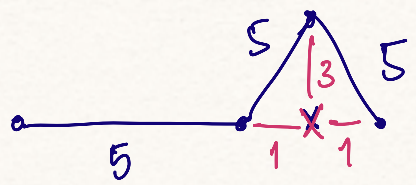
Review

G on vertices $v_1, \dots, v_n$. Every pair of vertices are connected.

$\hookrightarrow X \subseteq V$

$Z \supseteq X \} \; mst(Z) \leftarrow$ min Steiner tree on X if MST COST(Z) $\leq$ MST COST(X)

MST(original) ~ 15



Given a set of vertices (terminals), can we augment our graph in a way that new MST has a lower cost.

$G = (V, E)$ ← complete graph.

$X \subseteq Z \subseteq V$

$Y \sqcup X = Z$ ← terminals.

Y ← extra nodes

$Y \subseteq V \setminus X$

Brute force algo: $2^{n-k} \cdot m \log n \cdot c$.

- For each $Y \subseteq V \setminus X$:

  compute MST on $X \sqcup Y$.

  Store the cost

- Output the set Y that gives least cost.

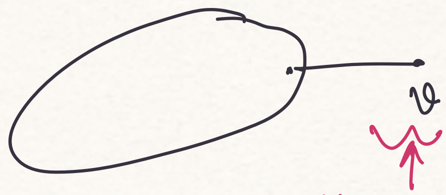
Suppose, we get a promise that at most k vertices need to be added

$$\binom{n-k}{k} \sim \frac{(n-k)!}{k!\,(n-2k)!} \qquad \underbrace{k\cdot\binom{n-k}{k}}_{k<\frac{n-k}{2}}\cdot\, m\log n\cdot c.$$

$$\left(\frac{n-k}{k}\right)^k \le \binom{n-k}{k} \le \left(\frac{e(n-k)}{k}\right)^k \sim n^{O(k)} \sim \underbrace{e\cdot\frac{n^k}{k^k}\left(1-\frac{k}{n}\right)^k}_{\le\, n^{(1+\varepsilon)k}}.$$

Claim: Any vertex from Y can have at least degree 3 in $\underbrace{MST(Z)}_{T}$ (if $MST(Z) \le MST(X)$).

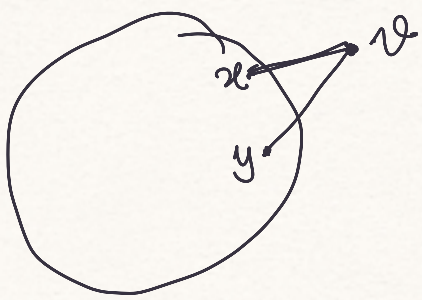
Say a vertex $v \in Y$ has a degree of 1



this node does not help.

In this case $MST(X \sqcup Y) >$ $MST((X\sqcup Y)\setminus\{v\})$

Say a vertex $v \in Y$ has a degree of 2 in T.



$w_{xy} \le w_{xv} + w_{yv}$

v is again "not needed".

Consider $MST((Y \sqcup X)\setminus\{v\})$

Using this, we get a bound on Y.

t vertices in Z

Sum of degrees in T = $2\times$ edges in the tree = $2(|Z|-1)$.

$$\sum_{x\in X} \deg_T(x) + \sum_{y \in Y} \deg_T(y)$$

$\deg_T(x) \geq 1$

$$\geq \sum_{x \in X} 1 + \sum_{y\in Y} 3$$

$$= |X| + 3|Y|$$

$$2(|X| + |Y| - 1) \geq |X| + 3|Y|$$

$\Rightarrow |Y| \leq |X| - 2$.

$\#$ Extra nodes $\leq k-2$.