

Assignment 3

Linear Algebra

August 22 2018

Problem 1. What do we mean by linear independence of a set of vectors? Explain using examples.

Problem 2. Let A be a 4×4 matrix with determinant 7. Give a proof or counterexample for each of the following :

- a) For some vector \mathbf{b} the equation $\mathbf{Ax} = \mathbf{b}$ has exactly one solution.
- b) some vector \mathbf{b} the equation $\mathbf{Ax} = \mathbf{b}$ has infinitely many solutions.
- c) For some vector \mathbf{b} the equation $\mathbf{Ax} = \mathbf{b}$ has no solution.
- d) For all vectors \mathbf{b} the equation $\mathbf{Ax} = \mathbf{b}$ has at least one solution.

Problem 3. Invert the following matrix using pen/pencil and paper. (Please dont use automated things - computer, calculators etc.)

$$\mathbf{I} = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 3 & -1 \\ 1 & 3 & -1 & 0 \\ 2 & 1 & 0 & -1 \end{bmatrix}$$

Problem 4. Let A , B , and C be any $n \times n$ matrices.

- a) Show that $\text{trace}(ABC) = \text{trace}(CAB) = \text{trace}(BCA)$.
- b) Is $\text{trace}(ABC) = \text{trace}(BAC)$. If you cant prove it, give a counterexample.

Problem 5. If B is invertible then show that $\text{rank}(A) = \text{rank}(AB) \leq \text{rank} AB$
(Hint: columns of AB are linear combinations of A)