Fiscal Inflation with Incomplete Information

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We quantify fiscal inflation through the lens of incomplete information between funded and unfunded fiscal shocks. Information friction naturally breaks the Ricardian equivalence for deficits to be inflationary. We identify the unfunded shock through a pair of interconnected short-term debt and inflation targets that vary over time. Qualitatively, incomplete information alters the effects of both monetary and fiscal shocks on inflation. Quantitatively, in response to an unfunded shock, the inflation responses are approximately 40%-64% lower compared to the case of full information. Both fiscal stimulus and supply shocks contribute significantly to COVID inflation. An unfunded tax cut can increase output and reduce debt burden, at the cost of higher lingering inflation.

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I. Introduction

The COVID-19 pandemic and the subsequent inflationary episode have sparked a renewed debate about the role of fiscal policy and its impact on inflation. The pandemic caused substantial disruptions to the economy, including supply chain shortages, shifts in preferences toward consumption and labor supply, and the unprecedented fiscal response, among others. Although there is a consensus that all of these shocks can contribute to the increase in prices in theory, there is still much controversy about the decomposition of supply- and demand-driven inflation in practice (see Giannone and Primiceri (2024) and Bernanke and Blanchard (2023)).

An important debate centers on the relative importance of fiscal inflation. The prompt fiscal response to the COVID crisis has been the largest in peacetime. From March 2020 to March 2021, the US government issued three rounds of fiscal stimulus (See Figure 1). In total, more than $5 trillion in federal tax cuts,

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spending increases, and stimulus checks have been spent to increase aggregate demand. These stimulus payments were coupled with large scale purchase of government debt by the central bank. These measures likely put the economy on a path to recovery, but the effects on inflation are unclear. Are fiscal expansions inflationary? If so, how large? Using similarly estimated medium-scale DSGE models, Bianchi, Faccini and Melosi (2023) finds that most of the COVID inflation is due to fiscal stimulus. In contrast, Smets and Wouters (2024) claims that supply shocks, not fiscal expansion, are the primary drivers of post-pandemic inflation.

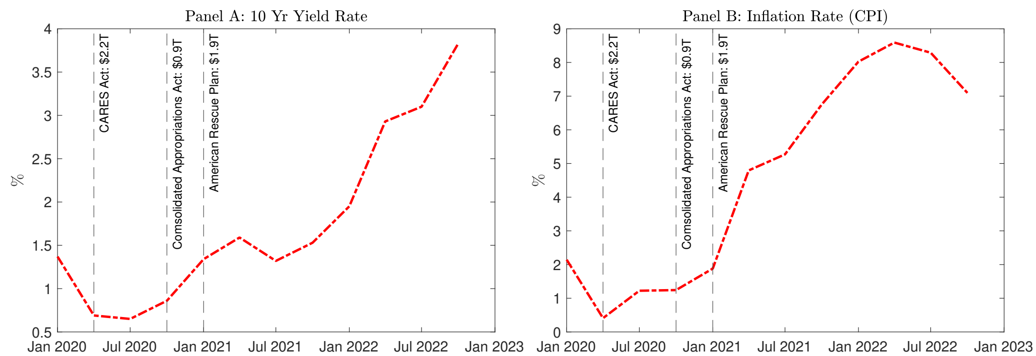

Figure 1. Bond Yield and COVID Inflation

Notes: Panel A shows the quarterly yield rate on 10-year treasury securities and Panel B shows the quarterly inflation rate as measured by CPI during the period of 2020:Q1 to 2022:Q4. Horizontal dotted lines mark the three fiscal stimulus payments enacted in response to COVID along with amount of disbursements in trillions of dollars.

Although there is a quantitative debate, the aforementioned literature has largely settled on the origin of potential fiscal inflation. According to the Fiscal Theory of Price Level (FTPL), as prices need to adjust to ensure that the real value of government debt equals the present value of future surpluses, fiscal policy will have a direct impact on inflation. The early influential work of Leeper (1991) proposes two separate regimes of policy behavior. In the monetary-led regime, the central bank controls inflation by adjusting the nominal interest rate more than one for one in response to inflation deviations from its target (i.e., the Taylor principle), and the fiscal authority ensures that it adjusts primary surpluses (i.e., raising taxes and cutting expenditure) properly to stabilize government debt. In contrast, in the fiscal-led regime, the fiscal authority does not adjust primary surpluses sufficiently. With an accommodating central bank that does not satisfy the Taylor principle, fiscal inflation can arise to reduce the real value of that debt. Such a debt-eroding process is central to the FTPL and can only happen in the fiscal-led regime. For the past three decades, the literature on FTPL has focused mainly on identifying fiscal/monetary policy regimes to infer the magnitude of fiscal inflation (Leeper, Traum and Walker, 2017; Bianchi and Melosi, 2017).

The recent literature (Bianchi, Faccini and Melosi, 2023; Smets and Wouters, 2024) has moved away from policy regimes and has taken a more unified approach to model fiscal inflation. In particular, Bianchi, Faccini and Melosi (2023) design an environment with both funded and unfunded fiscal shocks. The funded shock is backed by future primary surpluses and is similar to the policy behavior in the monetary-led regime. However, the unfunded shock is unbacked, mimicking the policy behavior in the fiscal-led regime. Consequently, while the former shock satisfies the Ricardian equivalence (Barro, 1974) and is noninflationary, the latter shock can generate substantial fiscal inflation once fully accommodated by the central bank. The coexistence of both shocks allows researchers to confront a quantitative DSGE model directly using standard linear Bayesian estimation techniques.

Both Bianchi, Faccini and Melosi (2023) and Smets and Wouters (2024) impose the assumption of Full Information Rational Expectations (FIRE) of all economic agents. In face of unprecedented government transfers and monetary accommodation, one immediate implication of the FIRE model is that economic agents should predict high inflation in the future, resulting in higher current inflation. However, as Figure 1 shows, neither the CPI inflation rate nor the 10-year treasury yield (as a proxy for future inflation) spiked right after the implementation of stimulus payments.[1] The delayed inflation response raises concerns about the validity of the full-information assumption in explaining the COVID inflation dynamics.

Moreover, the coexistence of both funded and unfunded fiscal shocks naturally introduces an information friction that has not been explored. Intuitively, since the two shocks rely on different monetary accommodations in the present and distinct fiscal backings in the future, it is likely households and firms cannot fully tell whether it is a funded shock, an unfunded shock, or a combination of both shocks when a fiscal change occurs today. This is particularly true for U.S. fiscal and monetary policy, as (i) politicians often are unable to clarify the future fiscal backings and (ii) the Federal Reserve rarely admits explicitly it is accommodating the fiscal authority. To capture this salient feature, we model the friction in the source of fiscal expansion through incomplete information. To connect directly to Bianchi, Faccini and Melosi (2023) and Smets and Wouters (2024), we work within the rational expectations paradigm.

The modeling of the funded fiscal shock is standard. To model the unfunded shock, we utilize a pair of time-varying short-term inflation and debt targets that are intertwined. Although the interpretation is slightly different, our approach is in full agreement with the original design of Bianchi, Faccini and Melosi (2023). For example, we map an analytical model to the simple endowment economy in Bianchi, Faccini and Melosi (2023) one-to-one. We show that the unfunded fiscal shock is essentially an exogenous shock to the debt target. When an unfunded

[1] The failure to anticipate the COVID-19 inflation in early 2020 was a widespread phenomenon, with economists and central bankers initially predicting a period of disinflation or deflation. For example, when inflation did start to rise in 2021, the Federal Reserve viewed it as transitory, a temporary result of supply shortages that would soon resolve.

shock increases the debt target, it also allows the government to reduce or delay adjustments of future primary surpluses. Moreover, the unfunded shock also needs to drive the inflation target so that the central bank is being "accommodating". We specify a persistent AR(1) debt target process. To discipline central bank accommodation, Bianchi, Faccini and Melosi (2023) considers a shadow economy in which the only shock is the unfunded fiscal shock. We show how the shadow economy can translate into an endogenous AR(1) inflation target in closed-form solutions.

Incomplete information relies on the fact that both the debt and inflation targets are hidden and cannot be perfectly observed. These targets reflect how tolerable policy makers are to short-term deviations of debt and inflation from their long-term goals. Although the U.S. has never formally adopted a numerical long-term debt target, the debt-to-GDP ratio has been rising steadily well before the COVID-19 pandemic. This trend indicates that the federal government is reluctant to stabilize the high debt burden, especially in the short term (Han, 2021). At the same time, while the Federal Reserve adopts a long-term inflation target of 2%, the central bank does not follow the strict inflation targeting rule in the short run (Ireland, 2007). Historically, the central bank considered a broad range of inflation indicators when setting the federal funds rate, and had adopted a deliberately vague "average inflation target" framework during COVID. Empirically, we find strong evidence supporting the presence of both hidden and time-varying policy targets.

Given the combination of a hidden target and a standard policy shock (either monetary or fiscal), the private sector (i.e. households and firms) faces a signal extraction problem. Since the unfunded shock enters the debt and inflation targets simultaneously, there is a subsequent informational interaction of monetary and fiscal policy. We follow the standard practice in the literature on incomplete information models (Blanchard, L'Huillier and Lorenzoni, 2013) and let households and firms use the optimal algorithm, the Kalman filter, to solve the linearized signal extraction problem.

We first illustrate the transmission mechanism in a simple endowment economy. We use analytical solutions to shed light on intuition and numerical calculations to highlight the importance of fiscal financing. Incomplete information leads to an initial under-reaction of inflation to the unfunded shock, as economic agents perceive that the shock is partially funded at the beginning. Households gradually figure out the nature of the shock, leading to a hump-shaped impulse response to inflation. Similarly, since households can also believe that a funded shock is partially unfunded, its effect on inflation is nonzero (i.e., Ricardian equivalence breaks down) and can be persistent. Lastly, as households confuse between an exogenous monetary policy shock and a changing inflation target driven by the unfunded shock, a transitory monetary policy shock can also generate some persistent inflation.

We then estimate a medium-scale DSGE model to quantify fiscal inflation with incomplete information. Information frictions arise because agents cannot perfectly distinguish unfunded fiscal shocks from funded fiscal or monetary policy shocks. Under incomplete information, an unfunded fiscal shock produces a smaller immediate inflation response but more persistent inflation dynamics, as households gradually infer that unfunded shocks are not backed by future fiscal adjustments. Quantitatively, fiscal inflation arising from unfunded shocks under incomplete information is approximately 40% to 64% lower than under full information, at the estimated 95th and 5th percentiles. Despite the presence of information friction, funded transfers and tax cuts generate negligible inflation. The decomposition of shock contributions to inflation under incomplete information indicates that, at its peak of 8.7% in 2022:Q2, inflation attributable to unfunded shocks was approximately 5.8%, while supply shocks resulted in an additional inflation of 3.4%. Although unfunded fiscal shocks remain a primary driver of post-COVID inflation in our model, their quantitative contribution is substantially smaller than the findings in Bianchi, Faccini and Melosi (2023), which attribute almost 100% of observed inflation between 2021 and 2022 to unfunded shocks. The information friction embedded in our model also better captures the observed dynamics of post-pandemic inflation expectation and bond yields, along with the gradual rise in inflation following the stimulus payments.

The final sections of the paper examine the contributions of monetary policy and unfunded tax cuts to inflation. As illustrated in the simple model, interest rate cuts become more inflationary under incomplete information because agents are unable to distinguish exogenous monetary policy shocks from endogenous changes in short-term inflation target. Quantitatively, we find that information friction generates a nearly 60% larger inflation response to an exogenous rate cut relative to a full-information benchmark. Thus, while incomplete information dampens the inflationary contribution of unfunded fiscal shocks, they simultaneously amplify the non-neutrality of monetary policy as agents attempt to infer the nature of observed signals. Lastly, incomplete information does not generate meaningful inflationary responses to funded tax shocks. Although funded labor and capital tax cuts stimulate output at the cost of higher debt, neither produces a significant effect on inflation. An unfunded tax cut, while being more stimulative and reducing the debt burden, can generate very persistent inflation that last years.

Related Literature This paper belongs to the vast literature on monetary-fiscal policy interactions (see Sargent and Wallace (1981), Leeper (1991), Sims (1994), and Woodford (2001)). We propose an informational interaction of monetary and fiscal policy and quantify fiscal inflation through the lens of incomplete information between funded and unfunded shocks. These shocks resemble the different monetary-fiscal policy behaviors in the regime-switching literature on FTPL (see Davig, Leeper and Walker (2010) and Bianchi and Melosi (2017)). Smets and Wouters (2024) estimates a medium-scale DSGE model, which allows for a time-invariant partial fiscal backing between funded and unfunded shocks.

They find that on average 80% of fiscal shocks are funded. We instead consider a dynamic signal extraction problem such that the degree of partial fiscal backing always depends on the underlying monetary-fiscal policy actions and the household expectations. Consequently, the degree of fiscal backing in our model is always state-dependent and time-varying.

In addition to incomplete information, other information channels or frictions can also play a significant role in generating fiscal inflation. We share the spirit of Eusepi and Preston (2018), who propose a theory of the fiscal foundations of inflation based on imperfect knowledge and recursive least square learning. Bassetto and Miller (2025) uses rational inattention and shows that when bond holders are more concerned with the possibility of a fiscally led regime, sudden inflation can occur. Angeletos, Lian and Wolf (2024) connects to the FTPL by considering a Heterogeneous Agent New Keynesian (HANK) model with a sufficiently slow fiscal adjustment. Since households are non-Ricardian in HANK models, fiscal deficits drive aggregate demand, and thus inflation. Interestingly, we arrive at a quantitatively similar result that fiscal inflation is reduced by about half compared to the full-information FTPL case.

II. A Simple Endowment Economy

We use a simple endowment economy (see Leeper (1991)) to highlight the key information friction. The one-period nominal bond B_t, issued by the government, is sold for Q_t. The gross nominal interest rate R_t is given by the inverse of Q_t, i.e., $R_t = 1/Q_t$. The steady-state gross nominal interest rate is $1/\beta$, where $\beta \in (0, 1)$ is the household's time discount factor. In logarithmic linearized form, the Fisher equation is

$$i_t = \mathbb{E}_t^{HH} \pi_{t+1}, \tag{1}$$

where i_t is the net nominal interest rate and π_t is inflation. The rational expectations operator $\mathbb{E}_t^{HH} = \mathbb{E}_t(\cdot | I_t^{HH})$ is conditional on the household's information set I_t^{HH}, which will be specified below.

Monetary policy follows a simple rule.

$$i_t = \phi_\pi(\pi_t - \pi_t^*), \tag{2}$$

where π_t^* stands for the central bank's short-term inflation target and can be varying over time (see Ireland (2007)).[2] The term $\pi_t - \pi_t^*$ defines the inflation gap. The parameter ϕ_π controls the strength with which the central bank reacts to its inflation gap and satisfies the Taylor principle (that is, $\phi_\pi > 1$). To focus solely on fiscal inflation, we do not introduce an exogenous monetary policy shock at this time. We also leave the task of specifying the inflation target π_t^* after

[2]The central bank's long-term inflation target π^* can still be time-invariant, For example, the Federal Reserve adopts a 2% long-term inflation target.

introducing the fiscal block.

The government budget constraint is

$$Q_t B_t + P_t S_t = B_{t-1},$$

where S_t stands for the primary surplus and $Q_t B_t / P_t$ defines the real market value of the debt. We normalize the endowment economy's output to one. The steady-state surplus-to-output ratio $S^* > 0$ determines the steady-state real market debt s_b^* that the government can finance in the long run. To see the point, imposing steady-state values in the government budget constraint leads to $s_b^* + S^* = s_b^*/\beta$. Consequently, $s_b^* = (\beta S^*)/(1-\beta) > 0$, where a larger steady-state primary surplus S^* supports a higher steady-state debt s_b^*.

Let s_t and $s_{b,t}$ denote the logarithmic deviations of the two variables from their corresponding steady-state values. The linearized government budget constraint (GBC) is

$$s_{b,t} = \beta^{-1} \left[s_{b,t-1} + i_{t-1} - \pi_t - (1-\beta) s_t \right]. \tag{3}$$

Recently, elevated public debt levels have been at the center of fiscal discussions. We consider a fiscal rule that incorporates a hidden time-varying short-term debt target (see also Bianchi and Melosi (2017) and Han (2021)). Let us first consider a log-linearized surplus rule

$$s_t = \gamma (s_{b,t-1} - s_{b,t}^*) + \eta_t,$$

where $s_{b,t}^*$ is the logarithmic deviation of the contemporaneous debt target from its steady-state value. The parameter $\gamma > 1$ controls the strength with which the government reacts to movements of realized debt $s_{b,t-1}$ from its time-varying target $s_{b,t}^*$. The η_t represents an exogenous fiscal shock.

We still need to specify the law of motion for the time-varying debt target $s_{b,t}^*$. To achieve this goal, we draw on recent literature on funded and unfunded fiscal shocks. In particular, Bianchi, Faccini and Melosi (2023) considers the following log-linearized fiscal rule.

$$s_t = \gamma (s_{b,t-1} - s_{b,t-1}^*) + \gamma_b s_{b,t-1}^* + (\varepsilon_t^U + \varepsilon_t^F),$$

with $0 < \gamma_b < 1 < \gamma$, $\varepsilon_t^F \sim N(0, \sigma_F^2)$, and $\varepsilon_t^U \sim N(0, \sigma_U^2)$. For reasons illustrated below, Bianchi, Faccini and Melosi (2023) labels ε_t^F the "funded" shock, and ε_t^U the "unfunded" fiscal shock. Equating the above two fiscal rules and imposing the restriction that $\eta_t \equiv \varepsilon_t^F$ give the law of motion for the time-varying debt target

$$s_{b,t}^* = (1 - \gamma_b/\gamma) s_{b,t-1}^* - \gamma_\tau^{-1} \varepsilon_t^U, \quad \varepsilon_t^U \sim N(0, \sigma_U^2); \tag{4}$$

which is a stationary AR(1) process with persistence $1 - \gamma_b/\gamma \in (0,1)$.

The crucial distinction between "funded" and "unfunded" fiscal shocks is that while future primary surpluses back the former and therefore are not inflationary (that is, ε_t^F satisfies Ricardian equivalence), the unfunded shock ε_t^U is unbacked by necessary future fiscal adjustments and must be inflationary. In Bianchi, Faccini and Melosi (2023), all economic agents have perfect knowledge of ε_t^F and ε_t^U. Furthermore, the central bank is willing to accommodate all fiscal inflation by adjusting its π_t^*, which can only arise from the unfunded shock ε_t^U.

We now specify the time-varying inflation target, π_t^*. Consistent with Bianchi, Faccini and Melosi (2023), we assume that the unfunded fiscal shock completely drives π_t^*,

$$\pi_t^* = \mathcal{P}(L)\varepsilon_t^U. \tag{5}$$

For now, we do not impose any restrictions on the serial correlations of π_t^* and only assume that it is a covariance stationary process, which is equivalent to not imposing any functional forms of $\mathcal{P}(L)$ and only requiring $\sum_{j=0}^{\infty} \mathcal{P}_j^2 < \infty$. Technically, $\mathcal{P}(L)$ is an analytical function.

A. *Making connections to Leeper (1991) and Bianchi, Faccini and Melosi (2023)*

Before introducing incomplete information, it is worth connecting to two papers in the literature which also consider similar endowment economies. Setting

$$\gamma_b = \gamma, \quad \mathcal{P}(L) = 0 \Rightarrow \textit{Leeper } (1991) \tag{6}$$

eliminates the time-varying debt and inflation targets. In Leeper (1991), two separate parameter spaces deliver the existence and uniqueness of a rational expectation equilibrium (REE). The first equilibrium is in the monetary-led regime, where nominal interest rates i_t respond more than one-to-one to deviations in inflation from its target ($\phi_\pi > 1$), and primary surpluses s_t respond strongly to deviations of debt to keep it on a stable path ($\gamma > 1$). The fiscal shock is funded in the first regime. The other equilibrium is in the fiscal-led regime, where i_t responds less than one to one to inflation ($0 \leq \phi_\pi < 1$), and s_t responds weekly to debt ($0 \leq \gamma < 1$). The fiscal shock in the second regime is unfunded. Leeper (1991) labels the two regimes as Active Money-Passive Fiscal (AMPF) and Passive Money-Active Fiscal (PMAF). Appendix A derives both the AMPF and PMAF solutions. As debt stability is achieved with sufficient fiscal adjustments, a distinct feature of the monetary-led regime is that fiscal policy is Ricardian and inflation is independent of the funded fiscal shocks. In the fiscal-led regime, since fiscal authority responds weakly to debt, inflation must adjust to surplus shocks to stabilize government debt. By design, there is only one type of fiscal shock, funded or unfunded, in either regime.

We share the same surplus and debt target rules as Bianchi, Faccini and Melosi (2023). In its simple model section, Bianchi, Faccini and Melosi (2023) considers the following policy rule in which the monetary authority reacts differently to funded and unfunded shocks

$$i_t = \phi_\pi(\pi_t - \pi_t^F) + \phi_F \pi_t^F. \tag{7}$$

In particular, the additional parameter ϕ_F satisfies $0 \leq \phi_F < 1$, and π_t^F is the inflation that would arise in a shadow economy in which the fiscal-led regime is always in place. The derivation in Appendix A shows that

$$\pi_t^F = \frac{\beta - 1}{1 - \phi_F L} \varepsilon_t^U,$$

which is an AR(1) process. Equating the two rules (7) and (2) implies $\pi_t^* = (1 - \phi_F/\phi_\pi)\, \pi_t^F$. The setting of the inflation target process is the following

$$\mathcal{P}(L) = \left(1 - \frac{\phi_F}{\phi_\pi}\right) \frac{\beta - 1}{1 - \phi_F L} \Rightarrow \textit{Bianchi, Faccini and Melosi (2023)} \tag{8}$$

leads to the simple model in Bianchi, Faccini and Melosi (2023).

Both papers assume a Full Information Rational Expectations (FIRE) environment. However, the immediate implication of the FIRE assumption suggests that the private sector should largely predict the COVID-19 inflation that would arise from large-scale fiscal stimuli. It contradicts the movements of bond yields in 2021, as few market participants anticipated the subsequent surge in inflation.

B. *Introducing incomplete information*

Let $\mathcal{M}$ denote the model structure which includes all the structural parameters and the equilibrium conditions. As in all rational expectations models, we first assume $\mathcal{M}$ is common knowledge for the household. Consequently, our information friction is different from the imperfect information channel considered in learning models (e.g. Eusepi and Preston (2018)).

Full-information models assume that economic agents can observe or learn all shocks (i.e., ε_t^F and ε_t^U) perfectly. As a benchmark, we first define the information set for the FIRE case as the following

$$I_t^{FI} = \{\varepsilon_{t-k}^F, \varepsilon_{t-k}^U, \mathcal{M} | k \geq 0\}. \tag{9}$$

The superscript "FI" denotes full information. The coexistence of funded and unfunded shocks, along with their vague clarifications from policymakers, naturally introduces incomplete information. We now formally introduce our concept of information friction while preserving the assumption of rational expectations.

Rewrite the two policy rules and define the monetary and fiscal signals $s_{m,t}$ and

$s_{f,t}$ as

$$s_{m,t} = \pi_t - i_t/\phi_\pi = \pi_t^* \tag{10}$$

$$s_{f,t} = s_t - \gamma s_{b,t-1} = -\gamma s_{b,t}^* + \varepsilon_t^F. \tag{11}$$

We assume that households make $\mathbb{E}_t^{HH}\pi_{t+1}$ after observing the entire history of the nominal interest rate $\{i_{t-k}|k \geq 0\}$ and inflation $\{\pi_{t-k}|k \geq 0\}$, they can also observe the entire history of surpluses $\{s_{t-k}|k \geq 0\}$ and realized real market debt $\{s_{b,t-k}|k \geq 0\}$. It follows that we can define the incomplete information set as

$$I_t^{II} = \{s_{f,t-k}, s_{m,t-k}, \mathcal{M}|k \geq 0\}, \tag{12}$$

where the superscript "II" stands for incomplete information.

C. An analytical case without any monetary policy shocks

So far, we have not introduced exogenous monetary policy shocks. Although it is certainly a simplification, such a modeling choice can deliver powerful insight when coupled with an appropriate assumption. To see the point, comparing the observables in I_t^{II} and I_t^{FI} implies that we can establish the household's signal extraction problem as

$$\underbrace{\begin{bmatrix} s_{f,t} \\ s_{m,t} \end{bmatrix}}_{\mathbf{s}_t} = \underbrace{\begin{bmatrix} 1 & \frac{1}{1-\rho L} \\ 0 & \mathcal{P}(L) \end{bmatrix}}_{M(L)} \begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^U \end{bmatrix} \tag{13}$$

where $M(L)$ is a mapping in the lag operator L that links the household signals $\mathbf{s}_t$ to the underlying shocks ε_t^F and ε_t^U. For ease of notation, we define $\rho = 1 - \gamma_b/\gamma$.

The signal extraction (13) makes it clear that if the mapping $M(L)$ is invertible, then households can perfectly learn the two types of fiscal shocks and $I_t^{II} = I_t^{FI}$. Consequently, the model dynamics becomes FIRE. The Riesz-Fisher Theorem (see Sargent (1987)) establishes a necessary and sufficient condition for the invertibility of the mapping $M(L)$. For invertibility to hold, we must have a nonzero determinant of the mapping (i.e. $\det M(z) = \mathcal{P}(z) \neq 0$ for all $z \in (-1,1)$.) For example, when π_t^* follows the stationary AR(1) process (8), there is no incomplete information between funded and unfunded fiscal shocks, even if the household's information set is given by (12).

In other words, to preserve incomplete information, $\mathcal{P}(L)$ must be non-invertible. A non-invertible $\mathcal{P}(L)$ requires at least one root $z \in (-1,1)$ such that $\mathcal{P}(z) = 0$. For simplicity, we assume that there is only one such root, denoted by λ. Assumption 1 below formalizes the condition.

ASSUMPTION 1: *Assume there is a unique $\lambda \in (-1,1)$, where $\lambda \neq 0$ such that $\mathcal{P}(z) = 0$. For all $z \in (-1,1)$ and $z \neq \lambda$, $\mathcal{P}(z) \neq 0$.*

Given Assumption 1 and the signal extraction problem (13), we can solve for the equilibrium inflation process analytically using the frequency domain techniques (see Kasa, Walker and Whiteman (2014) and Han, Ma and Mao (2022)). Appendix B shows that the incomplete-information inflation process follows

$$\pi_t^{II} = \frac{\Pi_{F,0}}{1-\lambda L}\varepsilon_t^F + \left[\frac{\mathcal{P}(\phi_\pi^{-1}) - \phi_\pi L\mathcal{P}(L)}{1-\phi_\pi L} + \frac{\Pi_{U,0} - \mathcal{P}(\phi_\pi^{-1})}{1-\lambda L}\right]\varepsilon_t^U. \tag{14}$$

where $\Pi_{F,0}$ and $\Pi_{U,0}$ are two endogenously determined constants. In contrast, the full-information inflation follows

$$\pi_t^{FI} = \frac{\mathcal{P}(\phi_\pi^{-1}) - \phi_\pi L\mathcal{P}(L)}{1-\phi_\pi L}\varepsilon_t^U. \tag{15}$$

We make several remarks about π_t^{II} and π_t^{FI}. While π_t^{FI} makes it clear that the funded shock ε_t^F satisfies the Ricardian equivalence and is non-inflationary in π_t^{FI}, it introduces an AR(1) term, $\Pi_{F,0}/(1-\lambda L)\varepsilon_t^F$, in π_t^{II}. The persistence of the AR(1) process is given by λ. Under incomplete information, even a transitory, funded fiscal shock can generate persistent inflation. The breakdown of Ricardian equivalence under incomplete information should not come as a surprise, as households cannot perfectly distinguish a funded shock from an unfunded fiscal shock. If the latter can generate some inflation, the former must also be inflationary with incomplete information. Second, compared to π_t^{FI}, which is fully driven by the unfunded shock ε_t^U, the same shock introduces an additional AR(1) term in π_t^{II}, which is given by $\frac{\Pi_{U,0}-\mathcal{P}(\phi_\pi^{-1})}{1-\lambda L}\varepsilon_t^U$. The additional term allows for a more complicated fiscal inflation process.

D. *An illustrative case with exogenous monetary policy shock*

The analytical result described above relies on a technical assumption (i.e., the non-invertibility of $\mathcal{P}(L)$) to preserve incomplete information. We now argue that information friction can arise naturally if there is an exogenous monetary policy shock e_t. Let the monetary policy rule be

$$i_t = \phi_\pi(\pi_t - \pi_t^*) + e_t, \quad e_t \sim N(0, \sigma_e^2). \tag{16}$$

Households can observe both the histories of i_t and π_t, but cannot separate the exogenous shock e_t from the endogenous inflation target π_t^*. All else being equal, we can establish the household's signal extraction problem between signals and the underlying shocks as

$$\begin{bmatrix} s_{f,t} \\ s_{m,t} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \frac{1}{1-\rho L} & 0 \\ 0 & \mathcal{P}(L) & -1/\phi_\pi \end{bmatrix}}_{M_2(L)} \begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^U \\ e_t \end{bmatrix}$$

Since there are three shocks $\{\varepsilon_t^F, \varepsilon_t^U, e_t\}$ and the private sector can observe only two signals $\{s_{f,t}, s_{m,t}\}$, the above non-square mapping $M_2(L)$ must be non-invertible and can preserve incomplete information naturally.

E. A numerical case with a meaningful fiscal financing scheme

So far, we have concentrated fiscal discussions on primary surpluses S_t. We now introduce a more realistic fiscal financing scheme to the simple endowment economy and show how incomplete information can alter the policy analysis numerically. The fiscal financing scheme also reveals the root of the problem of elevated public debt levels.

Consider $S_t = T_t - Z_t$ where T_t and Z_t are lump-sum taxes and transfers. The slow-moving increase in public debt begins long before the pandemic, mainly due to an aging population and higher mandatory payments of the Social Security, Medicaid, and Medicare programs (i.e., higher transfers Z_t). For this reason, we impose constant tax revenues $T_t \equiv T^*$. The steady-state transfers-to-output ratio satisfies $Z^* < T^*$, leading to a steady-state surplus-to-output ratio $S^* = T^* - Z^* > 0$. The log-linearized transfers rule follows

$$z_t = -\gamma_z(s_{b,t-1} - s_{b,t}^*) + \varepsilon_t^F, \quad \varepsilon_t^F \sim N(0, \sigma_F^2), \tag{17}$$

where $s_{b,t}^*$ is the debt target and $\gamma_z > 0$ is the debt response parameter.[3] The ε_t^F stands for the "funded" transfers shock. Given a predetermined $s_{b,t-1}$ and a zero funded shock, a higher $s_{b,t}^*$ increases transfers z_t. The shock of "unfunded" transfers, ε_t^U, drives a very persistent debt target $s_{b,t}^*$ as

$$s_{b,t}^* = \rho s_{b,t-1}^* + \varepsilon_t^U, \quad \varepsilon_t^U \sim N(0, \sigma_U^2), \tag{18}$$

with $\rho \in (0, 1)$ and $\rho \approx 1$.

We close the model by specifying the inflation target process $\mathcal{P}(L)$. Consistent with Bianchi, Faccini and Melosi (2023), we assume that there is an underlying shadow economy in which only the unfunded shock ε_t^U can drive the inflation target π_t^*. Appendix C shows that the inflation target implied by the shadow economy is an AR(1) process given by

$$\pi_t^* = \mathcal{P}(L)\varepsilon_t^U = \frac{\Phi}{1 - \phi_F L}\varepsilon_t^U, \tag{19}$$

where

$$\Phi = \left(1 - \frac{\phi_F}{\phi_\pi}\right)(1 - \beta)\frac{Z^*}{S^*}. \tag{20}$$

[3]Given the fiscal financing scheme, the primary surplus now follows $s_t = -\frac{Z^*}{S^*}z_t$. It follows that the restriction $\gamma > 1$ translates to $\gamma_z > S^*/Z^*$.

Compared to the inflation target process (8), the term $\beta - 1$ changes to $1 - \beta$ in (19) as we switch from a fiscal income (i.e., primary surplus) shock to a fiscal expenditure (i.e., transfers) shock. More importantly, the fiscal financing scheme introduces an additional term Z^*/S^* in Φ. The parameter Φ controls the magnitude of the increase in the inflation target to an unfunded shock, indicating the required strength of central bank accommodation to fiscal inflation. As the steady-state primary surplus $S^* = T^* - Z^*$ approaches zero, it greatly amplifies the central bank accommodation Φ implied by the shadow economy.

Throughout the paper, we focus on incomplete information on the private sector (i.e., households and firms). It remains to ask: Does the central bank have perfect information on the unfunded shock? We think it is unlikely that the answer is yes. The question provides another reason for introducing e_t in the monetary rule (16). In addition to the usual interpretation of e_t as a policy shock, we could also interpret e_t as a measurement or implementation error of the short-term inflation target, since the central bank may also be unable to observe ε_t^U perfectly.

We redefine the fiscal signal $s_{f,t} = z_t + \gamma_z s_{b,t-1} = \gamma_z s_{b,t}^* + \varepsilon_t^F$ so that

$$\begin{bmatrix} s_{f,t} \\ s_{m,t} \end{bmatrix} = \begin{bmatrix} 1 & \frac{\gamma_z}{1-\rho L} & 0 \\ 0 & \frac{\Phi}{1-\phi_F} & -1/\phi_\pi \end{bmatrix} \begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^U \\ e_t \end{bmatrix}. \tag{21}$$

Given the non-square signal extraction problem (21), there is generally no analytical solution. We solve the model following the algorithm in Blanchard, L'Huillier and Lorenzoni (2013). Let $\mathbf{y}_t$ denote the vector of endogenous state variables and $\mathbf{s}_t$ the exogenous state variables. The equilibrium conditions can be written as

$$\mathbf{F}\mathbb{E}_\mathbf{t}^\mathbf{HH}\mathbf{y_{t+1}} + \mathbf{G}\mathbf{y_t} + \mathbf{H}\mathbf{y_{t-1}} + \mathbf{M}\mathbf{s_t} + \mathbf{N}\mathbb{E}_\mathbf{t}^\mathbf{HH}\mathbf{s_{t+1}} = \mathbf{0}, \tag{22}$$

where $\mathbf{F}, \mathbf{G}, \mathbf{H}, \mathbf{M}, \mathbf{N}$ are coefficient matrices. Solving (22) gives a law of motion

$$\mathbf{y_t} = \mathbf{P}\mathbf{y_{t-1}} + \mathbf{Q}\mathbf{s_t} + \mathbf{R}\mathbf{s_{t|t}}. \tag{23}$$

The $\mathbf{s_{t|t}}$ denotes the perceived exogenous state variables of the households, which can be obtained from the Kalman recursion once we establish the household signal extraction problem (21) as a state-space model.[4]

For illustration, we fix a set of parameters as a benchmark. The time discount factor β is 0.99, suggesting a quarterly model. Empirically, the low frequency movements in transfers are very persistent. We set the persistence of the time-

[4]For technical details of the Kalman recursion, see Hamilton (1994). Guo and Han (2025) uses the same solution algorithm and studies how time-varying fiscal foresight uncertainty impacts government spending multipliers in an incomplete information setting.

varying debt target $\rho = 0.99$.[5] The other parameters are as follows:

$$\phi_\pi = 1.5, \quad \gamma_z = 0.2, \quad Z^* = 0.2, \quad S^* = 0.01,$$
$$\phi_F = 0.95, \quad \sigma_e = 0.25, \quad \sigma_U = 1, \quad \sigma_F = 1;$$

The steady-state surplus-to-output ratio is 1%, leading to a steady-state debt-to-output ratio $s_b^* = (\beta S^*)/(1-\beta) = 0.99$. The implied $\Phi \approx 0.07$, indicating that the central bank increases its inflation target by 7 basis points for a 1% unfunded transfer shock. The magnitudes of the monetary and fiscal shocks are chosen to be in line with existing estimates (see Smets and Wouters (2007) and Leeper, Plante and Traum (2010)).

Figure 2 plots the impulse responses of inflation (quarterly, non-annualized), nominal interest rate, real debt, and transfers to monetary and fiscal shocks (both funded and unfunded). With full information, a negative monetary policy shock $e_1 = -\sigma_e$ generates a one-time inflation. Since inflation is transitory and $\mathbb{E}_t^{HH}\pi_{t+1} = 0$, the nominal interest rates remain the same. Higher inflation causes the real market value of debt to decrease, leading to higher transfers. Due to Ricardian equivalence, the funded fiscal shock $\varepsilon_1^F = \sigma_F$ generates trivial (that is, identically zero) inflation responses. Only the unfunded fiscal shock $\varepsilon_1^U = \sigma_U$ can generate persistent inflation with full information.

All shocks can generate non-trivial and persistent inflation in the incomplete information model. This result is intuitive: When a transitory shock (either a monetary policy or a funded fiscal) hits the economy, incomplete-information households rationally attribute the movements in their observables $\{s_{m,t}, s_{f,t}\}$ to a linear combination of three shocks. The non-zero weight households assign to their *perceived* unfunded fiscal shock is the direct cause of the persistent inflation impulse responses. Consequently, the impulse responses of the nominal interest rate are also persistent to all three types of shock as it tracks $\mathbb{E}_t^{HH}\pi_{t+1}$ in the Fisherian model.

Similarly, when a persistent unfunded fiscal shock ($\varepsilon_1^U = \sigma_U$) hits the economy, households rationally believe that some of the observed changes in their signals $\{s_{m,t}, s_{f,t}\}$ are due to funded fiscal and monetary policy shocks. Since both perceived funded fiscal and monetary shocks cannot generate much inflation, the initial impact is small compared to the full-information model. Over time, households gradually determine the intrinsic nature of the underlying shocks, and the impulse responses of inflation and nominal interest rates converge to their full-information counterparts within two years. The two properties force inflation and nominal interest rates to display hump-shaped impulse responses to the unfunded fiscal shock. The hump-shaped pattern has become a hallmark of incomplete-information rational expectation models.

Next, we conduct a sensitivity analysis to demonstrate how key parameters affect inflation dynamics. Figure 3 plots the impulse responses of inflation when

[5] Bianchi, Faccini and Melosi (2023) sets $\gamma_b = 0$ in (4) so that the debt target is a unit root process.

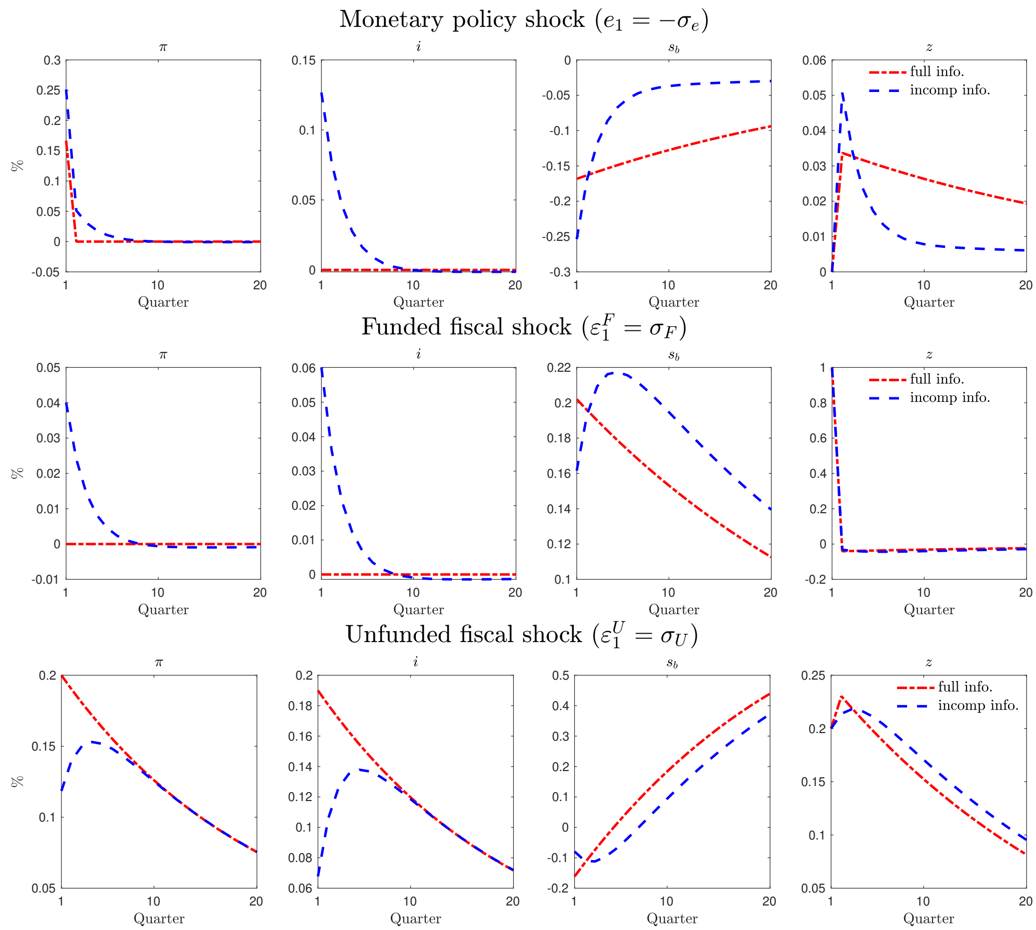

FIGURE 2. IMPULSE RESPONSES OF INFLATION (QUARTERLY, NON-ANNUALIZED), NOMINAL INTEREST RATE, REAL DEBT, AND LUMP-SUM SURPLUSES.

we vary the standard deviations of the shocks $\{\sigma_e, \sigma_F, \sigma_U\}$ one at a time. The other parameters are fixed at their benchmark values. These parameters enter the model dynamics multiplicatively only through their corresponding shocks in full-information models. With incomplete information, these standard deviations play an additional role as they enter the signal extraction problem of the household (21) and determine the weights that the household assigns to their perceived shocks. For example, when σ_e and σ_F are relatively small compared to σ_U, households will assign more weight to their perceived unfunded shock when the actual shock is $\varepsilon_1^U = \sigma_U$, with larger inflationary impulse responses (that is, the third column of Figure 3).

Figures 4 and 5 plot the impulse responses of inflation when varying the policy parameters $\{\phi_\pi, \phi_F, \gamma_z, S^*\}$ one at a time. As ϕ_π increases so that the central

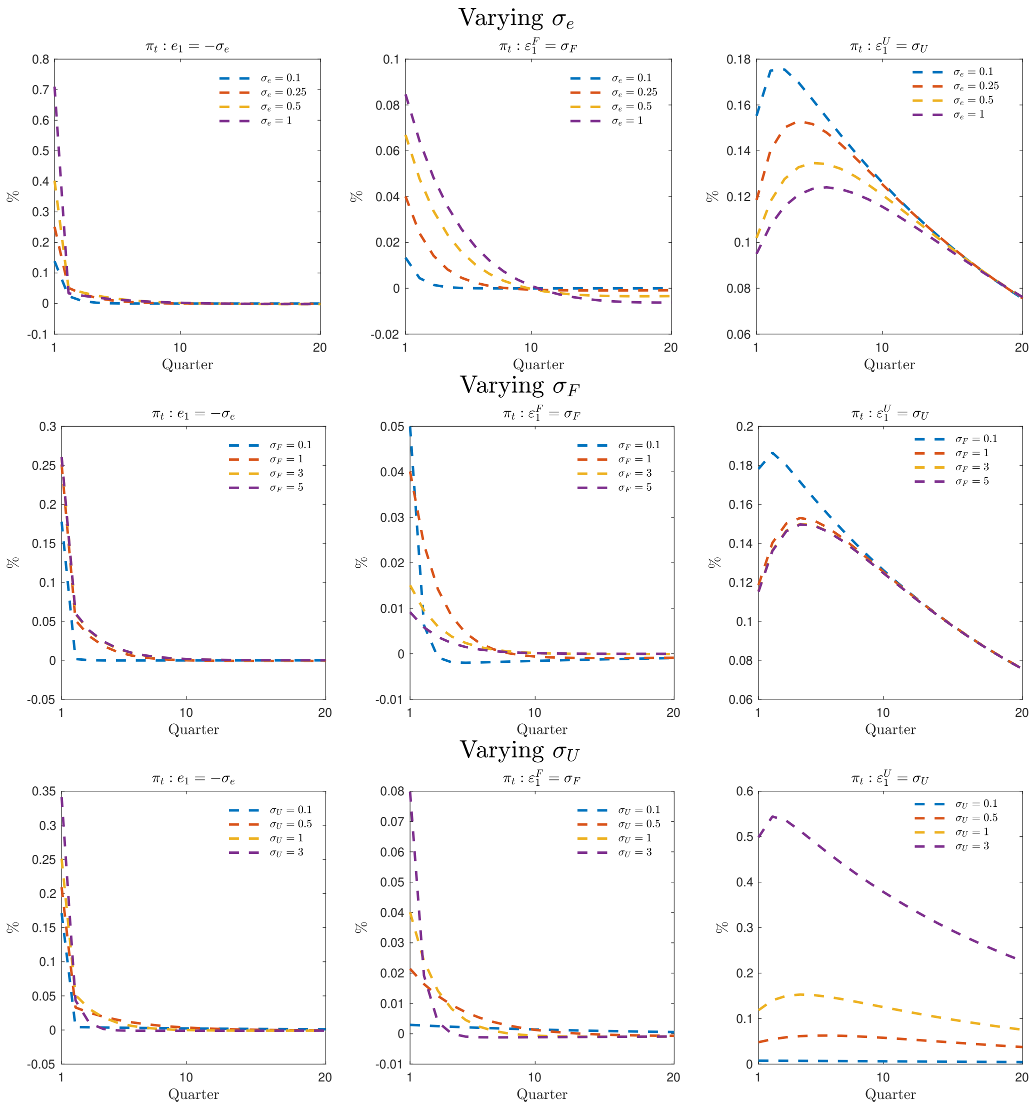

FIGURE 3. IMPULSE RESPONSES OF INFLATION (QUARTERLY, NON-ANNUALIZED) WHEN VARYING $\{\sigma_e, \sigma_F, \sigma_U\}$.

bank becomes more hawkish, a monetary cut or a funded transfer shock generates smaller inflationary effects. However, a larger ϕ_π also indicates a stronger central bank accommodation Φ of the unfunded shock, leading to a higher inflationary effect of the unfunded shock. The parameter ϕ_F mainly controls the persistence of fiscal inflation. When $\phi_F = 0$, the central bank only allows transitory fiscal inflation by allowing π_t^* to temporarily deviate from its long-term target. As ϕ_F approaches one, the fiscal inflation driven by the unfunded shock becomes more

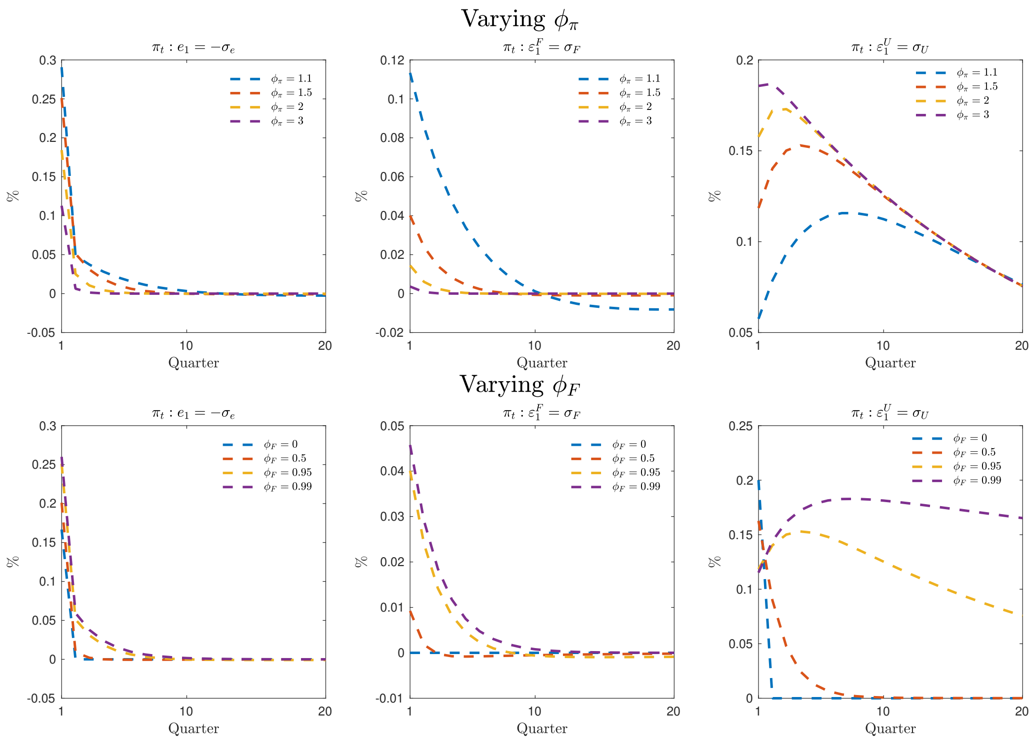

Figure 4. Impulse responses of inflation (quarterly, non-annualized) when varying monetary policy parameters $\{\phi_\pi, \phi_F\}$.

persistent. On the other hand, Φ represents the degree to which the central bank accommodates the unfunded fiscal shock ε_t^U by raising its short-term inflation target π_t^* and controls the magnitude of fiscal inflation. As S^* decreases from 1% to 0.1% so that Φ increases from 0.07 to 0.7, it greatly amplifies the impulse responses of inflation to unfunded shocks.

III. A Quantitative Model

We now embed the incomplete information structure into a medium-scale DSGE New Keynesian framework. Following Leeper, Traum and Walker (2017) and Bianchi, Faccini and Melosi (2023), the model includes a large set of real and nominal frictions and a rich fiscal block. These features are included to enhance the model's empirical fit and align with US business cycle dynamics. Since the model structure is standard, we describe here its main ingredients and defer the details and log-linearization to the Appendix D.

There are two types of households in the economy: savers and hand-to-mouth consumers. Both households are subject to external habit formation and a discount factor shock. They receive wage income by providing labor to firms and

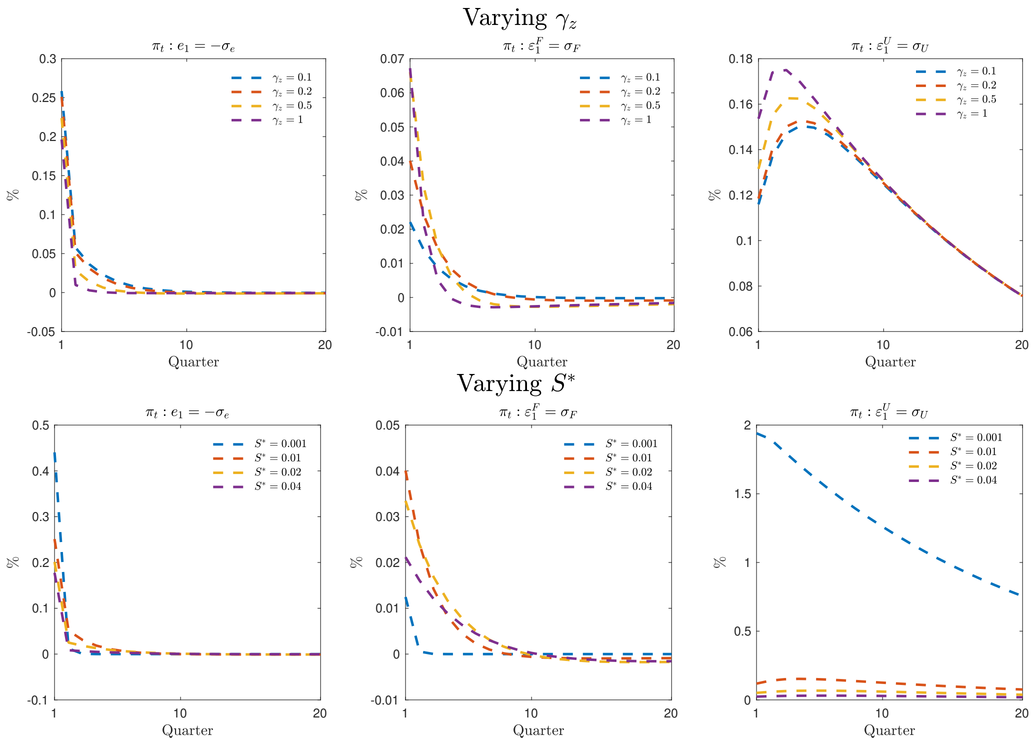

Figure 5. Impulse responses of inflation (quarterly, non-annualized) when varying fiscal policy parameters $\{\gamma_z, S^*\}$.

are subject to various taxes. They also receive lump-sum transfers from the government. Savers have access to short- and long-term government bonds and can also accumulate capital, subject to variable capacity utilization and adjustment costs in investment. Hand-to-mouth households consume all of their disposable, after-tax income and do not save. Both Saver (S) and Non-saver (N) households derive utility from consumption of the composite good and disutility from the supply of labor services:

$$U_t^i(j) = u_t^d \left(\ln \left(C_t^{*i}(j) - hC_{t-1}^{*i} \right) - \frac{L_t^i(j)^{1+\chi}}{1+\chi} \right), \qquad i \in \{S, N\} \tag{24}$$

where u_t^d is a shock from the discount factor, and $1/\chi$ is the Frisch elasticity of the labor supply.

Both households receive after-tax nominal income (W_t) and lump sum transfers from the government (Z_t). In addition, households can save by investing in a one-period government bond (B_t), a long-term nominal government bond (B_t^m), invest in physical capital (I_t), and receive dividends from firms (D_t). The budget constraint of saver households can be written as:

$$
\begin{aligned}
& P_t\left(1+\tau_{C,t}\right) C_t^S+P_t I_t^S+P_t^m B_t^m+R_{n,t}^{-1} B_t \\
= & \left(1+\rho P_t^m\right) B_{t-1}^m+B_{t-1}+\left(1-\tau_{L,t}\right) \int_0^1 W_t(l) L_t^S d l \\
& +\left(1-\tau_{K,t}\right) R_{K,t} \nu_t \bar{K}_{t-1}^S-\Psi\left(\nu_t\right) \bar{K}_{t-1}^S+P_t Z_t^S+D_t,
\end{aligned} \tag{25}
$$

where $\tau_{C,t}$ and $\tau_{L,t}$ denote the tax rates on consumption and labor income, respectively. P_t and P_t^m are the prices of one-year and long-term bonds. The arbitrage condition and the law of capital accumulation are described in Appendix D in more detail. The household maximizes the expected utility $\mathbb{E}_0^{HH}\sum_{t=0}^{\infty}\beta^t U_t^S$ subject to the sequence of budget constraints in equation (25) and the law of motion of capital accumulation.

Intermediate goods firms are subject to the Calvo Pricing and price indexation. To allow for balanced growth, labor-augmenting technology A_t follows an exogenous process that is stationary in the growth rate. There is a perfectly competitive sector of final good firms that produce the final consumption good Y_t by combining a unit measure of intermediate differentiated inputs. Intermediate firms produce goods according to the production function:

$$
Y_t(i)=K_t(i)^\alpha\left(A_t L_t(i)\right)^{1-\alpha}-A_t \Omega, \tag{26}
$$

where Ω is a fixed production cost that increases with the rate of technological advancement that increases labor A_t, and $\alpha\in[0,1]$ is the capital share. When setting prices, firms face Calvo-style price rigidity. Wages set by firms are subject to both sticky wages and wage indexation.

The monetary and fiscal policy blocks in our model differ from the canonical DSGE models in introducing both time-varying inflation and debt targets. First, monetary policy follows a generalized Taylor rule

$$
\hat{r}_t^n=\rho_r \hat{r}_{t-1}^n+\left(1-\rho_r\right)\left[\phi_\pi\left(\hat{\pi}_t-\hat{\pi}_t^*\right)+\phi_y \hat{y}_t\right]+u_t^m, \tag{27}
$$

which embeds interest rate inertia and satisfies the Taylor principle (that is, $\phi_\pi>1$). It also responds to the deviation of the inflation gap $\hat{\pi}_t-\hat{\pi}_t^*$ and the output $\hat{y}_t$. The existence of a persistent (that is, AR (1)) monetary policy shock, u_t^m, prevents households from learning the inflation target $\hat{\pi}_t^*$ perfectly from the monetary policy rule (27).

Fiscal policy consists of a set of expenditure, transfer, and tax rules. Let $s_{b,t}$ denote the real market debt-to-output ratio. The fiscal authority adjusts government spending $\hat{g}_t$, transfers $\hat{z}_t$, and tax rates on capital income $\hat{\tau}_k$ and labor income $\hat{\tau}_l$ as follows:

$$\hat{g}_t = \rho_G \hat{g}_{t-1} - (1-\rho_G)\left[\gamma_G\left(\hat{s}_{b,t-1} - \hat{s}^*_{b,t}\right) + \phi_{g,y}\hat{y}_t\right] + \varepsilon^g_t, \tag{28}$$

$$\hat{z}^b_t = \rho_Z \hat{z}^b_{t-1} - (1-\rho_Z)\left[\gamma_Z\left(\hat{s}_{b,t-1} - \hat{s}^*_{b,t}\right) + \phi_{z,y}\hat{y}_t\right] + \varepsilon^z_t, \tag{29}$$

$$\hat{\tau}_{J,t} = \rho_J \hat{\tau}_{J,t-1} + (1-\rho_J)\gamma_J\left[\hat{s}_{b,t-1} - \hat{s}^*_{b,t}\right] + \varepsilon^J_t, \text{ for } J \in \{k, l\}; \tag{30}$$

where γ_G, γ_Z, and $\gamma_J > 0$ are large enough so that the debt remains on a stable path. We abstract from the consumption tax $\hat{\tau}_c$ by fixing its value. As in the Fisherian economy, the time-varying debt target, $s^*_{b,t}$, follows a stationary AR(1) process:

$$\hat{s}^*_{b,t} = \rho_s \hat{s}^*_{b,t-1} + \varepsilon^U_t, \quad \varepsilon^U_t \sim N(0, \sigma^2_U),$$

with $\rho_s \in (0,1)$ and ε^U_t represents the unfunded fiscal shock. It should be noted that we do not let the tax rates respond to the output $\hat{y}_t$. This model choice follows Leeper, Traum and Walker (2017) and Bianchi, Faccini and Melosi (2023). It does not affect our main results.

To specify the inflation target π^*_t, Bianchi, Faccini and Melosi (2023) considers the shadow economy in which there is only an unfunded fiscal shock and policymakers follow a fiscal-led policy mix. All other model equations are identical across the actual and shadow economies. Due to the scale and complexity of the medium-scale DSGE model, Bianchi, Faccini and Melosi (2023) solves π^*_t numerically. However, for our purpose, a numerical π^*_t will not work as we have to put the household's signal extraction into a linear state-space form. For this purpose, we assume the shadow economy is the benchmark three-equation New Keynesian model for the production economy. We augment the model with a government budget constraint and a simple fiscal rule. We then solve the implied π^*_t analytically. Appendix E shows that, as in the Fisherian model, the inflation target is still an AR(1) process, given by

$$\hat{\pi}^*_t = \phi_F \hat{\pi}^*_{t-1} + \Phi \cdot \varepsilon^U_t \tag{31}$$

with $0 < \phi_F < 1$ and $\Phi > 0$.[6] While ϕ_F governs the persistence of fiscal inflation, the other parameter Φ controls the strength with which the central bank adjusts its short-term inflation target π^*_t to accommodate the unfunded shock ε^U_t.

All economic agents form rational expectations, subject to incomplete information on monetary and fiscal policy shocks. We do not introduce information friction for any other shocks in the economy to isolate the incomplete-information fiscal inflation.

[6]Considering a more complicated shadow economy may lead to a higher-order ARMA(p,q) inflation target process with $p > 1, q > 0$. We find that additional complexity may not be necessary. First, one could argue if it is indeed possible for the central bank to follow an almost perfect shadow economy like in Bianchi, Faccini and Melosi (2023). Second, we find that the AR(1) inflation target yield very similar estimation results to Bianchi, Faccini and Melosi (2023) under full information.

A. Empirical Analysis

The model is estimated using Bayesian techniques to match the following 12 observables for the US economy: real per capita GDP growth, real per capita consumption growth, real per capita investment growth, a measure of the hours gap, the effective federal funds rate, the growth of average weekly earnings, price inflation based on the GDP deflator, the growth of real government transfers, the growth of government expenditure, the government debt-to-GDP ratio, labor tax revenue, and capital tax revenue. The data construction is explained in more detail in the Appendix H.

Following Bianchi, Faccini and Melosi (2023), our sample period spans from 1960:Q1 to 2022:Q3. The estimation strategy to deal with the zero lower bound period after the financial crisis relies on two sub-samples, the first sample from 1960:Q1 to 2007:Q4 and the second subsample from 2008:Q1 to 2022:Q3. The measurement equations linking observables to model variables are shown in the Appendix H.H1. Consistent with Bianchi, Faccini and Melosi (2023) and Campbell et al. (2012), we discipline the second sample estimation by allowing agents' expectations of future interest rates to be informed by market forecasts. The latter subsample incorporates additional observables using overnight index swaps to measure the forecast of one to ten quarters ahead federal funds rate. Each sample is estimated in two steps; first we calculate the likelihood of our model and elicit the mode of posterior distribution. Second, we used MCMC with 500,000 simulations with burn-in of 20% to obtain the entire posterior distribution.

The economic agents in the model form rational expectations of inflation, real interest rate, bond price, investment, marginal utility of the household of the savers, capital rental rate, capital tax rate, marginal utility of investment, and real wages. We first estimate our model under the assumption that agents have full information about the shocks underlying the economy, which gives us our FIRE estimates.

We introduce incomplete information by assuming that all households, intermediate firms, and final good firms share the same information set, denoted by $\mathcal{I}_t^{HH}$. We assume that the agents can observe the entire history of the nominal interest rate $\{r_{t-k}^n | k \geq 0\}$, inflation $\{\pi_{t-k} | k \geq 0\}$, and output $\{y_{t-k} | k \geq 0\}$. The households can also observe the history of real-market debt $\{s_{b,t-k} | k \geq 0\}$, government spending and transfers $\{g_{t-k}, z_{t-k} | k \geq 0\}$, and the history of labor and capital tax rates $\{\tau_{k,t-k}, \tau_{l,t-k} | k \geq 0\}$. However, households cannot distinguish between exogenous shocks to fiscal and monetary policies and shocks to time-varying debt and inflation targets.

There are more policy shocks than signals. Thus, incomplete information arises naturally in our model. For example, rewriting the monetary rule (27) as

$$\hat{r}_t^n - \rho_r \hat{r}_{t-1}^n - (1-\rho_r)\left(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t\right) = \underbrace{-(1-\rho_r)\phi_\pi \pi_t^* + u_t^m}_{s_t^m}$$

indicates that the history of the variables on the right side, s_t^m, is also known to households. Similarly, rewriting the fiscal rules (28) and (29) as follows

$$\hat{g}_t - \rho_G \hat{g}_{t-1} + (1-\rho_G)\left[\gamma_G \hat{s}_{b,t-1} + \phi_{g,y}\hat{y}_t\right] = \underbrace{(1-\rho_G)\gamma_G \hat{s}^*_{b,t} + \varepsilon_t^g}_{s_t^g}$$

$$\hat{z}_t - \rho_Z \hat{z}_{t-1} + (1-\rho_Z)\left[\gamma_Z \hat{s}_{b,t-1} + \phi_{z,y}\hat{y}_t\right] = \underbrace{(1-\rho_Z)\gamma_Z \hat{s}^*_{b,t} + \varepsilon_t^z}_{s_t^z}$$

suggests that the two right-hand variables, s_t^g and s_t^z, are also in the household information set. Finally, the tax rules (30) can also be rewritten with signals where s_t^J is in the household information set. That is,

$$\hat{\tau}_{J,t} - \rho_J \hat{\tau}_{J,t-1} - (1-\rho_J)\,\gamma_J \hat{s}_{b,t-1} = \underbrace{-\,(1-\rho_J)\,\gamma_J \hat{s}^*_{b,t} + \varepsilon_t^J}_{s_t^J} \quad \forall J \in \{k, l\}$$

We do not introduce incomplete information about any other shocks in the model and assume that they can be observed perfectly by households. Although incomplete information certainly can exist among these non-policy shocks, we know from Chahrour and Ulbricht (2023) that there is an equivalence result between the incomplete information model and a full-information model, as long as these non-policy shocks do not interact with policy shocks. These exogenous shocks are intended to improve the empirical fit of the medium-scale DSGE model. Formally, we define the incomplete information set $\mathcal{I}_t^{HH}$ as

$$\mathcal{I}_t^{HH} = \{s_{t-k}^m, s_{t-k}^g, s_{t-k}^z, s_{t-k}^k, s_{t-k}^l, \mathcal{M} | k \geq 0\}.$$

The model is then re-estimated with incomplete information. Appendix F establishes the solution of the incomplete information rational expectations (IIRE) model as a state-space representation used in estimation.

B. Calibration and Prior Distributions

The priors for the parameters are shown in Table 1. Our choice of priors for all non-policy structural parameters and exogenous parameters follows Bianchi, Faccini and Melosi (2023) and Leeper, Traum and Walker (2017). The response of fiscal variables to debt follows a normal distribution with mean 0.1 and standard deviation 0.1. Our fiscal rule allows government spending and transfers to respond to fluctuations in output. The associated policy parameters are restricted to be positive using the Gamma distribution with mean 0.2 and standard deviation 0.05. The monetary policy parameters for inflation and output response follow a normal distribution with mean 1.5 and 0.5 respectively, and standard deviation 0.1. The AR(1) coefficients in the monetary and fiscal policy rules follow a beta distribution with mean 0.5 and standard deviation 0.1.

The prior parameter measuring the response of the short-term inflation target to the unfunded fiscal shock Φ is defined as a Gamma distribution with mean 0.2 and variance at 0.1. Our prior imposes the restriction that Φ must be greater than 0. Consistent with other policy rules, the AR(1) coefficient for the inflation target shock follows a beta distribution with mean 0.5 and standard deviation 0.1. The AR(1) coefficient for the time-varying debt target ρ_s is calibrated at 0.99, allowing for a very high persistence of the unfunded fiscal shock. The autocorrelation parameter ρ for all persistent shocks (except cost push) follows a Beta distribution with mean 0.5 and standard deviation 0.1.

We calibrate the remaining parameters and the steady state in the model to sample averages consistent with Bianchi, Faccini and Melosi (2023). The discount factor β is set at 0.99, the share of capital in the production function α at 0.33, the depreciation rate δ at 2.5%, the substitution elasticity between labor and between intermediate goods at 0.14 and the share of agents from hand to mouth μ at 0.11. The steady state of government expenditure to GDP ratio is set at 0.11, and the steady state of labor, capital, and consumption tax rates are set at 0.186, 0.218, and 0.023, respectively. Bianchi, Faccini and Melosi (2023) compute the average duration of government maturity to be 24 quarters.

IV. Estimation Results

We now present the results of our model estimation under the assumption of Full Information Rational Expectations (FIRE) and Incomplete Information Rational Expectations (IIRE). The posterior medians along with the 90%-credible intervals for all parameters from the first sample estimation (1960:Q1 to 2007:Q4) are presented in Table 1.[7] The incomplete information model performs better than the full information model in the estimation, measured by a higher logarithmic likelihood, -5038.5 for the FIRE estimate versus -4942.7 for the IIRE estimate.

A. FIRE versus IIRE: Non-policy shocks

Table 1 provides a direct comparison between the estimation results under FIRE and IIRE. Estimates are largely similar for most structural parameters and shock processes, including policy response coefficients. We find that the central bank is slightly more aggressive in its response to inflation deviations from its long-term steady state (measured by ϕ_π) under full information. The responses of fiscal variables to the debt target are higher under the FIRE estimation for government expenditure, but lower for tax rules.

It should be noted that, under both both FIRE and IIRE models, the data point to a very persistent inflation target π_t^* driven by ε_t^U. The estimated ϕ_F is close to 0.98 in both models. Since ϕ_F largely governs the persistence of fiscal inflation, the following impulse responses show that inflation would not converge

[7]The results of the second sample estimation (2008:Q1 to 2022:Q3) are reported in Appendix Table D1.

Parameter	Prior			FIRE				IIRE			
	Type	Mean	Std	Mode	Median	5%	95%	Mode	Median	5%	95%
Debt to GDP s_b	N	2.40	0.05	2.416	2.408	2.321	2.492	2.388	2.389	2.297	2.463
SS growth 100γ	N	0.5	0.05	0.402	0.361	0.350	0.394	0.469	0.504	0.451	0.584
SS inflation 100Π	N	0.5	0.05	0.489	0.517	0.502	0.537	0.509	0.553	0.499	0.603
Inverse Frisch ξ	G	2	0.25	2.391	2.364	2.351	2.390	2.410	2.375	2.341	2.408
Non-savers μ	B	0.11	0.01	0.088	0.083	0.075	0.092	0.088	0.088	0.077	0.100
Wage Calvo ω_w	B	0.5	0.1	0.714	0.715	0.698	0.730	0.722	0.710	0.693	0.729
Price Calvo ω_p	B	0.5	0.1	0.725	0.708	0.691	0.731	0.732	0.727	0.708	0.747
Cap util cost ψ	B	0.5	0.1	0.805	0.804	0.788	0.825	0.708	0.740	0.697	0.785
Invest adj cost	N	6	0.5	5.832	5.805	5.776	5.842	4.793	4.959	4.847	5.018
Wage index χ_w	B	0.5	0.2	0.038	0.035	0.032	0.038	0.058	0.048	0.035	0.059
Price index χ_p	B	0.5	0.2	0.092	0.100	0.096	0.103	0.026	0.014	0.006	0.028
Habits θ	B	0.5	0.2	0.929	0.929	0.923	0.935	0.917	0.913	0.903	0.922
C/G sub α_G	N	0	0.1	0.017	-0.010	-0.029	0.011	-0.010	-0.111	-0.188	-0.044
Taylor rule coeff ϕ_π	N	1.5	0.1	1.704	1.684	1.625	1.736	1.642	1.825	1.714	1.907
TR coeff on Y ϕ_y	N	0.5	0.1	0.004	0.003	0.000	0.008	0.002	0.001	0.000	0.004
Debt response γ_G	N	0.1	0.1	0.219	0.232	0.213	0.245	0.169	0.178	0.156	0.186
Debt response γ_{TK}	N	0.1	0.1	0.062	0.056	0.053	0.059	0.182	0.186	0.178	0.195
Debt response γ_{TL}	N	0.1	0.1	0.002	0.001	0.000	0.003	0.013	0.002	0.000	0.006
Debt response γ_z	N	0.1	0.1	0.009	0.007	0.006	0.008	0.001	0.007	0.004	0.008
Output response $\phi_{z,y}$	G	0.2	0.05	0.209	0.208	0.201	0.215	0.188	0.188	0.182	0.197
Output response $\phi_{g,y}$	G	0.2	0.05	0.177	0.189	0.182	0.193	0.182	0.187	0.176	0.199
Infl. target strength Φ	G	0.2	0.1	0.152	0.149	0.140	0.164	0.094	0.191	0.168	0.226
AR(1) Coefficients and Standard Deviations of Shocks											
AR(1) MP-rule ρ_r	B	0.5	0.1	0.767	0.754	0.738	0.779	0.706	0.730	0.703	0.759
AR(1) G-rule ρ_g	B	0.5	0.1	0.985	0.985	0.982	0.988	0.984	0.985	0.981	0.988
AR(1) Z-rule ρ_z	B	0.5	0.1	0.929	0.917	0.902	0.932	0.957	0.945	0.931	0.963
AR(1) TK-rule ρ_{TK}	B	0.5	0.1	0.957	0.966	0.960	0.973	0.950	0.952	0.941	0.962
AR(1) TL-rule ρ_{TL}	B	0.5	0.1	0.935	0.957	0.945	0.971	0.937	0.945	0.930	0.968
AR(1) Infl. target ϕ_F	B	0.5	0.1	0.978	0.979	0.974	0.983	0.989	0.987	0.982	0.991
Technology ρ_a	B	0.5	0.1	0.343	0.338	0.329	0.350	0.134	0.148	0.128	0.178
Preference ρ_b	B	0.5	0.1	0.377	0.395	0.383	0.404	0.677	0.692	0.660	0.742
Monetary ρ_m	B	0.5	0.1	0.340	0.342	0.321	0.361	0.365	0.371	0.349	0.416
Investment ρ_i	B	0.5	0.1	0.888	0.887	0.877	0.899	0.892	0.880	0.860	0.894
Risk Prem. ρ_{rp}	B	0.5	0.1	0.899	0.895	0.883	0.906	0.846	0.837	0.819	0.853
Cost-push ρ_{cp}	B	0.995	0.001	0.996	0.996	0.995	0.997	0.996	0.996	0.995	0.997
Transfer rho_z^u	B	0.5	0.05	0.374	0.397	0.352	0.410	0.376	0.401	0.313	0.453
Gov spending σ_g	I	0.5	0.2	1.880	1.783	1.733	1.869	1.882	1.823	1.767	1.900
Technology σ_a	I	0.5	0.2	1.274	1.395	1.331	1.448	1.199	1.214	1.126	1.288
Preference σ_b	I	0.5	0.2	4.999	4.998	4.994	5.000	4.990	4.981	4.953	4.998
Monetary σ_m	I	0.5	0.2	0.248	0.248	0.230	0.271	0.254	0.291	0.265	0.313
Investment σ_i	I	0.5	0.2	0.641	0.663	0.614	0.730	0.640	0.711	0.646	0.799
Wage σ_w	I	0.5	0.2	0.322	0.326	0.311	0.342	0.331	0.336	0.307	0.373
Price markup σ_p	I	0.5	0.2	0.182	0.189	0.174	0.201	0.174	0.186	0.161	0.208
Risk prem. σ_{rp}	I	0.5	0.2	0.408	0.400	0.381	0.426	0.494	0.528	0.478	0.578
Cost-push σ_{cp}	I	0.5	0.2	0.141	0.161	0.135	0.172	0.134	0.138	0.120	0.151
Transfer σ_z	I	0.5	0.2	3.435	3.427	3.415	3.451	3.496	3.465	3.391	3.529
Unfunded σ_U	I	0.5	0.2	0.385	0.377	0.366	0.394	0.374	0.301	0.243	0.354
Capital tax σ_{tk}	I	0.5	0.2	4.992	4.991	4.983	4.998	4.950	4.892	4.827	4.946
Labor tax σ_{tl}	I	0.5	0.2	2.679	2.589	2.565	2.610	2.674	2.631	2.528	2.788
GDP M.E. σ_y^{ME}	I	0.5	0.2	0.439	0.438	0.422	0.470	0.448	0.445	0.410	0.481

Table 1—Parameter Estimates Comparison: FIRE vs. IIRE

Notes: Table reports the mode, median, and confidence intervals from first sample estimation under FIRE and IIRE. Prior Type indicate the prior density function where N, G, B, and I stand for Normal, Gamma, Beta, and Inverse-Gamma respectively. The posterior median and confidence intervals are computed using MCMC with 500,000 simulations and 20% burn-in.

to zero to an unfunded fiscal shock, even after 20 years. The finding confirms one of the main results in Bianchi, Faccini and Melosi (2023), which also shows a very persistent fiscal inflation process.

Similar estimated posterior distributions for non-policy parameters across the two frameworks imply that the impulse responses of macroeconomic variables should be quantitatively similar for all non-policy shocks which are not subject to incomplete information. Appendix G presents the impulse responses of six key macro variables, output, inflation, nominal interest rate, real interest rate, expected inflation, and real marginal cost, to all non-policy shocks in the model. Since incomplete information is introduced only for policy shocks, responses to other shocks help reveal effects of differences in the estimated structural parameters. For these non-policy shocks, the effects on macro variables are nearly identical under both the FIRE and IIRE frameworks. A notable difference can be seen in the response to preference shock, which is estimated to be twice as persistent under IIRE (0.377 vs. 0.677), leading to a larger impact on inflation, expected inflation, and the real interest rate compared to FIRE.

B. *FIRE versus IIRE: Funded and unfunded fiscal shocks*

The main interest of our analysis lies with the behavior of the economy in response to an unfunded fiscal shock under FIRE and IIRE. Figure 6 presents the impulse response for output, inflation, and the real interest rate, under a one standard deviation unfunded fiscal shock ε_t^U or a one standard deviation funded transfer shock ε_t^z. The magnitude of the responses are percentage deviations from the steady state. Funded and unfunded fiscal shocks are clearly identified in the model, with the funded transfers shock having quantitatively negligible effects (less than 0.02%) on inflation and real interest rate under both FIRE and IIRE.

The real interest rate decreases in response to the unfunded shock, raising both inflation and output under both FIRE and IIRE estimates. On impact, output responds similarly to both estimates, reaching the maximum response in about four years. In the long run, output remains above its steady state level, with a modest but persistent difference (0.15%) between the responses of FIRE and IIRE after ten years. In contrast, the response to inflation is large and strikingly different between the two information frameworks. On impact, the annualized inflation response is significantly lower under incomplete information than under full information (0.1% vs. 0.5%), despite similar posterior distributions for all policy parameters under FIRE and IIRE.

Quantitatively, the response of inflation on impact under IIRE is 87% lower than under the FIRE model, as agents are initially unable to distinguish between funded and unfunded fiscal shocks. As the nature of the shock gradually becomes clear, inflation in response to an unfunded fiscal shock under IIRE reaches a maximum of 0.27%, compared to 0.52% under FIRE. Across parameter estimates at the 95% and 5% confidence intervals, the peak inflation response in the IIRE model remains between 40% and 64% lower than in the FIRE model. Overall,

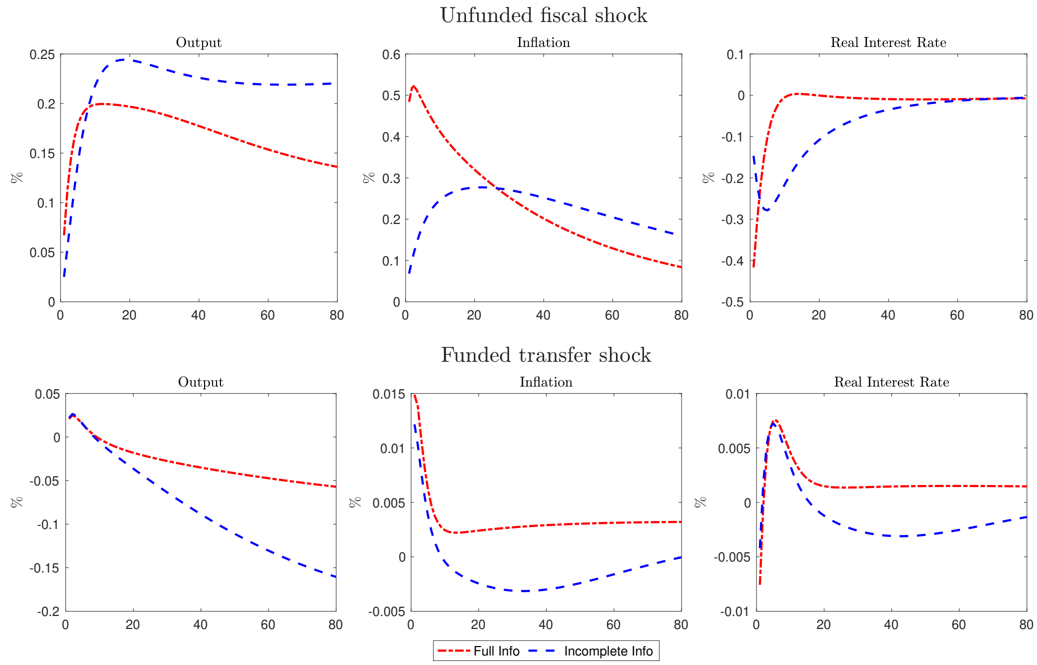

FIGURE 6. IMPULSE RESPONSES TO EXPANSIONARY FUNDED AND UNFUNDED FISCAL SHOCKS

Notes: Impulse response to unfunded fiscal shock (top panel) and funded transfer shock (bottom panel) for output, inflation (annualized), and real interest rate (annualized) under FIRE and IIRE. The size of shocks are fixed at one standard deviation expansionary shock, as estimated in the first sample. The units of response are percentage deviations from steady state.

these results highlight that assuming full information leads to substantially larger estimated contributions of fiscal shocks to inflation, while introducing information frictions significantly attenuates these effects.

The difference in response of inflation lies in how an unfunded shock transmits under full and incomplete information. Under full information, agents can perfectly separate unfunded shocks from other shocks to the economy. Consequently, inflation rises rapidly to pay for the unfunded shock. This increase in inflation is accommodated by monetary policy in the short run, as captured by the large decrease in real interest rate under FIRE. The interest rate increases as the central bank reacts more aggressively to inflation deviations, with deviations for the real interest rate reverting to 0 in approximately two years. The difference in the inflation response narrows as the inflation under IIRE reaches a peak of 0.33% in approximately four years. Given the information frictions, agents cannot immediately distinguish between a funded and an unfunded shock attributing some proportion of the observed shock to each type. As in the simple model, a hump-shaped response to inflation once again emerges as agents acquire more information about the nature of the shock.

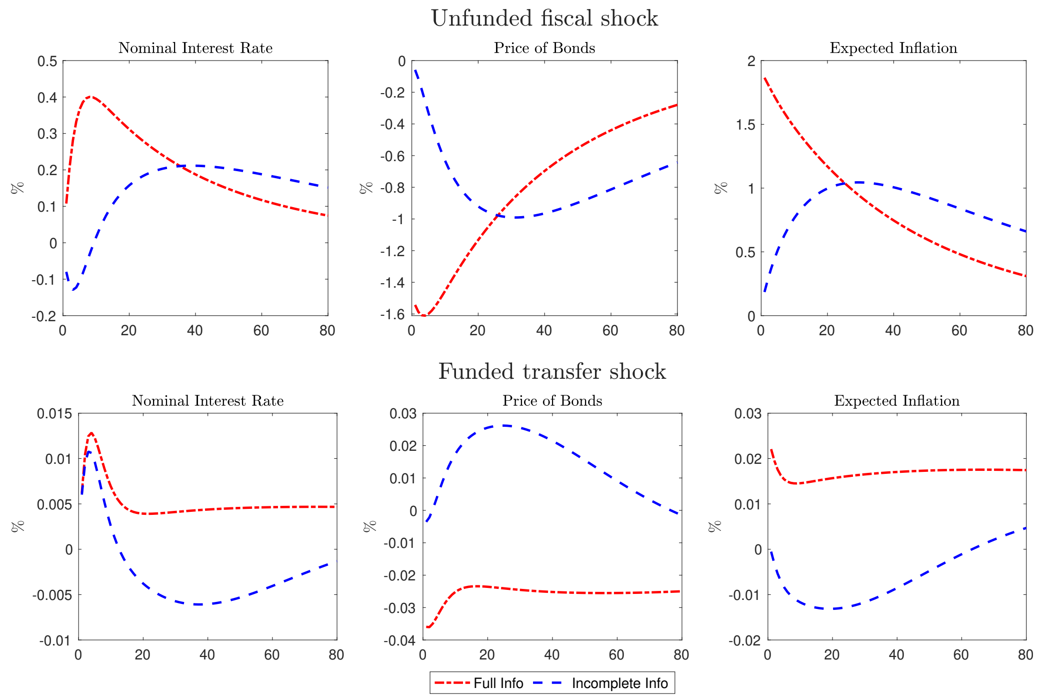

FIGURE 7. IMPULSE RESPONSES TO EXPANSIONARY FUNDED AND UNFUNDED FISCAL SHOCKS

Notes: Impulse response to unfunded fiscal shock (top panel) and funded transfer shock (bottom panel) for nominal interest rate (annualized), price of bonds, and expected inflation (annualized) under FIRE and IIRE. The size of shocks are fixed at one standard deviation expansionary shock, as estimated in the first sample. The units of response are percentage deviations from steady state.

Figure 7 presents the impulse response functions for nominal interest rate, price of bonds and expected inflation, to one percent unfunded fiscal and funded transfers shocks. Similarly to Figure 6, the magnitude of the responses of all nominal variables is near 0 for an expansionary funded transfer shock, while the unfunded fiscal shock creates significant effects. The nominal interest rate increases with inflation under both FIRE and IIRE with the persistence of response being higher under IIRE. Combined with higher inflation and interest rate, the decrease in bond prices on impact is nearly four times as significant under FIRE than under IIRE. As the nature of the policy shock becomes clearer under IIRE, bonds prices fall further, quantitatively reaching the same levels as FIRE but only after ten additional quarters. An unfunded shock not only affects current inflation but significantly raises five-year-forward inflation expectations. The expected inflation under FIRE is highest on impact and decays with time as the real value of debt decreases. The 5-year forward inflation expectation follows the same hump-shaped pattern as current inflation under incomplete information. The transmission mechanism remains the same. As economic agents learn about the nature of the fiscal shock over time, inflation expectations gradually increase for nearly five years after impact before slowly decaying.

In response to an unfunded fiscal shock, both the FIRE and IIRE models can generate hump-shaped impulse responses of the nominal interest rate; thus, they are equally successful in explaining a Federal Reserve that is seemingly "Behind the Curve", which is not raising nominal interest rates fast enough to fight against high inflation. The feature is mainly driven by the interest rate inertia parameter ρ_r in the FIRE model, while information friction plays an additional role in the IIRE model. However, the impulse responses of the bond price and expected inflation provide more direct evidence in favor of the IIRE model. Although the COVID-19 outburst caused initial market turmoils and price drops (more so in the stock market than in the bond market) in early to mid-March 2020, when the fiscal stimulus packages were implemented, bond prices recovered quickly and bond yields stabilized. This movement in bond prices is in contrast to the FIRE model's implication that there should be a large and immediate drop in bond prices if we assume a large share of unfunded fiscal shock in the CARES Act. Inflation expectations observed through various surveys and derived through market-based financial instruments in 2020 also show little support of the FIRE model and are more in agreement with the IIRE model's hump-shaped impulse responses.

C. Historical Decomposition

We now discuss the main drivers of historical inflation and real GDP growth under incomplete information. Figure 8 presents the decomposition of annualized inflation (Panel A) and real GDP growth (Panel B) over our sample period. The bars indicate the cumulative contributions from three sources: unfunded fiscal shock (dark blue), supply shocks (green), and other shocks and steady state (light blue). The solid red line plots realized inflation or GDP growth.[8] While the unfunded fiscal shock explains a significant portion of realized inflation, it contributes little to the observed real GDP growth rate.

The decomposition results illustrate how the model attributes observed inflation to unfunded fiscal shocks versus other shocks under incomplete information rational expectations (IIRE). Large fiscal stimulus became prominent in the United States in the mid-1960s after tax cuts went into effect in 1964, marking a sharp departure from the government's balanced budget philosophy. A second wave of expansionary policies emerged in the early 1970s, characterized by rising government transfers and a substantial increase in the money supply. The model suggests that unfunded fiscal shocks explained about one-third of the roughly

[8] "Supply shocks" include persistent cost push shock, transitory price markup shock, transitory wage markup shock, and technology shock. The category "Remaining Shocks" include all policy shocks except the unfunded fiscal shock-funded fiscal transfers, funded government expenditure, shock to labor tax revenue, shock to capital tax revenue, monetary policy shock, and remaining non-policy shocks including investment shock, preference shock, and risk premium shock, along with the steady state.

12% inflation observed in the mid-1970s.

At the same time, this period coincided with major oil price shocks that fueled inflation worldwide. The model attributes a significant share of the rapid inflationary surge to these supply-side disturbances, which explain roughly half of the inflation peak in 1974. Supply shocks play a central role not only in the sharp rise in inflation but also in its subsequent decline during this period. By contrast, the contribution of unfunded fiscal shocks is more gradual and begins to diminish in the late 1970s. However, inflation remained elevated, driven in part by a renewed surge in oil prices in 1981, which the IIRE framework again primarily attributes to supply shocks.

The unfunded tax cuts of the 1970s and early 1980s continued to influence inflation, although volatility during this period was increasingly driven by supply shocks. The final major oil price shock of the twentieth century struck in the late 1980s and early 1990s, pushing inflation to a peak of about 5%. The IIRE model captures this episode with spikes in contribution of supply shocks, while unfunded shock remained less important for inflation dynamics across this period. Inflation remained subdued through the rest of the 1990s and early 2000s, the period dubbed as great moderation. Fiscal policy during this period was also largely focused on deficit management (Blinder, 2022) while the remaining shocks, including expansionary monetary policy shocks, countered deflationary pressures from supply shocks. In particular, throughout this period of time, unfunded shocks played an insignificant role in GDP growth, which was mainly driven by supply shocks.

A fresh wave of tax cuts in the early 2000s and an increase in military expenditure raised inflation, which is reflected in the persistent positive contribution of unfunded fiscal shocks during this period. Following the financial crisis, unfunded fiscal shocks served as a counterforce to deflationary pressures from all other shocks. The contribution of unfunded fiscal shocks to inflation remained high until the pandemic, where it peaked in 2020 after the disbursement of the first round of stimulus payments under the CARES act.

However, in contrast to the FIRE model, under IIRE a combination of unfunded shocks and supply shocks contributed to the sharp increase in inflation post-pandemic. The FIRE model heavily attributes all inflationary pressures after the pandemic to unfunded fiscal shocks with all other shocks exerting deflationary effects (see Bianchi, Faccini and Melosi (2023)). However, the IIRE model suggests that part of the inflation increase in 2021 can be attributed to supply chain disruptions and semiconductor shortages that increased production costs for many consumer goods during the pandemic. The IIRE model attributes over 20% of observed inflation to a combination of supply shocks in the first quarter of 2022, with this contribution rising to nearly 40% at the peak of inflation in the second quarter before declining to 25% in the third quarter. Although the supply-side factor may have contributed to the deflation at the beginning of COVID-19 (e.g., Wu, Xie and Zhang (2024)), it later became an important

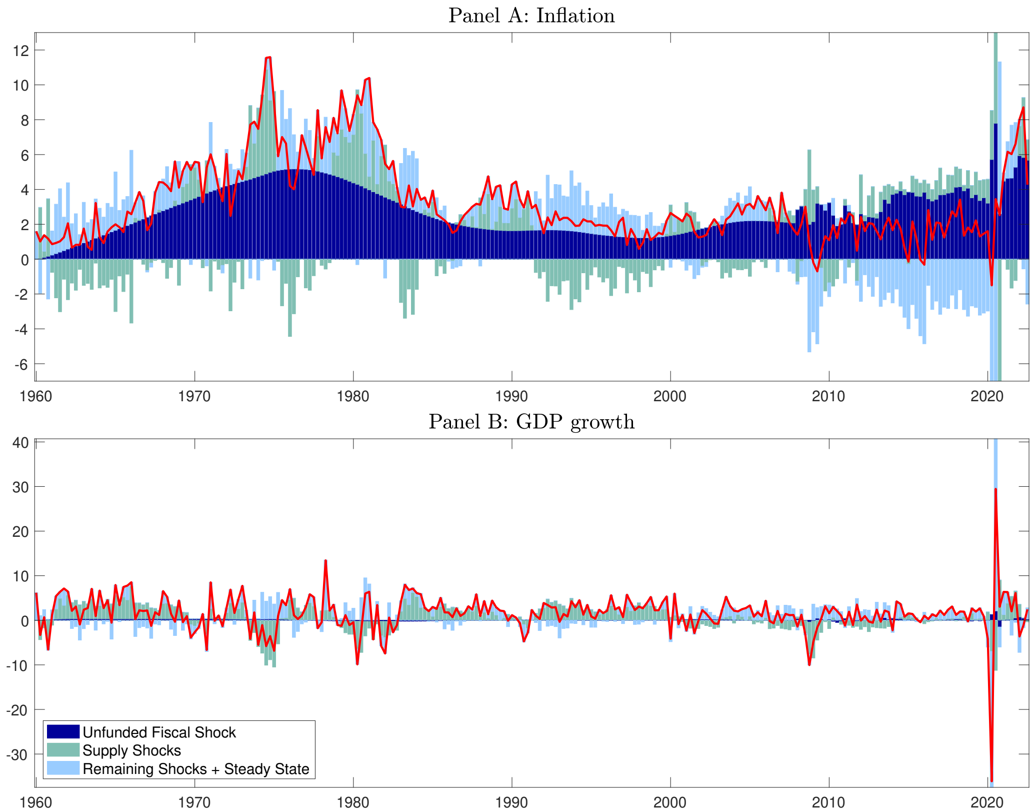

Figure 8. Historical Decomposition of Inflation and real GDP growth under Incomplete Information (1960Q1-2022Q3)

Notes: Figure shows the historical decomposition of inflation (Panel A) and GDP growth (Panel B) under Incomplete Information over our sample period from 1960:Q1 to 2022:Q3. The solid red line is the annualized inflation as measured by growth in GDP deflator. Dark blue bars are the contribution of unfunded fiscal shocks, green bars are the contribution of supply shocks which include cost push shock, price-markup shock, technology shock, and wage markup shock. The remaining light blue bar includes contribution of all other shocks and steady-state. Shocks are estimated using estimated mode for sample 1 until 2007:Q4, and estimated modes from sample 2 starting 2008:Q1.

driver of inflation during the COVID period. The global supply chain pressure index built by the New York Fed shows that the supply side pressures, including measures of manufacturing and transportation costs, are highly correlated with post-pandemic inflation, with the index peaking in December 2021. The index also registers a sharp decline in 2022, reaching the average levels by the beginning of 2023, likely easing inflationary pressure.[9] This dynamic is captured in our model by the sharp increase in contribution of supply shock post-pandemic with

[9]The Global Supply Chain Pressure Index can be viewed on the NY Fed's website: `https://www.newyorkfed.org/research/policy/gscpi#/interactive`.

the peak in 2022:Q2 and an equally significant decline in 2022:Q3. At the same time, unfunded fiscal shocks continued to contribute towards inflation supported by additional stimulus from the government. At its peak in 2022:Q2, the IIRE model attributes roughly 60% of the total inflationary contribution to unfunded shock and the remaining to supply shocks.

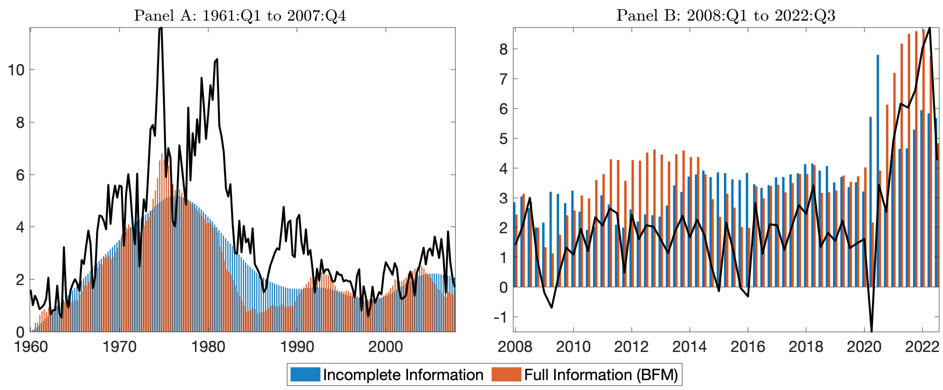

Figure 9. Comparing FIRE and IIRE fiscal inflation

Notes: Figure compares the contribution of unfunded fiscal shock to inflation under Incomplete Information (blue bar) with the decomposition in Bianchi, Faccini and Melosi (2023) (orange bar). Panel A shows the decomposition in our first sample (1960:Q1 to 2007:Q4) and Panel B covers the period between 2008:Q1 and 2022:Q3. The solid black line is the annualized inflation as measured by growth in GDP deflator.

The literature studying drivers of post-pandemic inflation is divided on the key source of inflation. In Bianchi, Faccini and Melosi (2023), unfunded fiscal shocks contribute more than 100% of the observed inflation, whereas non-policy shocks exert an offset, deflationary pressure. By contrast, Smets and Wouters (2024) argue that supply shocks were the primary contributors to pandemic-era inflation, with unfunded shocks playing a minor role. As an external validation exercise, Figure 9 compares our decomposition under incomplete information with the results in Bianchi, Faccini and Melosi (2023) for unfunded fiscal shocks. For most time periods, the two models yield similar results. However, while Bianchi, Faccini and Melosi (2023) attributes most of the inflation post-COVID to unfunded fiscal transfers, the model with information frictions highlights the role of supply shocks. Quantitatively, at its peak in 2022:Q2, the FIRE model by Bianchi, Faccini and Melosi (2023) attributes approximately 20% more inflation to unfunded shocks and 50% lower inflation to supply shocks than the IIRE model. Together with the estimation results, we argue that the assumption of full information can lead to an overestimation of inflation's response to unfunded shocks and, consequently, an underestimation of post-pandemic inflation to supply shocks. Our IIRE model shows that the information friction (i.e., incomplete information) is key to reconciling the findings of Bianchi, Faccini and Melosi (2023) and Smets and Wouters (2024).

D. *Monetary-Fiscal Interaction under Information Frictions*

A novel feature of our model is the interaction between monetary and fiscal policy in the presence of information frictions. The interaction is twofold: First, the unfunded shock drives both the short-term debt and inflation targets. Second, since economic agents cannot perfectly distinguish between funded and unfunded fiscal shocks, an exogenous monetary policy shock (e.g., an interest rate cut) can signal an increase in the central bank's short-term inflation target, leading to a higher and more persistent inflation response compared to the full-information (FIRE) case.

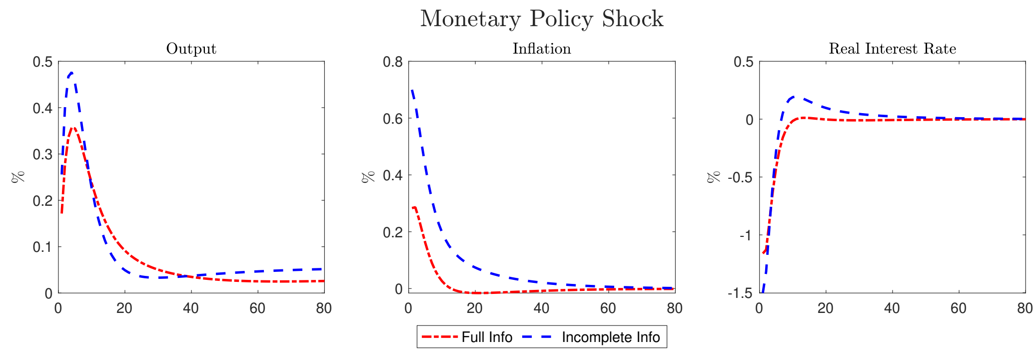

Figure 10. Impulse responses to expansionary monetary policy shock

Notes: Impulse response to monetary policy shock for three variables: output, inflation, and real interest rate, under full information and incomplete information estimation. The size of shocks are fixed at one standard deviation expansionary shock, as estimated in the first sample. The units are percentage deviations from steady-state.

In response to the COVID pandemic, the Federal Reserve cut its target for the federal funds rate by a total of 1.5 percentage points at its meetings on March 3 and 15, 2020. These cuts reduced the funds rate to a range of 0% to 0.25%. The time of the interest cuts coincided with the CARES Act, which was signed into law on March 27. We now highlight the point that the interest rate cut may also contribute to a higher inflation under incomplete information. Figure 10 shows the impulse response of output, inflation, and interest rate to an exogenous rate cut $\varepsilon_1^m = -\sigma_m$. Inflation and output increase, while real interest rates fall under both FIRE and IIRE. However, under IIRE, the effects on inflation are notably stronger. Annualized inflation increases by nearly 0.8 percentage points on impact and remains elevated. In contrast, under FIRE the inflationary response is more muted, with inflation increasing by only 0.3 percentage points initially. Quantitatively, the peak of the inflation response under FIRE is between

54% and 61% lower than under IIRE at the 5% and 95% credible sets of the parameter estimates. As agents learn to distinguish between monetary policy shocks and unfunded fiscal shocks, inflation under IIRE gradually declines. The higher inflation under IIRE leads to a larger decline in the real interest rate and a correspondingly stronger output response, rendering interest rate cuts more expansionary than in the full-information case. Quantitatively, the information interaction generates a 50% larger output response at its peak relative to the FIRE model.

E. *Tax multipliers: Funded versus unfunded*

We now shift our focus to quantifying the response to tax shocks in our model. Recall that incomplete information affects all policy shocks, including shocks to labor and capital tax rate. Figure 11 reports the impulse responses of output, inflation, and the debt-to-GDP ratio to a one-standard-deviation expansionary capital or labor tax shock (i.e., a tax cut) with incomplete information. For comparison, we also present results for the unfunded fiscal shock. Despite the information frictions, funded capital and labor tax cuts have only negligible effects on inflation compared to the effects under the unfunded shock. When both tax cuts are funded, the real debt-to-GDP ratio increases steadily, stabilizing at about 0.7 percentage points above its steady state. By contrast, in response to an unfunded fiscal shock, output increases persistently and the real interest rate decreases, lowering the borrowing cost (Figure 6), resulting in a persistent decrease in the debt-to-GDP ratio.

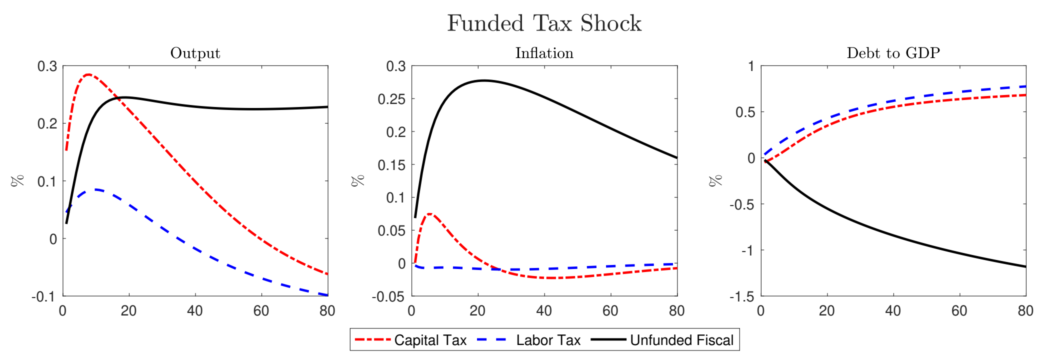

Figure 11. Impulse responses to expansionary funded labor tax, capital tax, and unfunded fiscal shock

Notes: Impulse response to funded labor tax, capital tax, and unfunded fiscal shock for three variables: output, inflation, and debt to GDP ratio, under incomplete information estimation. The size of shocks are fixed at one standard deviation expansionary shock, as estimated in the first sample. The units are percentage deviations from steady-state.

In the short run, an expansionary capital tax shock in the model generates a significant increase in output that is 25% higher than that of an unfunded fiscal shock at its peak. To capture the dynamics of output over time in response to capital and labor tax shocks, we compute the present value multipliers (Table 2) following Leeper, Traum and Walker (2011).[10] As funded tax cuts increase output, their multipliers are negative. Capital tax shocks generate an output multiplier of 0.45 on impact and a present value multiplier greater than one after 10 quarters. The multiplier for the labor tax shock is modest: 0.14 on impact and a maximum multiplier of 0.4 reached after 25 quarters. Our findings are consistent with the empirical literature on fiscal multipliers, which shows that capital tax cuts are more expansionary than labor tax cuts (Romer and Romer, 2010).

Variable	Impact	5 quarters	10 quarters	25 quarters
Labor Tax Shock	-0.141	-0.215	-0.292	-0.399
Capital Tax Shock	-0.445	-0.764	-1.013	-1.441
Unfunded Fiscal Shock	4.521	4.681	4.938	5.859

Table 2—Present Value Multipliers of labor tax, capital tax, unfunded fiscal shock

Since the unfunded fiscal shock also enters the tax rules through the debt target, the government can also implement tax cuts supported by the unfunded fiscal shock. A positive unfunded shock reduces tax rates and increases output, leading to positive multipliers. Although the expansionary effect on output is small in the short run, the total tax revenue also responds only weakly to an unfunded shock, resulting in a tax multiplier of 4.5 on impact. In the long run, an unfunded tax cut can significantly increase output and reduce the debt burden, leading to a very large multiplier close to six. The size of the unfunded fiscal multiplier is quantitatively similar to the deficit-financed tax multiplier documented in Mountford and Uhlig (2009). Their maximum deficit-financed tax multiplier, peaking at 5.25 in the 12th quarter, is close to our 25-quarter unfunded fiscal shock multiplier, 5.86.

A caveat remains in place: inflation remains persistently high and does not converge to zero, even after 20 years. While higher public debt along with future primary surpluses finances the funded tax cuts, it is certainly not the case for the unfunded tax cuts. Essentially, it is higher inflation, which dilutes real debt, that finances the unfunded tax cuts.

V. Conclusion

This paper presents a novel informational interaction of monetary and fiscal policy within the larger class of DSGE models to analyze fiscal inflation. We

[10] Present value multipliers are calculated as the ratio of the present value of changes in output to the present value of changes in total tax revenue: $E_t \frac{\sum_{j=0}^{k}(\Pi_{i=0}^{j} R_{t+i}^{-1})\Delta Y_{t+j}}{\sum_{j=0}^{k}(\Pi_{i=0}^{j} R_{t+i}^{-1})\Delta T_{t+j}^{total}}$.

achieve this through the lens of incomplete information between funded and unfunded fiscal shocks. We augment monetary and fiscal policy rules with a pair of interconnected inflation and debt targets that vary over time. The unfunded fiscal shock drives both short-term targets, altering policy implications.

We incorporate our adjusted policy rules and information friction in a New Keynesian framework and estimate our model using macroeconomic data. Historical decomposition shows that unfunded fiscal shocks significantly contributed to inflation during the 1970s and in the COVID pandemic. However, full-information models can overestimate fiscal inflation. We highlight the roles of supply shocks and interest rate cuts in the case of COVID inflation and explore the macroeconomic impacts of unfunded tax cuts.

Throughout the paper, we follow the existing literature and utilize a shadow economy to discipline the central bank's accommodation to fiscal inflation. It remains to ask: Given a slow-moving and ever-increasing debt burden, what is the optimal monetary policy to accommodate the unfunded fiscal shocks? We focus on the incomplete information of the private sector and do not model the central bank's expectations to unfunded shocks explicitly. Under what circumstances should the central bank address the issue of fiscal sustainability? Should the central bank wait and see, or take a preemptive approach? We leave these policy issues for future research.

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APPENDIX

Recovering AMPF and PMAF dynamics in Leeper (1991)

This section recovers the classic *Active Monetary-Passive Fiscal* (AMPF) and *Passive Monetary-Active Fiscal* (PMAF) inflation dynamics of the simple model in Leeper (1991). The only shock in the economy is the fiscal shock η_t. The equilibrium conditions are

(A1)	Fisher equation:	$i_t = \mathbb{E}_t^{HH}\pi_{t+1}$
(A2)	Monetary policy:	$i_t = \phi_\pi \pi_t$
(A3)	Gov. budget constraint:	$s_{b,t} = \beta^{-1}\left[s_{b,t-1} + i_{t-1} - \pi_t - (1-\beta)s_t\right]$
(A4)	Fiscal policy:	$s_t = \gamma s_{b,t-1} + \eta_t$

Denote the inflation dynamics $\pi_t = \Pi(L)\eta_t = \sum_{j=0}^{\infty}\Pi_j\eta_t$. We utilize frequency domain techniques (see Kasa, Walker and Whiteman (2014)) to derive the functional form of $\Pi^{FI}(L)$. Combining the Fisher equation (A1) and the Taylor rule (A2) and applying the Wiener-Kolmogorov formula lead to

$$L^{-1}\left[\Pi(L) - \Pi_0\right] = \phi_\pi \Pi(L).$$

Rearranging the terms yields the following results

$$\Pi(L) = \frac{\Pi_0}{1-\phi_\pi L}. \tag{A5}$$

If $\phi_\pi > 1$ (i.e. active monetary policy), the above equation defines a stationary inflation process if and only if $\Pi_0 = 0$. It follows $\Pi(L) = 0$ and $\pi_t = 0$ under active monetary policy. Combining the government budget constraint (A3) and the fiscal rule (A4) and plugging in $i_t = 0, \pi_t = 0$ yield

$$s_{b,t} = \beta^{-1}\left[1-(1-\beta)\gamma\right]s_{b,t-1} - \beta^{-1}\left[(1-\beta)\right]\eta_t. \tag{A6}$$

If $0 < \beta^{-1}\left[1-(1-\beta)\gamma\right] < 1$, or equivalently, $\gamma > 1$ (that is, passive fiscal policy), then $s_{b,t}$ defined by (A6) is always a stationary process. We summarize the AMPF inflation and debt dynamics as

(A7)

$$\textbf{AMPF:}\quad \pi_t = 0, \quad s_{b,t} = -\frac{\beta^{-1}(1-\beta)}{1-\beta^{-1}\left[1-(1-\beta)\gamma\right]L}\eta_t, \text{ when } \phi_\pi > 1, \gamma > 1.$$

If $0 < \phi_\pi < 1$ (i.e., passive monetary policy), then equation (A5) always defines a stationary AR(1) inflation process, where Π_0 is a free parameter. Since $i_t =$

$\phi_\pi \Pi_0/(1-\phi_\pi L)\varepsilon_t, \pi_t = \Pi_0/(1-\phi_\pi L)\varepsilon_t$, it follows that

$$i_{t-1} - \pi_t = \frac{\Pi_0 \phi_\pi L}{1-\phi_\pi L}\eta_t - \frac{\Pi_0}{1-\phi_\pi L}\eta_t = -\Pi_0 \eta_t.$$

Combining the government budget constraint (A3) and the fiscal rule (A4) and plugging in $i_{t-1} - \pi_t = \Pi_0 \eta_t$ yield

$$s_{b,t} = \beta^{-1}\left[1-(1-\beta)\gamma\right] s_{b,t-1} - \beta^{-1}\left[\Pi_0 + (1-\beta)\right]\eta_t. \tag{A8}$$

If $0 < \gamma < 1$ so that $\beta^{-1}\left[1-(1-\beta)\gamma\right] > 1$ (that is, active fiscal policy), we can rewrite equation (A8) using the lag operator L as

$$\left(1-\beta^{-1}\left[1-(1-\beta)\gamma\right]L\right) s_{b,t} = -\beta^{-1}\left[\Pi_0 + (1-\beta)\right]\eta_t \tag{A9}$$

Evaluating the above equation at $L = z_1 = \beta\left[1-(1-\beta)\gamma_\tau\right]^{-1}$ pins down

$$\Pi_0 = -(1-\beta),$$

leading to PMAF inflation and debt dynamics as

$$\textbf{PMAF:} \quad \pi_t = \frac{\beta - 1}{1-\phi_\pi L}\eta_t, \quad s_{b,t} = 0, \text{ when } 0 < \phi_\pi < 1, 0 < \gamma < 1. \tag{A10}$$

Deriving the analytical inflation process

The mapping between the household's observables and the underlying shocks is

$$\begin{bmatrix} f_t^* \\ \pi_t^* \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \frac{1}{1-\rho L} \\ 0 & \mathcal{P}(L) \end{bmatrix}}_{M(L)} \begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^U \end{bmatrix} \tag{B1}$$

We have assumed that $z = \lambda$ is the only root of $\mathcal{P}(z)$ inside of $z \in (-1, 1)$. Following Rozanov (1967)'s terminology, the existence of such a root implies the representation (B1) is non-fundamental to the household, and a fundamental representation needs to be derived before forming households' expectations. A fundamental representation consists of a fundamental mapping, denoted by $M^*(L)$, and its associated fundamental innovations. To derive $M^*(L)$, the principal is to utilize the Blaschke factor matrix to flip the root of $\mathcal{P}(z)$ from $|z| < 1$ to $|z| > 1$. Let $\mathcal{B}_\lambda(L)$ be

$$\mathcal{B}_\lambda(L) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1-\lambda L}{\lambda - L} \end{bmatrix} \tag{B2}$$

The fundamental mapping $M^*(L)$ is given by

$$M^*(L) = M(L)\mathcal{W}_\lambda \mathcal{B}_\lambda(L), \tag{B3}$$

where $\mathcal{W}_\lambda$ is an orthonormal matrix given by

$$\mathcal{W}_\lambda = \begin{bmatrix} B_1 & A_1 \\ B_2 & A_2 \end{bmatrix} = \frac{1}{\sqrt{\sigma_U^2 + (1-\rho\lambda)^2\sigma_F^2}} \begin{bmatrix} \sigma_F(1-\rho\lambda) & -\sigma_U \\ \sigma_U & \sigma_F(1-\rho\lambda) \end{bmatrix}.$$

Given $M^*(L)$, the associated fundamental innovations are

$$\begin{bmatrix} \tilde{\varepsilon}_t^F \\ \tilde{\varepsilon}_t^U \end{bmatrix} = \mathcal{B}_\lambda(L^{-1})\mathcal{W}_\lambda' \begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^U \end{bmatrix} \tag{B4}$$

Calculating $\mathcal{B}_\lambda(L)$ and $\mathcal{W}_\lambda$ gives

$$M^*(L) = M(L) \begin{bmatrix} B_1 & A_1 \frac{1-\lambda L}{\lambda - L} \\ B_2 & A_2 \frac{1-\lambda L}{\lambda - L} \end{bmatrix}, \tag{B5}$$

$$\begin{bmatrix} \tilde{\varepsilon}_t^F \\ \tilde{\varepsilon}_t^U \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \\ A_1 \frac{\lambda - L}{1-\lambda L} & A_2 \frac{\lambda - L}{1-\lambda L} \end{bmatrix} \begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^U \end{bmatrix} \tag{B6}$$

Combining the Fisher equation and the monetary policy rule gives a

$$\pi_t - \pi_t^* = \phi_\pi^{-1} \mathbb{E}_t^{HH} \pi_{t+1} \tag{B7}$$

Under FIRE, it can be shown that the inflation process π_t^{FI} is given by a Hansen-Sargent formula

$$\pi_t^{FI} = \frac{\mathcal{P}(\phi_\pi^{-1}) - \phi_\pi L \mathcal{P}(L)}{1 - \phi_\pi L} \varepsilon_t^U. \tag{B8}$$

Denote the incomplete information inflation process

$$\pi_t^{II} = \Pi_F(L)\varepsilon_t^F + \Pi_U(L)\varepsilon_t^U. \tag{B9}$$

We now utilize the fundamental representation to derive $\mathbb{E}_t^{HH}\pi_{t+1}$. It follows that

$$\begin{aligned}
\mathbb{E}_t^{HH}\pi_{t+1} &= L^{-1}\left[\Pi_F(L)B_1 + \Pi_U(L)B_2 - (\Pi_F(0)B_1 + \Pi_U(0)B_2)\right]\tilde{\varepsilon}_t^F \\
&\quad + L^{-1}\left[\frac{1-\lambda L}{\lambda - L}(\Pi_F(L)A_1 + \Pi_U(L)A_2) - \frac{1}{\lambda}(\Pi_F(0)A_1 + \Pi_U(0)A_2)\right]\tilde{\varepsilon}_t^U \\
&= L^{-1}\left[\Pi_F(L) - \Pi_F(0)\right]\varepsilon_t^F + L^{-1}\left[\Pi_U(L) - \Pi_U(0)\right]\varepsilon_t^U \\
&\quad + \frac{1-\lambda^2}{\lambda(1-\lambda L)}\left[\Pi_F(0)A_1 + \Pi_U(0)A_2\right]\left(A_1\varepsilon_t^F + A_2\varepsilon_t^U\right)
\end{aligned}$$

Plugging the above $\mathbb{E}_t^{HH}\pi_{t+1}$ into (B7) and matching the coefficients in front of ε_t^F lead to

$$L\Pi_F(L) = \phi_\pi^{-1}\left[\Pi_F(L) - \Pi_F(0)\right] + \phi_\pi^{-1}\frac{1-\lambda^2}{\lambda(1-\lambda L)}\left[\Pi_F(0)A_1 + \Pi_U(0)A_2\right]A_1 L$$

Imposing $L = \phi_\pi^{-1}$ on both sides gives the first restriction

$$\phi_\pi \Pi_F(0) = \frac{1-\lambda^2}{\lambda(1-\lambda\phi_\pi^{-1})}\left[\Pi_F(0)A_1 + \Pi_U(0)A_2\right]A_1. \tag{B10}$$

It follows that

$$\Pi_F(L) = \frac{\Pi_F(0)}{1-\lambda}.$$

Similarly, matching the coefficients in front of ε_t^U in(B7) yields

$$L\Pi_U(L) - L\mathcal{P}(L) = \phi_\pi^{-1}\left[\Pi_U(L) - \Pi_U(0)\right] + \phi_\pi^{-1}\frac{1-\lambda^2}{\lambda(1-\lambda L)}\left[\Pi_F(0)A_1 + \Pi_U(0)A_2\right]A_2 L$$

Imposing $L = \phi_\pi^{-1}$ on both sides leads to the second restriction

$$\phi_\pi\left[\Pi_U(0) - \mathcal{P}(\phi_\pi^{-1})\right] = \frac{1-\lambda^2}{\lambda(1-\lambda\phi_\pi^{-1})}\left[\Pi_F(0)A_1 + \Pi_U(0)A_2\right]A_2 \tag{B11}$$

It follows that

$$\Pi_U(L) = \frac{\mathcal{P}(\phi_\pi^{-1}) - \phi_\pi L\mathcal{P}(L)}{1-\phi_\pi L} + \left[\Pi_U(0) - \mathcal{P}(\phi_\pi^{-1})\right]\frac{1}{1-\lambda L}.$$

Finally, it should be noted that $\Pi_F(0)$ and $\Pi_U(0)$ are not free parameters. Instead, they are the unique solution to the linear system (B10)-(B11).

The shadow endowment economy with $S_t = T^* - Z_t$

Consistent with Bianchi, Faccini and Melosi (2023), this section considers a shadow economy in which the only shock is the unfunded transfer shock ε_t^U. The equilibrium conditions are

$$\begin{aligned}
&\text{Fisher equation:} && i_t = \mathbb{E}_t^{HH}\pi_{t+1}\\
&\text{Monetary policy:} && i_t = \phi_F \pi_t\\
&\text{Gov. budget constraint:} && s_{b,t} = \beta^{-1}\left[s_{b,t-1} + i_{t-1} - \pi_t - (1-\beta)s_t\right]\\
&\text{Primary surplus:} && s_t = -\frac{Z^*}{S^*}z_t\\
&\text{Unfunded transfers:} && z_t = -\gamma_z s_{b,t-1} + \varepsilon_t^U
\end{aligned}$$

Denote π_t^F as the inflation that would arise in the shadow economy in which the fiscal-led regime is always in place (i.e., we restrict $0 < \frac{Z^*}{S^*}\gamma_z < 1$). Repeating the PMAF derivation as in Appendix A implies

$$\pi_t^F = \frac{1-\beta}{1-\phi_F L}\frac{Z^*}{S^*}\varepsilon_t^U.$$

Consequently, the inflation target implied by the shadow economy is

$$\pi_t^* = \left(1 - \frac{\phi_F}{\phi_\pi}\right)\pi_t^F = \left(1 - \frac{\phi_F}{\phi_\pi}\right)\frac{1-\beta}{1-\phi_F L}\frac{Z^*}{S^*}\varepsilon_t^U.$$

The Medium-scale DSGE Model Environment

D1. Households

There are two types of households in the economy, and their measures sum up to one. Among these households, a fraction of μ are hand-to-mouth consumers, and the remaining $1-\mu$ are savers.

Savers

Savers, each indexed by j, derive utility from the consumption of a composite good $C_t^{*S}(j)$, which comprises private consumption $C_t^S(j)$ and government consumption G_t such that $C_t^{*S}(j) = C_t^S(j) + \alpha_G G_t$. The parameter α_G governs the substitutability between private and government consumption. When α_G is negative (positive), these goods are complements (substitutes). External consumption habits imply that the utility is derived relative to the previous period value of aggregate saver consumption of the composite good hC_{t-1}^{*S}, where $h \in [0,1]$ is the habit parameter. Saver households also derive disutility from the supply of differentiated labor services from all their members, indexed by l, $L_t^S(j) = \int_0^1 L_t^S(j,l)dl$. The period utility function is given by

$$U_t^S(j) = u_t^d\left(\ln\left(C_t^{*S}(j) - hC_{t-1}^{*S}\right) - \frac{L_t^S(j)^{1+\chi}}{1+\chi}\right), \tag{D1}$$

where u_t^d is a discount factor shock, and $1/\chi$ is the Frisch elasticity of the labor supply.

Savers accumulate wealth in the form of physical capital $\bar{K}_t^S$. The law of motion for physical capital is given by

$$\bar{K}_t^S(j) = (1-\delta)\bar{K}_{t-1}^S(j) + u_t^i\left[1 - s\left(\frac{I_t^S(j)}{I_{t-1}^S(j)}\right)\right]I_t^S(j), \tag{D2}$$

where δ is the depreciation rate, u_t^i is a shock to the marginal efficiency of investment and s denotes an investment adjustment cost function that satisfies the properties $s\left(e^{\varkappa}\right)=s'\left(e^{\varkappa}\right)=0$ and $s''\left(e^{\varkappa}\right)>0$, where $\varkappa$ is a drift parameter capturing the logarithm of the growth rate of technology in steady state.

Households derive income from renting effective capital $K_t^S(j)$ to intermediate firms. Effective capital is related to physical capital according to the following law of motion,

$$K_t^S(j)=\nu_t(j)\bar{K}_{t-1}^S, \tag{D3}$$

where $\nu_t(j)$ is the capital utilization rate. In steady state, the utilization rate $\nu(j)$ is 1. The cost of utilizing one unit of physical capital is given by the function $\Psi\left(\nu_t(j)\right)$ that satisfies the following properties: $\Psi(1)=0$ and $\frac{\Psi''(1)}{\Psi'(1)}=\frac{\psi}{1-\psi}$, where $\psi\in[0,1)$. We denote the gross capital rental rate as $R_{K,t}$ and the capital rental income tax rate as $\tau_{K,t}$.

The household can also save by purchasing one-period government bonds in zero net supply and a more general portfolio of long-term government bonds in non-zero net supply. The one-period bonds promising a nominal payoff B_t at time $t+1$ can be purchased at the present discounted value $R_{n,t}^{-1}B_t$, where $R_{n,t}$ is the gross nominal interest rate set by the central bank. The long-term bond B_t^m mimics a portfolio of bonds with average maturity m and duration $(1-\beta\rho)^{-1}$, where $\rho\in[0,1]$ is a constant decay rate. This bond can be purchased at a price P_t^m, determined by the arbitrage condition

$$R_{n,t}=\mathbb{E}_t^{HH}\left[\left(1+P_{t+1}^m\right)/P_t^m\right]e^{-u_t^{rp}},$$

where the wedge u_t^{rp} can be interpreted as a risk premium shock.

In each period, households receive nominal labor income after tax, after-tax revenues from renting capital to firms, lump sum transfers from the government (Z_t^S), and dividends from firms (D_t). Households spend to consume and invest in physical capital and bonds. Omitting the index j to simplify the notation, we can write the nominal budget constraint for the saver household as

$$\begin{aligned}
&P_t\left(1+\tau_{C,t}\right)C_t^S+P_tI_t^S+P_t^mB_t^m+R_{n,t}^{-1}B_t\\
&=\left(1+\rho P_t^m\right)B_{t-1}^m+B_{t-1}+\left(1-\tau_{L,t}\right)\int_0^1 W_t(l)L_t^S dl\\
&\quad+\left(1-\tau_{K,t}\right)R_{K,t}\nu_t\bar{K}_{t-1}^S-\Psi\left(\nu_t\right)\bar{K}_{t-1}^S+P_tZ_t^S+D_t,
\end{aligned} \tag{D4}$$

where $W_t(l)$ denotes the wage rate faced by all household members, and $\tau_{C,t}$ and $\tau_{L,t}$ denote the tax rates on consumption and labor income, respectively. The household maximizes the expected utility $\mathbb{E}_0^{HH}\sum_{t=0}^{\infty}\beta^tU_t^S$ subject to the sequence of budget constraints in equation (D4) and the law of motion of capital accumulation (D2).

For ease of notation, we drop the index j in the following. The first-order optimality conditions (FOCs) concerning consumption, labor supply, one-period bond, investment, capital, and capital utilization are

$$\text{(D5)} \qquad (\partial C_t^{*S}) \quad \Lambda_t^S = u_t^d (C_t^{*S} - hC_{t-1}^{*S})^{-1}$$

$$\text{(D6)} \qquad (\partial L_t^S) \quad L_t^{S,\chi} = \Lambda_t^S (1-\tau_{L,t}) \frac{W_t^h}{P_t}$$

$$\text{(D7)} \qquad (\partial B_t) \quad \Lambda_t^S = \beta R_{nt} \mathbb{E}_t^{HH} \left[\frac{\Lambda_{t+1}^S}{\pi_{t+1}} \right]$$

$$\text{(D8)} \quad (\partial I_t) \quad 1 = Q_t^k \mu_t \left[1 - s\left(\frac{I_t}{I_{t-1}}\right) - s'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} \right] + \beta \mathbb{E}_t^{HH} \left\{ \frac{\Xi_{t+1}^K}{\Lambda_t^S} \left[Q_{t+1} u_{t+1}^i s'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 \right] \right\}$$

$$\text{(D9)} \quad (\partial \bar{K}_t) \; Q_t = \beta \mathbb{E}_t^{HH} \left\{ \frac{\Xi_{t+1}^K}{\Lambda_t^S} \left[((1-\tau_{K,t}) r_{K,t} \nu_{t+1} - \Psi(\nu_{t+1})) + Q_{t+1}(1-\delta) \right] \right\}$$

$$\text{(D10)} \qquad (\partial \nu_t) \quad (1-\tau_{K,t}) r_{K,t} = \Psi'(\nu_t)$$

where Λ_t^S and Ξ_{t+1}^K are the Lagrange multipliers associated with the budget and capital accumulation constraints, respectively, and $Q_t = \frac{\Xi_t^K}{\Lambda_t^S}$ is the Tobin's Q and equals one in the absence of adjustment costs.

Hand-to-Mouth Households

In every period, hand-to-mouth households derive disposable, after-tax income from labor supply and government transfers and consume all of them. They provide differentiated labor services and set their wage equal to the average wage that is optimally chosen by savers, as described below. Hand-to-mouth households face the same tax rates on consumption and labor income as savers. The specification of period-by-period utility for hand-to-mouth households is the same as that of savers, that is,

$$\text{(D11)} \qquad U_t^N(j) = u_t^d \left(\ln\left(C_t^{*N}(j) - hC_{t-1}^{*N}\right) - \frac{L_t^N(j)^{1+\chi}}{1+\chi}) \right).$$

Their budget constraint is

$$\text{(D12)} \qquad (1+\tau_{C,t})\, P_t C_t^N = (1-\tau_{L,t}) \int_0^1 W_t(l) L_t^N(l) dl + P_t Z_t^N,$$

where the superscript N indicates the variables for hand-to-mouth households.

Hand-to-mouth households maximize the discounted utility $\mathbb{E}_0^{HH} \sum_{t=0}^{\infty} \beta^t U_t^N$

subject to the budget constraint (D12). The F.O.Cs are

$$(\partial(C_t^{*N})) \quad \Lambda_t^N = u_t^d(C_t^{*N} - hC_{t-1}^{*S})^{-1}, \tag{D13}$$

$$(\partial L_t^N) \quad L_t^{S,\chi} = \Lambda_t^N(1-\tau_{L,t})\frac{W_t}{P_t}. \tag{D14}$$

D2. Firms

Final good producers

There is a perfectly competitive sector of final good firms that produces the homogeneous good Y_t in time t by combining a unit measure of intermediate differentiated inputs using aggregation technology

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\eta_t^p+u_t^p}}\, di\right)^{1+\eta_t^p+u_t^p}, \tag{D15}$$

where η_t^p is the *i.i.d.* price mark-up shock. The variable u_t^p is a cost push shock and is assumed to follow a near-unit-root process. The highly persistent cost-push shock captures other external forces, such as international trade, that can generate low-frequency movements of inflation. Profit maximization yields the demand function for intermediate goods as

$$Y_t(i) = Y_t\left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\eta_t^p+u_t^p}{\eta_t^p+u_t^p}}, \tag{D16}$$

where $P_t(i)$ is the price of the differentiated good i and P_t is the price of the final good.

Intermediate good producers

There is a unit measure of intermediate firms that produce goods according to the production function

$$Y_t(i) = K_t(i)^\alpha \left(A_t L_t(i)\right)^{1-\alpha} - A_t\Omega, \tag{D17}$$

where Ω is a fixed cost of production that grows with the rate of labor-augmenting technological progress A_t, and $\alpha \in [0,1]$ is the capital share. The labor-augmenting technological progress, A_t, follows an exogenous process that is stationary in its growth rate. Specifically, we assume that $a_t = \ln\left(A_t/A_{t-1}\right) - \varkappa = u_t^a$. Intermediate firms rent capital and labor from perfectly competitive capital and labor markets, respectively. As described in the following, L_t is a bundle of all the differentiated labor services supplied in the economy, aggregated into a homogeneous input by a labor agency. Intermediate firms' cost minimization implies the same nominal marginal cost for all firms

$$MC_t = (1-\alpha)^{\alpha-1}\alpha^{-\alpha}(R_{K,t})^{\alpha} W_t^{1-\alpha} A_t^{-1+\alpha}. \tag{D18}$$

Intermediate producers reset prices in the spirit of the Calvo pricing. At time t, a firm i can optimally reset its price with probability ω_p. Otherwise, it adjusts the price with partial indexation to the previous period's inflation rate according to the rule

$$P_t(i) = (\Pi_{t-1})^{\xi_p}(\Pi)^{1-\xi_p} P_{t-1}(i), \tag{D19}$$

where $\xi_p \in [0,1]$ is a parameter, $\Pi_{t-1} = P_{t-1}/P_{t-2}$, and Π denotes the aggregate rate of inflation at steady state. Intermediate producers that are allowed to reset their price maximizes the expected discounted stream of nominal profits,

$$\max \mathbb{E}_t^{Firm} \sum_{s=0}^{\infty} (\beta\omega_p)^s \frac{\Lambda_{t+s}^S}{\Lambda_t^S} \left[\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\xi^p} \Pi^{1-\xi^p} \right) P_t(i) Y_{t+s}(i) - MC_{t+s} Y_{t+s}(i) \right], \tag{D20}$$

subject to the demand function (D16), with Λ_t^S denoting the marginal utility of the savers.

The FOC is given by

$$\max \mathbb{E}_t^{Firm} \sum_{s=0}^{\infty} (\beta\omega_p)^s \frac{\Lambda_{t+s}^S}{\Lambda_t^S} Y_{t+s}(i) \left[\frac{-1}{\eta_t^p + u_t^p} X_{t,s}^P P_t(i) + \frac{1+\eta_t^p + u_t^p}{\eta_t^p + u_t^p} MC_{t+s} \right] = 0, \tag{D21}$$

where

$$X_{t,s}^P = \begin{cases} 1 & \text{for } s = 0 \\ \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\xi^p} \Pi^{1-\xi^p} \right) & \text{for } s = 1, \ldots, \infty \end{cases} \tag{D22}$$

D3. Wage Settings

Both savers and hand-to-mouth households supply a unit measure of differentiated labor service indexed by l. In each period, a saver household has probability ω_w to optimally re-adjust the wage rate that applies to all of its workers, $W_t(l)$. If the wage cannot be re-optimized, it will be increased at the geometric average of the steady-state rate of inflation Π and of last period inflation Π_{t-1}, according to the rule

$$W_t(l) = W_{t-1}(l) \left(\Pi_{t-1} e^{\varkappa}\right)^{\xi_w} \left(\Pi e^{\xi}\right)^{1-\xi_w}, \tag{D23}$$

where $\xi_w \in [0,1]$ captures the degree of nominal wage indexation. All households, including both savers and non-savers, sell their labor service to a representative, competitive agency that transforms it into an aggregate labor input, according to the technology

$$L_t = \left(\int_0^1 L_t(l)^{\frac{1}{1+\eta_t^w}} dl \right)^{1+\eta_t^w}, \tag{D24}$$

where η_t^w is an *i.i.d.* exogenous wage mark-up shock. The agency rents labor type $L_t(l)$ at a price $W_t(l)$ and sells a homogeneous labor input to the intermediate producers at a price W_t. The static profit maximization problem yields the labor demand function

$$L_t(l) = L_t \left(W_t(l)/W_t \right)^{-(1+\eta_t^w)/\eta_t^w}. \tag{D25}$$

Labor unions use this marginal rate of substitution as the cost of labor services in their negotiations with labor packers. The markup above the marginal disutility is distributed to the households. For those that can adjust, the problem is to choose a wage $W_t(l)$ that maximizes the discounted total wage income in the future subject to (D23) and (D25),

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \frac{\Lambda_{t+s}^S P_t}{\Lambda_t^S P_{t+s}} \left[W_{t+s}(l) - W_{t+s}^h \right] L_{t+s}(l). \tag{D26}$$

The FOC becomes

(D27)

$$\mathbb{E}_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \frac{\Lambda_{t+s}^S P_t}{\Lambda_t^S P_{t+s}} L_{t+s}(l) \left[\left(X_{t,s}^W W_t(l) - W_{t+s}^h \right) \left(-\frac{1+\eta_{w,t+s}}{\eta_{w,t+s}} \right) - X_{t,s}^W W_t(l) \right] = 0,$$

where

$$X_{t,s}^W = \begin{cases} 1 & \text{for } s = 0 \\ \left(\prod_{k=1}^s \Pi_{t+k-1}^{\xi^w} \Pi^{1-\xi^w} \right) & \text{for } s = 1, \ldots, \infty \end{cases} \tag{D28}$$

D4. Monetary and Fiscal Policy

We have assumed that the government supplies one-period bonds in zero net supply, and both types of households receive the same amount of transfers. It follows that the government's nominal budget constraint is

$$P_t^m B_t^m + \tau_{K,t} R_{K,t} K_t + \tau_{L,t} W_t L_t + \tau_{C,t} P_t C_t = \left(1 + \rho P_t^m\right) B_{t-1}^m + P_t G_t + P_t Z_t, \tag{D29}$$

where C_t and Z_t denote aggregate consumption and total transfers, respectively. Their expressions are the following

$$C_t = \mu C_t^N + (1-\mu)C_t^S, \tag{D30}$$

$$Z_t = \int_0^1 Z_t(j)dj. \tag{D31}$$

The budget constraint (D29) implies that the fiscal authority finances government expenditures, transfers, and the rollover of expiring long-term debt by issuing new long-term debt obligations as well as by raising labor, capital, and consumption taxes.

The aggregate resource constraint is given by

$$Y_t = C_t + I_t + G_t + \Psi_t(\nu_t)\bar{K}_{t-1}. \tag{D32}$$

We rescale the variables entering the fiscal rules as $g_t = G_t/A_t$ and $z_t = Z_t/A_t$. For each variable $x_t, \widehat{x}_t$ denotes the percentage deviation from its balanced-growth steady state.

Let $s_{b,t} = (P_t^m B_t^m)/(P_t Y_t)$ denote the real market debt-to-GDP ratio. The fiscal authority adjusts government spending $\hat{g}_t$, transfers $\hat{z}_t$, and tax rates on capital income, labor income, and consumption $\hat{\tau}_J, J \in \{k, l, c\}$ as follows:

$$\hat{g}_t = \rho_G \hat{g}_{t-1} - (1-\rho_G)\left[\gamma_G\left(\hat{s}_{b,t-1} - \hat{s}_{b,t}^*\right) + \phi_{g,y}\hat{y}_t\right] + \varepsilon_t^g, \tag{D33}$$

$$\hat{z}_t^b = \rho_Z \hat{z}_{t-1}^b - (1-\rho_Z)\left[\gamma_Z\left(\hat{s}_{b,t-1} - \hat{s}_{b,t}^*\right) + \phi_{z,y}\hat{y}_t\right] + \varepsilon_t^z, \tag{D34}$$

$$\hat{\tau}_{J,t} = \rho_J \hat{\tau}_{J,t-1} + (1-\rho_J)\gamma_J\left[\hat{s}_{b,t-1} - \hat{s}_{b,t}^*\right] + u_t^J, \text{ for } J \in \{k, l\}; \tag{D35}$$

$$\hat{\tau}_{c,t} = \rho_c \hat{\tau}_{c,t-1}; \tag{D36}$$

where γ_G, γ_Z, and $\gamma_J > 0$ are large enough to guarantee that the debt remains on a stable path. The time-varying debt target, $s_{b,t}^*$, follows a stationary AR(1) process

$$\hat{s}_{b,t}^* = \rho_s \hat{s}_{b,t-1}^* + \varepsilon_t^U, \quad \varepsilon_t^U \sim N(0, \sigma_U^2),$$

with $\rho_s \in (0,1)$. ε_t^U is the unfunded fiscal shock that increases the government's debt target.

Finally, the central bank follows the Taylor rule and adjusts the short-term interest rate R_t^n in response to inflation deviations and the output gap. The linearized monetary policy rule is the following:

$$\hat{r}_t^n = \rho_r \hat{r}_{t-1}^n + (1-\rho_r)[\phi_\pi(\hat{\pi}_t - \hat{\pi}_t^*) + \phi_y \hat{y}_t] + u_t^m \tag{D37}$$

where u_t^m is the monetary policy shock and $\phi_\pi > 1$ satisfies the Taylor principle. We introduce a time-varying inflation target which follows a stationary AR(1) process, that is

$$\hat{\pi}_t^* = \phi_F \hat{\pi}_{t-1}^* + \Phi\varepsilon_t^U, \quad \Phi > 0$$

D5. Specifying Shocks and Introducing Incomplete Information

Throughout the paper, we use the notation u_t^x to denote a persistent shock and η_t^x to denote a transitory shock. For each $x \in \{d, i, rp, p, a, g, z, m\}$, we specify a stationary AR(1) process for the persistent shock u_t^x as

$$u_t^x = \rho_x u_{t-1}^x + \varepsilon_t^x, \quad \varepsilon_t^x \sim N(0, \sigma_x^2)$$

with $\rho_x \in (0,1)$. We let the transitory shock $\eta_t^p \sim N(0, \sigma_{\eta,p}^2)$ and $\epsilon_t^U \sim N(0, \sigma_U^2)$ follow an *i.i.d.* process.

We now introduce incomplete information by assuming all households, intermediate firms, and final good firms share the same information set, denoted by $\mathcal{I}_t^{HH}$. We assume agents can observe the entire histories of nominal interest rate $\{r_{t-k}|k \geq 0\}$ and inflation $\{\pi_{t-k}|k \geq 0\}$. The households can also observe history of surpluses $\{\tau_{t-k}|k \geq 0\}$, the realized real market debt $\{s_{b,t-k}|k \geq 0\}$, and the histories of labor, capital, and consumption tax rates $\{\tau_{J,t-k}|k \geq 0\}$. However, households cannot distinguish between the exogenous shock to fiscal and monetary policy and the shock to time-varying debt and inflation targets.

There are now four shocks to fiscal and monetary policy rules, but only three signals and incomplete information arise naturally in the model. The monetary policy rule (D37) and the fiscal rules (D33) and (D34) indicate that policy variables can also serve as signals to households. For instance, rewriting the monetary rule (D37) as

$$\hat{r}_t^n - \rho_r \hat{r}_{t-1}^n - (1-\rho_r)\left(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t\right) = \underbrace{-(1-\rho_r)\phi_\pi \pi_t^* + u_t^m}_{s_t^m}$$

indicates the history of the right side variables, s_t^m, is also known to households. Similarly, rewriting the fiscal rules (D33) and (D34) as

$$\hat{g}_t - \rho_G \hat{g}_{t-1} + (1-\rho_G)\left[\gamma_G \hat{s}_{b,t-1} + \phi_{g,y}\hat{y}_t\right] = \underbrace{(1-\rho_G)\gamma_G \hat{s}_{b,t}^* + \varepsilon_t^g}_{s_t^g}$$

$$\hat{z}_t - \rho_Z \hat{z}_{t-1} + (1-\rho_Z)\left[\gamma_Z \hat{s}_{b,t-1} + \phi_{z,y}\hat{y}_t\right] = \underbrace{(1-\rho_Z)\gamma_Z \hat{s}_{b,t}^* + \varepsilon_t^z}_{s_t^z}$$

suggests that the two right side variables, s_t^g and s_t^z, are also in the household's information set. Additionally, agents cannot distinguish between shocks to time-varying debt targets and shocks to the tax rules. Finally, the tax rules (D35) can also be rewritten with signals where s_t^J are in the household's information set. We do not include shock to consumption tax rate in our estimation.

$$\hat{\tau}_{J,t} - \rho_J \hat{\tau}_{J,t-1} - (1-\rho_J)\gamma_J \hat{s}_{b,t-1} = \underbrace{-(1-\rho_J)\gamma_J \hat{s}_{b,t}^* + \varepsilon_t^j}_{s_t^J} \quad \forall J \in \{k, l\}$$

We do not introduce incomplete information to any other shocks in the model and assume that they can be observed perfectly by households. These shocks are intended to improve the empirical fit of the medium-scale DSGE model. Formally, we define the incomplete information set $\mathcal{I}_t^{HH}$ as

$$\mathcal{I}_t^{HH} = \{s_{t-k}^m, s_{t-k}^g, s_{t-k}^z, s_{t-k}^k, s_{t-k}^l \mathcal{M} | k \geq 0\}.$$

D6. Deriving the log-linearized equilibrium conditions

To make the model stationary, we de-trend the non-stationary variables, accounting for the unit root in the labor-augmenting technology process. We define the following variables: $y_t = \frac{Y_t}{A_t}, c_t^* = \frac{C_t^{*S}}{A_t}, c_t^S = \frac{C_t^S}{A_t}, c_t^N = \frac{C_t^N}{A_t}, k_t = \frac{K_t}{A_t}, g_t = \frac{G_t}{A_t}, z_t = \frac{Z_t}{A_t}, b_t = \frac{P_t^m B_t^m}{P_t A_t}, s_{b,t} = \frac{P_t^m B_t^m}{P_t Y_t}, w_t = \frac{W_t}{P_t A_t}$, and $\lambda_t^S = \Lambda_t^S A_t$. In what follows, $e^{\varkappa}$ denotes the steady-state growth of the technology process. That is, $e^a = e^{\varkappa}$.

Production function:

$$\hat{y}_t = \frac{y + \Omega}{y} \left[\alpha \hat{k}_t + (1 - \alpha) \hat{L}_t \right]. \tag{D38}$$

Capital-labor ratio:

$$\hat{r}_{K,t} - \hat{w}_t = \hat{L}_t - \hat{k}_t. \tag{D39}$$

Marginal cost:

$$\hat{m}_t = \alpha \hat{r}_{K,t} + (1 - \alpha) \hat{w}_t. \tag{D40}$$

Phillips curve:

$$\hat{\pi}_t = \frac{\beta}{1 + \xi_p \beta} \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\xi_p}{1 + \xi_p \beta} \hat{\pi}_{t-1} + \kappa_p \hat{m}_t + \kappa_p \hat{tp}_t, \tag{D41}$$

where $\kappa_p = [(1 - \beta\omega_p)(1 - \omega_p)] / [\omega_p(1 + \beta\xi_p)]$.

Saver household's FOC for consumption:

$$\hat{\lambda}_t^S = \hat{u}_t^d - \frac{h}{e^z - h} \hat{u}_t^a - \frac{e^z}{e^z - h} \hat{c}_t^{*S} + \frac{h}{e^z - h} \hat{c}_{t-1}^{*S} - \frac{\tau^C}{1 + \tau^C} \hat{\tau}_t^C \tag{D42}$$

Cost-Push shock Process:

$$\hat{tp}_t = \hat{u}_t^p + \hat{\eta}_t^p \tag{D43}$$

Public/private consumption in utility:

$$\hat{c}_t^* = \frac{c^S}{c^S + \alpha_G g}\hat{c}_t^S + \frac{\alpha_G g}{c^S + \alpha_G g}\hat{g}_t \tag{D44}$$

Euler equation:

$$\hat{\lambda}_t^S = \hat{r}_{n,t} + \mathbb{E}_t\hat{\lambda}_{t+1}^S - \mathbb{E}_t\hat{\pi}_{t+1} - \mathbb{E}_t\hat{u}_{t+1}^a + \hat{u}_t^{rp} \tag{D45}$$

The maturity structure of debt is

$$\hat{r}_{n,t} + \hat{P}_t^m = \frac{P_m}{1+P_m}\mathbb{E}_t\hat{P}_{t+1}^m - \hat{u}_t^{rp} \tag{D46}$$

where $P_m = \frac{\rho}{R_n - \rho}$.

Saver household's FOC for capacity utilization:

$$\hat{r}_{K,t} - \frac{\tau_K}{1-\tau_K}\hat{r}_{K,t} = \frac{\psi}{1-\psi}\hat{v}_t. \tag{D47}$$

Saver household's FOC for capital:

(D48)
$$\hat{q}_t = \mathbb{E}_t\hat{\pi}_{t+1} - \hat{r}_{n,t} + \beta e^{-\varkappa}(1-\tau_K)r_K\mathbb{E}_t\hat{r}_{K,t+1} - \beta e^{-\varkappa}\tau_K r_K\mathbb{E}_t\hat{\tau}_{K,t+1} + \beta e^{-\varkappa}(1-\delta)\mathbb{E}_t\hat{q}_{t+1} - \hat{u}_t^{rp}$$

Saver household's FOC for investment:

$$\hat{i}_t = \frac{1}{(1+\beta)se^{2\varkappa}}\hat{q}_t + \hat{u}_t^i + \frac{\beta}{1+\beta}\mathbb{E}_t\hat{i}_{t+1} - \frac{1}{1+\beta}\hat{u}_t^a + \frac{\beta}{1+\beta}\mathbb{E}_t\hat{u}_{t+1}^a + \frac{1}{1+\beta}\hat{i}_{t-1} \tag{D49}$$

Effective capital:

$$\hat{k}_t = \hat{v}_t + \hat{\bar{k}}_{t-1} - \hat{u}_t^a. \tag{D50}$$

Law of motion for capital:

$$\hat{\bar{k}}_t = (1-\delta)e^{-\varkappa}\left(\hat{\bar{k}}_{t-1} - \hat{u}_t^a\right) + \left[1-(1-\delta)e^{-\varkappa}\right]\left[(1+\beta)se^{2\varkappa}\hat{u}_t^i + \hat{i}_t\right]. \tag{D51}$$

Hand-to-mouth household's budget constraint:

$$\tau_C C_N\hat{\tau}_{C,t} + (1+\tau_C)C_N\hat{C}_t^N = (1-\tau_L)wL\left(\hat{w}_t + \hat{L}_t\right) - \tau_L wL\hat{\tau}_{L,t} + z\hat{z}_t. \tag{D52}$$

Wage equation:

$$\hat{w}_t = \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}\mathbb{E}_t\hat{w}_{t+1} - \kappa_w\left[\hat{w}_t - \chi\hat{L}_t - \hat{d}_t + \lambda_t^S - \frac{\tau_L}{1-\tau_L}\hat{\tau}_{L,t}\right] + \frac{\chi_w}{1+\beta}\hat{\pi}_{t-1}$$

(D53)

$$- \frac{1+\beta\chi_w}{1+\beta}\hat{\pi}_t + \frac{\beta}{1+\beta}\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\chi_w}{1+\beta}\hat{u}^a_{t-1} - \frac{1+\beta\chi_w - \rho_a\beta}{1+\beta}\hat{u}^a_t + \kappa_w\eta^w_t$$

where $\quad \kappa_w \equiv [(1-\beta\omega_w)(1-\omega_w)] \,/\, \left[(1+\beta)\omega_w\left(1+\frac{(1+n^w)\xi}{\eta^w}\right)\right]$.

Aggregate households' consumption:

$$c\hat{c}_t = c^S(1-\mu)\hat{c}^S_t + c^N\mu\hat{c}^N_t. \tag{D54}$$

Aggregate resource constraint:

$$y\hat{y}_t = c\hat{c}_t + i\hat{i}_t + g\hat{g}_t + \psi'(1)k\hat{k}_t. \tag{D55}$$

Government budget constraint:

(D56)

$$\begin{aligned}&\frac{b}{y}\hat{b}_t + \tau_K r_K\frac{k}{y}\left[\hat{\tau}_{K,t} + \hat{r}_{K,t} + \hat{k}_t\right] + \tau_L w\frac{L}{y}\left[\hat{\tau}_{L,t} + \hat{w}_t + \hat{L}_t\right] + \tau_C\frac{C}{y}\left(\hat{\tau}_{C,t} + \hat{c}_t\right)\\ &= \frac{1}{\beta}\frac{b}{y}\left[\hat{b}_{t-1} - \hat{\pi}_t - \hat{P}^m_{t-1} - \hat{u}^a_t\right] + \frac{b}{y}\frac{\rho}{ye^{\varkappa}}\hat{P}^m_t + \frac{g}{y}\hat{g}_t + \frac{z}{y}\hat{z}_t,\end{aligned}$$

where we define $b_t = \frac{P^m_t B_t}{P_t A_t}$ so that $s_{b,t} = \frac{P^m_t B_t}{P_t Y_t} = \frac{b_t}{y_t}$. It follows that

$$s_{b,t} = \hat{b}_t - \hat{y}_t. \tag{D57}$$

Fiscal rules:

$$\hat{g}_t = \rho_G\hat{g}_{t-1} - (1-\rho_G)\left[\gamma_G(\hat{s}_{b,t-1} - \hat{s}^*_{b,t}) + \phi_{g,y}\hat{y}_t\right] + \hat{u}^g_t \tag{D58}$$

$$\hat{z}_t = \rho_Z\hat{z}_{t-1} - (1-\rho_Z)\left[\gamma_Z(\hat{s}_{b,t-1} - \hat{s}^*_{b,t}) + \phi_{z,y}\hat{y}_t\right] + \hat{u}^z_t \tag{D59}$$

$$\hat{\tau}_{L,t} = \rho_L\hat{\tau}_{L,t-1} + (1-\rho_L)\gamma_L[\hat{s}_{b,t-1} - \hat{s}^*_{b,t}] + \hat{u}^l_t \tag{D60}$$

$$\hat{\tau}_{K,t} = \rho_K\hat{\tau}_{K,t-1} + (1-\rho_K)\gamma_K[\hat{s}_{b,t-1} - \hat{s}^*_{b,t}] + \hat{u}^k_t \tag{D61}$$

where the time-varying debt target follows an AR(1) process

$$s^*_{b,t} = \rho_s s^*_{b,t-1} + \epsilon^U_t \tag{D62}$$

Monetary Rule:

$$\hat{r}_{n,t} = \rho_r\hat{r}_{n,t-1} + (1-\rho_r)\left[\phi_\pi(\hat{\pi}_t - \hat{\pi}^*_t) - \phi_y\hat{y}_t\right] + u^m_t \tag{D63}$$

Time-varying inflation target:

$$\hat{\pi}^*_t = \phi_F \hat{\pi}^*_{t-1} + \Phi \epsilon^U_t \tag{D64}$$

D7. Parameters From Second Sample Estimation

Parameter	**FIRE**				**IIRE**			
	Mode	Median	5%	95%	Mode	Median	5%	95%
Gov spending σ_g	2.3343	2.349	2.330	2.371	2.4075	2.420	2.413	2.426
Technology σ_a	3.6055	3.605	3.593	3.610	3.3287	3.314	3.309	3.320
Preference σ_b	4.9976	4.994	4.990	4.998	4.997	4.997	4.995	5.000
Monetary σ_m	0.1995	0.191	0.185	0.197	0.1505	0.154	0.144	0.162
Investment σ_i	3.3801	3.366	3.355	3.387	2.949	2.959	2.947	2.969
Wage σ_w	1.1923	1.164	1.153	1.182	0.5428	0.540	0.520	0.551
Price markup σ_p	0.0791	0.081	0.077	0.087	0.2599	0.272	0.269	0.282
Risk prem. σ_{rp}	2.7622	2.758	2.751	2.769	4.5083	4.516	4.506	4.526
Cost-push σ_{cp}	0.5478	0.526	0.496	0.546	0.4787	0.459	0.441	0.474
Transfer σ_z	4.9955	4.998	4.995	5.000	4.9876	4.991	4.985	4.994
Unfunded σ_U	0.5063	0.520	0.515	0.531	1.7962	1.843	1.829	1.858
Capital tax σ_{tk}	4.9955	4.992	4.991	4.994	4.9924	4.990	4.989	4.991
Labor tax σ_{tl}	3.0293	3.038	3.030	3.047	2.9954	2.988	2.975	3.010
GDP M.E. σ_y^{ME}	1.7591	1.766	1.738	1.782	1.7213	1.752	1.737	1.774
Debt to GDP M.E. σ_b^{ME}	4.9881	4.997	4.991	5.000	4.9864	4.988	4.986	4.991
Breakeven inflation σ_π^{5yr}	0.5844	0.574	0.566	0.600	0.2044	0.196	0.188	0.203

Table D1—Parameter Estimates From Second Sample, 2008 Q1 -2022 Q3

The Justification of The Inflation Target

The short term inflation target in the DSGE model follows an AR(1) process with unfunded shock. In this section we show that this specification of the inflation target is equivalent to the solution derived from analytically solving a shadow economy with a fiscal led regime.

E1. The Analytical Solutions to The Shadow Economy

Suppose that the shadow economy is a stylized 3-equation New Keynesian model augmented with a government budget constraint and a fiscal rule. The linearized equilibrium conditions are as follows.

$$i_t = \phi_F \pi_t, \qquad 0 < \phi_F < 1, \tag{E1}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t, \tag{E2}$$

$$x_t = \mathbb{E}_t x_{t+1} - \sigma \left(i_t - \mathbb{E}_t \pi_{t+1} \right), \tag{E3}$$

$$\hat{z}_t = \gamma_z \, sb_{t-1} + \eta_t, \tag{E4}$$

$$sb_t = \beta^{-1} \Big(sb_{t-1} + i_t - \pi_t + (1-\beta)\lambda \hat{z}_t \Big), \tag{E5}$$

where z_t denotes the transfer, s_t denotes the primary surplus, and $\lambda = \frac{Z_{ss}}{S_{ss}}$, the ratio of steady-state transfer to steady-state primary surplus. Substituting (E4) into (E5), we have

$$\begin{aligned} sb_t &= \beta^{-1} \Big(sb_{t-1} + i_t - \pi_t + (1-\beta)\lambda \hat{z}_t \Big) \\ &= \beta^{-1} \Big(sb_{t-1} + i_t - \pi_t + (1-\beta)\lambda (\gamma_z sb_{t-1} + \eta_t) \Big). \end{aligned} \tag{E6}$$

Using the monetary rule $i_t = \phi_F \pi_t$:

$$sb_t = \beta^{-1} \Big(\underbrace{\big[1 + (1-\beta)\lambda\gamma_z\big]}_{A_z} sb_{t-1} - \underbrace{(1-\phi_F)}_{\text{since } \phi_F \pi_t - \pi_t} \pi_t + \underbrace{(1-\beta)\lambda}_{C_z} \eta_t \Big). \tag{E7}$$

Define$A_z \equiv 1 + (1-\beta)\lambda\gamma_z$ and $C_z \equiv (1-\beta)\lambda$. Multiplying by β on both sides of the linearized budget constraint yields:

$$\beta sb_t \ = \ A_z sb_{t-1} - (1-\phi_F)\pi_t + C_z \eta_t. \tag{E8}$$

E2. Guess-and-verify Solutions

To obtain analytical solutions for the shadow economy, we use the guess-and-verify solution. Guess that the equilibrium variables are linear in lagged debt and the transfer shock:

$$\pi_t = \pi_s \, sb_{t-1} + \pi_\eta \, \eta_t, \tag{E9}$$

$$x_t = x_s \, sb_{t-1} + x_\eta \, \eta_t, \tag{E10}$$

$$sb_t = s_s \, sb_{t-1} + s_\eta \, \eta_t. \tag{E11}$$

Because η_t is i.i.d., we have $\mathbb{E}_t sb_{t+1} = s_s sb_t$, $\mathbb{E}_t \pi_{t+1} = \pi_s sb_t$, and $\mathbb{E}_t x_{t+1} = x_s sb_t$.

Substitute (E9)–(E11) into (E8):

$$\beta \big(s_s sb_{t-1} + s_\eta \eta_t \big) = A_z sb_{t-1} - (1-\phi_F) \big(\pi_s sb_{t-1} + \pi_\eta \eta_t \big) + C_z \eta_t. \tag{E12}$$

Matching coefficients gives

$$\beta s_s = A_z - (1 - \phi_F)\pi_s, \tag{E13}$$

$$\beta s_\eta = -(1 - \phi_F)\pi_\eta + C_z. \tag{E14}$$

Similarly, by substituting (E9) and (E10) into (E2), we have:

$$\pi_s sb_{t-1} + \pi_\eta \eta_t = \beta \pi_s sb_t + \kappa(x_s sb_{t-1} + x_\eta \eta_t). \tag{E15}$$

Using $sb_t = s_s sb_{t-1} + s_\eta \eta_t$ and matching coefficients yield

$$\pi_s = \beta \pi_s s_s + \kappa x_s, \tag{E16}$$

$$\pi_\eta = \beta \pi_s s_\eta + \kappa x_\eta. \tag{E17}$$

Finally, combining (E3), and (E1) and substituting (E9) and (E11) yields:

$$(1 - s_s)x_s = \sigma \pi_s (s_s - \phi_F), \tag{E18}$$

$$\phi_F \pi_\eta \sigma - \pi_s s_\eta \sigma - s_\eta x_s + x_\eta = 0. \tag{E19}$$

From (E18), we have

$$x_s = \frac{\sigma \pi_s (s_s - \phi_F)}{1 - s_s}, \qquad s_s \neq 1. \tag{E20}$$

E3. Solving for The Debt Dynamics

Since sb_{t-1} is a state variable, we need to solve the law of motion for sb_t before we proceed to the analytical solutions of π_t. Combine (E16) with (E20) and assuming $\pi_s \neq 0$ we have

$$(1 - s_s)(1 - \beta s_s) = \kappa\sigma(s_s - \phi_F). \tag{E21}$$

This yields the quadratic

$$\beta s_s^2 - (1 + \beta + \kappa\sigma)s_s + (1 + \kappa\sigma\phi_F) = 0, \tag{E22}$$

where the two roots are

$$s_s^{(\pm)} = \frac{1 + \beta + \kappa\sigma \pm \sqrt{(1 + \beta + \kappa\sigma)^2 - 4\beta(1 + \kappa\sigma\phi_F)}}{2\beta}. \tag{E23}$$

To avoid explosive debt dynamics requires the root to be stable and hence we have:

$$s_s = \frac{1 + \beta + \kappa\sigma - \sqrt{(1 + \beta + \kappa\sigma)^2 - 4\beta(1 + \kappa\sigma\phi_F)}}{2\beta}. \tag{E24}$$

With s_s, we can solve for π_s and x_s from (E13) and (E16), respectively:

$$\pi_s = \frac{A_z - \beta s_s}{1 - \phi_F} \tag{E25}$$

and from (E16) we can get,

$$x_s = \frac{1 - \beta s_s}{\kappa}\,\pi_s. \tag{E26}$$

Define $r \equiv \frac{C_z}{A_z} = \frac{(1-\beta)\lambda}{1+(1-\beta)\lambda\gamma_z}$, where $A_z \neq 0$. Guess that $\pi_\eta = r\pi_s$ and $s_\eta = rs_s$, we solve π_η and s_η from (E13), which yields

$$\pi_\eta = \frac{C_z}{A_z}\pi_s = \frac{(1-\beta)\lambda}{1+(1-\beta)\lambda\gamma_z} \cdot \frac{A_z - \beta s_s}{1 - \phi_F}, \tag{E27}$$

and

$$s_\eta = \frac{C_z}{A_z}s_s = \frac{(1-\beta)\lambda}{1+(1-\beta)\lambda\gamma_z}s_s. \tag{E28}$$

Then, we have

$$sb_t = s_s sb_{t-1} + s_\eta\eta_t = s_s\frac{\pi_t - \pi_\eta\eta_t}{\pi_s} + s_\eta\eta_t. \tag{E29}$$

The one-period-ahead inflation is

$$\begin{aligned} \pi_{t+1} &= \pi_s sb_t + \pi_\eta\eta_{t+1} \\ &= s_s\pi_t + \big(\pi_s s_\eta - s_s\pi_\eta\big)\eta_t + \pi_\eta\eta_{t+1}. \end{aligned} \tag{E30}$$

Since $\pi_s s_\eta - s_s\pi_\eta = 0$, we have

$$\pi_{t+1} = s_s\pi_t + \pi_\eta\eta_{t+1}, \tag{E31}$$

which is AR(1) consistent with specification of equation (31) in the quantitative model.

State-space representation for estimation

This section establishes the incomplete information model solution as a state-space representation used in the estimation. We follow the notation and algorithm of Blanchard, L'Huillier and Lorenzoni (2013) closely. The signal extraction problem is defined by a pair of equations

$$\mathbf{x_t} = \mathbf{A}\mathbf{x_{t-1}} + \mathbf{B}\nu_\mathbf{t}, \tag{F1}$$

$$\mathbf{s_t} = \mathbf{C}\mathbf{x_t} + \mathbf{D}\nu_\mathbf{t}, \tag{F2}$$

where ν_t is an n_ν-dimensional vector of mutually independent *i.i.d.* shocks. The dimensions of exogenous variables $\mathbf{x_t}$ and signals $\mathbf{s_t}$ are n_x and n_s. Let $\mathbf{y_t}$ denote a vector of endogenous state variables of size n_y. The economic model can be described in terms of the stochastic difference equation

$$\mathbf{F}\mathbb{E}_\mathbf{t}[\mathbf{y_{t+1}}] + \mathbf{G}\mathbf{y_t} + \mathbf{H}\mathbf{y_{t-1}} + \mathbf{M}\mathbf{s_t} + \mathbf{N}\mathbb{E}_\mathbf{t}[\mathbf{s_{t+1}}] = \mathbf{0}, \tag{F3}$$

where $\mathbb{E}_t(\cdot)$ is the rational expectations operator under incomplete information and $\mathbf{F}, \mathbf{G}, \mathbf{H}, \mathbf{M}, \mathbf{N}$ are matrices of parameters. The solution of the model can be described as

$$\mathbf{y_t} = \mathbf{P}\mathbf{y_{t-1}} + \mathbf{Q}\mathbf{s_t} + \mathbf{R}\mathbf{x_{t|t}}, \tag{F4}$$

where the matrices $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ can be found by solving the three matrix equations

$$\mathbf{F}\mathbf{P}^\mathbf{2} + \mathbf{G}\mathbf{P} + \mathbf{H} = \mathbf{0}, \quad (\mathbf{F}\mathbf{P} + \mathbf{G})\mathbf{Q} + \mathbf{M} = \mathbf{0},$$
$$(\mathbf{F}\mathbf{P} + \mathbf{G})\mathbf{R} + [\mathbf{F}(\mathbf{Q}\mathbf{C} + \mathbf{R}) + \mathbf{N}\mathbf{C}]\,\mathbf{A} = \mathbf{0}.$$

We know from the Kalman recursion that the law of motion of $\mathbf{x_{t|t}}$ can be written as

$$\mathbf{x_{t|t}} = \mathbf{K}\mathbf{C}\mathbf{A}\mathbf{x_{t-1}} + (\mathbf{I} - \mathbf{K}\mathbf{C})\mathbf{A}\mathbf{x_{t-1|t-1}} + \mathbf{K}(\mathbf{C}\mathbf{B} + \mathbf{D})\nu_\mathbf{t},$$

where $\mathbf{K}$ is the Kalman gain matrix of size $n_x \times n_s$. It follows that we can write the model solution $\mathbf{y}_t$ as

$$\mathbf{y_t} = \mathbf{P}\mathbf{y_{t-1}} + (\mathbf{Q} + \mathbf{R}\mathbf{K})\mathbf{C}\mathbf{A}\mathbf{x_{t-1}} + \mathbf{R}(\mathbf{I} - \mathbf{K}\mathbf{C})\mathbf{A}\mathbf{x_{t-1|t-1}} + (\mathbf{Q} + \mathbf{R}\mathbf{K})(\mathbf{C}\mathbf{B} + \mathbf{D})\nu_\mathbf{t}$$

Define the extended state variables as $\left[\mathbf{y_t}, \mathbf{x_t}, \mathbf{x_{t|t}}\right]'$. The state-space representation of the incomplete information model solution can be written as

$$\begin{bmatrix} \mathbf{y_t} \\ \mathbf{x_t} \\ \mathbf{x_{t|t}} \end{bmatrix} = \begin{bmatrix} \mathbf{P} & (\mathbf{Q} + \mathbf{R}\mathbf{K})\mathbf{C}\mathbf{A} & \mathbf{R}(\mathbf{I} - \mathbf{K}\mathbf{C})\mathbf{A} \\ \mathbf{0} & \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}\mathbf{C}\mathbf{A} & (\mathbf{I} - \mathbf{K}\mathbf{C})\mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{y_{t-1}} \\ \mathbf{x_{t-1}} \\ \mathbf{x_{t-1|t-1}} \end{bmatrix} + \begin{bmatrix} (\mathbf{Q} + \mathbf{R}\mathbf{K})(\mathbf{C}\mathbf{B} + \mathbf{D}) \\ \mathbf{B} \\ \mathbf{K}(\mathbf{C}\mathbf{B} + \mathbf{D}) \end{bmatrix} \nu_\mathbf{t}.$$

Impulse Response for all other shocks

This section presents the impulse response for the remaining shocks in the model. Each figure presents the IRFs for six variables- output, inflation, nominal interest rate, real interest rate, expected inflation, and real marginal cost, under full information and incomplete information. Figure G1 shows the impulse response for funded fiscal shock. A decrease in fiscal spending decreases production and causes deflation. The presence of hand-to-mouth agents in our model allows us to have small inflation effects from funded fiscal spending under both FIRE and incomplete information (IIRE).

We also present results for tax shocks in our fiscal block, i.e. an expansionary labor-tax shock and capital-tax shock. A one-standard-deviation negative (expansionary) labor tax shock is deflationary and reduces expected inflation under both FIRE and incomplete information (Figure G2). Cut in capital tax rate expands output and increases inflation, both of which effects are more prominent under IIRE. Output increases due to the fall in real marginal cost, which expands investment.

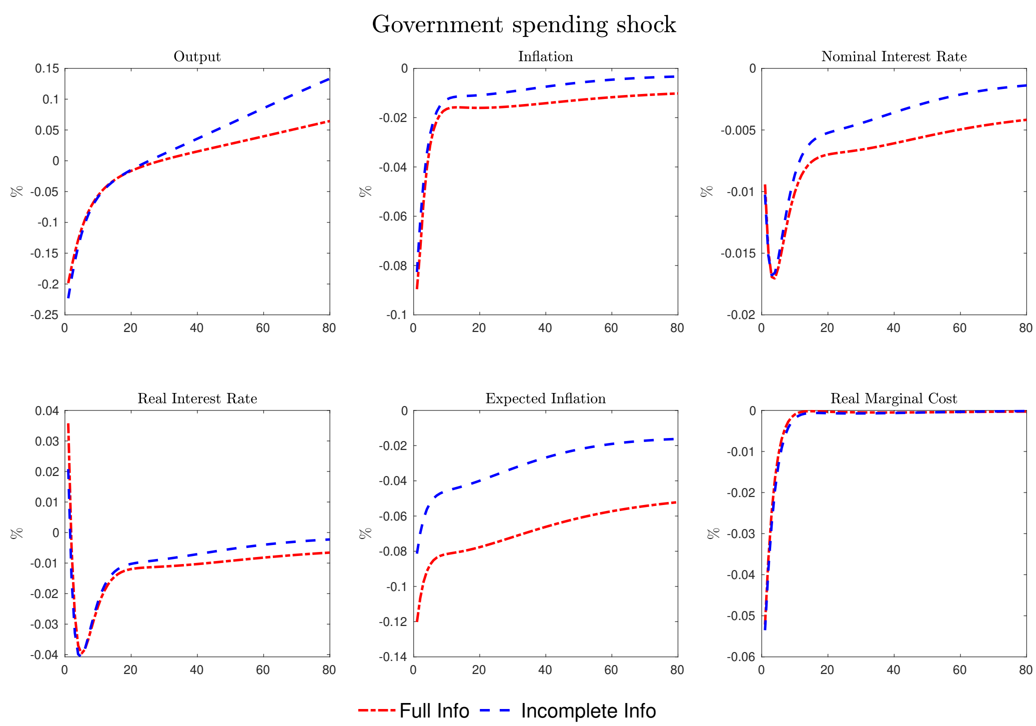

Figure G1. Impulse responses to a one standard deviation government spending shock

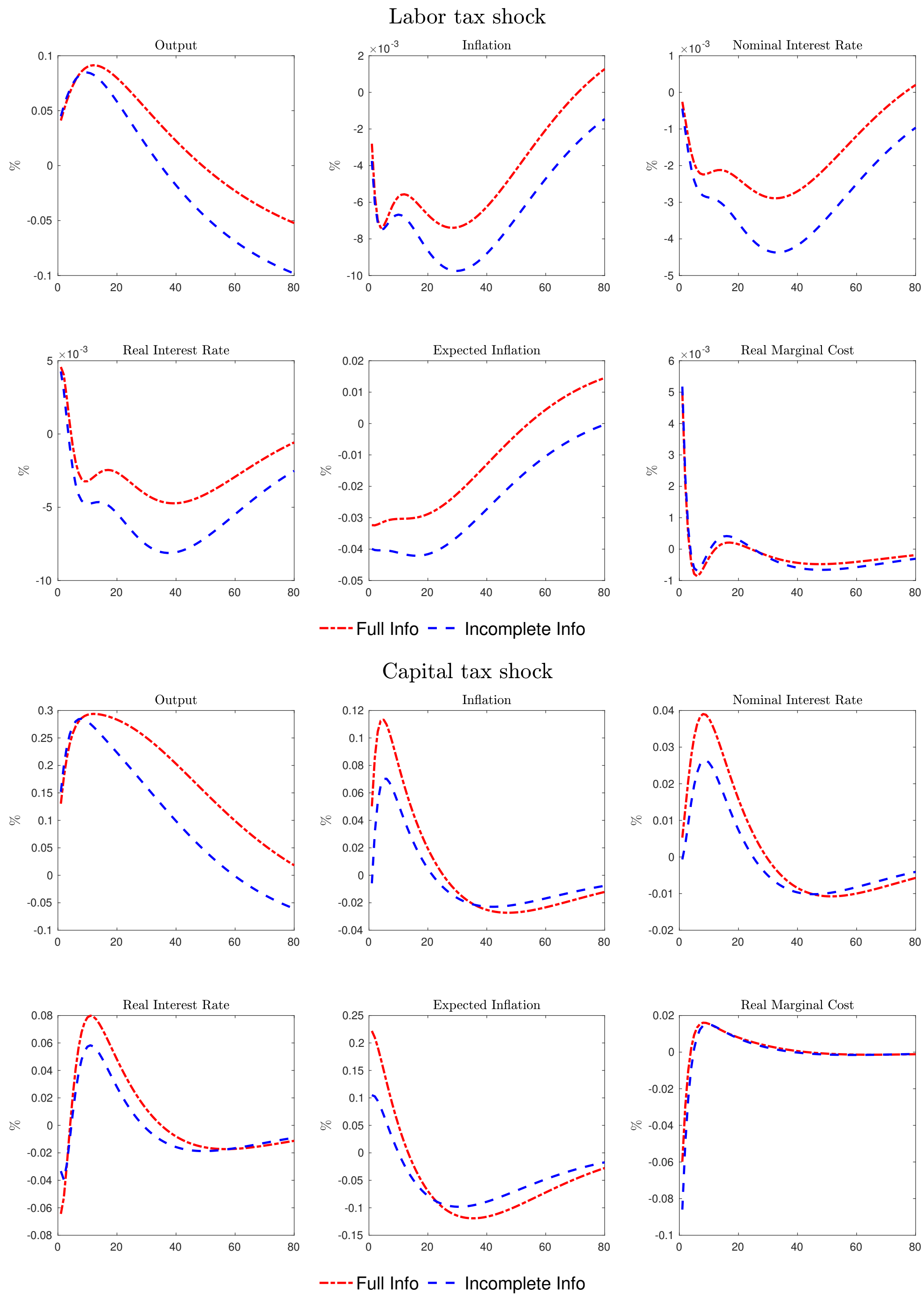

Figure G2. Impulse responses to a one standard deviation labor and capital tax shock

We do not introduce incomplete information with respect to any other shocks in our model. The response of macro variables to a one standard deviation risk premium shock (G3), technology shock, investment-specific technology shock (Figure G4), preference shock, wage mark-up shock (Figure G5), cost push shock, and price mark-up shock (G6) are largely similar under FIRE and IIRE.

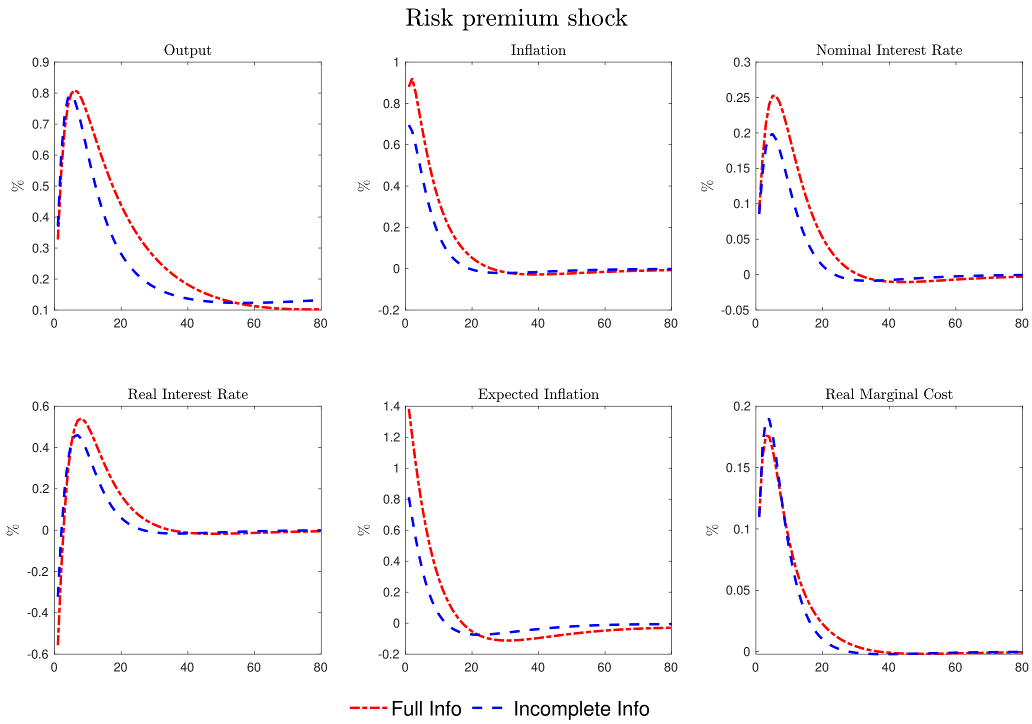

Figure G3. Impulse responses to a one standard deviation risk premium and wage markup shock

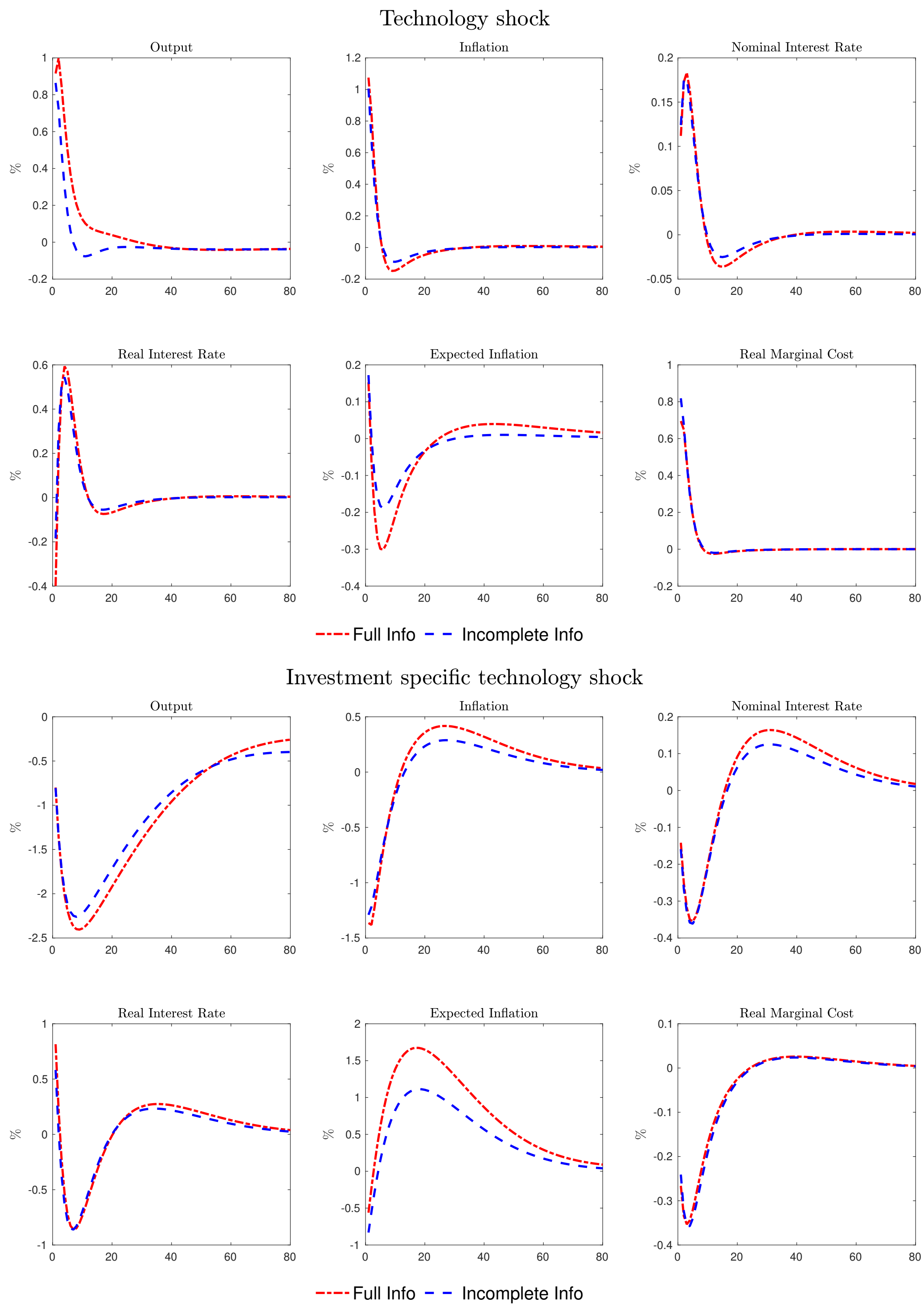

Figure G4. Impulse responses to a one standard deviation technology and investment specific technology shock

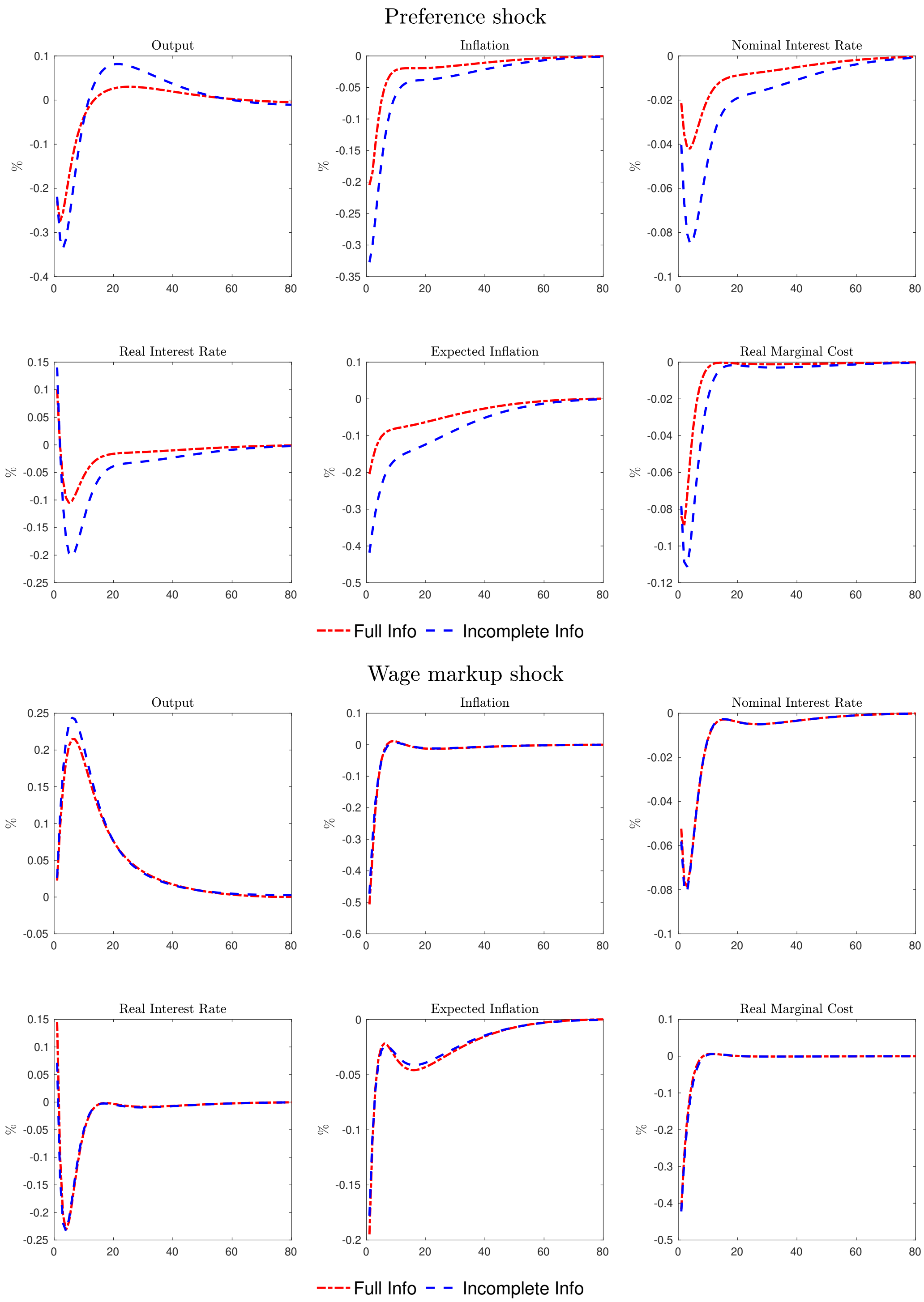

FIGURE G5. IMPULSE RESPONSES TO A ONE STANDARD DEVIATION PREFERENCE AND WAGE MARKUP SHOCK SHOCK

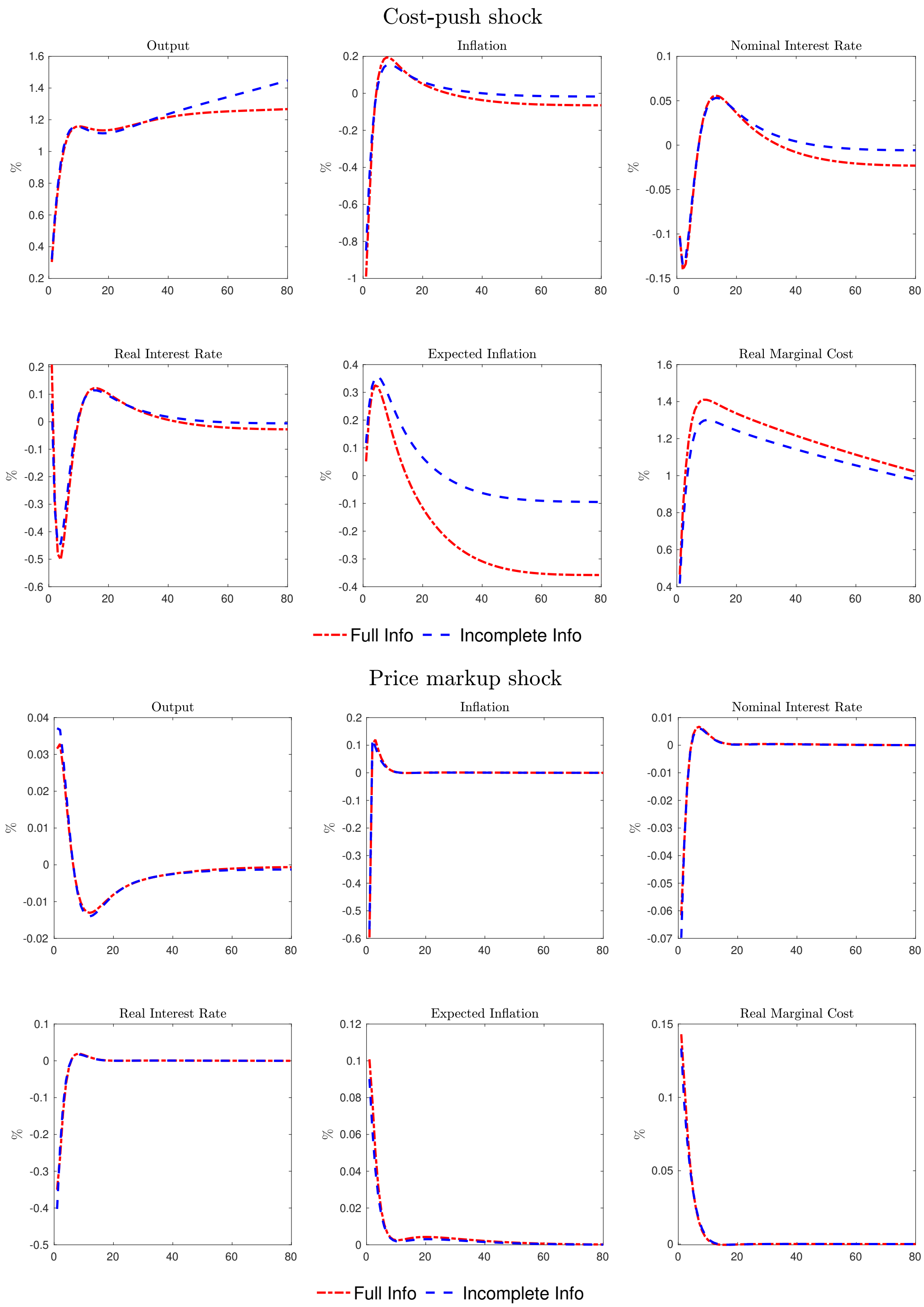

FIGURE G6. IMPULSE RESPONSES TO A ONE STANDARD DEVIATION COST PUSH AND PRICE MARKUP SHOCK

Data Construction

GDP, Consumption, and Investment: All national accounting variables are obtained from National Income and Product Account (NIPA) tables and converted to real values by dividing with the respective price indices with base year 2012:Q3. Consumption is the sum of personal expenditure in non-durable goods and services. Investment is the sub of gross private domestic investment and personal consumption expenditure in durable goods. We scale the real variables with population and convert into growth rates using the formula in Section H.H1.

Hours worked and the average weekly earning: We follow Bianchi, Faccini and Melosi (2023) in constructing the hours gap. We use average weekly hours of all non-farm workers, convert it to per capita using the population index (base year 2012:Q3) and take a difference of hours per capita from its trend which is computed as a fourth degree polynomial. Average weekly wages are constructed by dividing wage and salary accrual with average hours of non-farm workers. Wages are then converted to index with base year 2012:Q3, divided by GDP deflator, and converted to growth rates.

Interest rate and inflation: Interest rates are measured as the effective federal funds rate, inflation is computed as growth rate of GDP deflator index (BY 2012:Q3), and inflation expectations is measured using the 5-year breakeven inflation rate. In our second sample estimation, we also incorporate interest rate expectations measured by overnight index swaps for one to ten quarters ahead federal funds rate.

Government expenditure and transfers: We construct the government expenditure variable as a sum of gross government consumption expenditure and gross government investment expenditure. The nominal variables are converted to real value by dividing with the government expenditure implicit price deflators. To construct an index of government transfers we add together government social benefits, current transfer payments (incl. grants-in-aid to state and local government) and divide by GDP deflator. All real quantities are converted to growth rate after being normalized by the population index.

Revenue from labor and capital tax: Data to calculate revenue from labor and capital tax are obtained from NIPA tables. Following Leeper, Plante and Traum (2010) we first calculate the personal tax rate as:

$$\tau_p = \frac{IT}{WS + PRI/2 + CI}$$

where IT is the personal current tax revenues (Table 3.2, line 3), WS is the wage and salary accrual (Table 1.12, line 3), PRI is the proprietors's income (Table

1.12, line 9), and CI is the capital income defined as the sum of rental income (Table 1.12, line 12), corporate profits (Table 1.12, line 13), interest income (Table 1.12, line 18), and $PRI/2$. Using the personal income tax rate, the labor and capital tax revenue is calculated as:

$$T_L = \tau^p(WS + PRI/2) + CSI$$

$$T_K = \tau^p CI + CT$$

where CSI are contributions to government social insurance (NIPA Table 3.2, line 10) and CT are taxes on corporate income (Table 3.2, line 8).

H1. Observables

All nominal series are converted to real values using GDP deflator indexed to 100 in 2012:Q3 and normalized using civilian non-institutional population (obtained from BLS) indexed to 100 in 2012:Q1 such that:

$$x = ln\left(\frac{x}{\text{Popindex}}\right) \times 100$$

where x is the GDP, consumption, investment, average weekly earnings, government transfers, government consumption and investment expenditure, labor tax revenue, and capital tax revenue. All above series are then converted to growth rates.

The measurement equations are then defined as

$$X_t = C + HY_t + K\Xi_t$$

where Ξ is the vector of measurement errors. The observables are related to the model variables through the following equations:

$$\begin{bmatrix} \text{dlOutput}_t \\ \text{dlCons}_t \\ \text{dlInv}_t \\ \text{dlWage}_t \\ \text{dlGovSpend}_t \\ \text{dlTransfers}_t \\ \text{lGovDebt}_t \\ \text{lHours}_t \\ \text{lInflation}_t \\ \text{lFedFunds}_t \\ \text{dlLaborTaxRevenue}_t \\ \text{dlCapitalTaxRevenue}_t \end{bmatrix} = \begin{bmatrix} 100e^{\gamma} \\ 100e^{\gamma} \\ 100e^{\gamma} \\ 100e^{\gamma} \\ 100e^{\gamma} \\ 100e^{\gamma} \\ \log(s_b) \times 100 \\ 0 \\ \pi^* \\ \log(R_s) \times 100 \\ 100e^{\gamma} \\ 100e^{\gamma} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} + \hat{u}_t^a \\ \hat{c}_t - \hat{c}_{t-1} + \hat{u}_t^a \\ \hat{i}_t - \hat{i}_{t-1} + \hat{u}_t^a \\ \hat{w}_t - \hat{w}_{t-1} + \hat{u}_t^a \\ \hat{g}_t - \hat{g}_{t-1} + \hat{u}_t^a \\ \hat{z}_t - \hat{z}_{t-1} + \hat{u}_t^a \\ \hat{b}_t \\ \hat{L}_t \\ \hat{\pi}_t \\ \hat{R}_t \\ \hat{T}_t^L - \hat{T}_{t-1}^L + \hat{u}_t^a \\ \hat{T}_t^K - \hat{T}_{t-1}^K + \hat{u}_t^a \end{bmatrix}$$