**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans- *μ* = 45 minutes, *σ* = 8 minutes.

Let ‘X’ is time required for car drop off.

To complete the task on time we required P(X<=50), since cat work begin after 10 minutes of Drop Off.

the probability that the service manager cannot meet his commitment will be P(X>50).

P(X>50) = 1-P(X<=50)

Z = (X-*μ)/σ*

P(X<=50) = P(Z<= (50-45)/8) = P(Z<=0.625)=0.7323

P(X>50) = 1-0.7323 = 0.267

Ans- B

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans- Ans- *μ* = 38 minutes, *σ* = 6 minutes.

Let ‘X’ is no of employees.

a.Probability of employees greater than age of 44= P(X>44)

P(X>44) = 1-P(X<=44)

Z = (X-*μ)/σ*

P(X<=44) = P(Z<= (44-38)/6) = P(Z<=1)=0.8413

P(X>44) = 1-0.8413 = 0.1587

So, the probability of number of employees between 38-44 years of age = P(38<Z>44) = 0.8413-0.5 = 0.3413

So, More employees at the processing center are older than 44 than between 38 and 44 is True.

b. Probability of employees less than age of 30 = P(X<30).

Z = (30-38)/6 = -1.33

P(-1.33,Z) = 0.09176

So, the number of employees with probability 0.912 of them being under age 30= 0.0912\*400=36.48

So, the second statement is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans- *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2)

As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

So,

2X1~ N(2 µ,4 σ^2) and

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 µ, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans- The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area = 1-0.99=0.01

The range would be 0.01/2 = +/- 0.005

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z = (X-*μ)/σ*

X = Z \* *σ +μ*

*Probability of 0.005 in Z = -2.57*

X = -(-2.57 \*100+20) = 151.4

X = (-2.57 \*100+20) = 48.5

Ans- D

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans- Mean = 7+5=$12

12\*45= Rs.540

Std = 3+4=$7

7\*45=Rs. 315

1. For 95% probability, range = stats.norm.interval(0.95,540,315) = (-77.38865513011706, 1157.388655130117)
2. To find 5th Percentile, we use the formula X=μ + Zσ; wherein from z table, 5 percentiles = -1.64

X= 540+(-1.64)\*(315)= 23.4

1. Probability of Division 1 making a loss P(X<0)

stats.norm.cdf(0,5,3) =0.047

Probability of Division 2 making a loss P(X<0)

stats.norm.cdf(0,7,4) = 0.040

Probability of Division 1 making a loss in a given year is more than Division 2.