Presentation for **Introduction to Cosmology** - *Barbara Ryden*

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Introduction

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- The units of measurement in cosmological scales are

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      (Distance between earth and Sun)

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- **Distance:**
  - $$1AU = 1.50 \times 10^{11} m$$
    (Distance between earth and Sun)
  - $$1pc = 3.09 \times 10^{16} m$$
    (Distance at which 1 arcsecond subtents 1 AU)

  A common metric is Mpc (Mega 10^6 parsecs)

Introduction

Units

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    (Luminosity of sun)
- **Time:**
  - $$1yr = 3.16 \times 10^{7} seconds$$
    (One Year on Earth) A common metric is Gyr (10^{9} years)

Introduction

Units

- **Energy:**

Introduction

Units

- **Energy:**

    - $$1eV = 1.6 \times 10^{-19} J$$

      (Columb Energy of a charged electron) A common measure is MeV used to get rest energy (10^6 eV)

Introduction

Plank's Units

Based on universal constants like $G, c, \hbar$, which remain constant throughout the universe

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- $$E_p = M_p c^2 = 1.96 \times 10^{9} J$$

Fundamental Observations

Fundamental Observations

Sky is Dark

Heinrich Olber came up with the observation that the Sky is dark, when it really shouldn't be. If we assume universe to be infinitely long, and indefinitely long, then using calculated parameters like n (number density of stars), L_* luminosity of an average star, Radius of star R_*, etc, we can see that luminosity of night sky far outweighs the luminosity of night sky by 10 trillion.
Start with the fact that we need only one start with R_* to block our view to the rest of the stars behind it. Say this star is at distance λ

$$1 = nV$$

$$1 = n\pi r^2 \lambda$$

Fundamental Observations

Sky is Dark

This is called Olber's paradox and there are various explanations to this effect:

- **The space is not transparent and there are particles that absorb light.** This can't be true because any body that absorbs light will eventually start emitting light like BBR.

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- **Assumption that Universe has existed forever is wrong.**: This supports to a big bang model, rather than a steady state one.

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- **The space is not transparent and there are particles that absorb light.** This can't be true because any body that absorbs light will eventually start emitting light like BBR.
- **Assumption that Universe is infinitely large is wrong.**
- **Assumption that Universe has existed forever is wrong.**: This supports to a big bang model, rather than a steady state one.
- **If universe were to be constantly expand, then the brightness of the light source would decrease as it reaches us.** This doesn't explain why we don't see stars in all directions though.

Fundamental Observations

Universe is Isotropic and Homogenous

In small scales its not. In the orders of 100 Mpc though we can consider universe to be both "Isotropic" and "Homogenous"

Fundamental Observations

Early Observations

Early astronomers and philosophers like Galileo and Coppernicus disproved the geocentric model and adopted the heliocentric one. But according to Coppernicus, even the heliocentric model doesn't explain the speciality of our place.
This leads us to the Coppernican principle that when built on top of it leads us to the Cosmological principle.
"There is nothing special or privileged about our location in the universe." - Coppernicus
"We're not only not number one, but we are next to none" - Cosmological Principle.

Fundamental Observations

Redshift is proportial to Distance

Redshift is caused due to Doppler's effect in EM waves which cause a shift in the emitted wavelength versus the observed

$$\frac{v}{c} \approx z = \frac{\lambda_{ob} - \lambda_{em}}{\lambda_{em}}$$

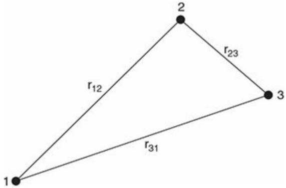
We can use the doppler shift to explain that, objects which move farther away produce a red shift and the ones, coming closer cause blue shift. Scientists like Lemaitre, observed that 37 of 42 observed objects were redshifted rather than blueshifted. Hubble was another scientists who calculated that there is a linear relation between the redshifts (z) and the distances of the objects from Earth.

$$z = \frac{H_0}{c} r$$

Using $\frac{v}{c} \approx z$, we get

$$v = H_0 r$$

Does this mean that all particles are moving away from us? That would violate the cosmological principle.



$$r_{12} = a(t) r_{12}(t_0)$$

$$v_{12} = \frac{dr_{12}}{dt} = \dot{a}(t) r_{12}(t_0) = \frac{\dot{a}}{a} r_{12}(t)$$

Thus,

$$\frac{v_{12}(t)}{r_{12}(t)} = \frac{\dot{a}}{a} = H_0$$

This means the universe has expanded in linear fashion. Thus we can estimate the lifetime of universe by $t = \frac{r}{v} = H_0^{-1}$ of any object. This is a good estimated of the lifetime of universe, $t_0 = 14.38 Gyr$.

Fundamental Observations

Different Types of Particles

I will cover only photon and neutrino.

- **Neutrino:** This is mostly a relativistic particle but has rest mass energy of about $10^{-2}eV$. There are three types of neutrinos, and each of these type have three mass states (m_1, m_2, m_3). Due to quantum occilations the rest energy keeps changing and this energy can be calculated.

$$(m_2^2 - m_1^2)c^4 \approx 7.5 \times 10^{-5}$$

$$(m_3^2 - m_2^2)c^4 \approx 2.4 \times 10^{-3}$$

But this doesn't give us an estimate of individual mass states. Approximations can be made by calculating the rest energy of sum of $(m_e + m_t + m_u) = (m_1 + m_2 + m_3)$, which comes out to be between $(0.057, 0.3)eV$.

- **Photon:** Even though they have no mass, they do have energy density which can be calculated using the blackbody function:

$$\epsilon(f)df = \frac{8\pi h}{c^3} \frac{f^3 df}{e^{\frac{hf}{kT}} - 1}$$

We can use it to calculate the energy and the number density by integrating over all frequencies.

$$\epsilon_\gamma = \alpha T^4$$

Where $\alpha = \frac{\pi^2}{15} \frac{k^4}{\hbar^3 c^3}$,

$$n_\gamma = \beta T^3$$

Where $\beta = \frac{2.4041}{\pi^2} \frac{k^3}{\hbar^3 c^3}$,
One great thing about radiation is that we can calculate the temperature corresponding to a give frequency using

$$E_{mean} = hf_{mean} = 2.7kT$$

Fundamental Observations

Comsological Microwave Background

There have been other models being proposed too which go against the fact that universe is expanding linearly, one such is Steady State model.

$$\frac{dr}{dt} = H_0 r$$

$$r = e^{H_0 t}$$

Another postulate of this model is that the Mass density of the universe is constant. But if universe is expanding and density is constant, that would mean new matter is being produced, but that violates various laws of physics. But this model got disproved once the CMB waves were discovered and it gave an accurate representation

It was observed that there is an isotropic background of microvawe radiation all over the universe and the wavelength of this corresponds to that of microwaves, thus temperature can be calculated by above relation as $T_0 = 2.7255K$. This can be again used to calculate the energy density and the number density as.

$$\epsilon_\gamma = 4.175 \times 10^{-14} MeV/m^3, n_\gamma = 4.107 \times 10^8 m^{-3}$$

It is said that CMB is a relic to the past when the radiation was so dominant, that it used to render the universe opaque and matter was ionized. Then this energy density fell off to a point where the energy of these photons couldn't photoionize matter anymore.
We can start with the second law of thermodynamics and actually calculate the dependence of temperature of CMB to the expansion coefficient

$$dQ = dE + PdV$$

Newton Vs. Einstein

Newton Vs. Einstein

Newton's Laws

According to newton's laws we know that

$$F = m_i a$$

where m_i is inertial mass.
Newton also proposed the Gravitational Law:

$$F = -\frac{GM_g m_g}{r^2}$$

Equivalence law states that the property that determines, how strongly an object is pulled by gravity, also tells us how it accelerates by ANY force. Proof is simple observations made by Galileo on objects on earth.
Using Guass's law and other vector algebra we can define a quantity that measures how much Gravity acts on a given point in space using **Poisson's Law**:

$$\nabla^2 \Phi = 4\pi G \rho$$

This quantity is called the Gravitational potential Φ, and is divergence of the acceleration due to gravity of a test mass,

$$\vec{a} = -\vec{\nabla} \cdot \vec{\Phi}$$

This Poission's equation basically relates Gravity with mass density.

Newton Vs. Einstein

Special Theory of Relativity

Postulates:

- In all intertial frames (where laws of physics are valid), the basic laws of physics are same.

This means say in two different frames, the displacement of light in both of these frames must be equal.

$$c^2t^2 = x^2 + y^2 + z^2$$

$$c^2t'^2 = x'^2 + y'^2 + z'^2$$

Thus we have to use the lorentz tranformation instead:

$$x' = \gamma(x - vt), t' = \gamma(t - vx/c^2), y' = y, z' = z$$

Where $\gamma = (1 - v^2/c^2)^{-1/2}$

Newton Vs. Einstein

Special Theory of Relativity

Postulates:

- In all intertial frames (where laws of physics are valid), the basic laws of physics are same.
- If above is true, then for all the frames Maxwell's laws are true and we calculate the speed of light to be c.

This means say in two different frames, the displacement of light in both of these frames must be equal.

$$c^2t^2 = x^2 + y^2 + z^2$$

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So here, we need to find a quantity that is constant accross these intertial frames. This quantity is called spacetime (minkowski metric)

$$ds^2 = ct^2 - dx^2 - dy^2 - dz^2$$

or simply

$$ds^2 = ct^2 - dl^2$$

Light is said to travel a null geodesic path, which is in accordance with the fermat's principle that, light always takes the path with shortest time (now spacetime). Thus $ds^2 = 0$

Newton Vs. Einstein

General Theory of Relativity

The above calculations are considering that there is no curvature in space and there is no gravity acting.
Einstein used the Equivalence principle to explain that light must bend due to gravity even though it has no mass.
Thus Einstein concludes that it is not mass that makes things bend but the Energy. Mass and energy for non-relativistic objects are interconvertible using the $E = mc^2$, and Energy is interconvertible with momentum for relativistic objects as $E = pc$

Newton Vs. Einstein

Defining Curvature

There are two types of uniform curvatures, postive and negetive (taking 2-D for example):

- Uniform Positive Curvature:

$$\alpha + \beta + \gamma = \pi + A/R_0^2$$

When we measure a distance *dl* along this curved surface, then interms of spherical coordinates it can be written as:

$$dl^2 = dr^2 + R^2 sin^2(r/R) d\theta^2$$

- Uniform Negetive Curvature:

$$\alpha + \beta + \gamma = \pi - A/R_0^2$$

When we measure in spherical coordinates, the value comes out to be

$$dl^2 = dr^2 + R^2 sinh^2(r/R) d\theta^2$$

Newton Vs. Einstein

Defining Curvature

When we try to scale this to 3-D we get:
Flat: Uniform positive curvature gives:

$$dl^2 = dr^2 + r[d\theta^2 + sin^2\theta d\phi^2]$$

Uniform positive curvature gives:

$$dl^2 = dr^2 + R^2 sin^2(r/R)[d\theta^2 + sin^2\theta d\phi^2]$$

Negetive positive curvature gives:

$$dl^2 = dr^2 + R^2 sinh^2(r/R)[d\theta^2 + sin^2\theta d\phi^2]$$

We can generalize this by substituiting variables like $S_k(r)$ and $d\Omega$. Where

$$d\Omega = d\theta^2 + sin^2\theta d\phi^2$$

$$S_k(r) = \begin{cases} R\sin(r/R) & (\kappa = 1) \\ r & (\kappa = 0) \\ R\sinh(r/R) & (\kappa = -1) \end{cases}$$

Which makes the distance equation as:

$$dl^2 = dr^2 + S_k(r)^2 d\Omega^2$$

Newton Vs. Einstein

Robertson-Walked Metric

This metric is a curved generalization to the Minkowski metric given by

$$ds^2 = -c^2 dt^2 + a(t)^2[dr^2 + S_k(r)^2 d\Omega^2]$$

This is important because it takes into account that, the expansion coefficient determines how the universe moves.

Newton Vs. Einstein

Proper Distance

When we measure distance in a curved / expanding universe we don't take into account the error which is caused because of the expansion coefficient. Proper distance does.

$$ds^2 = a(t)^2[dr^2 + S_\kappa(r)^2 d\Omega^2]$$

Taking angular component to be zero if we observe object directly.

$$ds = a(t)dr$$

$$d_p(t) = a(t)r$$

If you observe this proves hubble's law:

$$v_p(t_0) = H_0 d_p(t_0)$$

Newton Vs. Einstein

Expansion and red Shift

We can use the fact that photons follow geodesic. In the robertson-walker equation we have:

$$0 = -cdt^2 + a(t)^2(dr)^2$$

$$a(t).dr = c.dt$$

$$\int_{t_e}^{t_o} \frac{dt}{a(t)} = \int_0^r dr = r$$

Integrating between time of observation and emission.

If we were to calculate for the next crest of the wave we will still get r. Thus by subtracting we get.

$$\int_{t_e}^{t_o} \frac{dt}{a(t)} = \int_{t_e+\lambda_e/c}^{t_o+\lambda_o/c} \frac{dt}{a(t)}$$

$$\int_{t_e}^{t_e+\lambda_e/c} \frac{dt}{a(t)} = \int_{t_o}^{t_o+\lambda_o/c} \frac{dt}{a(t)}$$

Using the fact that expansion of universe barely happens when the crest propogates we use:

$$\frac{\lambda_e}{a(t_e)} = \frac{\lambda_o}{a(t_o)}$$

$$1 + z = \frac{1}{a(t_e)}$$

Cosmic Dynamics

Cosmic Dynamic

Einstien's Field Equation

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

The G term is called Einstein's metric tensor (describes the curvature of universe at all x,y,z,t) and the T term is called the stress-energy metric tensor.

Cosmic Dynamic

Friedmann's equation

An equation that links the Expansion of universe, Curvature and the Mass-energy was required including terms $a(t), R_0, \kappa, \epsilon(t)$.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\epsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$

Friedmann proved this for General theory of relativity, but we will prove it using the Newtonian Way.
To get to this, start with this take, a sphere that is contracting because of its own action of gravity. Acceleration can be given as

$$\frac{d^2 R_s(t)}{dt^2} = -\frac{GM_s}{R(t)^2}$$

Cosmic Dynamic

Critical Density

We can calculate the energy density using above relation for a good estimate case senario where $\kappa = 0$, and $H(t) = \frac{\dot{a}}{a}$ is the hubble parameter. We get

$$\epsilon_c(t) = \frac{3c^2}{8\pi G} H(t)^2$$

For the current time, we can substituite the Hubble's constant and get $\epsilon_{c,0} = 4870 MeV/m^3$ which also gives us mass density if you divide by c^2.
We define Density parameter to be that of ratio with the critical density for any universe as:

$$\Omega(t) = \frac{\epsilon(t)}{\epsilon_c(t)} = 1 + \frac{\kappa c^2}{R_0^2 a(t)^2 H(t)^2}$$

Using we get the relation to finally get rid of κ:

$$\frac{\kappa}{R_0^2} = \frac{H_0^2}{c^2}(\Omega_0 - 1)$$

Cosmic Dynamic

Fluid Equation and Acceleration Equation

We know that Friedmann equation in the end of the day comes from energy conservation, so we can use the first law of thermodynamics to get a similar equation.
We start with

$$\dot{E} + P\dot{V} = 0$$

and finally get:

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) = 0$$

We can use this in the friedmann equation to get the acceleration equation.

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3c^2}(\epsilon + 3P)$$

Cosmic Dynamic

Equations of State

We need another set of equations to solve the friedmann equation: This equation called called equation of state is a relation between the pressure used in fluid equation and energy density

$$P = w\epsilon$$

This w can vary with the substance used. For $w = 0$ for matter (can be solved by using ideal gas equation) and for radiation the $w = 1/3$ comes from statistical mechanics and boson nature.

Cosmic Dynamic

Cosmological constant

Einstein first introduced this to explain the static nature of universe but cosmologists now use it because the default calculation of hubble's constant are very underestimated. When added to the friedmann equation, the cosmological constant looks like

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\epsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2} + \frac{\Lambda}{3}$$

Its as if cosmological constant had its own energy density which doesn't very with time or expansion.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}(\epsilon(t) + \epsilon_\Lambda) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$

If we were to call, $\epsilon_k = \epsilon + \epsilon_\Lambda$, that would mean to keep the fluid equation constant we need. $w = -1$, which means there is an attrative force.

This is same as what Einstein did to add Λ to the Newton's equation. (acceleration equation, when einstein's universe is static)

$$0 = -\frac{4\pi G}{3} + \frac{\Lambda}{3}$$

When we solve for friedmann equation too (with static universe) we get.

$$R_0 = \frac{c}{\Lambda^{1/2}}$$

Model Universes

Model Universes

Evolution of energy density

$$P = w\epsilon; \quad \dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) = 0$$

By solving the state equation and the fluid equation alone, we can come up with a relation

$$\epsilon_i(a) = \epsilon_{i,0} a^{-3(1+w_i)}$$

The can be multiple explanations for why energy density decreases

- For matter, we have a $\epsilon \propto a^{-3}$, taking the value of E_{mean} to be rest mass energy and constant we have that, the number density decreases cubically

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- For Lambda matter, we have no dependence that means energy remains constant

Model Universes

Solved Friedmann equation

Now if we use the relation we got by solving equation of state and fluid equation in the friedmann equation we get:

$$\dot{a}^2 = \frac{8\pi G}{3c^2} \sum \epsilon_i a^{-(1+3w_i)} - \frac{\kappa c^2}{R_0}$$

Assuming the universe contains, Curvature, Matter, Radiation and Comsmological Constant.

$$\dot{a}^2 = \frac{8\pi G}{3c^2} (\epsilon_\Lambda + \frac{\epsilon_r}{a^4} + \frac{\epsilon_m}{a^3}) - \frac{\kappa c^2}{R_0}$$

Using the relation in 4th chapter about curvature and using the definition of $\epsilon_{c,0} = \frac{3c^2}{8\pi G} H_0^2$ and $\frac{\kappa c^2}{R_0^2} = H_0^2(\Omega_0 - 1)$

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

Model Universes

General Procedure to solve the Model Universe

- Using the friedman equation find out a relation between a(t) and t.

The first part is usually the hardest part for more complex universes, but it is simple for, empty universe (direct proportion), single flat component universe (compare powers on the RHS and LHS).
by comparing powers we get:

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3+3w}$$

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- The get the $d_p(t_o)$ in terms of z instead of time.
- finally divide by 1+z to calculate $d_p(t_e)$.

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by comparing powers we get:

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3+3w}$$

Model Universes

Empty Universes

When universe is empty (no energy density) the friedmann equation can be written as:

$$\dot{a}^2 = -\kappa c^2 / R_0^2$$

Values $\kappa = -1$, gives an interesting case where universe is expanding linearly.

$$\dot{a}^2 = c^2 / R_0^2$$

$$a = \pm ct / R_0$$

Finding $\frac{\dot{a}}{a} = H_0 = 1/t_0$, which is original estimation of hubble's constant that hubble performed.

The horizon distance is infinite and the proper distance in-terms of redshift comes out to be:

$$d_p(t_0) = ct_0 \int_{t_e}^{t_o} \frac{dt}{a(t)} = \frac{c}{H_0} ln(1+z)$$

Model Universes

Single component

A single component universe the friedmann equation can be written as:

$$\dot{a}^2 = \frac{8\pi G \epsilon_0}{3c^2} a^{-(1+3w)}$$

Thus solving for relation between a and t.

$$a(t) = \left(\frac{t}{t_0}\right)^{2/(3+3w)}$$

Using above relations we get some good usable relations like:

$$d_p(t_0) = \frac{c}{H_0}\frac{2}{1+3w}[1-(1+z)^{-(1+3w)/2}]$$

$$d_{Hor}(t_0) = \frac{c}{H_0}\frac{2}{1+3w}$$

Model Universes

Single component

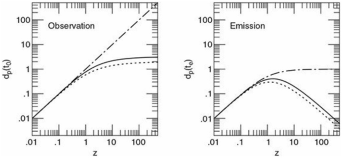
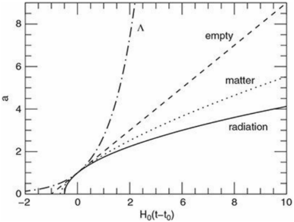


Figure 5.9 The proper distance to a light source with redshift z, in units of the Hubble distance, c/H_0. The left panel shows the distance at the time of observation; the right panel shows the distance at the time of emission. The bold solid line indicates the Benchmark Model. For comparison, the dot-dash line indicates a flat, lambda-only universe, and the dotted line a flat, matter-only universe.



Model Universes

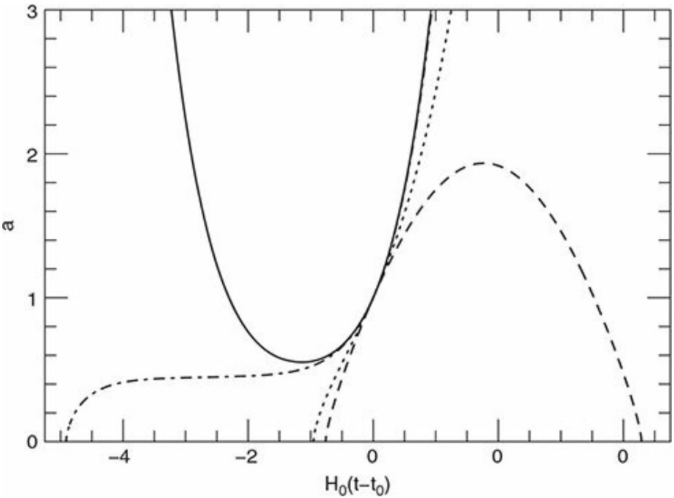
Multi component

It becomes difficult to integrate and find exact relations in multi component universe, hence most of the analysis is done frome the reduced version of friedmann equation itself.

$$\frac{H^2}{H_0^2} = \underbrace{\frac{\Omega_{r,0}}{a^4}}_{Radiation} + \underbrace{\frac{\Omega_{m,0}}{a^3}}_{Matter} + \underbrace{\Omega_{\Lambda,0}}_{Cosmologicalconstant} + \underbrace{\frac{1-\Omega_0}{a^2}}_{Curvature}$$

Where $\Omega_0 = \Omega_{r,0} + \Omega_{m,0} + \Omega_{\Lambda,0}$; and $\Omega_0 = 1$ only if flat universe is assumed
If we vary the density parameters the values, we keep observing different dynamics in the universe

Example (matter + lambda + curvature):



Model Universes

Benchmark Universes

This is a good estimation to our current universe with the following parameters.

Photons:	$\Omega_{\gamma,0} = 5.35 \times 10^{-5}$
Neutrinos:	$\Omega_{\nu,0} = 3.65 \times 10^{-5}$
Total radiation:	$\Omega_{r,0} = 9.0 \times 10^{-5}$
Baryonic matter:	$\Omega_{\text{bary},0} = 0.048$
Nonbaryonic dark matter:	$\Omega_{\text{dm},0} = 0.262$
Total matter:	$\Omega_{m,0} = 0.31$
Cosmological constant:	$\Omega_{\Lambda,0} \approx 0.69$

Important epochs

Radiation–matter equality:	$a_{rm} = 2.9 \times 10^{-4}$	$t_{rm} = 0.050$ Myr
Matter–lambda equality:	$a_{m\Lambda} = 0.77$	$t_{m\Lambda} = 10.2$ Gyr
Now:	$a_0 = 1$	$t_0 = 13.7$ Gyr

Measuring the Cosmological Parameters

Measuring the Cosmological Parameters

Taylor Expansion of expansion coefficient

If we taylor expand a(t) and consider only 2 degree terms

$$a(t) = a(t_0) + \dot{a}(t - t_0) + \frac{1}{2}\ddot{a}(t - t_0)^2$$

We can define some constants like $H_0 = \frac{\dot{a}}{a}$, $q_0 = -\frac{\ddot{a}}{aH_0^2} = \frac{1}{2}\sum_i \Omega_i(1 + 3w_i)$ (from the acceleration eq).

$$a(t) = 1 + H_0(t - t_0) + \frac{1}{2}q_0 H_0^2(t - t_0)^2$$

similarly inverse of this can be estimated as:

$$a(t)^{-1} = 1 - H_0(t - t_0) + (1 + q_0/2)H_0^2(t - t_0)^2$$

This second value is good because we can now calculate the proper distance

Proper distance in terms of the lookback time $(t_0 - t_e)$ is:

$$d_p(t_0) = c\int_{t_e}^{t_o} \frac{dt}{a(t)} = c\int_{t_e}^{t_o} (1 - H_0(t - t_0) + (1 + q_0/2)H_0^2(t - t_0)^2).dt$$

The $t = t_0$ term cancels out, and only taking upto 2nd degree terms we get.

$$d_p(t_0) \approx c(t_o - t_e) + cH_0/2(t_0 - t_e)^2$$

But the lookback time, is not a good metric that can be easily measured. Thus we need to find a way to relate lookback time to redshift.

Measuring the Cosmological Parameters

Proper Distance

If we put $t = t_e$ in the equation.

$$a(t_e)^{-1} = 1 - H_0(t_e - t_0) + (1 + q_0/2)H_0^2(t_e - t_0)^2$$

And then use the equation $\frac{a(t_0)}{a(t_e)} = 1 + z$, we get

$$z = H_0(t_0 - t_e) + (1 + q_0/2)H_0^2(t_0 - t_e)^2$$

By inverting the variables we get:

$$(t_0 - t_e) = H_o^{-1}\left[z - \frac{1 + q_0}{2}z^2\right]$$

Thus the final estimated proper in terms of red-shift distance after removing higher degree terms is given by:

$$d_p(t_0) = \frac{c}{H_0}z\left[1 - \frac{1 + q_o}{2}z\right]$$

Measuring the Cosmological Parameters

Since we cannot just measure the propper distance from the earth, it would be useful to relate it to some known quantities like:

- Standard Candle (Luminosity distance)

$$d_L = \sqrt{\frac{L}{4\pi f}} = d_p(1+z)$$

Thus proper distance needs to be corrected by a factor (1+z)

$$d_L(t_o) = \frac{c}{H_0} z \left[1 - \frac{1-q_o}{2} z\right]$$

- Standard Yardstick (Angular diameter distance)

$$d_A = \frac{l}{\partial\theta} = d_p/(1+z)$$

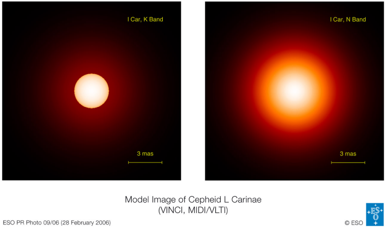
Thus proper distance needs to be corrected by a factor 1/(1+z)

$$d_A(t_o) = \frac{c}{H_0} z \left[1 - \frac{3+q_o}{2} z\right]$$

Measureing the Cosmological Parameters

How to actually calculate

Astronomers usually choose the luminosity distance instead of the angular diameter because of lack of proper yardsticks. We have extensively used stars called Cepheids whose luminosity keeps flucutating in periods, and also IA supernova stars to calculate higher distances since they act as good Standard candles with high luminosity.



It was observed that the luminosity observed by a human eye is in logarithmic scale, which also has a historical significance of Greeks dividing stars into different apparent brightness scales.

1. Thus the **apparent magnitude** of a light source is defined in terms of the source's bolometric flux

$$m = -2.5 \log_{10}(f/f_x)$$

2. Similarly we define **absolute magnitude** of a light source is defined wrt. to the apparent magnitude when its distance $d_L = 10pc$

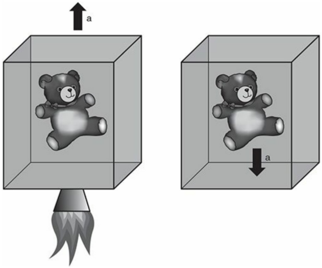
$$M = -2.5 \log_{10}(L/L_x)$$

Thus we can find luminosity distance using these metrics as:

$$m - M = 5log_{10}\left(\frac{d_L}{1Mpc}\right) + 25$$

Was it Fun?

Yes, ryden has a very creative of explaining things, and gives enough time to appreciate historical aspects too, and always includes metaphors and poems.



around the year 1880 by the astronomer Samuel Langley
measure solar radiation. As expressed more poetically
limerick:

> Oh, Langley devised the bolometer:
> It's really a kind of thermometer
> Which measures the heat
> From a polar bear's feet
> At a distance of half a kilometer.[2]

More prosaically, given the technical difficulties of m

Thank You