Week 1 presentation

Quantum Machine learning beyond kernel methods

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Framework of Quantum Machine Learning Models:

- **Three unified paradigms:** Explicit, implicit (kernel-based), and data re-uploading models all function as linear models in quantum feature spaces
- **Linear model:** model that can be described by a linear function in quantum hilbert space

$$f(x) = tr(\rho(x)O) = \langle \rho(x), O \rangle$$

$$\rho(x) - \text{feature encoded quantum state} \qquad O \in \mathcal{F} - \text{ Hermitian observable}$$

Explicit model	Expectation values are used directly to define a labeling function	$O_\theta = V(\theta)^\dagger O V(\theta)$ $V(\theta)\ -\ \text{variational unitary}$
Implicit model (kernel method)	inner products of encoded data points are used to assign labels	$f_{\alpha,D}(x) = \sum_{m=1}^{M} \alpha_m K\left(x, x^{(m)}\right)$ $O_{\alpha,D} = \sum_{m=1}^{M} \alpha_m \rho(x^m)$

Data re-uploading model:

- Generalization of Explicit model with more number of alternating encoding layers and variational Unitaries

$$f_{\theta}(x) = Tr(\rho_{\theta} O_{\theta}) \quad ; \quad \rho_{\theta}(x) = U(x,\theta)|0\rangle\langle 0|U^{\dagger}(x,\theta) \qquad U(x,\theta) = U_L(x)\prod_{l=1}^{L-1} V_l(\theta) U_l(x)$$

- In a Hilbert space with larger dimension, there exists a mapping from a family of data re-uploading models to an equivalent family of explicit models

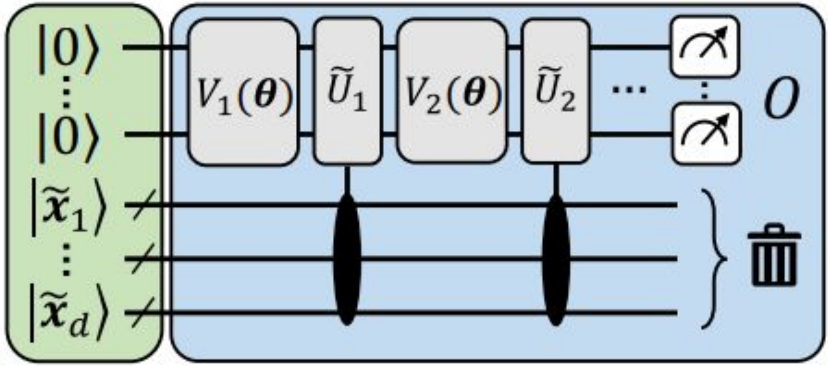

Fig: explicit model approximating a data re-uploading circuit. The circuit acts n working qubits and dp encoding qubits. Pauli-X rotations encode bit-string $\tilde{x}_i = |b_0, \ldots, b_{p-1}\rangle \in \{0,1\}^p$ Because of precision error $\varepsilon = 2^{-p}$, this mapping is approximate.

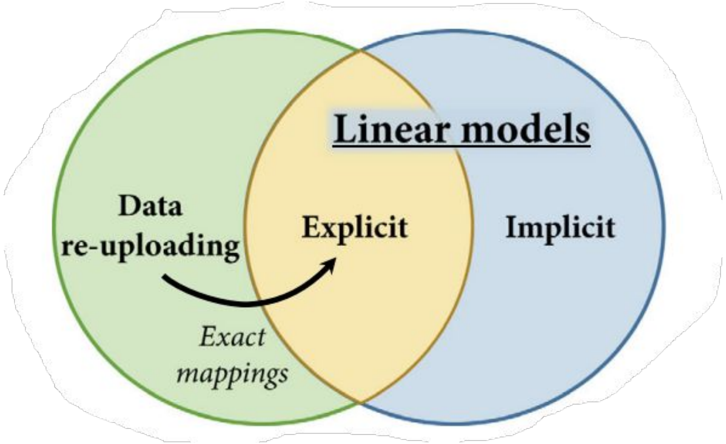

Fig: Data re-uploading models as a generalization of linear quantum models, of polynomial-size can be realized by a polynomial-size explicit linear model.

- For D the number of encoding gates used by the data re-uploading model, the equivalent explicit model uses additional qubits and gates of $\mathcal{O}\left(D\log_e\left(D/\delta\right)\right)$

- **Representer theorem:** states that Implicit quantum models are guaranteed to achieve a lower (or equal) regularized training loss than any explicit quantum model using the same feature encoding.

- Implicit model being more expressive, has a poor generalization performance.

Resource requirements

- Efficiency of a quantum model in solving a learning task is measured in terms of the number of qubits and the size of the training set it requires to achieve a non-trivial expected loss.

Exponential Resource Separations:

We consider the task of learning Parity function acting on d qubits input vectors x: which returns parity of a subset components of **x**.

$$\text{To obtain: } \mathbb{E}_{\mathbb{A}}\left[\inf_{\mathrm{f}}|\mathrm{f}-\mathrm{g}_{\mathrm{A}}|^{2}_{\mathrm{L}^{2}(\mathrm{D}_{\mathrm{A}})}\right]=\varepsilon<\frac{1}{2}$$

Model	Resources	
	Qubits	Data points
Re-uploading	1	$\mathcal{O}(\log d)$
Explicit	$\Omega(d)$	$\mathcal{O}(\log d)$
Implicit	$\Omega(d)$	$\Omega(2^d)$

Fig: resource requirements as a function of input dimension d. Here the data re-uploading model uses single qubit with d encoding gates

Implications for NISQ-Era Quantum Advantage:

- Beyond kernels methods, Quantum advantage likely stems from data re-uploading/explicit models

Benefits:

- **Sublinear scaling:** Efficient learning with limited data for structured tasks.
- Single-qubit dominance: Data re-uploading outperforms larger circuits on specific NISQ tasks