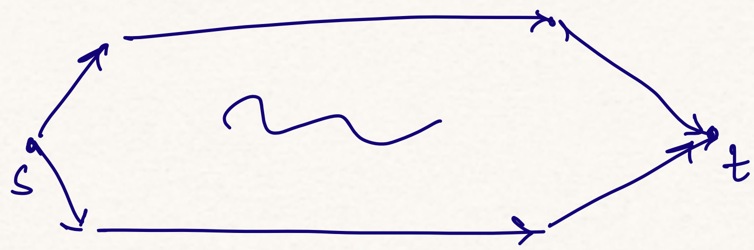
Network Flows - I

Setting: $G=(V,E)$ ↑ source to sink graph, DAG

Capacities $c: E \to \mathbb{N}$.



Want:

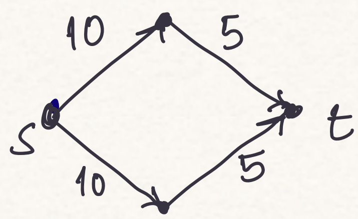
The max flow that can be routed from s to t.
(subj to capacity constraints).

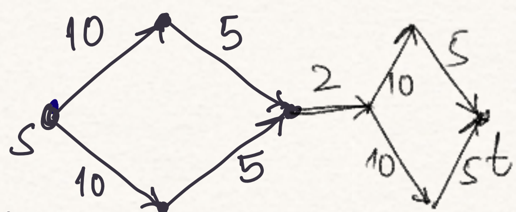
- for an edge $(\vec{u,v}) \in E$, we want $f(u\to v) \le c(u\to v)$.

$$\left.\begin{array}{l} c(u\to v)=0 \\ f(u\to v)=0 \end{array}\right\} \text{ if } (\vec{u,v}) \notin E$$

- $$\max\left(\sum_{v\in V} f(s\to v)\right).$$

$$\max \sum_{v\in V} f(v\to t)$$



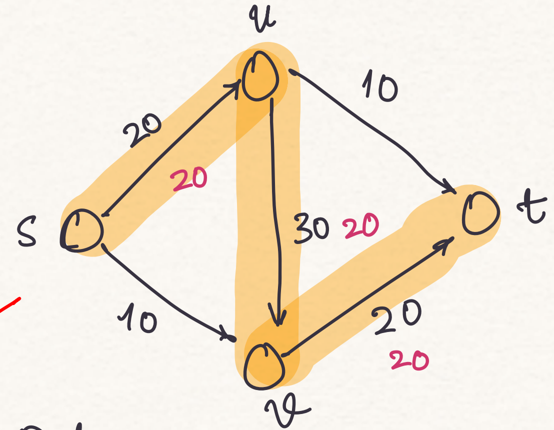


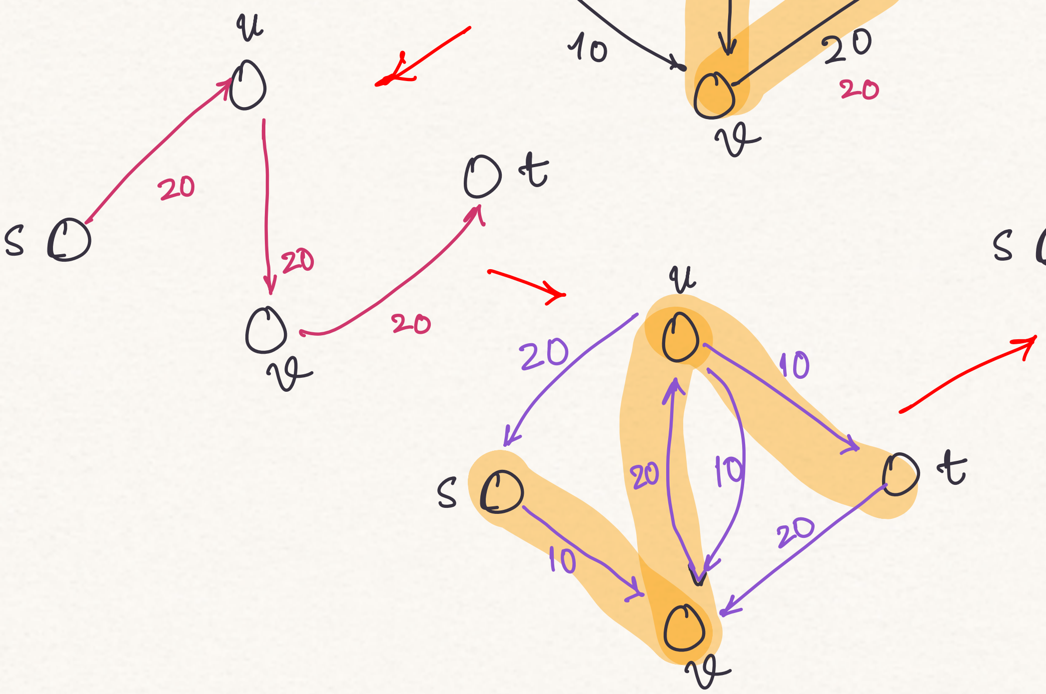
- Conservation of flow at every node v

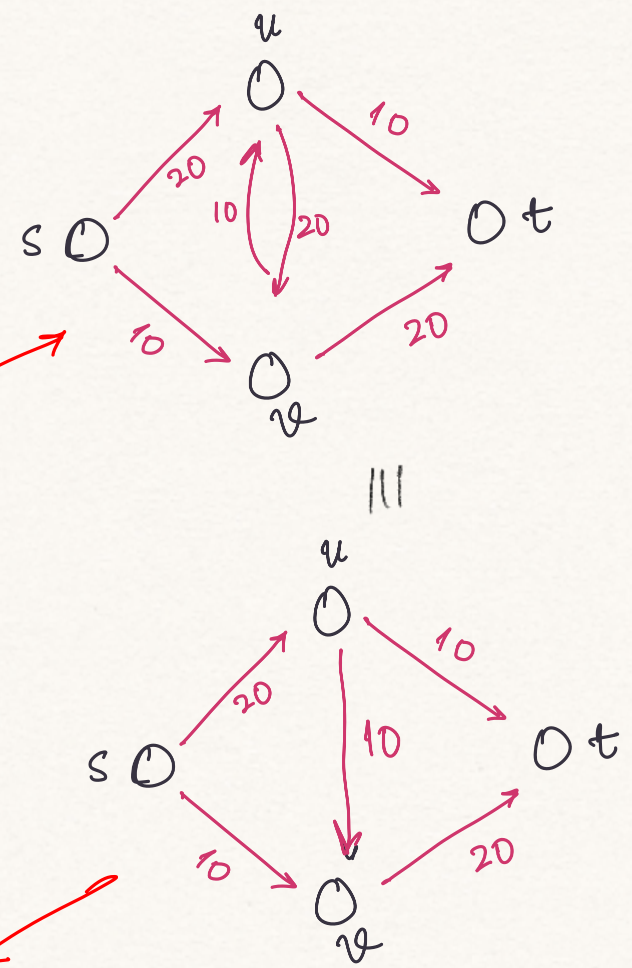
$$\sum_{w\in V} f(w\to v) = \sum_{x\in V} f(v\to x).$$

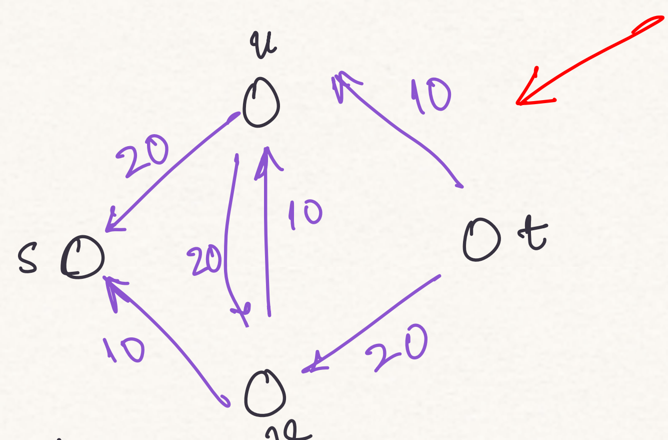
Example:

- Find a path from s-t
- Find the min capacity on this path.

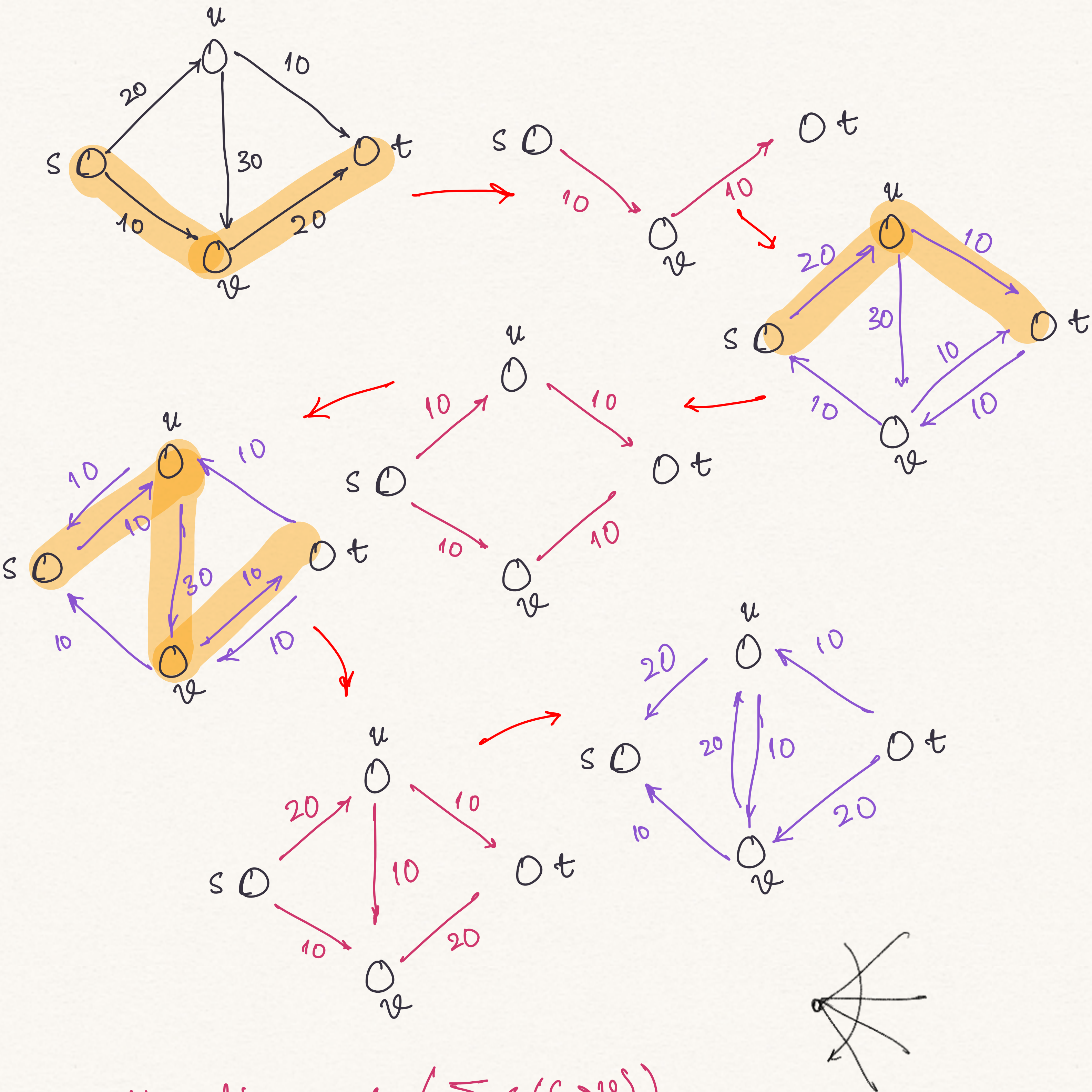








1. Given a graph w/ capacities, find a s-t path and compute the bottleneck capacity (call it F).
2. Route F units from s-t along this path.
3. Construct a residual capacity graph and set it as G
4. Repeat until no more s-t paths can be found.



$$\#\ \text{iterations} \le \left(\sum_{v \in V} c(s \to v)\right)$$

$$\text{Time} \le \left(\sum_{v \in V} c(s \to v)\right) \cdot O(mn)$$