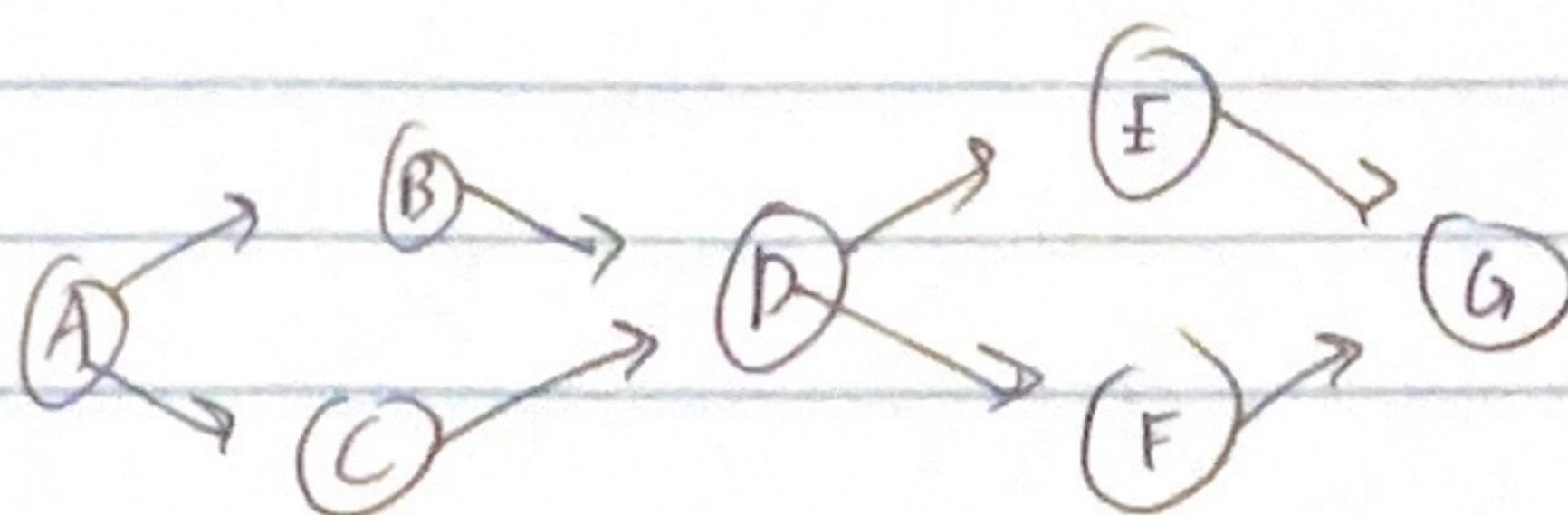


1.) let  $G$  be the graph



let  $u$  and  $v$  represent A and B.  
finding path from  $u$  to  $v$

Step 1: Topological sort (linear time)

A B C D E F G.

Step 2: apply forward propagation

$T_S =$

1	1	1	1+1=2	2	2
A	B	C	D	E	F
↑	↑	↑	↑	↑	↑

If our pointer is pointing at  $u$

then mark the  $u$  node with 1 as a 2 stack  
we look at A and B as say  $1 \rightarrow 1$ .

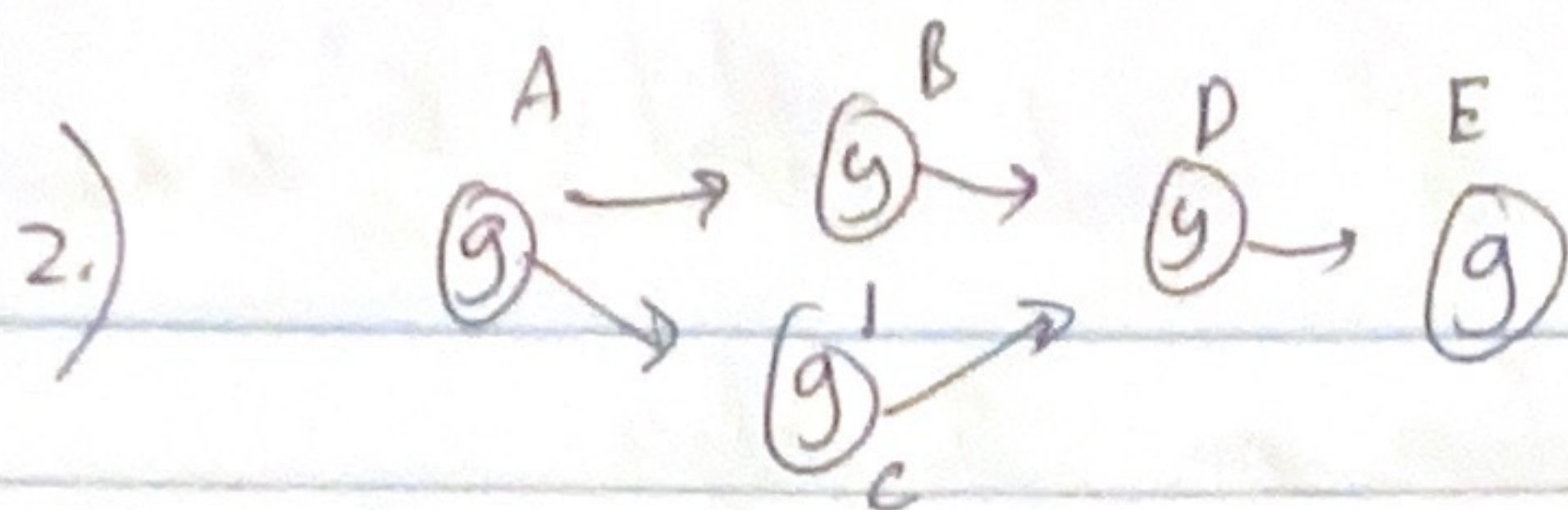
$B=1$ .

then check. A to C since we got to  
another node  $1 \rightarrow 1$   $C=1$

we keep repeating for B and D, C to D  
and D to E  $\rightarrow E=2$ .

$\therefore$  there are two paths to F from A.





~~but~~ we are using the same algorithm as problem but in this we are tracking multiple ways using an auxiliary data structure

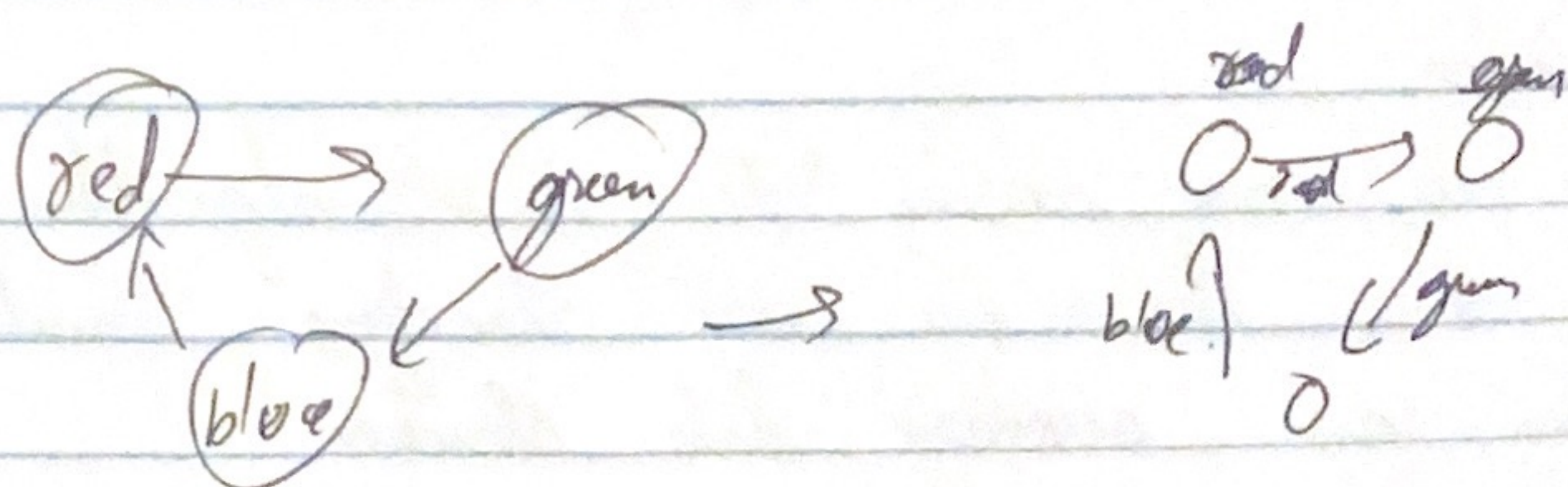
there are three columns in our table and each node will hold one column.  
by using forward propagation, we will make sure that each table holds the final results.

now for  $9 \rightarrow 9$  act sum the paths in the node,  
we get  $1+1=2$  paths.



3.)

convert  $G$  into  $G'$  such that  $G'$  is a graph with colours on the edges.



now use FA and cartesian product machine.

$G$  (colour on edge)  
 $U, V$  (node on  $1s$ )  
 $T$  (regular expression)

Suppose:  $M_1$ : recognize the graph  $G$ 's and node  $U, V$   
 $M_2$ : recognize at regular expression  $T$ .

now find the cartesian product  $M$ .

$M_1 \times M_2 = M$  such that  $M$  colour sequence satisfying  $T$  from  $U$  to  $V$ .



now we have two cases:

①  $M$  is a DAG here we use the Alg from proof below.

②  $M$  is not a DAG so we apply SCC

1) remove every edge, keep the nodes.

2) run DFS again, starting w/ nodes that are most recently finished

mark 2nd edges that are still remains  
remove non-marked edges.

4) Remains nodes still connects are part of  
SCC  
the rest are their own SCC.

DFS will show SCCs. If a node has  $> 1$  node return  $\infty$ .



g)

count = 0

start with either 0, " or 1101, then we  
either have 1101 to go to final state or  
we repeat whole count ++