Ring : $k=4$ neighbours.

$$P(k) = \delta(k-4)$$

Now, choose a random node, i.

In the first hop ($l=1$) it has 4 nodes
In the second hop ($l=2$) it has connection to next 4 nodes. Let's assume there is maxhop: l_{max} hop (highest distance)

So, $$1 + \sum_{l=1}^{l_{max}} 4 \approx N \quad \text{(why approx.)} \longrightarrow (1)$$

$$\Rightarrow \quad 1 + 4 l_{max} = N$$
$$l_{max} \approx \frac{N}{4} \longrightarrow (2)$$

For one particular node (Sum of all distances for $N \to \infty$)

$$4\left(1+2+\cdots+\frac{N}{4}\right) = 4 \times \frac{\frac{N}{4}\left(\frac{N}{4}+1\right)}{2}$$

$$\therefore \langle l \rangle = \frac{N}{2} \frac{4 \times \frac{N}{4}\left(\frac{N}{4}+1\right)}{\cancel{2} \times \frac{N(N-1)}{2}} = \frac{1}{4} \frac{N(N+4)}{N(N-1)} \times \frac{N}{2}$$

$$\approx \frac{N}{2 \times 4}$$

For N nodes

For double counts

(See pages 2 to 4 for ring)

~~For~~ For a ring (degree 2)

For a chosen node $\underline{1}$

	Dist
To node 2:	1
To node 3:	2
$\vdots$	

$$\begin{cases} \text{To node } \frac{N}{2}+1 : \quad \frac{N}{2} & \text{if } N \text{ is even} \\ \text{To node } \frac{N+1}{2} : \quad \frac{N-1}{2} & \text{if } N \text{ is odd} \end{cases}$$

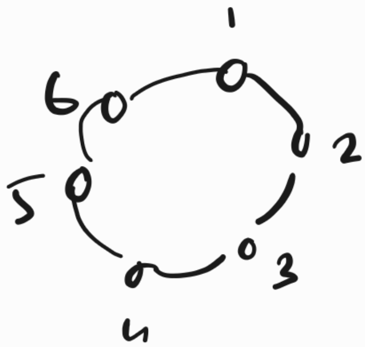


Fig.1

The distance then decrease due to the periodic boundary condition. For example, the distance to node N is 1

Calculate the sum of distances from a single node:

if N is even

For 4th move (it will not be multiplied by 2)

$$2\times\left(1+2+3+\dots+\frac{N}{2}-1\right)+\frac{N}{2}$$

1-2 or 1-6 (CF is 1)

1-3 or 1-5

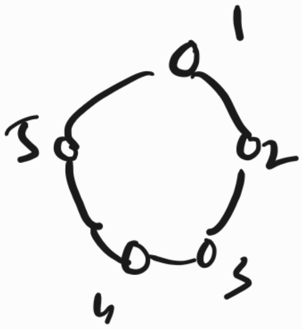
1 to 4

$$= 2\times\frac{\left(\frac{N}{2}-1\right)\left(\frac{N}{2}-1+1\right)}{2}+\frac{N}{2}$$

$$= \frac{N(N-2)}{4}+\frac{N}{2}$$

$$= \frac{N^2-2N+2N}{4} = \frac{N^2}{4}$$

If N is odd



$$2\times\left(1+2+3+\dots+\frac{N-1}{2}\right)$$

$$= \frac{2\times\left(\frac{N-1}{2}\right)\left(\frac{N-1}{2}+1\right)}{2} = \frac{(N-1)(N+1)}{4} = \frac{N-1}{4}$$

Calculate the total sum of all shortest paths

Therefore, the total sum of shortest paths is N times the sum for a single node. However, each path is counted twice (once for i to j and the other j to i), so the sum must be divided by 2:

If N is even: $$\frac{N\left(\frac{N^2}{4}\right)}{2} = \frac{N^3}{8}$$

If N is odd: $$\frac{N\left(\frac{N^2-1}{4}\right)}{2} = \frac{N(N^2-1)}{8}$$

Therefore the <u>avg. path length</u>

For even: $$\frac{N^3}{8} \Big/ {}^{N}C_2 = \frac{N^3 \times 2}{8 \times N(N-1)}$$

$$= \frac{N^2}{4(N-1)} \sim \frac{N}{4}$$

For odd

$$\frac{N(N^2-1) \times 2}{8 \times N(N-1)} = \frac{N+1}{4}$$

$$\sim \frac{N}{4}$$

What about degree : k

$$\boxed{\langle l \rangle \approx \frac{N}{2K}}$$

Avg. pathlength of chain

A — B — C — D — E $\quad N=5$

pathlength of 1 : AB / BC / CD / D-E

L_1 : $N-1$

Pathlength of 2 : $\overline{AC}$ / $\overline{BD}$ / $\overline{CD}$

L_2 : $N-2$

Path length of 3 : $\tilde{AD}$ / $\tilde{BE}$

L_3 : $N-3$

Path length of 4 : $L_4 = N-4$

. . . .

Thus avg. path length :

$$\langle l \rangle = \frac{1\times(N-1) + 2(N-2) + 3(N-3) + \cdots + (N-1)\times(N-(N-1))}{{}^N C_2}$$

$$\langle l \rangle = \frac{N(1+2+\cdots+N-1) - (1^2+2^2+3^2 \cdots + (N-1)^2)}{{}^N C_2}$$

$$= \frac{1}{{}^N C_2}\left[\frac{N\times N(N-1)}{2} - \frac{(N-1)N(2N-2+1)}{6}\right]$$

$$\langle l \rangle = \frac{2}{2}\frac{N(N-1)}{N(N-1)}\left[N - \frac{2N-1}{3}\right]$$

$$\langle l \rangle = \frac{3N-2N+1}{3} = \frac{N+1}{3} \approx \frac{N}{3}$$