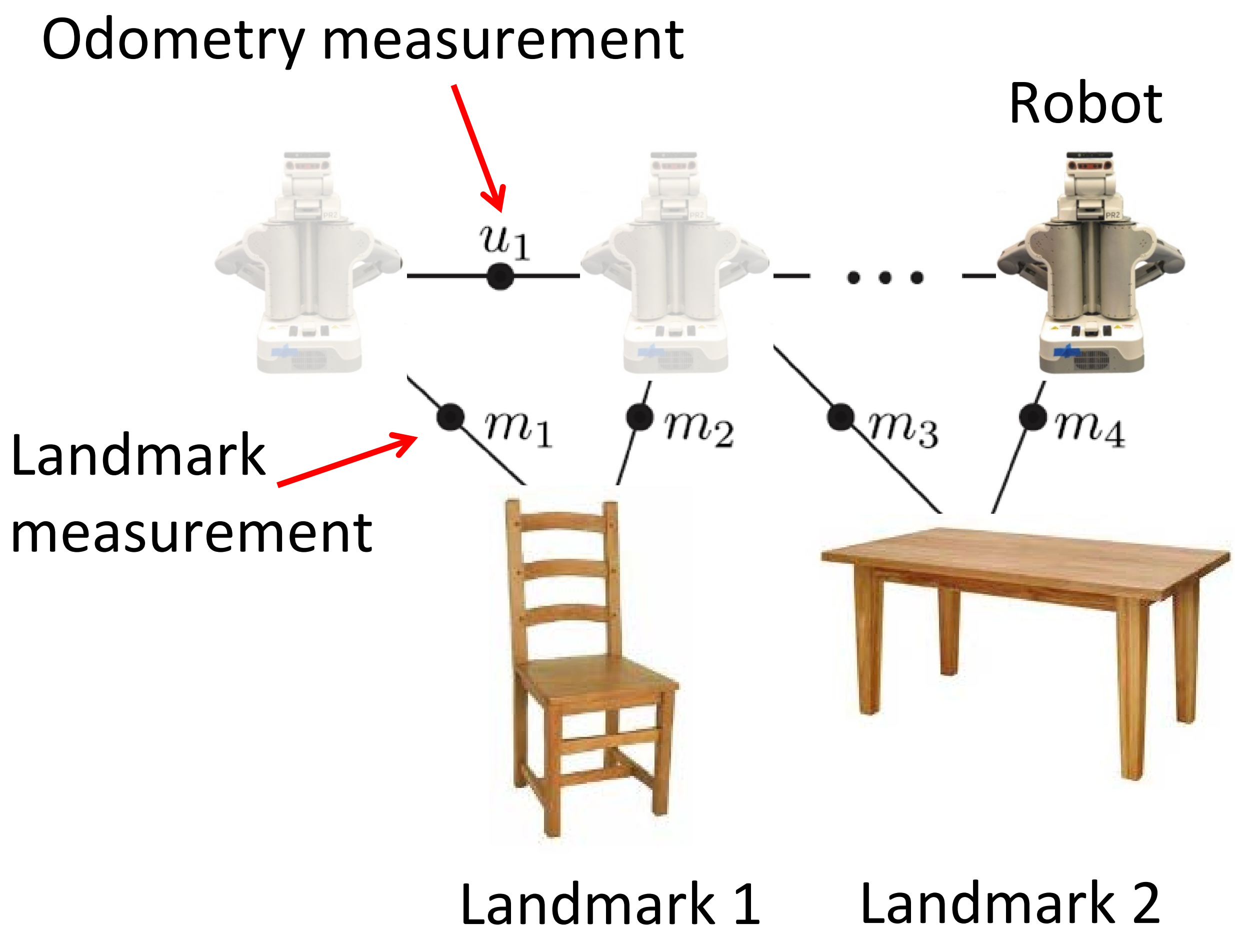
Onboard sensors:

- Wheel odometry
- Inertial measurement unit (gyro, accelerometer)
- Sonar
- Laser range finder
- Camera
- RGB-D sensors

The Mapping Problem (t=1)

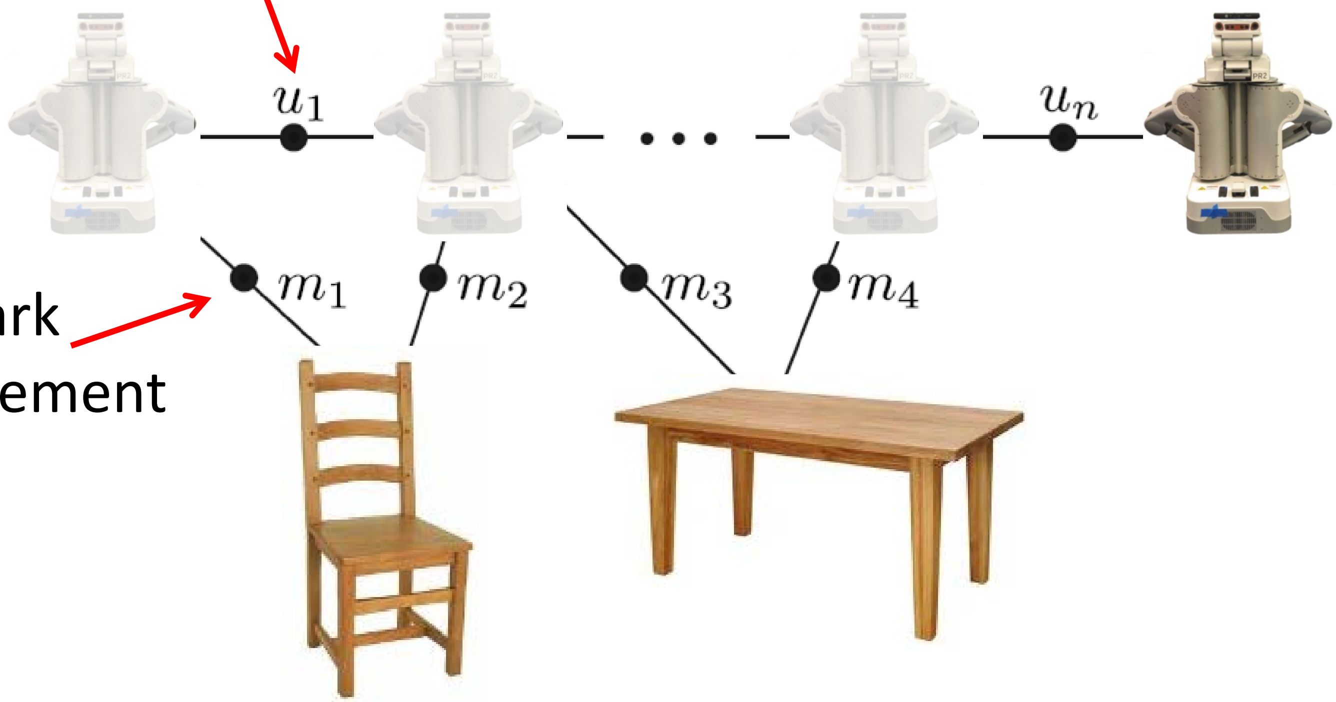


The Mapping Problem (t=n-1)



The Mapping Problem (t=n)

Odometry measurement

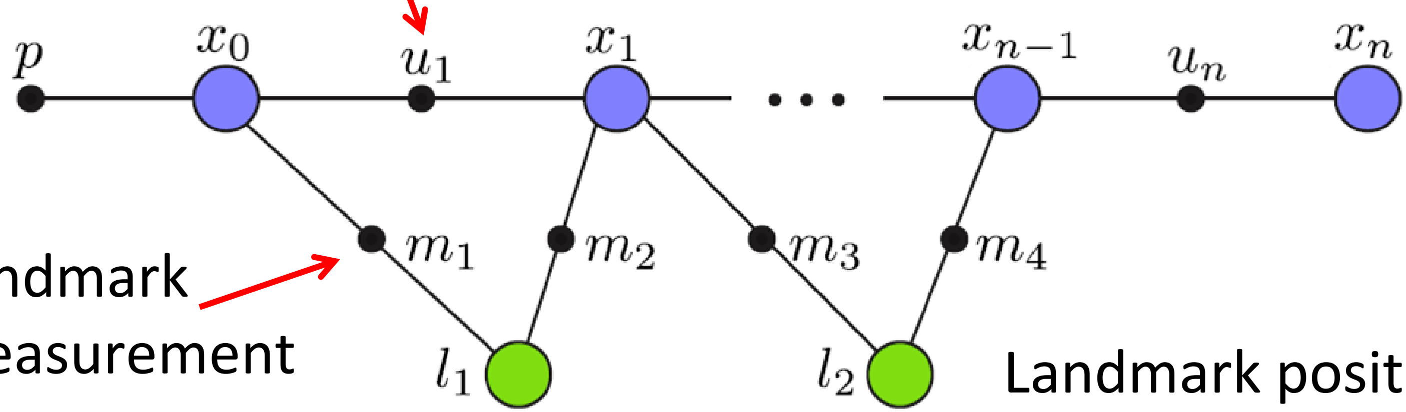


Landmark measurement

Mapping problem is incremental !

Factor Graph Representation

Odometry measurement

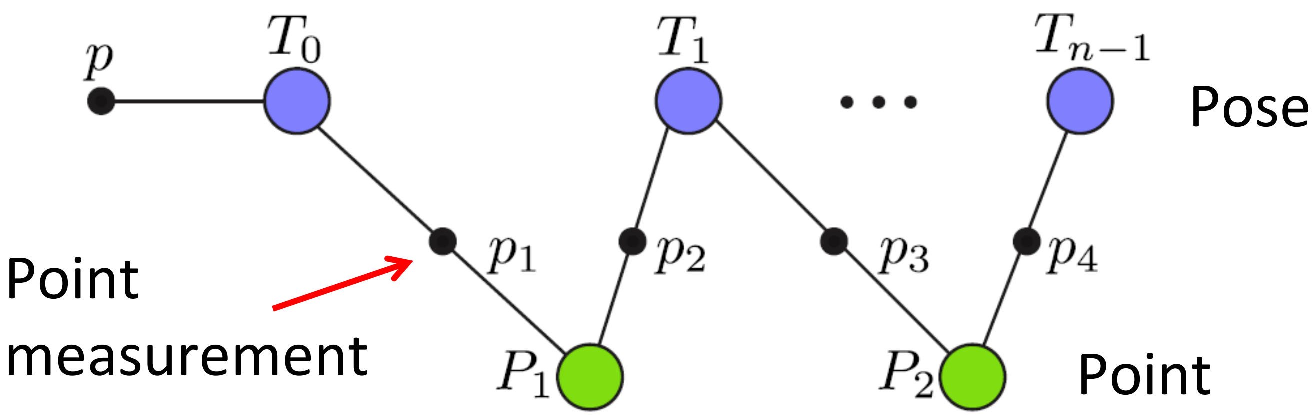


Robot pose

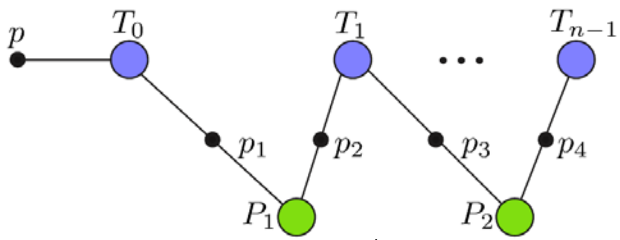
Landmark measurement

Landmark position

Factor Graph Representation

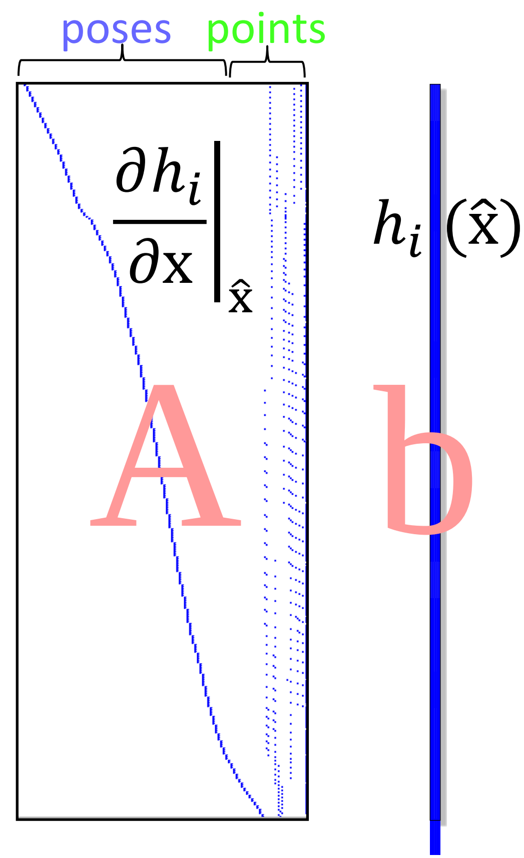


Nonlinear Least-Squares



Gaussian noise

$$\operatorname{argmin}_{\mathrm{x}} \sum_i \|h_i(\mathrm{x})\|_{\Xi}^{\;2}$$



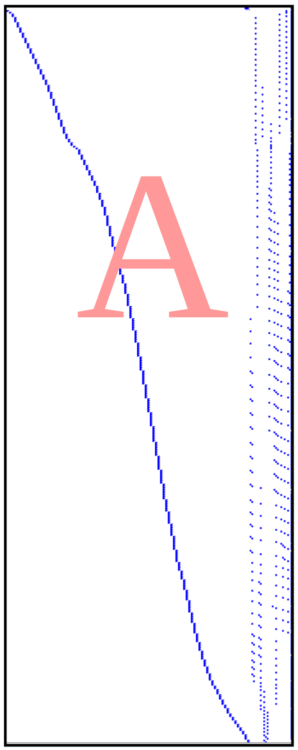
Repeatedly solve linearized system (GN)

$$\operatorname{argmin}_x \|Ax - b\|^2$$

$$A = \begin{bmatrix} F_{11} & & G_{11} & & \\ F_{12} & & & G_{12} & \\ F_{13} & & & & G_{13} \\ & F_{21} & G_{21} & & \\ & F_{22} & & G_{22} & \\ & F_{23} & & & G_{23} \end{bmatrix}, \quad x = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}, \quad b = \begin{bmatrix} b_{11} \\ b_{12} \\ b_{13} \\ b_{14} \\ b_{15} \\ b_{16} \end{bmatrix}$$

Solving the Linear Least-Squares System

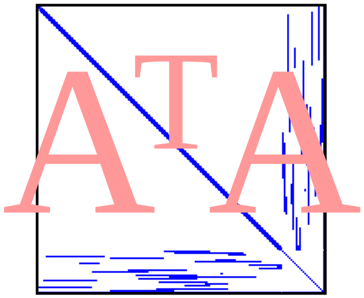
Solve: $\operatorname{argmin}_x \|Ax - b\|^2$



Measurement Jacobian

Normal equations

$$A^T A x = A^T b$$



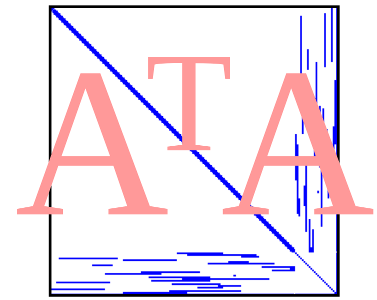
Information matrix

Solving the Linear Least-Squares System

- Can we simply invert A^TA to solve for x?

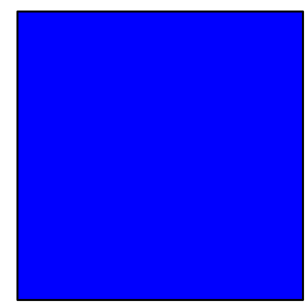
Normal equations

$$A^T Ax = A^T b$$



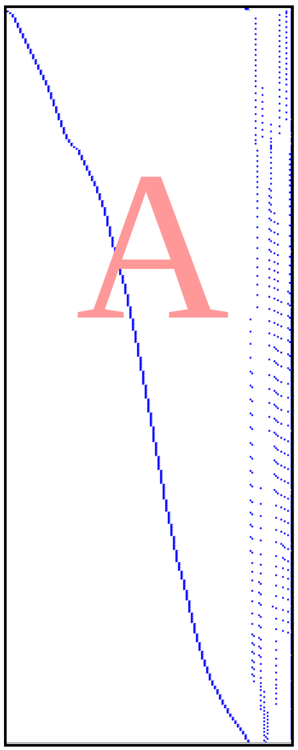
Information matrix

- Yes, but we shouldn't…
  The inverse of A^TA is dense -> O(n^3)
- Can do much better by taking advantage of sparsity!



Solving the Linear Least-Squares System

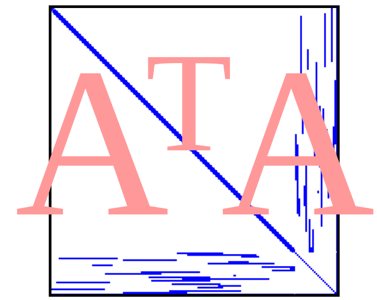
Solve: $\operatorname{argmin}_x \|Ax - b\|^2$



Measurement Jacobian

Normal equations

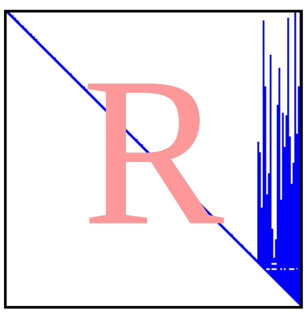
$$A^T A x = A^T b$$



Information matrix

Matrix factorization

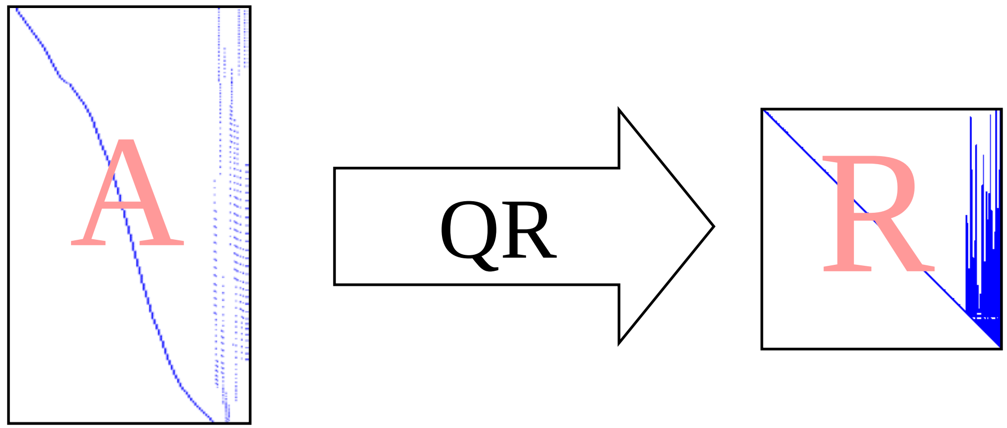
$$A^T A = R^T R$$



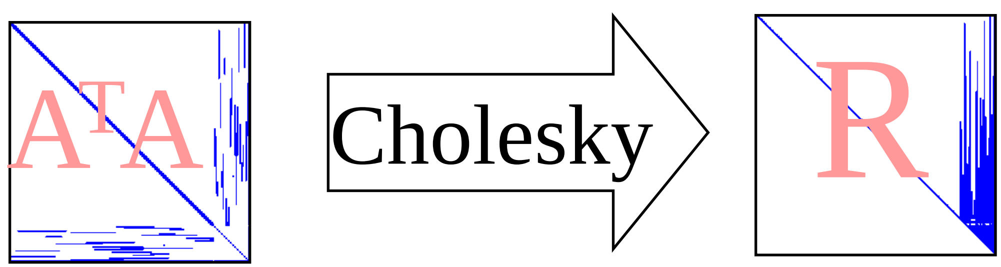
Square root information matrix

Matrix – Square Root Factorization

- QR on A: Numerically More Stable



- Cholesky on A^TA: Faster

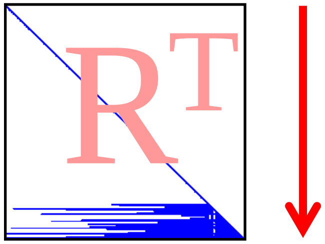


Solving by Backsubstitution

After factorization: $R^T R\ x = A^T b$

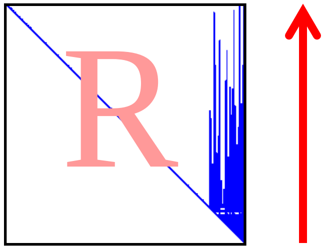
- Forward substitution

  $R^T y = A^T b$, solve for y

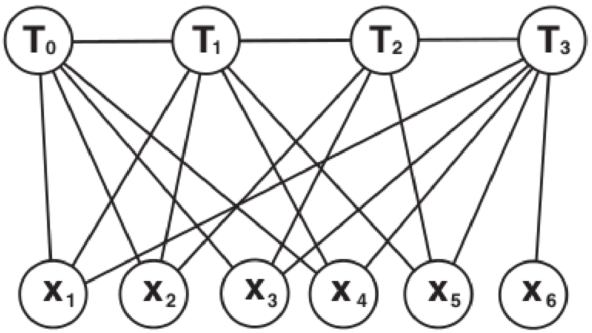


- Backsubstition

  $R\ x = y$, solve for x



Full Bundle Adjustment

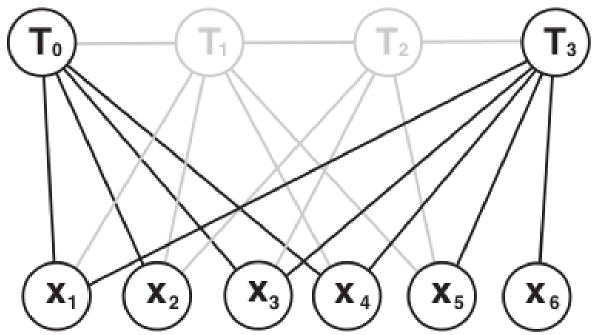


From Strasdat et al, 2011 IVC
"Visual SLAM: Why filter?"

- Graph grows with time:
  - Have to solve a sequence of increasingly larger BA problems
  - Will become too expensive even for sparse Cholesky

F. Dellaert and M. Kaess, "Square Root SAM: Simultaneous localization and mapping via square root information smoothing," IJRR 2006

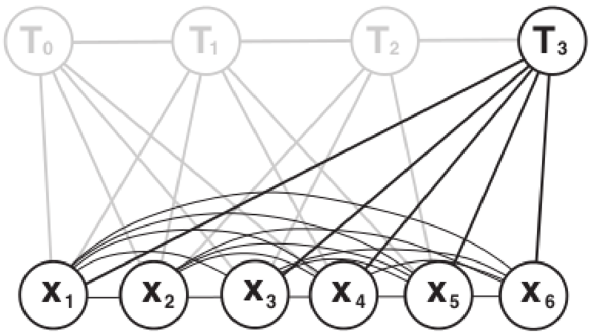
Keyframe Bundle Adjustment



- Drop subset of poses to reduce density/complexity
- Only retain "keyframes" necessary for good map

- Complexity still grows with time, just slower

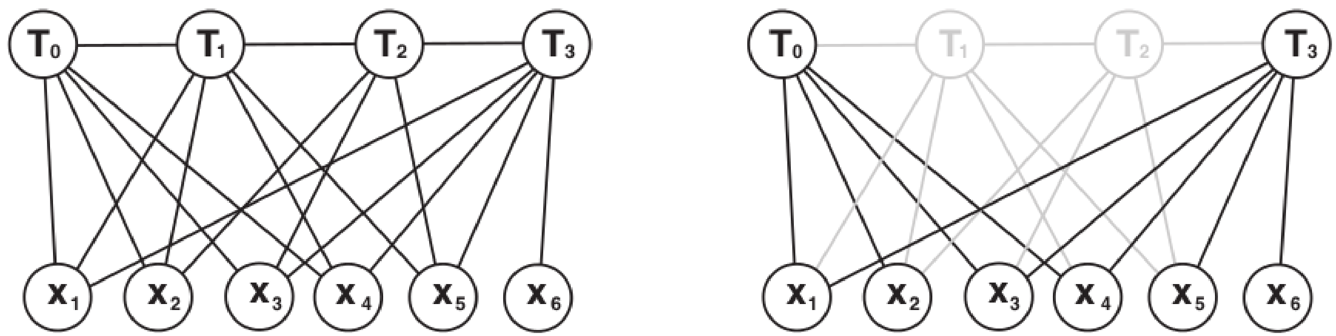
Filter



- Keyframe idea not applicable: map would fall apart
- Instead, marginalize out previous poses
  - Extended Kalman Filter (EKF)
- Problems when used for Visual SLAM:
  - All points become fully connected -> expensive
  - Relinearization not possible -> inconsistent

Incremental Solver

- Back to full BA and keyframes:



- New information is added to the graph
- Older information does not change
- Can be exploited to obtain an efficient solution!

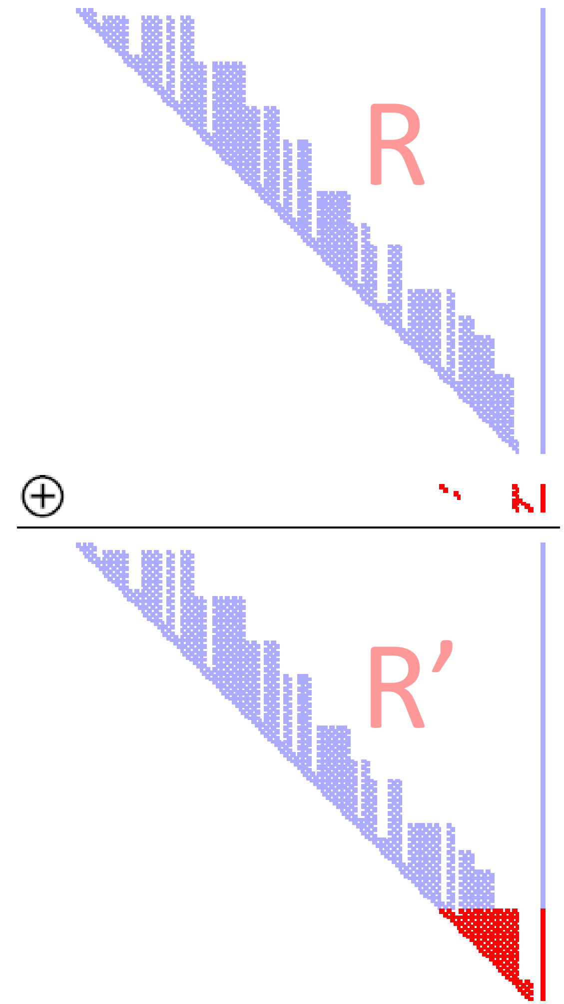
Incremental Smoothing and Mapping (iSAM)

Solving a growing system:

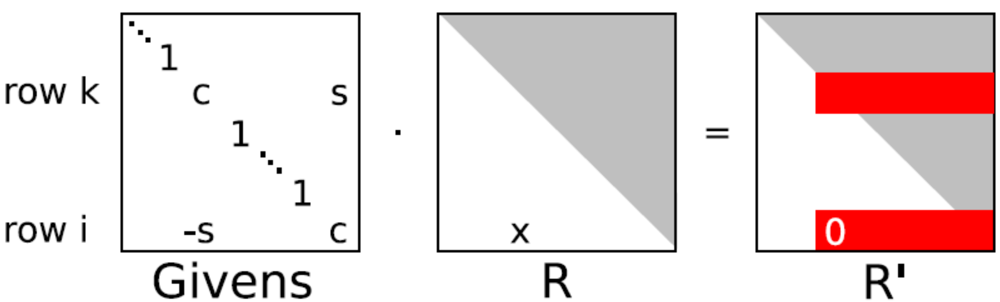
- R factor from previous step
- How do we add new measurements?

Key idea:

New measurements ->



- Append to existing matrix factorization
- “Repair” using Givens rotations



[Kaess et al., TRO 08]

Incremental Smoothing and Mapping (iSAM)

Update and solution are O(1)

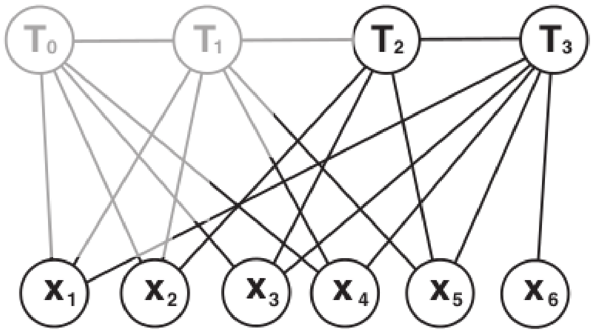
Are we done?

BA is nonlinear…

iSAM requires periodic batch factorization to relinearize

Not O(1), we need iSAM2!

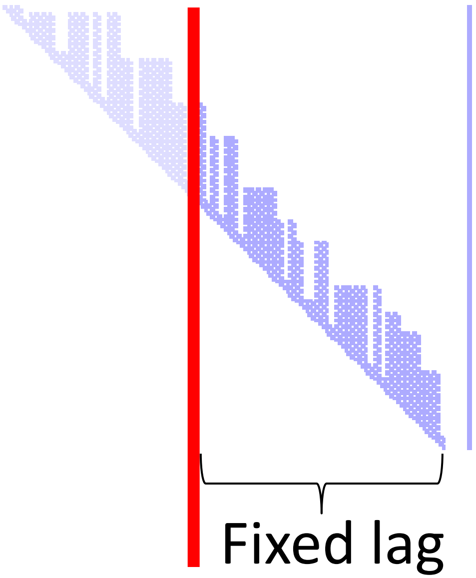
Fixed-lag Smoothing



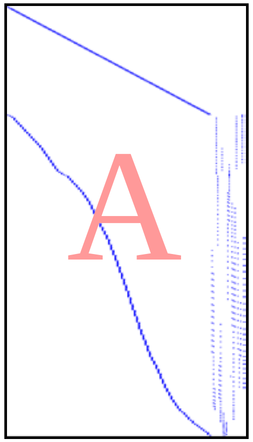
plus some fillin

- Marginalize out all but last n poses and connected landmarks
  - Relinearization possible
- Linear case
- Nonlinear: need iSAM2

Cut here to marginalize

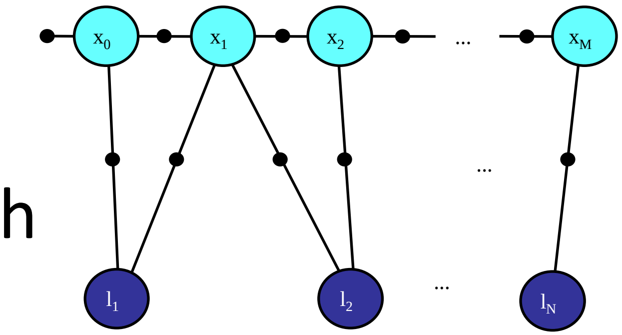


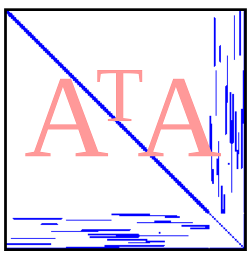
Matrix vs. Graph



Measurement Jacobian

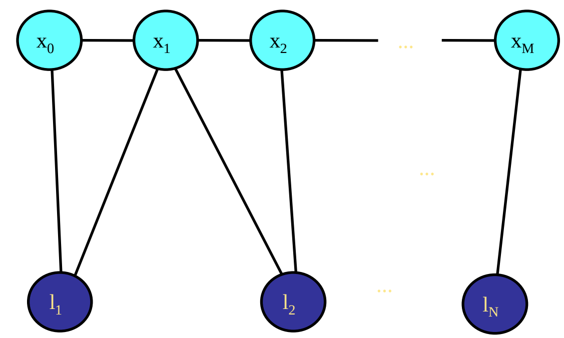
Factor Graph

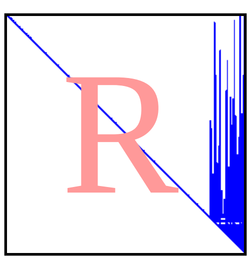




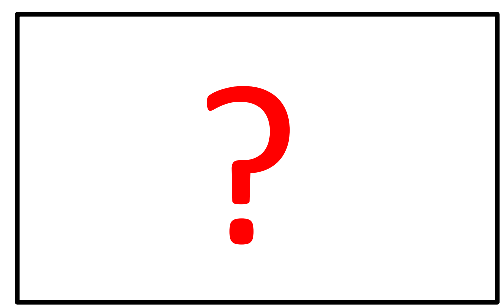
Information Matrix

Markov Random Field



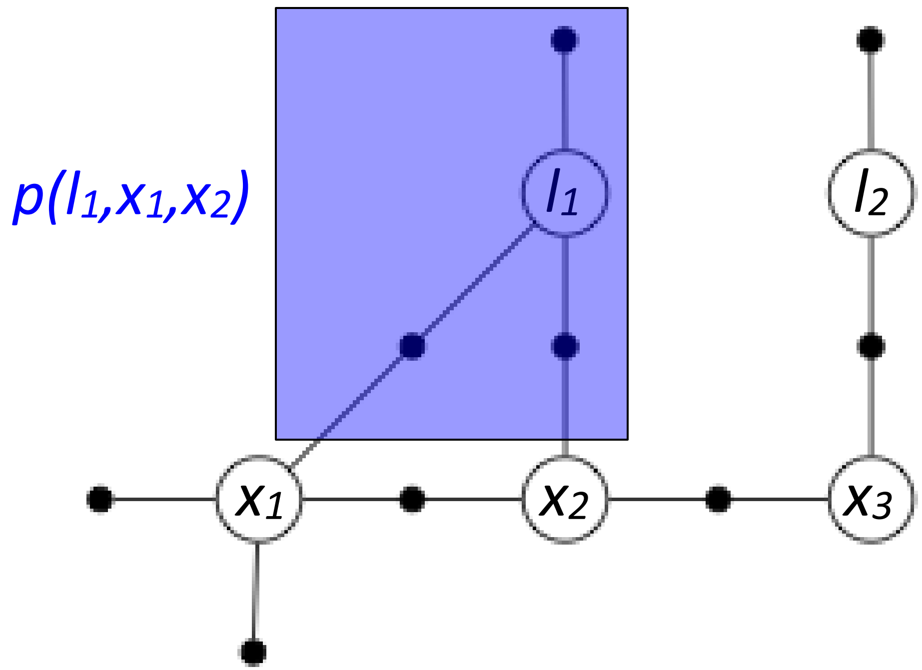


Square Root Inf. Matrix



iSAM2: Variable Elimination – Small Example

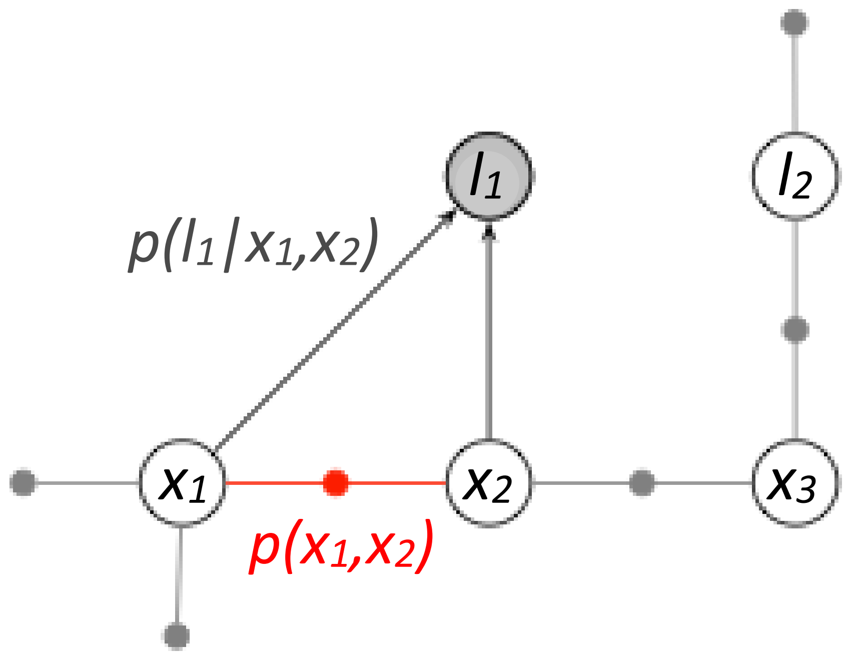
- Choose ordering: l_1, l_2, x_1, x_2, x_3
- Eliminate one node at a time



$$p(l_1,x_1,x_2) = p(l_1|x_1,x_2)\, p(x_1,x_2)$$

iSAM2: Variable Elimination – Small Example

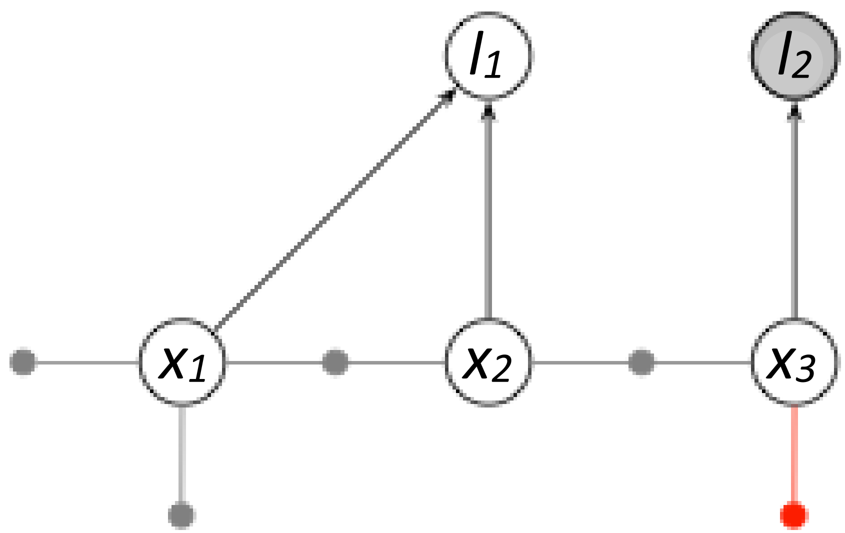
- Choose ordering: l_1, l_2, x_1, x_2, x_3
- Eliminate one node at a time



$$p(l_1,x_1,x_2) = p(l_1|x_1,x_2)\, p(x_1,x_2)$$

iSAM2: Variable Elimination – Small Example

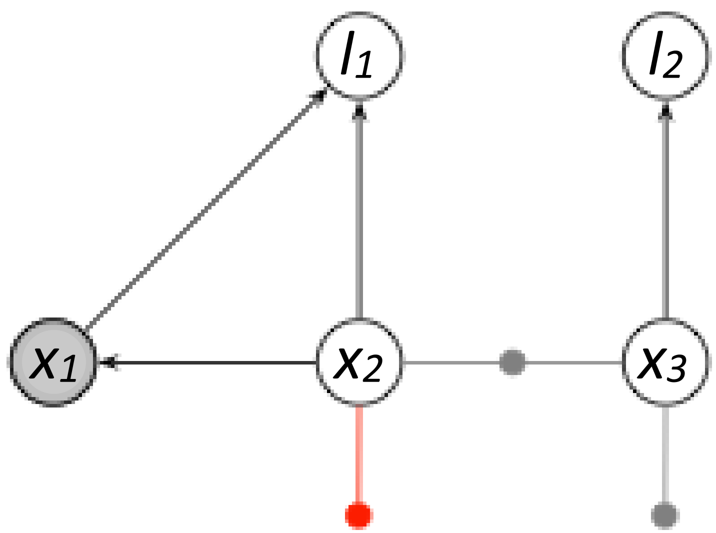
- Choose ordering: l_1, l_2, x_1, x_2, x_3
- Eliminate one node at a time



$p(l_2,x_3) = p(l_2|x_3)\, p(x_3)$

iSAM2: Variable Elimination – Small Example

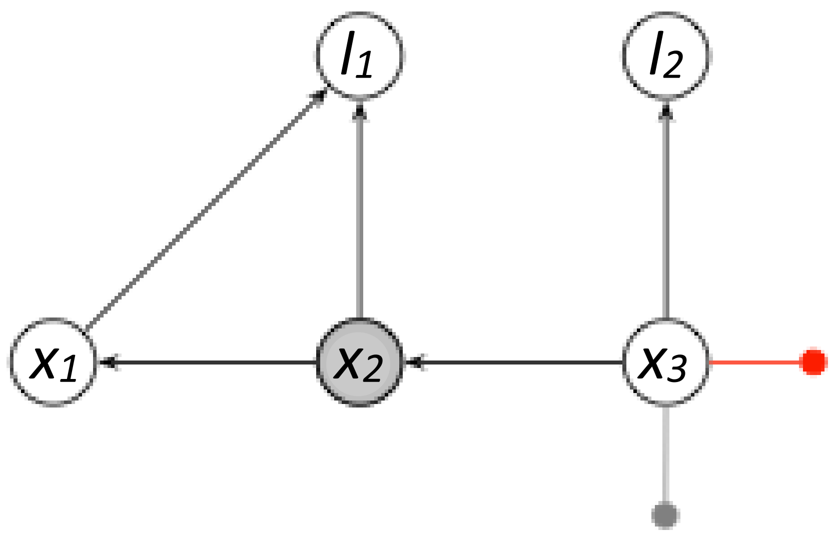
- Choose ordering: l_1, l_2, x_1, x_2, x_3
- Eliminate one node at a time



$$p(x_1,x_2) = p(x_1|x_2)\,p(x_2)$$

iSAM2: Variable Elimination – Small Example

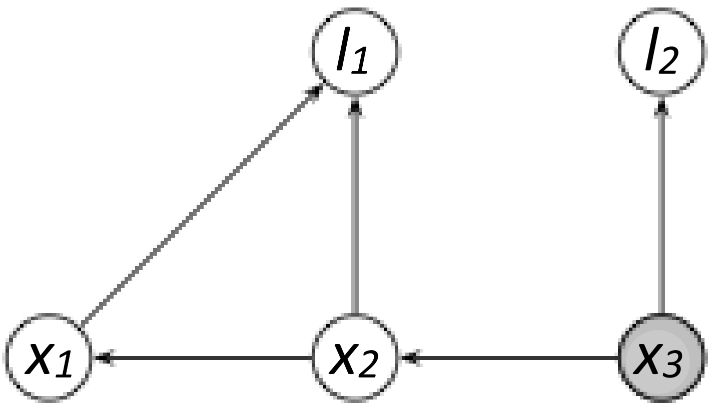
- Choose ordering: l_1, l_2, x_1, x_2, x_3
- Eliminate one node at a time



$$p(x_2,x_3) = p(x_2|x_3)\, p(x_3)$$

iSAM2: Variable Elimination – Small Example

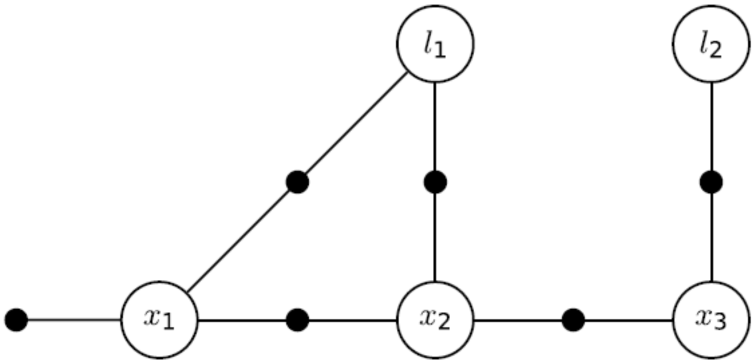
- Choose ordering: l_1, l_2, x_1, x_2, x_3
- Eliminate one node at a time



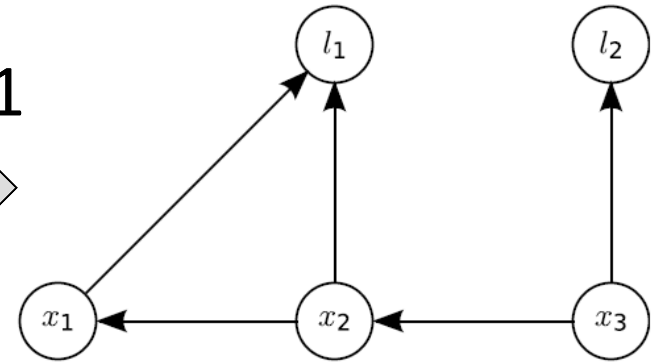
$p(x_3)$

[Kaess et al., IJRR 12]

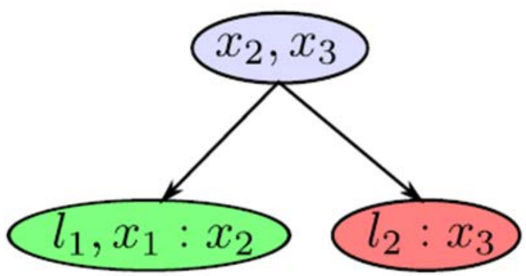
iSAM2: Bayes Tree Data Structure

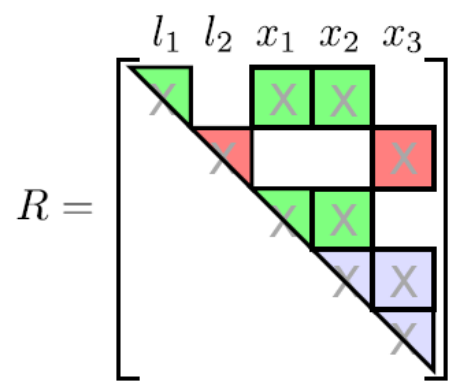


Step 1



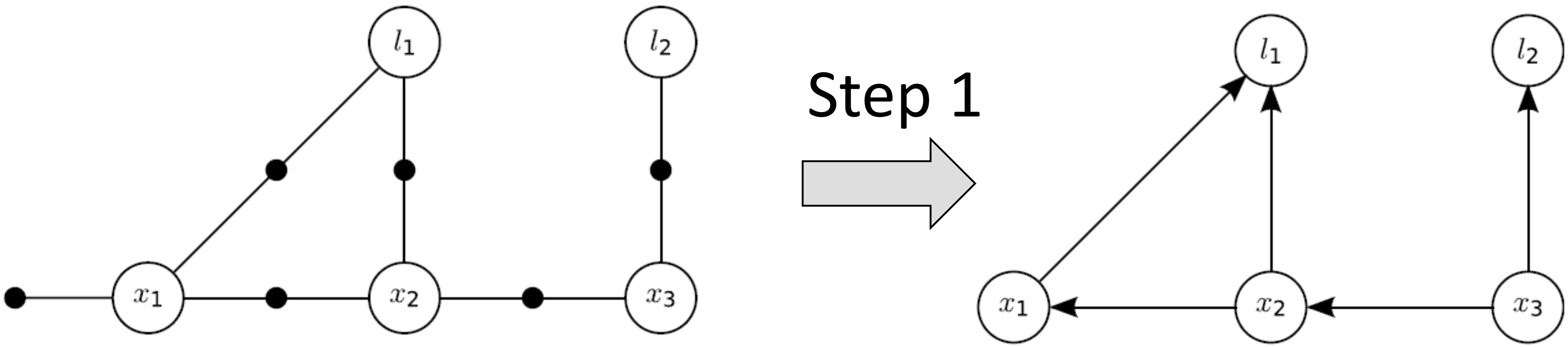
Step 2: Find cliques in reverse elimination order:



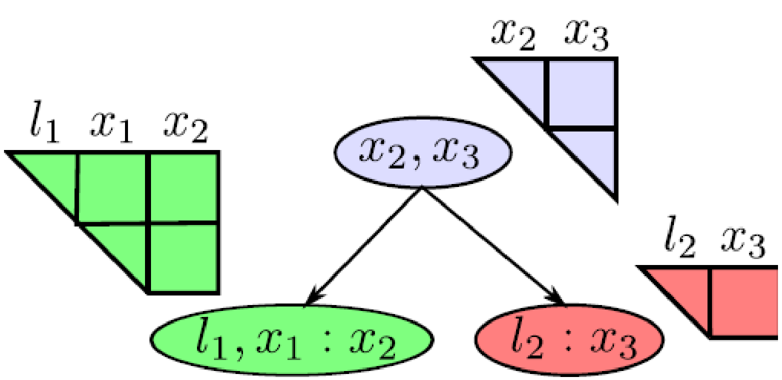


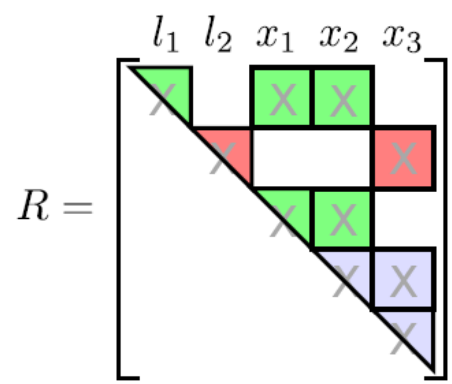
[Kaess et al., IJRR 12]

iSAM2: Bayes Tree Data Structure



Step 2: Find cliques in reverse elimination order:

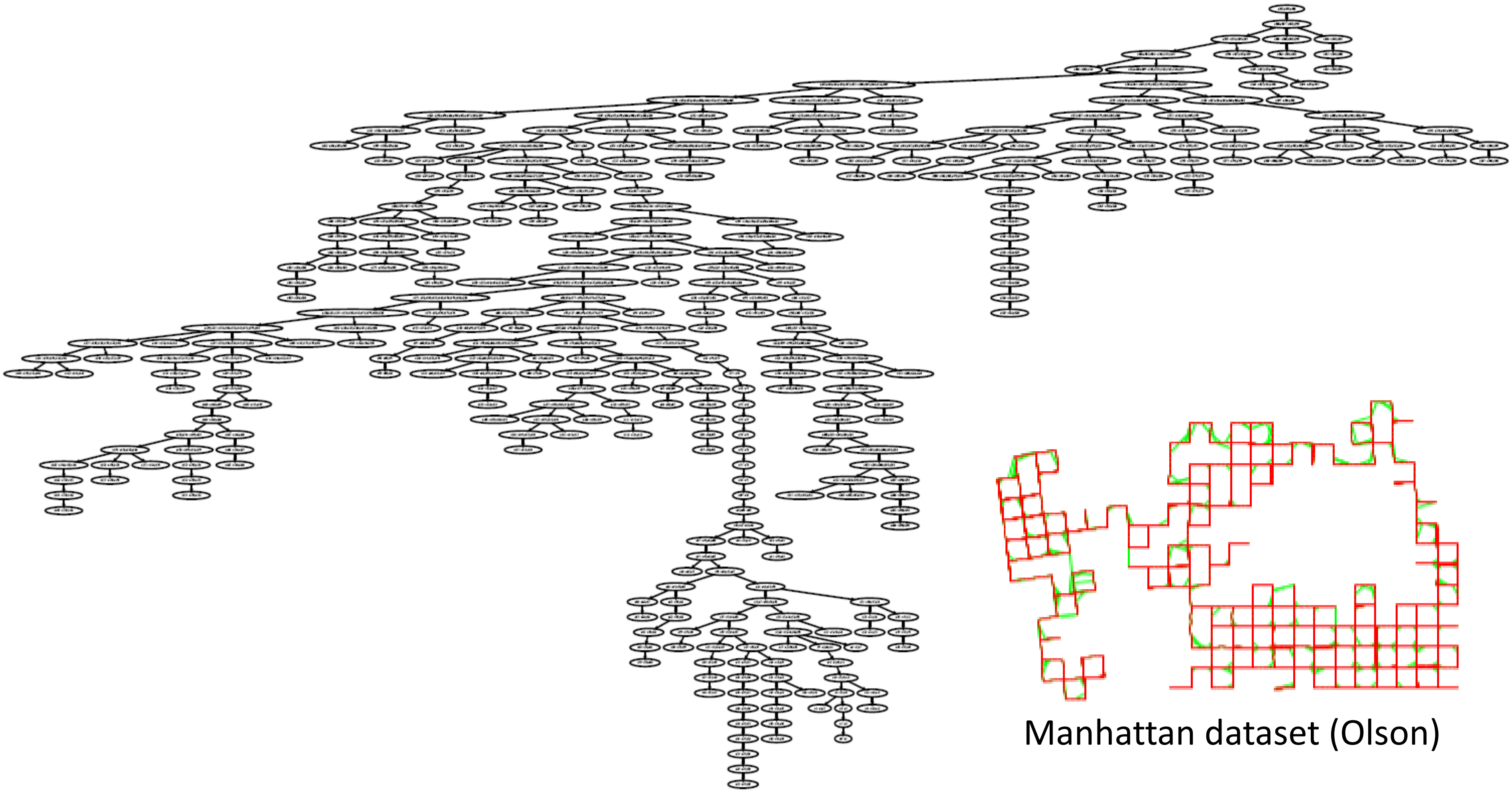




$$P(x_j|S_j) \propto \exp\left\{-\frac{1}{2\sigma^2}\left(x_j + rS_j - d\right)^2\right\}$$

[Kaess et al., IJRR 12]

iSAM2: Bayes Tree Example

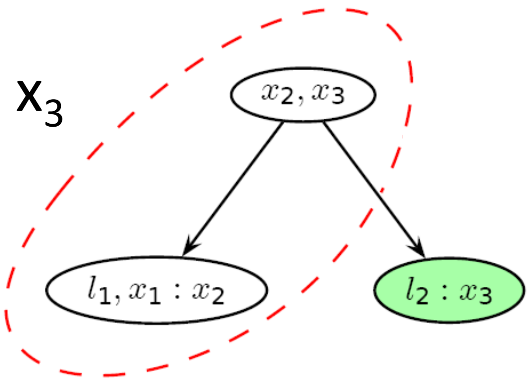


Manhattan dataset (Olson)

How to update with new measurements / add variables?

iSAM2: Updating the Bayes Tree

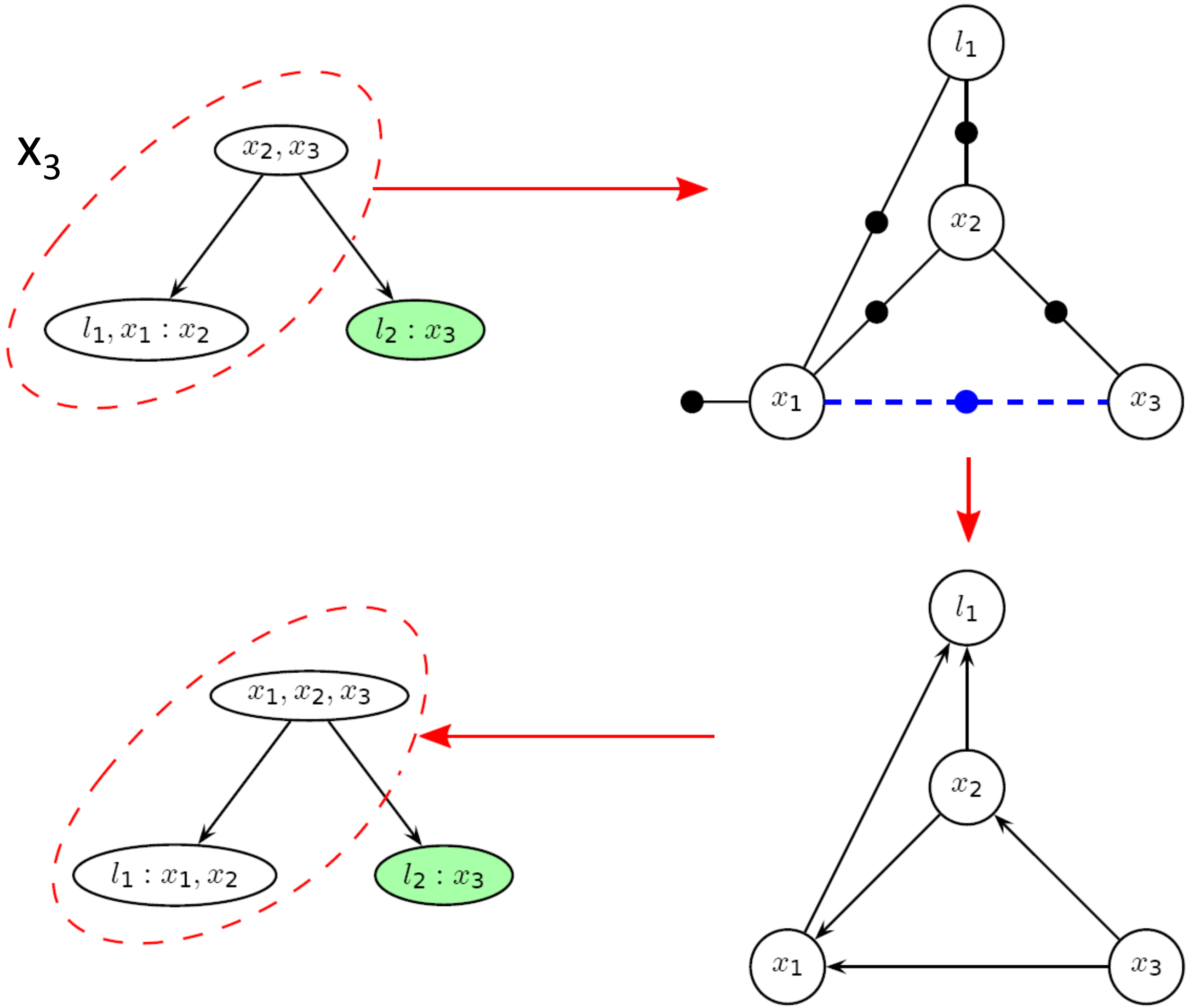
Add new factor between x_1 and x_3



[Kaess et al., IJRR 12]

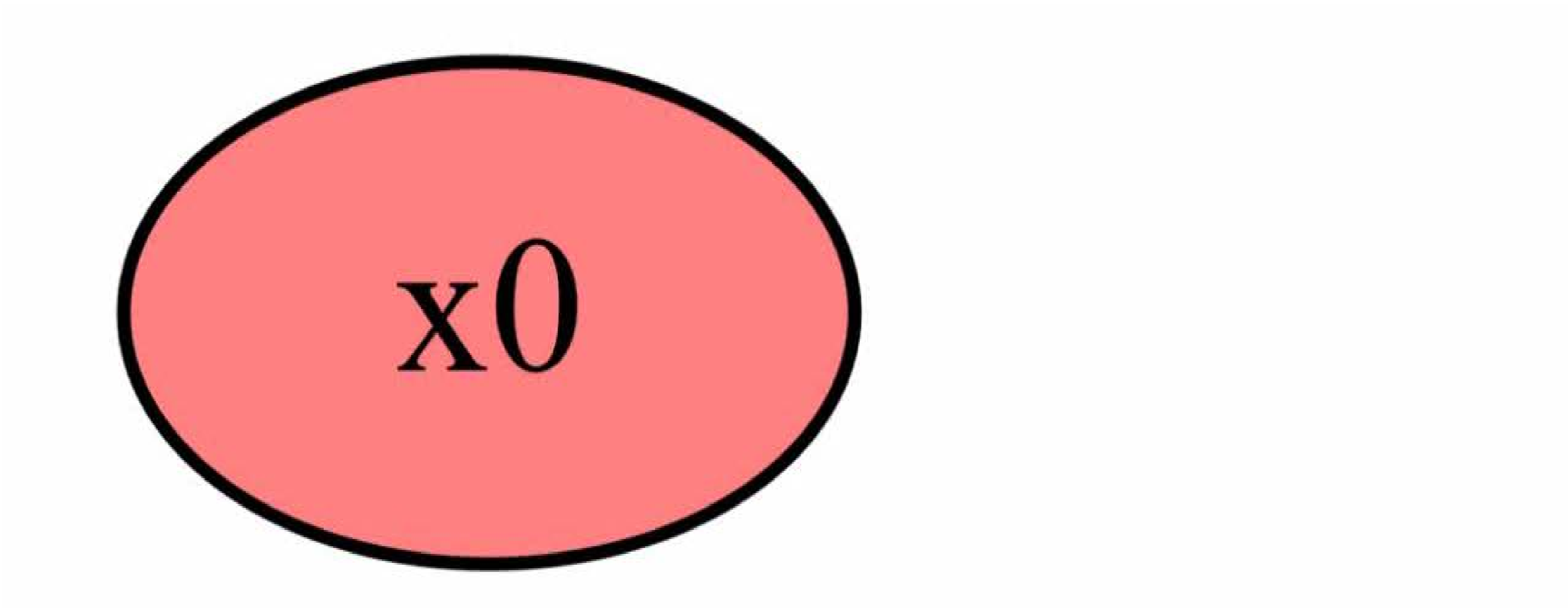
iSAM2: Updating the Bayes Tree

Add new factor between x_1 and x_3



[Kaess et al., IJRR 12]

iSAM2: Bayes Tree for Manhattan Sequence



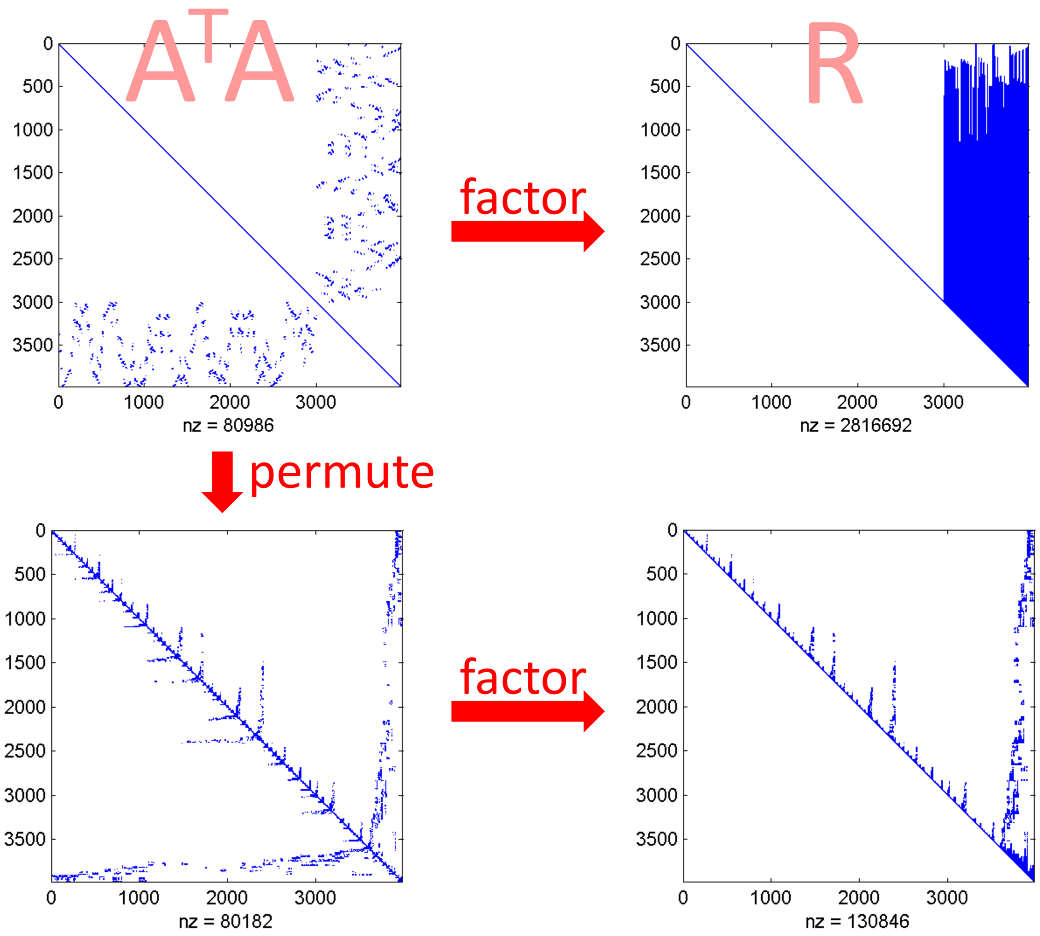
Backup slides

Relevant Publications

- D. Rosen, M. Kaess, and J. Leonard, “RISE: An incremental trust-region method for robust online sparse least-squares estimation,” IEEE Trans. on Robotics (TRO), 2014, to appear.
- M. Kaess, H. Johannsson, R. Roberts, V. Ila, J. Leonard, and F. Dellaert, “iSAM2: Incremental smoothing and mapping using the Bayes tree,” Intl. J. of Robotics Research (IJRR), vol. 31, no. 2, pp. 217–236, Feb. 2012.
- M. Kaess, V. Ila, R. Roberts, and F. Dellaert, “The Bayes tree: An algorithmic foundation for probabilistic robot mapping,” in Intl. Workshop on the Algorithmic Foundations of Robotics (WAFR), Singapore, Dec. 2010, pp. 157–173.
- M. Kaess, A. Ranganathan, and F. Dellaert, “iSAM: Incremental smoothing and mapping,” IEEE Trans. on Robotics (TRO), vol. 24, no. 6, pp. 1365–1378, Dec. 2008.
- F. Dellaert and M. Kaess, “Square Root SAM: Simultaneous localization and mapping via square root information smoothing,” Intl. J. of Robotics Research (IJRR), vol. 25, no. 12, pp. 1181–1204, Dec. 2006.

Retaining Sparsity: Variable Ordering

Fill-in depends on elimination order:



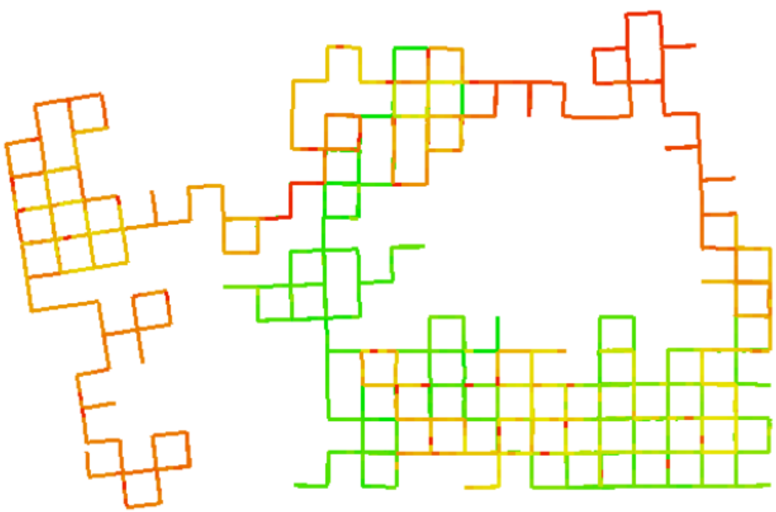
Default ordering (poses, landmarks)

Ordering based on COLAMD heuristic [Davis04] (best order: NP hard)

Variable Reordering – Constrained COLAMD

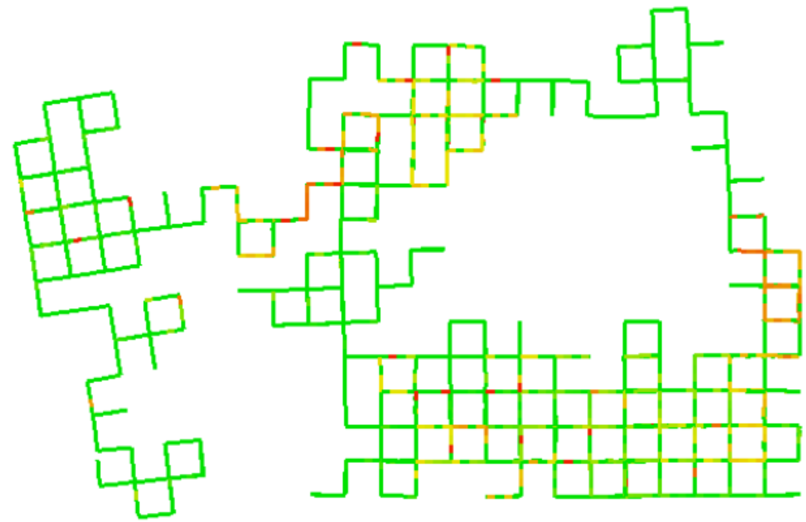
Greedy approach

Arbitrary placement of newest variable



Constrained Ordering

Newest variables forced to the end



Number of affected variables:

Much cheaper!